Longevity Risk Management for Individual Investors using MSP

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Section 1

INTRODUCTION
Financial Planning with Longevity Risk (1/2)

• The uncertainty in the life expectancy plays a critical role in the individual financial planning.

• Historical data suggests the average life expectation becomes longer as time goes on.

• As the life expectancy becomes longer, the original investment policy set to be optimal at the initial time point becomes sub-optimal.

• Thus, this longevity risk should be addressed carefully. Otherwise, the investors could face unpleasant results, which could have been prevented with careful analysis.
Financial Planning with Longevity Risk (2/2)

- Kim et al. (2012) have developed a simulation-based model to incorporate longevity risks in the financial planning for individual.
  - Under the longevity risk, they modeled the optimal portfolio of the traditional asset classes (stocks and bonds) along with the life insurance and the pension benefits for individual investors.
  - They showed that it is not likely to find a generalized linear rule for optimal investment and insurance policy under longevity risk.

- Therefore, a systematic approach is required to address the longevity risk management problems.

- In this paper, we develop MSP models for the optimal investment model including the life insurance and pension benefits.
Section 2

MODEL DESCRIPTION
Model Description

• The modeling assumptions of this study is similar to the simulation-based model by Kim et al. (2012).
  – The reason is that we want to compare the results from this MSP study and the simulation-based approach.

• Consider a couple who are about to retire under the following investment environment:
  – Only husband carries a pension plan. After his death, no more cash in-flow will be generated.
  – In addition to pension benefit, they have a certain amount of saving.
  – Annual expenses are carefully projected.
  – Their goal is to cover these expenses.

• Financial instruments under consideration
  – Benefits from pension plan: randomness caused by the husband’s death
  – Life insurance
  – $I$ asset classes ($I = 1$ representing cash)
Basic Settings (1/3)

• Time
  – Target planning period $T = \{0, 1, \ldots, \tau, \tau + 1\}$
  – Investment, insurance premium and borrowing decisions occur at the last instant of each time stage

• Asset classes
  – Asset classes are defined by the set $A = \{1, 2, \ldots, I\}$
    • Asset class 1 representing cash
    • Other asset classes can include broad investment groupings such as:
      – Stocks, Long-term government, Corporate bonds, Foreign equity, Etc.
    • The asset classes should track well-defined market segments.
    • Ideally, the co-movements between pairs of asset returns would be relatively low so that diversification can be done across the asset classes
  – Dividends and interest payments are reinvested in the originating asset class
Basic Settings (2/3)

• Life Insurance
  – The life insurance is J-year long.
  – If the policy owner dies within J years after the initial contract, they will receive a designated amount of benefits.
  – The couple could decide how much to invest in the life insurance.

• Premium vs. benefit
  – If the policy owner dies within the coverage period, the beneficiary receives the face amount according to the conversion ratio (CR).
    • Face value = CR * annual premium for first 10-year sub period.
  – CR decreases at every renewal of the policy
    • The base CR is set to make the expected total premium payment and the expected benefit to be equal according to the given mortality rates.
Basic Settings (3/3)

- Uncertainty
  - As with most implemented ALM models, uncertainty is depicted by a set of distinct realizations, called scenarios, $s \in S$.
  
  - The scenarios may reveal identical values for the uncertain quantities up to a certain period.
  
  - Namely, they share common information history up to that period by means of a scenario tree.

- Decisions: how much to invest in each asset class and insurance?
Investment Policy (1/2)

• Assumption: the couple is rational, and smart.

• Insurance investment policy
  – Since the couple doesn’t value surplus after their deaths, they buy insurance only for the husband.
  – However, if the wife dies before the husbands does, they stop paying the life insurance premium.
  – The couple will buy insurance that will provide the coverage proportionate to their expected remaining life spans.
  – CRs are calculated so that the coverage is proportionate to the expected life time for the husband given a fixed amount of premium.
  – Thus, we assume that they are paying fixed amount of premium throughout their lives before one of the couple dies.
    • There is a difference between the remaining life spans of the husband and the wife, thus leading to different CR structures required for the coverage proportionate to the expected life of the wife
    • However, the difference is not large when expected life spans for the couple are set to be similar, so we employ the policy above for simplicity.
Investment Policy (2/2)

- **Surplus investment policy**
  - Surplus will be reinvested to the portfolio of stock, bond, insurance.

- **If their wealth becomes negative, they simply borrow money from bank at the cash rate.**
  - They make no further investment, meaning that their wealth growth is 0.
  - We assume that they still pay insurance premium.
Parameters

- For each $i \in A$, $t \in T$, and $s \in S$, define the following parameters,

  \[ r_{i,t,s} = 1 + \rho_{i,t,s}, \text{ where } \rho_{i,t,s} \text{ is the rate of return for asset } i, \text{ in period } t, \text{ under scenario } s. \]

  \[ \tau_s^{m(w)} \text{ Final period at the beginning of which the husband (wife) is alive under scenario } s. \]

  \[ \tau_{2,s} = \max \left( \tau_s^w, \tau_s^m \right) \text{ for all } s \in S. \]

  \[ c r_{t,s} \text{ Conversion ratio for the insurance contract in period } t, \text{ under scenario } s. \text{ (Set to 0 for } t \neq \tau_s^m) \]

  \[ \pi_s \text{ Probability that scenario } s \text{ occurs - } \sum \pi_s = 1. \]

  \[ x_{i,0,s} \text{ Amount allocated to asset class } i, \text{ at the end of period } 0, \text{ under scenario } s, \text{ before first rebalancing} \]

  \[ \beta_{t,s} \text{ Cost of borrowing for period } t, \text{ under scenario } s. \]

  \[ y_{t,s}^{TG} \text{ Living expenses made in period } t \text{ (assumed to be at the beginning), under scenario } s. \]

  \[ y_{t,s}^{DB} \text{ Amount of DB pension benefits received at the beginning of period } t, \text{ under scenario } s. \text{ Assumed to be equal to } y_{t,s}^{TG} \text{ for } t \leq \tau_s^m, \text{ and } 0 \text{ otherwise.} \]

  \[ \sigma_{i,t} \text{ Transaction costs for rebalancing asset } i, \text{ period } t \text{ (symmetric transaction costs are assumed).} \]

  \[ p_s \text{ Probability of the scenario } s \in S. \]

  \[ UB \text{ Maximum investment in the insurance.} \]
Decision Variables

- For each $i \in A$, $t \in T$, and $s \in S$, define the following decision variables,
  
  $x_{i,t,s}$: Amount allocated to asset class $i$, at the beginning of period $t$, under scenario $s$, after rebalancing.

  $x_{i,t,s}^{TA}$: Total amount of assets, at the beginning of period $t$, under scenario $s$, after rebalancing.

  $x_{i,t,s}^{\rightarrow}$: Amount allocated to asset class $i$, at the end of period $t$, under scenario $s$, before rebalancing.

  $x_{i,t,s}^{BUY}$: Amount of asset class $i$ purchased for rebalancing in period $t$, under scenario $s$.

  $x_{i,t,s}^{SELL}$: Amount of asset class $i$ sold for rebalancing in period $t$, under scenario $s$.

  $y_{i,t,s}^{PREM}$: Insurance premium paid at the beginning of period $t$, under scenario $s$.

  $y_{i,t,s}^{INS}$: Amount of insurance payment received at the beginning of period $t$, under scenario $s$.

  $y_{i,t,s}^{BORR}$: Amount borrowed at the beginning of period $t$, under scenario $s$.

  $y_{i,t,s}^{DEBT}$: Amount of debt paid back at the beginning of period $t$, under scenario $s$. 
Objective Function

- The objective function is the expected utility function based on the following two values.
  - Expected final wealth
    \[ Z_1 = \sum_{s \in S} p(s) x^{TA}_{\min(\tau_{2,s}, \tau+1), s} \]
  - Expected average life-time borrowing to meet the living expenses
    \[ Z_2 = \sum_{s \in S} p(s) \sum_{t=1}^{\min(\tau_{2,s}, \tau+1)} y^{Borr}_{t,s} / \min(\tau_{2,s}, \tau + 1) \]
  - Expected utility
    \[ \alpha Z_1 - (1 - \alpha) Z_2, \text{ where } \alpha \in [0,1] \]
- The couple want to maximize their expected utility.
Constraints

- Inventory balance equations

$$x_{t,s} = x_{t-1,s} + x^{\text{BUY}}(1 - \sigma_{i,t}) - x^{\text{SELL}_{i,t,s}} , \quad \forall \ s \in S; \ i \neq 1, \ t = 1, \ldots, \tau_{2,s}$$

- Cash flow balance equations

$$x_{1,s} = x_{1-1,s} + \sum_{i=1}^{\text{SELL}} x^{\text{BUY}}(1 - \sigma_{i,t}) - \sum_{i=1}^{\text{BUY}} y_{i,s} + y^{\text{DB}_{t,s}} - y^{\text{TG}_{t,s}} + y^{\text{INS}_{t,s}} - y^{\text{PREM}_{t,s}} + y^{\text{BORR}_{t,s}} - y^{\text{DEBT}_{t,s}} , \quad \forall \ s \in S; \ t = 1, \ldots, \tau_{2,s}$$

- Maximum investment in the insurance

$$y^{\text{PREM}}_{t,s} \leq UB , \quad \forall \ s \in S; \ t = 1, \ldots, \tau_{2,s}$$

- Non-negativity constraints
  - All decision variables are non-negative.
Longevity Risk Management for Individual Investors via MSP

Optimization Model

Maximize \[ \alpha Z_1 - (1 - \alpha)Z_2 \]  

Subject to:

\[ \sum_{i \in A} x_{t,s} = x_{t,s} \]  
\[ \forall s \in S; \quad t = 1, \ldots, \tau + 1 \]  

\[ x_{t,s}^{TA} = \sum_{i \in A} x_{t,s}^{L,s} + \begin{cases} y_{t+1,s}^{INS} - y_{t+1,s}^{DEBT}, & \text{if } t + 1 = \min(\tau_{s}, \tau + 1) \\ 0, & \text{otherwise} \end{cases} \]  
\[ \forall s \in S; \quad t = 1, \ldots, \tau + 1 \]  

\[ y_{t,s}^{\text{INS}} = \alpha r_{t,s}^{\text{PREM}} \]  
\[ \forall s \in S; \quad t = 1, \ldots, \tau_{s} \]  

\[ y_{t,s}^{\text{DEBT}} = \beta_{t,s} y_{t,s}^{\text{BORR}} \]  
\[ \forall s \in S; \quad t = 1, \ldots, \tau_{s} \]  

\[ x_{i,t,s} = x_{i,t-1,s} + x_{i,t,s}^{BUY} (1 - \sigma_{i,t}) - x_{i,t,s}^{SELL} \]  
\[ \forall s \in S; \quad i \neq 1, \; t = 1, \ldots, \tau_{s} \]  

\[ x_{i,t,s} = x_{i,t-1,s} + \sum_{i=1} x_{i,t,s}^{SELL} (1 - \sigma_{i,t}) - \sum_{i=1} x_{i,t,s}^{BUY} y_{i,t,s} - y_{i,t,s}^{T9} + y_{i,t,s}^{INS} + y_{i,t,s}^{PREM} + y_{i,t,s}^{BORR} - y_{i,t,s}^{DEBT} \]  
\[ \forall s \in S; \quad t = 1, \ldots, \tau_{s} \]  

\[ y_{t,s}^{\text{PREM}} \leq UB \]  
\[ \forall s \in S; \quad t = 1, \ldots, \tau_{s} \]  

\[ x_{i,t,s} = x_{i,t,s}^{PREM} = x_{i,t,s}^{T9} = y_{i,t,s}^{BORR} = y_{i,t,s}^{\text{SOVR}} \quad \forall s \text{ and } s' \text{ with identical past up to time } \tau_{s} \]  

All decision variables are non-negative.
Section 3

SCENARIO GENERATION
Uncertainty Modeling: Asset Returns

- We assume that there are two asset classes, stocks and bonds. Asset returns are generated using the two regime model used in the previous MC-based paper.
  - Two regimes can occur: normal and crash

- Parameter assumptions

<table>
<thead>
<tr>
<th></th>
<th>Normal regime</th>
<th>Crash regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected return</td>
<td>[10% 4%]</td>
<td>[-5% 6.5%]</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>Volatility</td>
<td>[15% 10%]</td>
<td>[20% 15%]</td>
</tr>
</tbody>
</table>

- Transition probability = [0.9, 0.1; 0.4, 0.6]
  - Stationary distribution = [0.8, 0.2].
  - Thus, the crash regime comes once every 5 years.
  - Unconditional expected return = [7% 4.5%]
Uncertainty Modeling: Couple’s Life Time

- **Life time**
  - The mortality table (2008) in National Vital Statistics Reports is used to generate couple’s life time without longevity risk.
  - To generate scenario trees with longevity risk, we assume that the couple’s mortality probability decreases or increases by 20%.
  - Increase of mortality rate by 20% means that the couple lives shorter.
Scenario Generation (1/3)

- Clustering method
  - We want to link this MSP-based research to MC-based research.
  - In addition, tail events should be generated in scenario tree.
  - To achieve these goals, we employ the clustering method which is introduced in Dupacova et al. (2000). The two clustering methods, parallel and sequential simulation, are introduced in Gulpinar et al. (2004).
  - We use the sequential simulation method because our model has too many clustering constraints to apply the parallel method. If the parallel method is used, we may be not able to find a scenario tree satisfying clustering conditions.
Scenario Generation (2/3)

- **Algorithm**
  - Step 1: Create a one-stage scenario fan for asset returns and couple’s life time with \( N \) nodes by using MC-simulation.
  - Step 2: Randomly choose distinct seed scenarios. We group the remaining scenarios by these seeds.
  - Step 3: Group each rest scenario with the seeds which is the most similar to the scenario. The clusters should satisfy some conditions (see next slide). We use the Euclidean distance to compare the similarity between scenarios.
  - Step 4: Do step 2 and 3 \( M \)-times to find seed scenarios having minimum distance from rest scenarios.
  - Step 5: After finding the optimal seed scenarios in step 4, find the representative value of each cluster. We employ an average value of scenarios in the cluster.
  - Step 6: Do step 1 to 5 at every stage and at every cluster.
Scenario Generation (3/3)

• Conditions at step 3
  – Condition 1
    • In the same cluster, the scenarios should have the same regime. Therefore, the cluster which seed is a normal regime should have scenarios with a normal regime.
  – Condition 2
    • The relative sizes of the clusters should not be extreme. We define that a ratio of largest/smallest cluster size should be smaller than 20.
  – Condition 3
    • After finding the representative value of each cluster, we should check for an arbitrage opportunity. The first and the second type of arbitrage opportunities are tested using the optimization method introduced in Klaassen (2002).
Section 4

CASE STUDY: RETIREMENT PLANNING
Parameter Assumptions

• In this case study, assume that the couple’s life span is
  – They are both 60 years old.
  – They can live up to 100 years old.
  – The remaining lives of the husband and the wife are assumed to be independent.

• Wealth
  – Annual projected expenses: $50K
  – Annual pension benefit: $50K
  – Current saving: $200K

• Three representative choices of utility for further analysis
  – Conservative: $\alpha = 0$
  – Balanced: $\alpha = 0.0005$
  – Aggressive: $\alpha = 0.001$

• Assume that the life insurance is 10-year long.
Average Portfolio Wealth over Whole Scenario Tree

- When the couple’s mortality rates are not changed (H = 0%, W = 0%)
Average Portfolio Wealth over Whole Scenario Tree

• When the husband lives longer (H = -20%, W = 0%)
Average Portfolio Wealth over Whole Scenario Tree

- When the husband lives shorter (H = 20%, W = 0%)
Utility under the Optimal Policy - Balanced
Utility under the Optimal Policy - Aggressive

Mortality Rate Changes-Wife (%)
Mortality Rate Changes-Husband (%)

Expected Utility
Observations

- Longevity risk is induced by:
  - Shorter life expectation for husband
  - Longer life expectation for wife

- Under the optimal policy, the expected utility decreases as
  - Life expectation for husband decreases
  - Life expectation for wife increases
Optimal Investment on Bonds - Conservative

Mortality Rate Changes-Wife (%)

Mortality Rate Changes-Husband (%)

Investment on Stocks ($K)
Optimal Investment on Bonds - Balanced

- Mortality Rate Changes - Wife (%)
- Mortality Rate Changes - Husband

Investment on Bonds ($K)

- 0%
- 20%
- -20%
- 0%
- 20%

- 50%
- 60%
- 70%
- 80%
- 90%
- 100%
Optimal Investment on Stocks - Conservative
Optimal Investment on Stocks - Balanced
Optimal Investment on Stocks - Aggressive
Observations

• For the conservative utility:
  – the portfolio becomes more aggressive (higher weights on stock) as the longevity risk increases.

• For the balanced utility:
  – the optimal strategy becomes more aggressive as:
    • Longevity risk ↑ (expected life for husband ↓, wife ↑) and,
    • Longevity risk ↓ (expected life for husband ↑, wife ↓)

• For the aggressive utility:
  – the optimal strategy becomes more aggressive as:
    • Longevity risk ↓ (expected life for husband ↑, wife ↓)

• Interpretation:
  – Risk-averse couple manages downside risk by investing heavily in stocks.
  – As the longevity risk diminishes, risk-seeking couple doesn’t need to worry about the downside risk any more. Therefore, they seek for higher surplus to increase their utility by heavily betting on stocks.
Optimal Insurance Investment - Conservative
Optimal Insurance Investment - Aggressive
Observations

• The life expectation parameter for the husband is the dominant factor.
• As the longevity risk increases, the couple seeks for higher utility by cutting the downside risk with the insurance.
Section 5

CONCLUSION
Conclusion

• Longevity risks could impose a critical constraint in investment management practice.

• While our example focuses on the retirement planning problem of a couple, it could be easily extended to various domains.

• Portfolio construction policy coupled with insurance investment strategy should be carefully analyzed.

• It is unlikely to find a generalized linear rule for optimal policy. Thus, an MSP works well to address the longevity risk management problems.

• Since the parameters related to longevity risk are uncertain, we should put more efforts on obtaining robust solutions.
REFERENCES
References


