Optimal capital allocation and strategic portfolio selection for a large P/C insurer

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Outline

1. P/C portfolios stochastic optimization
2. Investment risk exposure
3. Case study and numerical evidences
4. Conclusions
Introduction

The financial environment for global property and casualty (P/C) insurers has changed dramatically in recent years mainly due to:

- record P/C claims,
- historically low fixed income and real estate returns,
- ongoing introduction of regulatory, risk-based, capital requirements (Solvency II)
- increasing market competition.

Jointly or individually those pressures have determined increasing liquidity deficits in core P/C activity, stressed technical and financial scenarios and stimulated integration between insurance, investment and risk management divisions of insurance corporations.
We analyse the specific problem of an investment manager subject to technical and maximum risk exposure constraints employing a dynamic portfolio strategy over a 10 year horizon. Key to the analysis:

- the introduction of risk-adjusted performance measures as medium and long-term investment targets,
- the enlargement of the investment universe to include alternative investments,
- the control of risk capital exposure over time,
- the sensitivity of short, medium and long term goals to stressed insurance and financial scenarios.
Asset and liability scenarios

Uncertainty is modeled through scenario trees. For given tree structure, random price $\rho$ and cash returns $\xi$ for all asset classes are generated through a set of econometric models and exogenously by a corporate economic scenario generator.

The asset universe includes:

- fixed-income asset classes: Treasury 1-3, 3-5, 5-7, 7-10 and 10+ year maturity indices; 10 year Securitised, IG and SG Corporate Bond indices; 5 and 10 year inflation-linked fixed income indices;
- equity assets: MSCI EMU (Public) Equity index and Private equity;
- real estates: indirect GPR Europe real estate index and Direct real estate;
- alternative investments: Infrastructure Cyclical and Defensive; Renewable energy and Commodities.

All insurance liabilities are inflation adjusted and generated exogenously by the company insurance division.
Objective function

For $t \in \mathcal{T}$, $n \in \mathcal{N}_t$ the following profit variables are considered in the objective function:

- $V^1_n = (\Pi^f_n + \Pi^t_n)$ is the sum of technical and financial operating profits before taxes – the **operating profit**

- $V^2_n = (\Pi^f_n - \Phi_n)\phi - \kappa K^f_n$ defines the surplus investment value – the **IVC** – generated by the portfolio manager and

- From $V^1$ we denote with $V^3_n = V^1_n \phi$ the company cumulative profit after taxes and for given **RoRAC** target $\tilde{z}$ we denote with $\tilde{V}^3 = \tilde{z}K_n$ the target company return.

$\Phi_n$ represents the cost of funding faced by the investment manager. Three targets $\tilde{V}_j$ for $j = 1, 2, 3$ are set by the management at the 1, 3 and 10 year horizon respectively.
Optimization problem

We solve the following stochastic program

\[
\max_{x \in X} \left\{ (1 - \alpha) \sum_j \lambda_j E[V^j_n | \Sigma_n] - \alpha \sum_j \lambda_j E[\tilde{V}^j - V^j_n | \Sigma_n] \right\}
\]  

(1)

s.t for all \( n \in \mathcal{N}_t, t \in \mathcal{T} \)

\[
X_n = X_n^-(1 + \rho_n) + X_n^+ - X_n^- + z_n
\]  

(2)

\[
z_n = X_n^- - X_n^+ + R_n - L_n - C_n + X_n^- \xi_n + z_n^- (1 + r_n)
\]  

(3)

\[
\Pi^f_n = X_n^- \xi_n + \sum_{m \in a(n)} X^-_{m,n} \gamma_{m,n} + \Pi^f_{n-}
\]  

(4)

\[
X_n^+ + X_n^- = \vartheta X_n^- (1 + \rho_n)
\]  

(5)

\[
K^f_n \leq X_n - \Lambda_n - K^t_n
\]  

(6)

\[
\sum_i X^i_n l_i \leq l(X_n)
\]  

(7)
P/C premiums $R_n$ are treated as exogenous and driven by market competition. The investment manager does not have a reinsurance option. Furthermore:

- $L_n$ the insurance claims and $C_n$ the associated operational costs are inflation adjusted.
- $\Gamma_n := \frac{L_n + C_n}{R_n}$ denotes a combined ratio monitored by the company to assess the insurance business efficiency.
- $\Lambda_n$ are statutory inflation-adjusted reserves allocated at time $t_n$.
- $K_n^t$ denotes the actuarial risk capital to hedge against unexpected losses from the core P/C business. We assume $K_n^t = \Lambda_n \times \kappa^t$ where $\kappa^t$ is a predetermined constant risk multiplier.
- $K_n^t$ the investment risk capital estimated at node $n$. $K_n = K_n^f + K_n^t$ defines the node $n$ company global risk exposure.
Investment risk capital

\[
K_n^f = K_n^{f_1} + K_n^{f_2} \quad \text{(8)}
\]

\[
K_n^{f_1} = \left( \Delta^A_{n-} - \Delta^\Lambda_{n-} \right) dr_n(t_n - t_{n-}) + K_n^{f_1} \quad \text{(9)}
\]

\[
K_n^{f_2} = \sqrt{\sum_{i \in A} \sum_{j \in A} X_{i,n-} X_{j,n-} k_{ij}(t_n - t_{n-})} + K_n^{f_2} \quad \text{(10)}
\]

where \( \Delta^A_n = \sum_i \sum_{m \in a(n)} x_{i,m,n} \Delta_{i,n} \) and \( \Delta^\Lambda_n = \sum_{T > t_n} \Lambda_{n,T} \times (t - t_n) \) are the asset and liability durations in node \( n \). \( K_n^{f_1} \) and \( K_n^{f_2} \) define the interest rate and market risk exposure of the portfolio. 

\( \hat{K}_n^f := X_n - \Lambda_n - K_n^t \) represent a maximum risk exposure for the portfolio manager whose aim is to maximise the portfolio expected return per unit risk capital.
Correlations

In (8), $k_{ij} = k_i \cdot k_j \cdot \rho_{ij}$, where $k_i$ is the $i$-th asset risk-charge and $\rho_{ij}$ is the correlation between assets $i$ and $j$.

We consider three cases: $\rho = 1$ for independent risk factors, $\rho := \{\rho_{ij}\}$ a Solvency II compliant matrix and $\rho := 1$ a correlation matrix of all 1 for perfect positive correlation.

The following relationships hold $\forall n \in \mathcal{N}_t$, $t \in \mathcal{T}$ and are taken into account in the results’ validation:

$$K_{n^2}^f(I) \leq K_{n^2}^f(\rho) \leq K_{n^2}^f(1) \quad (11)$$

The equation for $K_{n^2}^f$ with $\rho = 1$ becomes linear.
The stochastic program (1) to (7) is solved under alternative specifications of the objective function:

- For $K^f_n = K^f_m + \left[ K^f_n + \sum_i X^i_m k_i(t_n - t_m) \right]$, the SLP deterministic equivalent is solved with CPLEX dual simplex method.

- For $K^f_n = K^f_m + \left[ K^f_n + \sum_i \sum_j X^i_m X^j_m k_{ij}(t_n - t_m) \right]$ we adopt the CPLEX QCP solver. After solution we recover the risk capital original equation and analyse the risk capital dynamics and associated risk-adjusted reward measures.
Case study

We present a set of results focusing on:

- the generation and solution times of a 768 scenario problems
- the first stage implementable decisions under the SLP and SQCP formulations
- the investment risk capital optimal absorption under different risk factor correlation assumptions along specific scenarios
- the key variables sensitivity to increasing P/C loss ratios and risk capital constraints
- the trade-off between portfolio liquidity and alternative investments
- the targets attainability under the SLP and SQP approaches

The results have been collected on a HP workstation with Intel(R) Core i3-3220-CPU at 3.30Ghz processor and 5 Gb of RAM running Windows 8 O.S. and 453 GB of hard disk.
We analyse a 768 scenario problem with 1, 3 and 10 years targets for operating profit, IVC and RoRAC. The investment policy is constrained by a minimum 30% investment in Treasuries and maximum 50% investment in TIPS and corporates, 25% on equity, 20% on alternatives and commodities. A maximum 30% turnover is allowed from stage to stage. We present results for $\alpha = 1$ in the objective function.

The model has been instatiated in GAMS with scenarios and GUI and main program in Matlab. We generate a deterministic equivalent in Gams and select the solver for the large-scale MPS file from GAMS. As mentioned CPLEX solution algorithms have been adopted.
### Results

#### Table: P/C problem dimension and solution time

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<th>SCENARIO TREE</th>
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Optimal portfolio strategy

Figure: Optimal portfolios – linear and quadratic 768 scenario SPs
Default boundary and targets achievement

Figure: SLP Targets achievement likelihood and risk capital hedge
Default boundary and targets achievement

Scenario Analysis Comments

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Risk Cap. bound

Figure: SQP Targets achievement likelihood and risk capital hedge

G. Consigli
P/C ALM via dynamic stochastic optimization
Optimal risk capital allocation

Investment Risk Capital Bounds
SLP 768 scenarios

Investment Risk Capital Bounds
SQP 768 scenarios
Sensitivity to loss ratios and risk capital limits

Figure: 768 scenario SLP optimal loss ratio-risk capital trade-off
Portfolio liquidity and alternative investments
The need by the Insurance company of a unified approach to core insurance business, investment management and risk management has motivated the present development.

The mathematical model combines accounting and regulatory equations with the emerging financial and risk-management philosophy in the insurance business.

The combination of capital allocation constraints and risk-adjusted performance measurement supporting trade-off and stress-testing analysis had a relevant impact on the company strategic planning potential.

Increasing emphasis goes into solution analysis.

Scenario generation over a long term horizon is critical to the efficiency of the suggested optimal policy.