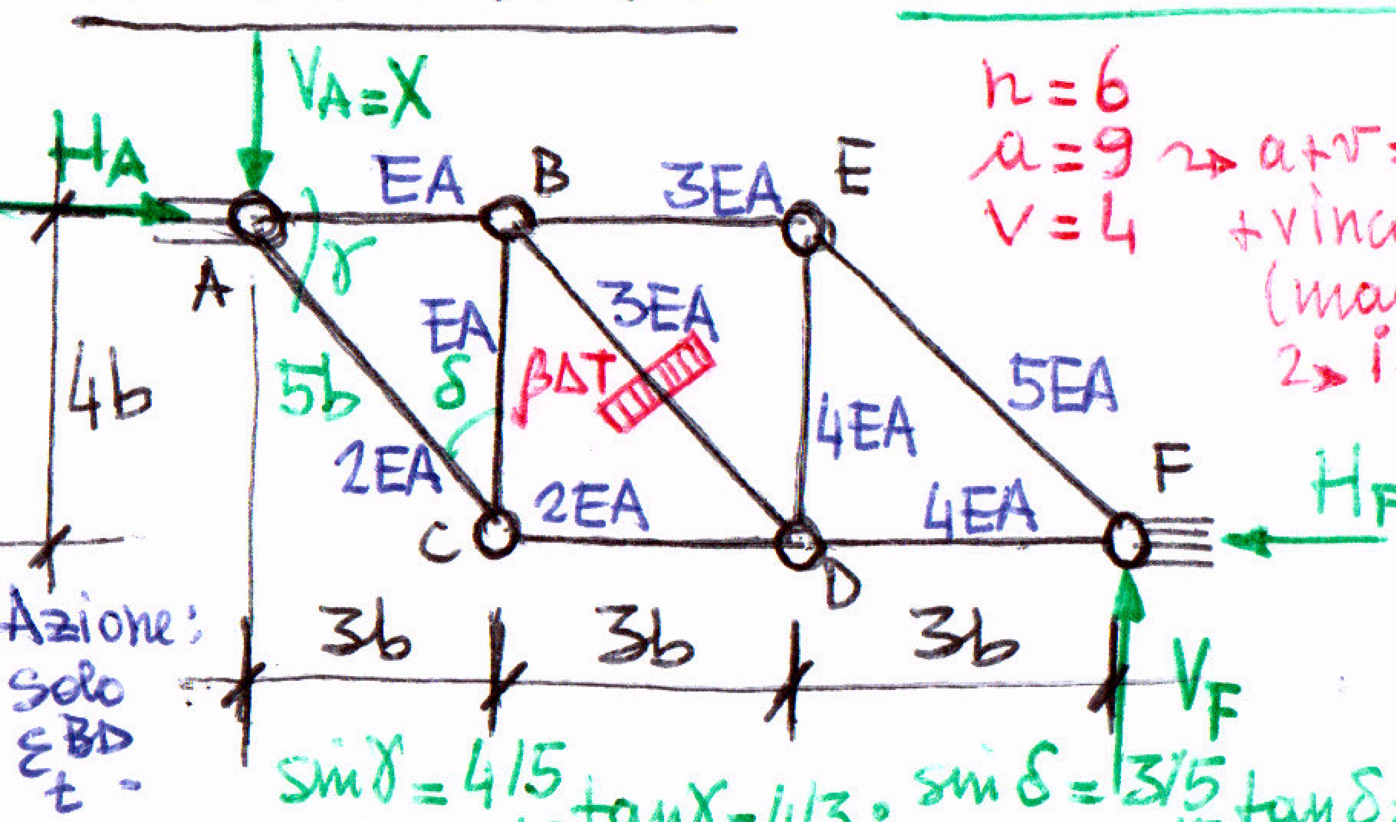
 erizzi@unibg.it



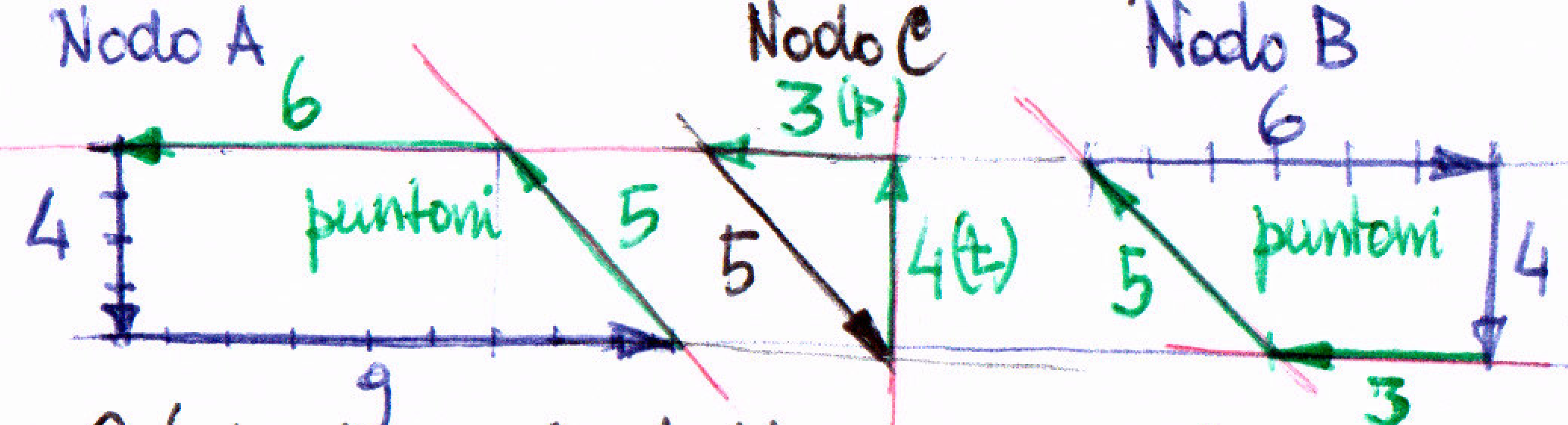
$\sin\gamma = 4/5$, $\cos\gamma = 3/5$, $\tan\gamma = 4/3$; $\sin\delta = 3/5$, $\cos\delta = 4/5$, $\tan\delta = 4/3$

$n = 6$
$a = 9 \Rightarrow a + v = 13 > 12 = 2n$
$v = 4$ + vincoli ben posti (maglie triangolari)
$\Rightarrow$ 1 iperstatica ($I = 1$)

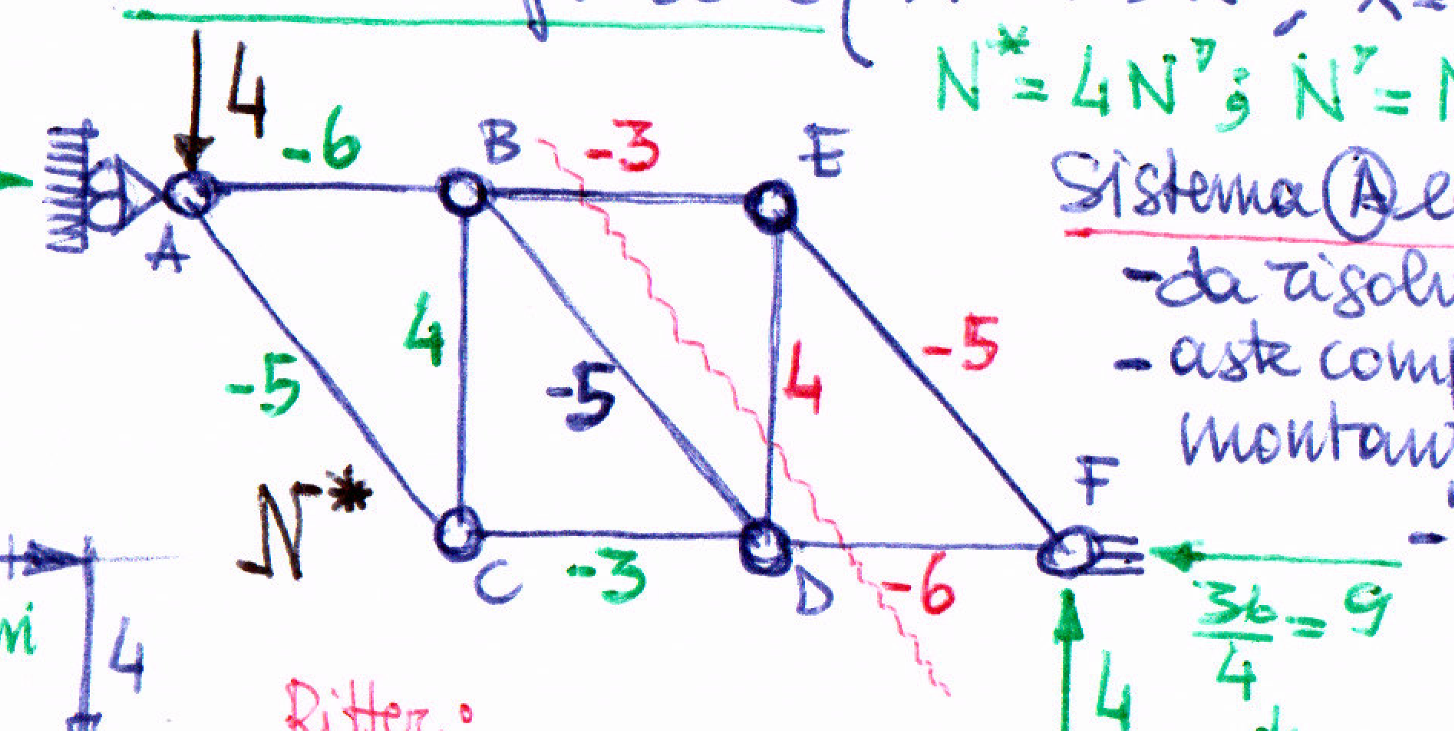
Eq.ni di equilibrio:

- $\Sigma F_x = 0 \rightarrow H_F = H_A \rightarrow H_A = \frac{9}{4} X$
- $\Sigma F_y = 0 \rightarrow V_F = \boxed{V_A = X}$
- $\Sigma M_A = 0 \rightarrow V_F\, 9\cancel{b} = H_F\, 4\cancel{b} \rightarrow H_F = \frac{9}{4} X$

Equil. ai nodi:

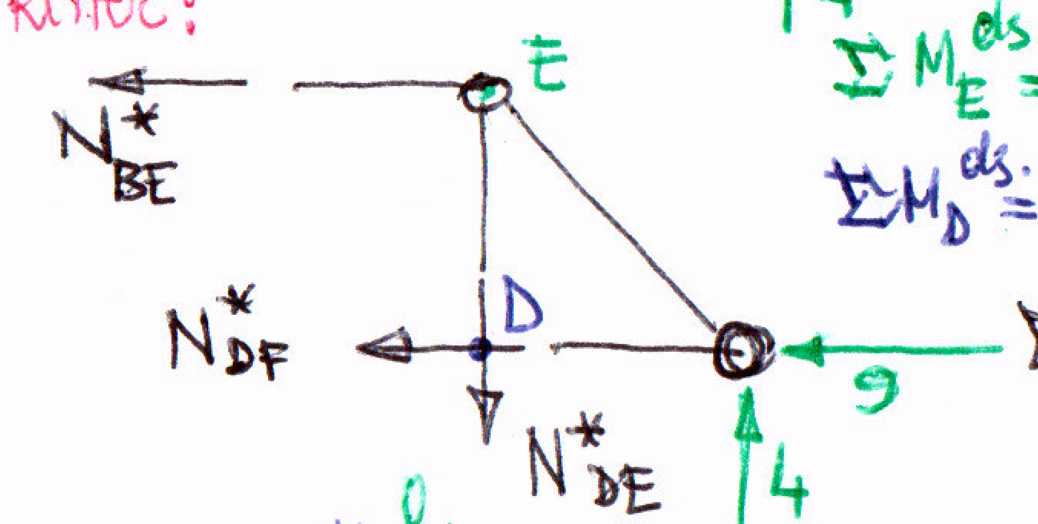


• Struttura fittizia ($X = 1 \rightarrow N'$; $X = 4 \rightarrow N^*$)
$N^* = 4N'$; $N' = N^*/4$



Sistema (A) equil. –
– da risolvere coi metodi visti
– aste compresse, salvo i montanti verticali.
– "simmetria" rispetto all'asta BD.

Ritter:



$\Sigma M_E^{ds.} = 0 \Rightarrow N_{DE}^* = \frac{4\cdot 3 - 9\cdot 4}{4} = -6\,(p)$

$\Sigma M_D^{ds.} = 0 \Rightarrow N_{BE}^* = -\frac{4\cdot 3}{4} = -3\,(p)$

$\Sigma F_y = 0 \Rightarrow N_{DE}^* = 4\,(t)$

• Struttura principale isostatica ($X = 0$) $\Rightarrow N_0 = 0$ (struttura isostatica priva di forze).

• Struttura reale (Sistema (B)): $N = N_0 + XN' = N_0 + XN^*/4$

– Scrittura del PLV (con $N' = N^*/4$):

$$1\cdot \underset{v_A = 0}{0} = \int_{str} N' \cdot \frac{\cancel{N_0}^{\,0} + XN'}{EA}\, ds + \int_B^D N' \alpha\beta\Delta T\, ds \Rightarrow X = -\frac{\sum_i \frac{N_i' \cancel{N_{0i}}^{\,0} l_i}{EA_i} + N_{BD}' \alpha\beta\Delta T\, 5b}{\sum_i \frac{N_i'^2 l_i}{EA_i}} = -\frac{\sum_i \frac{N_i^* \cancel{N_{0i}}^{\,0} l_i}{EA_i} + N_{BD}^* \alpha\beta\Delta T\, 5b}{\sum_i \frac{N_i^{*2} l_i}{EA_i}}\ 4$$

(congruenza)

$N' = N^*/4$

$\frac{\frac{1}{4}}{\frac{1}{4}\frac{1}{4}}$ semplif.

– Idem, con $N^* = 4N'$:

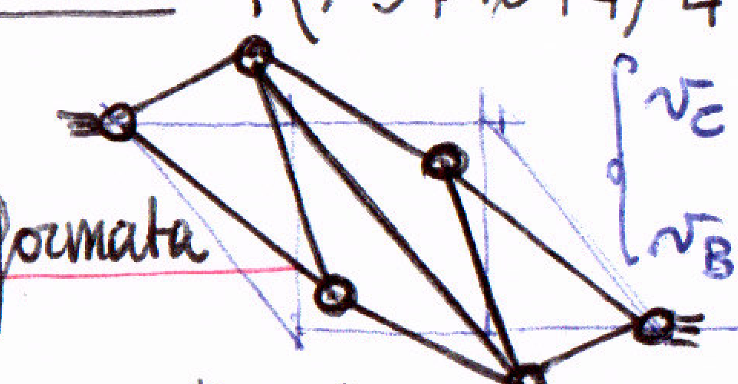
$$4\cdot 0 = \int_{str} N^* \cdot \frac{\cancel{N_0}^{\,0} + XN'}{EA} ds + \int_B^D N^* \alpha\beta\Delta T ds = \int_{str} \cancel{4} N' \cdot \frac{\cancel{N_0}^{\,0} + XN'}{EA} ds + \int_B^D 4 N' \alpha\beta\Delta T ds \quad \text{idem come sopra}$$

$$\begin{cases} u_B = \Delta l_{AB} = -\frac{432}{352}\beta\alpha\Delta T b = -\frac{27}{22}\beta\alpha\Delta T b\,; & u_D = \frac{108}{352}\beta\alpha\Delta T b = \frac{27}{88}\ldots \\ u_E = \Delta l_{AB} + \Delta l_{BE} = -\frac{504}{352}\beta\alpha\Delta T b = -\frac{63}{44}\beta\alpha\Delta T b\,; & u_C = \frac{216}{352}\ldots = \frac{27}{44}\ldots \end{cases}$$

– A denominatore si ottiene:

$$\sum_i \frac{N_i^{*2} l_i}{EA_i} = \frac{b}{EA}\left(\frac{25\cdot 5}{2} + \underbrace{\frac{36\cdot 3}{\cdot} + 16\cdot 4}_{188} + \frac{9\cdot 3}{2} + \frac{25\cdot 5}{3} + \frac{9\cdot 3}{3} + \frac{16\cdot 4}{4} + \frac{36\cdot 3}{4} + \frac{25\cdot 5}{5}\right) = \frac{b}{EA}\left(\frac{25\cdot 5\cdot 3 + 9\cdot 3\cdot 3 + 25\cdot 5\cdot 2}{6} + (9\cdot 3 + 16 + 4)\cdot 4 + 9 + 27 + 25\right) = \text{def.}$$

$$= \frac{b}{EA}\left(\frac{25^2 + 9^2}{6} + 47\cdot 4 + 61\right) = \frac{b}{EA}\left(\frac{625 + 81}{6} + 249\right) = \frac{b}{EA}\,\frac{706 + 1494}{6} = \frac{b}{EA}\,\frac{2200}{6} = \frac{b}{EA}\,\frac{1100}{3}$$



$$\begin{cases} v_C = \frac{537}{352}\beta\alpha\Delta T b \\ v_B = \frac{921}{352}\beta\alpha\Delta T b \end{cases}$$

– Quindi:

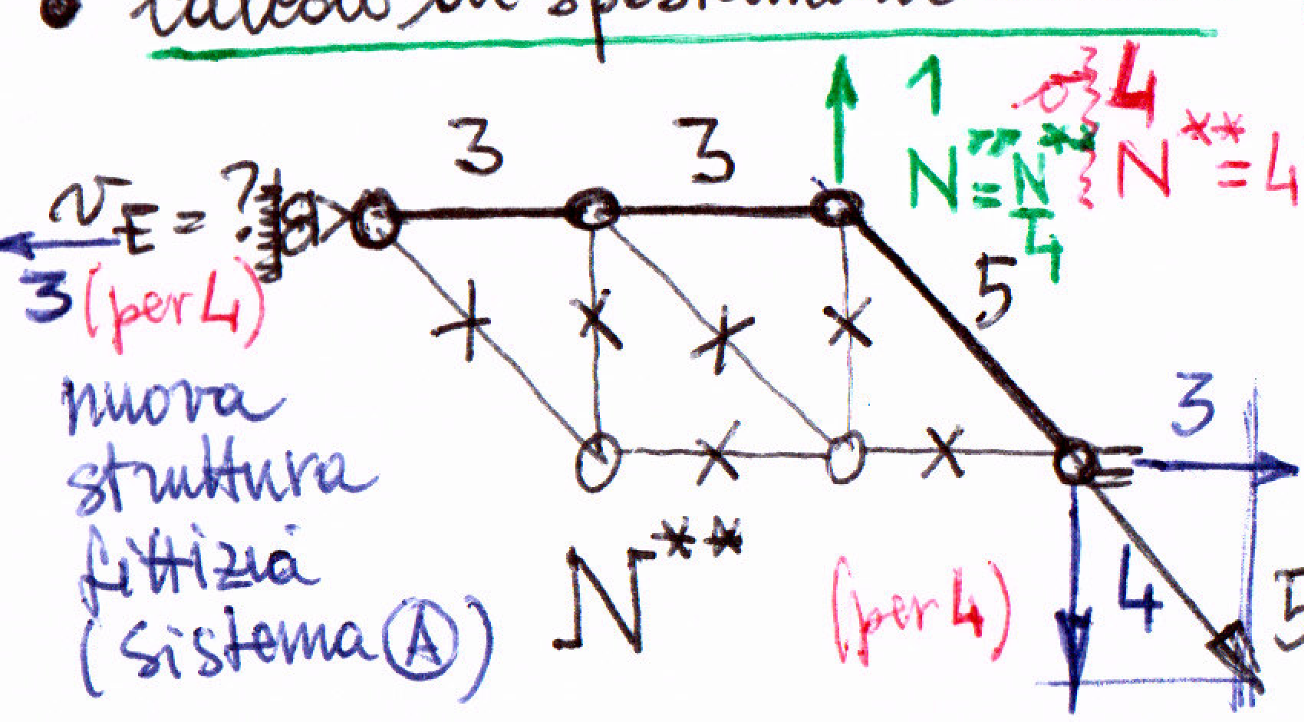
$$X = -\frac{(-5)\alpha\beta\Delta T\, 5\cancel{b}}{\frac{\cancel{b}}{EA}\frac{1100}{3}}\,4 = \frac{\cancel{100}\cdot 3}{\cancel{1100}}\alpha\beta\Delta T EA = \boxed{\frac{3}{11}\alpha\beta\Delta T EA = X}$$

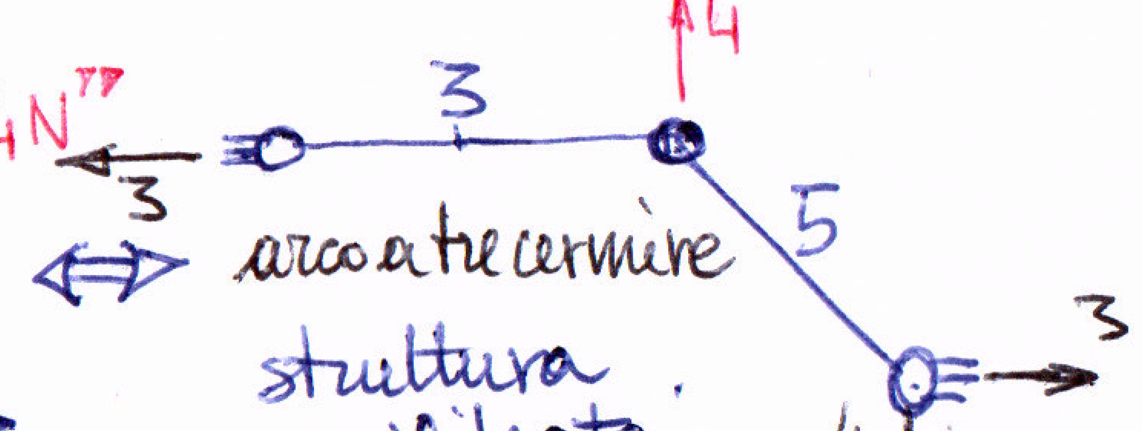
Azioni assiali finali:

$$N_i = \frac{X}{4} N_i^* = \frac{3}{44}\alpha\beta\Delta T EA\, N_i^*$$

(azioni N_i^* già rapp., riscalate di $\frac{X}{4}$)
aste compresse, salvo i montanti
$\Rightarrow$ effetto del ΔT positivo e delle cerniere in A e F. defor.

• Calcolo di spostamenti nodali (tramite nuova applicazione del PLV)





Scrittura del PLV:

$$1\cdot v_E = \int_{str} N'' \frac{N ds}{EA} + \cancel{N_{BD}''}^{\,0} \alpha\beta T 5b$$

$$v_E = \frac{1}{4}\sum_i N_i^{**} \frac{X}{4} N_i^* \frac{l_i}{EA_i} = \frac{1}{16} X \sum_i N_i^* N_i^{**} \frac{l_i}{EA_i}$$

$$= \frac{1}{16}\frac{3}{11}\alpha\beta\Delta T EA\,\frac{b}{EA}\left(-3\cdot 6\cdot 3 - \frac{3\cdot 3\cdot \cancel{3}}{\cancel{3}} - \frac{5\cdot 5\cdot \cancel{5}}{\cancel{5}}\right)$$

$$= -\frac{1}{\cancel{16}_{\,2}}\frac{3}{\cancel{11}}\alpha\beta\Delta T b\,(\underbrace{63 + 25}_{\cancel{88}\ 8}) = \boxed{-\frac{3}{2}\beta\alpha\Delta T b = v_E}$$

• A valle di v_E, calcolo di v_D semplificato:

$v_E = v_D + \Delta l_{DE} \Rightarrow v_D = v_E - \Delta l_{DE}$

con $\Delta l_{DE} = \frac{N_{DE} l_{DE}}{EA_{DE}} = \frac{N_{DE} 4b}{4EA}$

$= \frac{X}{4} 4\frac{b}{EA} = \frac{3}{11}\beta\alpha\Delta T b$

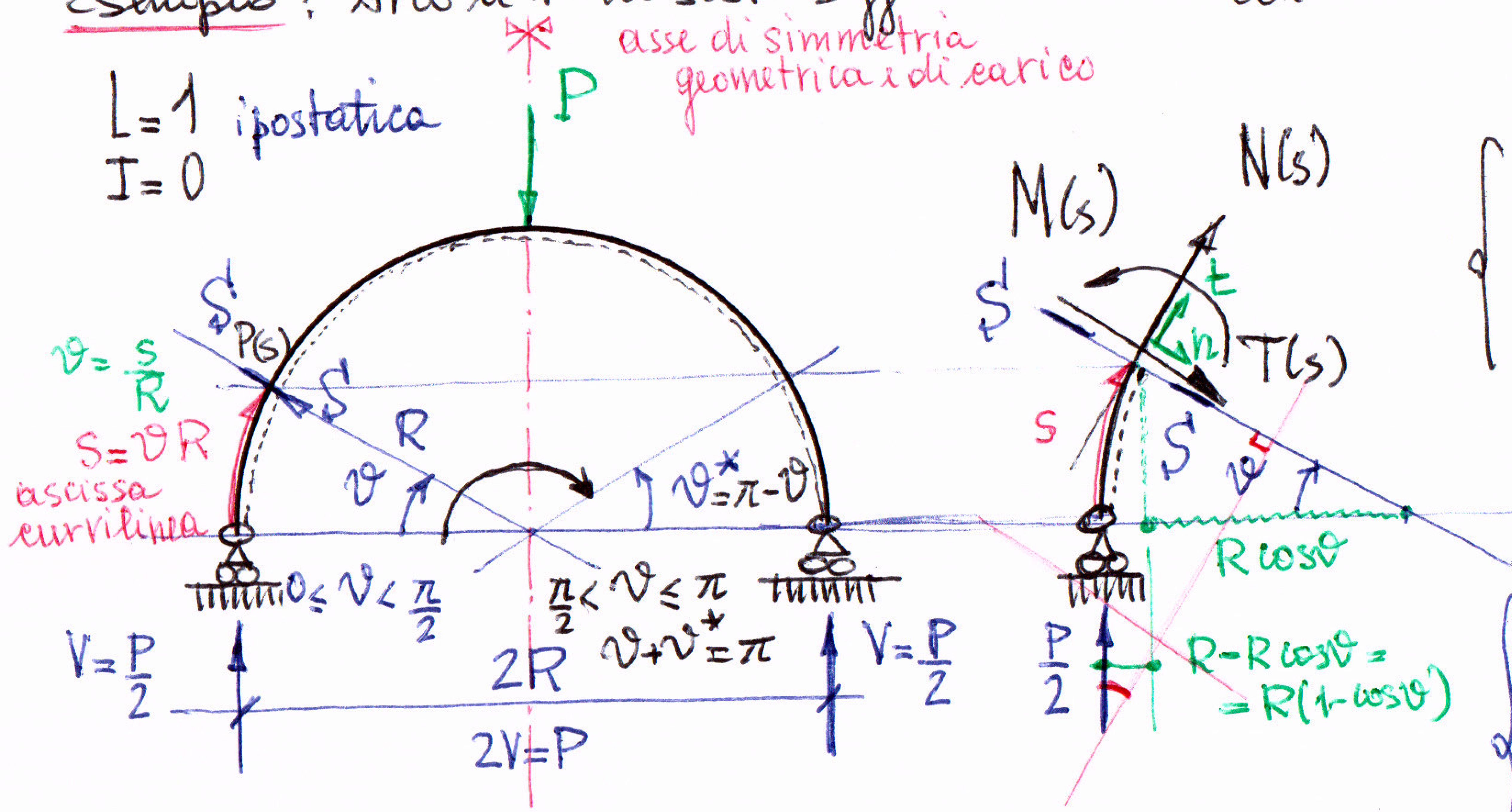
$|v_E| < |v_D|$, $\Delta l_{ED} > 0$

$v_D = -\left(\frac{3}{2} + \frac{3}{11}\right)\beta\alpha\Delta T b = -\frac{3}{22}\cdot 13\,\beta\alpha\Delta T b = -\frac{39}{22}\beta\alpha\Delta T b$

Azioni interne in aste curve (circolari in particolare) → raggio di curvatura costante $r(s)=R$

erizzi@unibo.it

Esempio: Arco a tutto sesto soggetto a carico concentrato in chiave (a spinta orizzontale eliminata).



Azioni interne all'ascissa s

$N(s)$ azione assiale o normale (agente nella direz. t tangente all'asta)
$T(s)$ azione tagliante o taglio (" " " n normale ")
$M(s)$ azione flettente o momento flettente

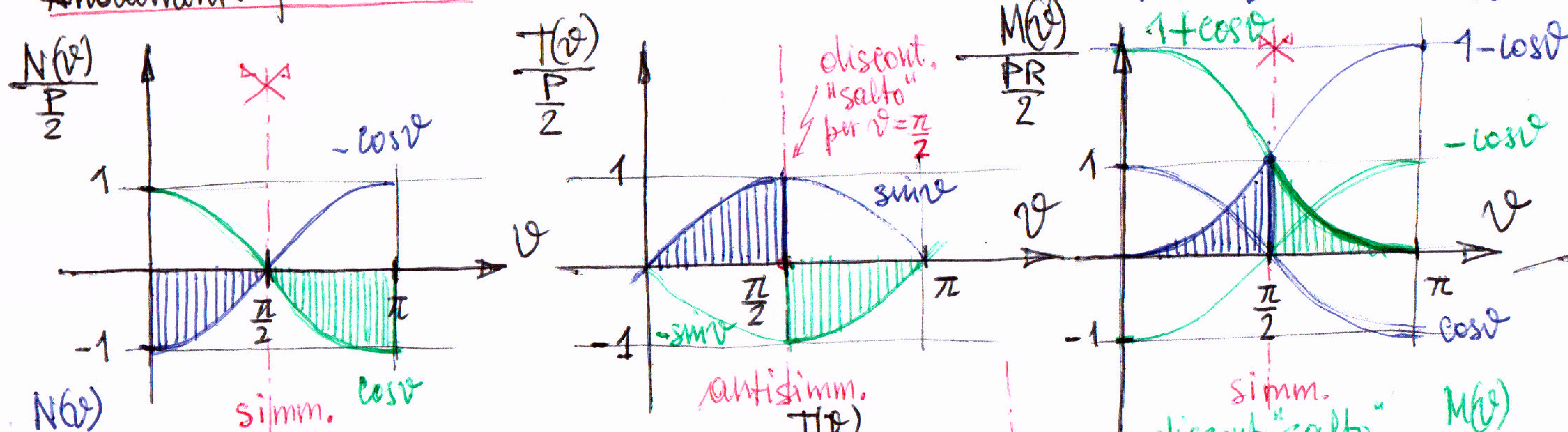
o $N(\vartheta)$, $T(\vartheta)$, $M(\vartheta)$

direzioni locali in P(s)

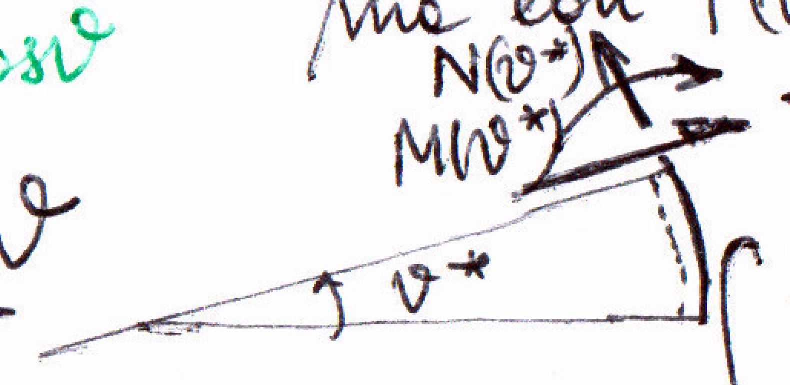
Equazioni di equilibrio: $0 \le \vartheta < \frac{\pi}{2}$

	$\vartheta = 0$	$\vartheta = \frac{\pi}{2}^-$	$\vartheta = \frac{\pi}{2}^+$	$\vartheta = \pi$
$\sum F_t^{sn.} = 0 \Rightarrow N(\vartheta) = -\frac{P}{2}\cos\vartheta$	$-\frac{P}{2}$	0	0	$-\frac{P}{2}$
$\sum F_n^{sn.} = 0 \Rightarrow T(\vartheta) = \frac{P}{2}\sin\vartheta$	0	$\frac{P}{2}$	$-\frac{P}{2}$	0
$\sum M_s^{sn.} = 0 \Rightarrow M(\vartheta) = \frac{P}{2}R(1-\cos\vartheta)$	0	$\frac{PR}{2}$	$\frac{PR}{2}$	0

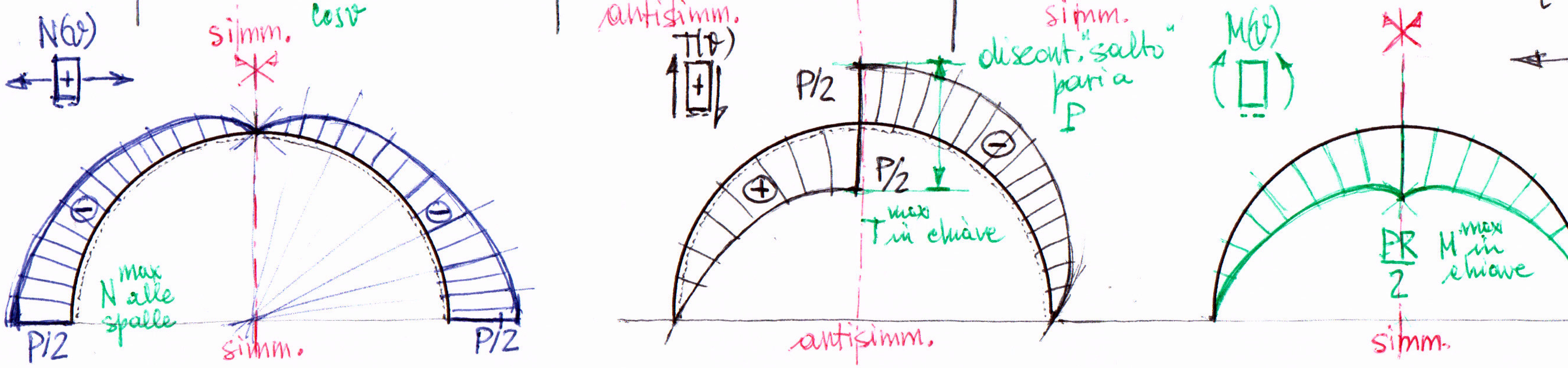
Andamenti funzionali:



• Per $\frac{\pi}{2} < \vartheta \le \pi$, vedi $\vartheta^* < \frac{\pi}{2}$ simile a ϑ ma con $T(\vartheta^*)$ cambiato di segno: $\vartheta^* = \pi - \vartheta$



$$\begin{cases} N(\vartheta^*) = -\frac{P}{2}\cos\vartheta^* = \frac{P}{2}\cos\vartheta \\ T(\vartheta^*) = -\frac{P}{2}\sin\vartheta^* = -\frac{P}{2}\sin\vartheta \\ M(\vartheta^*) = \frac{PR}{2}(1-\cos\vartheta^*) = \frac{PR}{2}(1+\cos\vartheta) \end{cases}$$



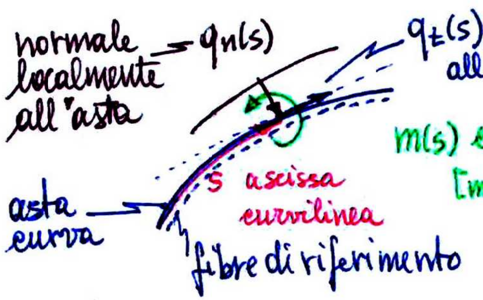
← DIAGRAMMI N, T, M (disegnati sulla struttura stessa): curvatura dell'andamento funzionale sommata alla curvatura della linea d'asse —

N.B.: risulta
$$\begin{cases} N'(\vartheta) = T(\vartheta) \\ T'(\vartheta) = -N(\vartheta) \\ M'(\vartheta) = R\,T(\vartheta) \end{cases}$$
è generale? v. prox. lez.

 – Equazioni indefinite di equilibrio delle aste curve (v. Belluzzi, Vol. 2, p. 154)

lrizzi@unibg.it

- Carichi distribuiti potenzialmente presenti:



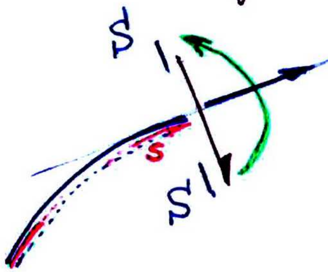
$[q]=\frac{[F]}{[L]}$

$m(s)$ coppie distr.

$[m]=\frac{[M]}{[L]}=[F]$

ci si chiede come carichi ed azioni sono correlati (staticamente)

- Azioni interne nella generica sezione S-S:

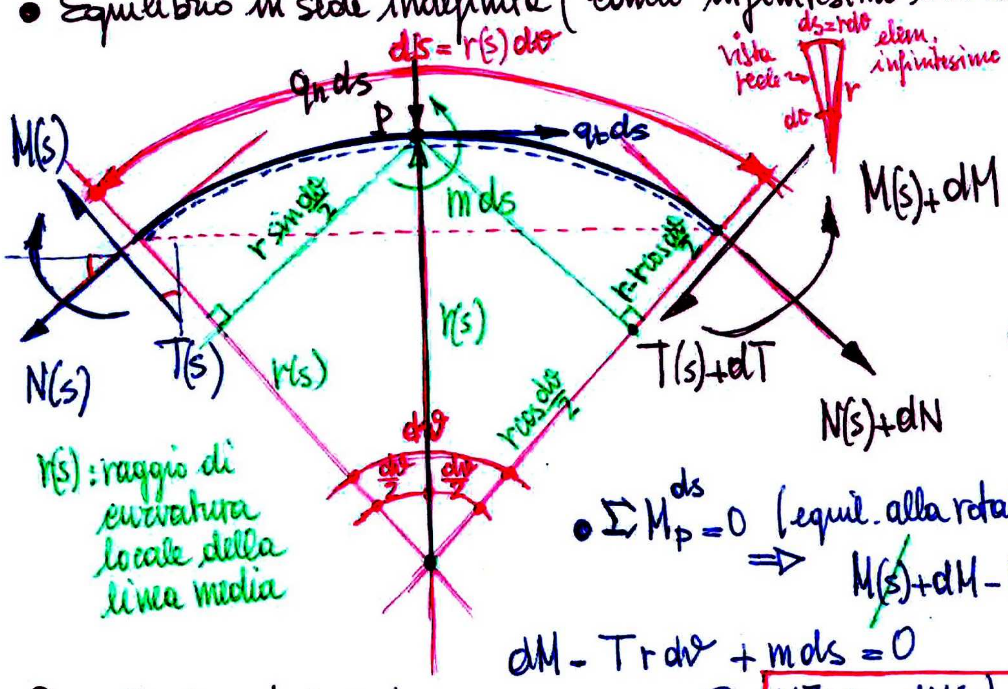


$N(s)$ Azione assiale

$T(s)$ Azione tagliante o taglio

$M(s)$ Azione flettente o momento flettente

- Equilibrio in sede indefinita (concio infinitesimo di asta curva):



$r(s)$: raggio di curvatura locale della linea media

- $\sum F_t^{ds}=0 \Rightarrow [\cancel{N(s)}+dN-\cancel{N(s)}]\underbrace{\cos\frac{d\vartheta}{2}}_{\sim 1}-\cancel{2}T\underbrace{\sin\frac{d\vartheta}{2}}_{\sim\frac{d\vartheta}{2}}-\underbrace{dT\sin\frac{d\vartheta}{2}}_{0\,\sim\, dT\frac{d\vartheta}{2}\text{ infinit. del 2° ord.}}+q_t\underbrace{ds}_{r(s)d\vartheta}=0$

$\frac{dN}{ds}-\frac{T\,d\vartheta}{r\,d\vartheta}+q_t\frac{ds}{ds}=0 \;\Rightarrow\; \boxed{N'(s)=\frac{dN(s)}{ds}=-q_t(s)+\frac{T(s)}{r(s)}}$.

nuovo termine rispetto alle aste rettilinee

Analogamente:

- $\sum F_n^{ds}=0 \Rightarrow [\cancel{T(s)}+dT-\cancel{T(s)}]\underbrace{\cos\frac{d\vartheta}{2}}_{\sim 1}+2N\underbrace{\sin\frac{d\vartheta}{2}}_{\sim\frac{d\vartheta}{2}}+\underbrace{dN\sin\frac{d\vartheta}{2}}_{0\,\sim\, dN\frac{d\vartheta}{2}}+q_n\,ds=0$ $(r\to\infty)$

$dT+N\,d\vartheta+q_n\,ds=0 \;\Rightarrow\; \boxed{T'(s)=\frac{dT(s)}{ds}=-q_n(s)-\frac{N(s)}{r(s)}}$ ← N, T accoppiate

- $\sum M_P^{ds}=0$ (equil. alla rotaz. rispetto a P)

$\Rightarrow \cancel{M(s)}+dM-\cancel{M(s)}-\cancel{2}T\,r\underbrace{\sin\frac{d\vartheta}{2}}_{\sim\frac{d\vartheta}{2}}-\underbrace{dT\,r\sin\frac{d\vartheta}{2}}_{0\,\sim\, dT\,d\vartheta\,\frac{r}{2}}-[\cancel{N(s)}+dN-\cancel{N(s)}]\,r\underbrace{\left(1-\cos\frac{d\vartheta}{2}\right)}_{\sim 0}+m\,ds=0$

$\overset{1\to 0}{}$

$dM-T\,r\,d\vartheta+m\,ds=0$

$\Rightarrow \boxed{M'(s)=\frac{dM(s)}{ds}=-m(s)+T(s)}$ come per aste rettilinee

Nel caso di aste rettilinee ($r(s)\to\infty$) si riottiene:

$$\begin{cases} N'(x)=-q_t(x)\\ T'(x)=-q_n(x)\\ M'(x)=-m(x)+T(x)\end{cases}$$

e $M''(x)=-(m'(x)+q_n(x))$

Per ulteriore derivazione:

$M''(s)=-m'(s)+T'(s)=$

$=-m'(s)-q_n(s)-\frac{N(s)}{r(s)}$

$=-(m'(s)+q_n(s))\boxed{-\frac{N(s)}{r(s)}}$ – termine aggiuntivo per $N(s)$ rispetto al caso di aste rettilinee $(r(s)\to\infty)$

- per $m=0$ $M'=T$

- per $q_t=0,\ q_n=0$

$N'=\frac{T}{r},\ T'=-\frac{N}{r}$

Aste circolari: $r(s)=R$ $\quad s=R\vartheta,\ \vartheta=\frac{s}{R}$

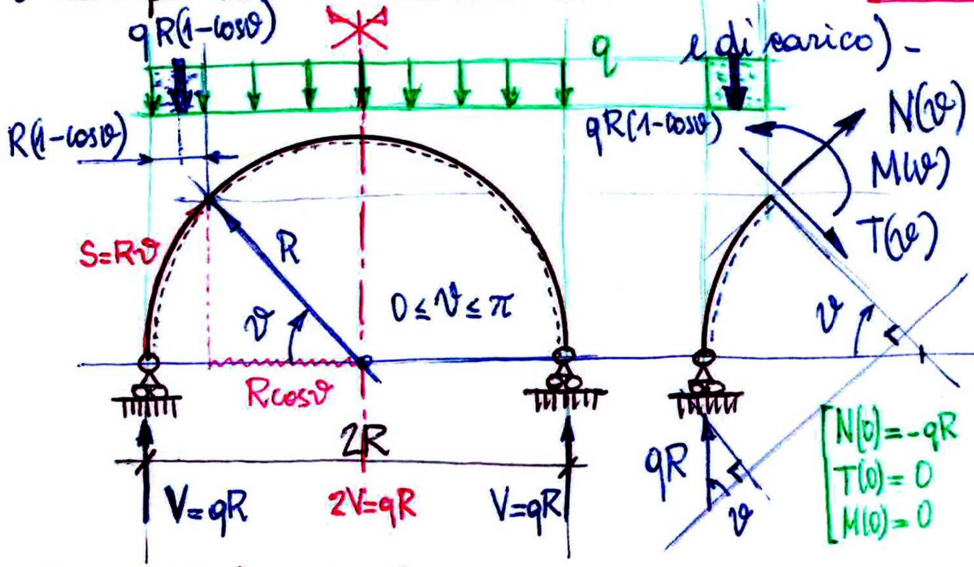
$\frac{d}{ds}=\frac{1}{R}\frac{d}{d\vartheta}$

$$\begin{cases} N'(s)=-q_t(s)+\frac{T(s)}{R}\\ T'(s)=-q_n(s)-\frac{N(s)}{R}\\ M'(s)=-m(s)+T(s)\end{cases}$$ e $M''(s)=-(m'(s)+q_n(s))-\frac{N(s)}{R}$

$$\begin{cases} N'(\vartheta)=-q_t(\vartheta)R+T\\ T'(\vartheta)=-q_n(\vartheta)R-N\\ M'(\vartheta)=-m(\vartheta)R+TR\end{cases}$$ e $M''(\vartheta)=-m'(\vartheta)R-q_n(\vartheta)R^2-N(\vartheta)R$

- Esempio di arco circolare a tutto sesto con carico uniformemente distribuito (configurazione simmetrica, geometrica e di carico) -

egidio.rizzi@unibg.it



- Azioni interne alla generica ascissa curvilinea $s=R\vartheta$, o anomalia $\vartheta = \frac{s}{R}$ - Determinazione per equilibrio diretto:

$$\begin{cases} N(\vartheta) = -\cancel{qR\cos\vartheta} + qR(\cancel{1}-\cos\vartheta)\cos\vartheta = \boxed{-qR\cos^2\vartheta = N(\vartheta)} \to N'(\vartheta) = qR\sin 2\vartheta = -q_t R + T \\ T(\vartheta) = \cancel{qR\sin\vartheta} - qR(\cancel{1}-\cos\vartheta)\sin\vartheta = \frac{qR}{2}\,2\sin\vartheta\cos\vartheta = \boxed{\frac{qR}{2}\sin 2\vartheta = T(\vartheta)} \to T'(\vartheta) = qR\cos 2\vartheta = -q_n R - N \\ M(\vartheta) = qR\,R(1-\cos\vartheta) - qR(1-\cos\vartheta)\frac{R(1-\cos\vartheta)}{2} = \\ = \frac{qR^2}{2}(1-\cos\vartheta)(2-1+\cos\vartheta) = \frac{qR^2}{2}(1-\cos\vartheta)(1+\cos\vartheta) \\ = \frac{qR^2}{2}(1-\cos^2\vartheta) = \boxed{\frac{qR^2}{2}\sin^2\vartheta = M(\vartheta)} \end{cases}$$

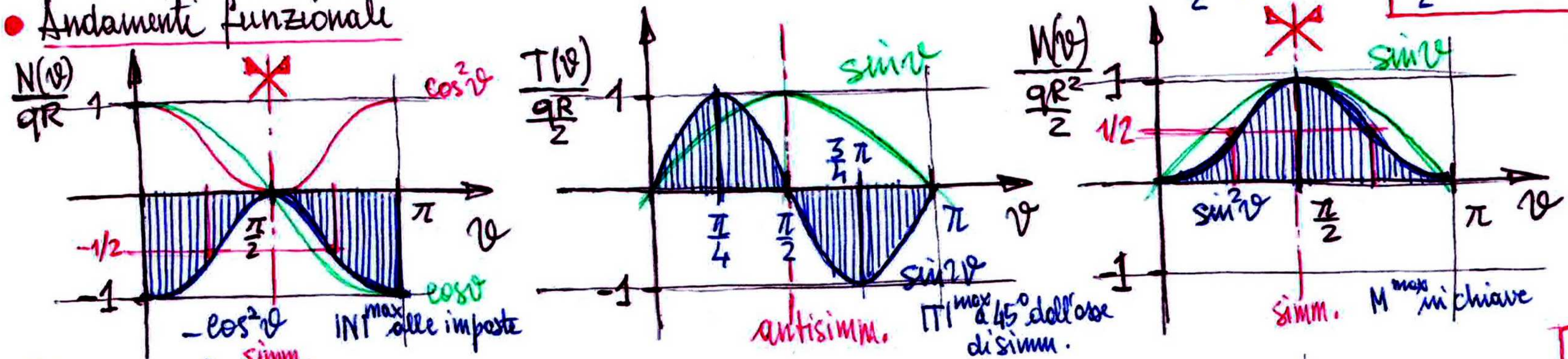
$M'(\vartheta) = \frac{qR^2}{2}\sin 2\vartheta = T(\vartheta)R$

$M''(\vartheta) = qR^2\cos 2\vartheta = -q_n R^2 - NR$

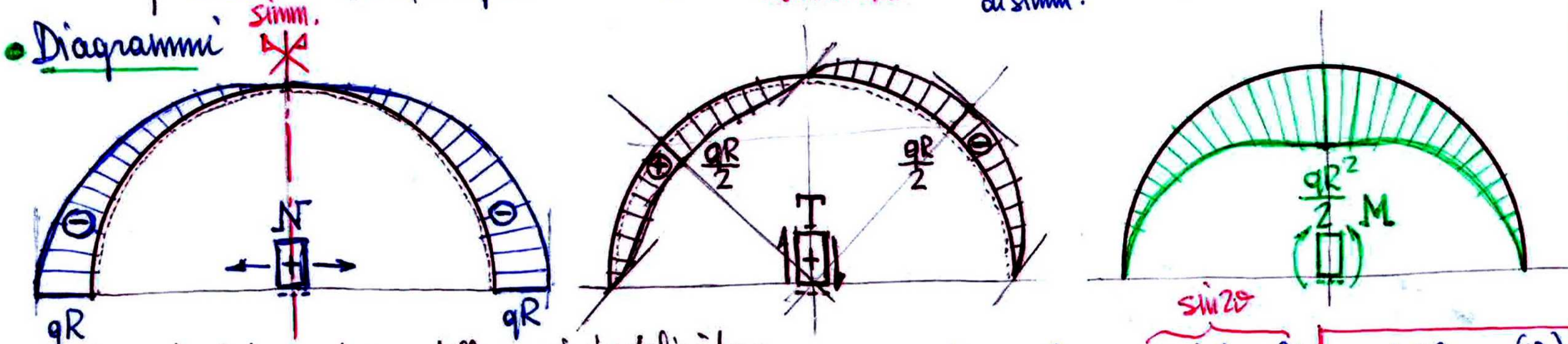
Valori caratteristici

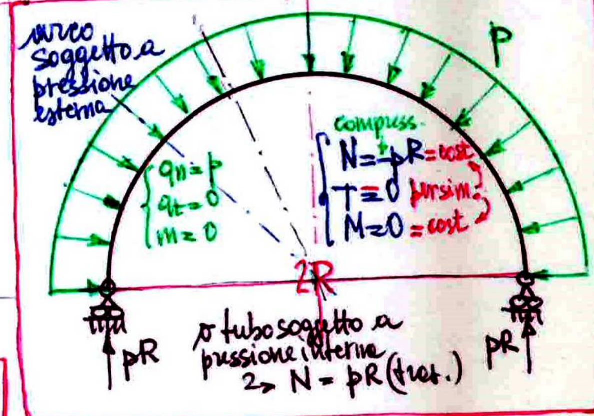
ϑ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
N	$-qR$	$-\frac{qR}{2}$	0	$-\frac{qR}{2}$	$-qR$
T	0	$\frac{qR}{2}$	0	$-\frac{qR}{2}$	0
M	0	$\frac{qR^2}{4}$	$\frac{qR^2}{2}$	$\frac{qR^2}{4}$	0

- Andamenti funzionali



- Diagrammi

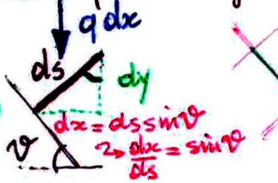




- Oppure, da integrazione delle eq.ni indefinite:

$$\begin{cases} N'(\vartheta) = -q_t R + T \\ T'(\vartheta) = -q_n R - N \\ M'(\vartheta) = -\cancel{m}R + TR \end{cases} \quad \text{con} \quad M(0)=0$$

$N'(0) = T(0) = 0,\ T'(0) = -N(0) = qR$



$$\begin{cases} q_t = -q\frac{dx}{ds}\cos\vartheta = -\frac{q}{2}\overbrace{2\sin\vartheta\cos\vartheta}^{\sin 2\vartheta} = \boxed{-\frac{q}{2}\sin 2\vartheta = q_t(\vartheta)} \\ q_n = q\frac{dx}{ds}\sin\vartheta = q\sin\vartheta\sin\vartheta = \boxed{q\sin^2\vartheta = q_n(\vartheta)} \end{cases}$$

$$T'' = -q_n' R - (-q_t R + T) \Rightarrow T'' + T = (q_t - q_n')R = -\frac{3}{2}qR\sin 2\vartheta \Rightarrow T = \underbrace{A\sin\vartheta + B\cos\vartheta}_{\text{integr. generale omogenea assoc.}} + \underbrace{\frac{qR}{2}\sin 2\vartheta}_{\text{integr. part.}}$$

$$\begin{cases} N = -q_n R - T' \\ M = \int_0^{\vartheta} TR\, d\vartheta \end{cases}$$

c.c. $T(0) = B = 0$, $T'(0) = A + qR = qR \Rightarrow A = 0$