Long-term Bank Balance Sheet Management
Estimation and Simulation of Risk Factors

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Summary

1. Motivation
2. One-period model
3. Multiperiod model
4. Evolution of the balance sheet
5. Estimation of the model
6. Comparison with Vašíček credit model
7. Simulation of risk scenarios
8. Conclusion
1. Motivation

- Bank has short-term liabilities and long-term assets.
- Assets are funded by:
  - Equity – can absorb losses but shareholders demand good return on equity (ROE).
  - Core deposits – pay typically low interest rate but growth is limited.
  - Wholesale funding – can be a good way of expanding the balance sheet, but are more volatile.
- Main decision of bank is deciding the expansion of the balance sheet
  - Balance sheet too big -> more risk
  - Balance sheet too small -> ROE becomes smaller
1. Motivation

- Decisions of the expansion of the balance sheet should be:
  - Long-term (assets have a long maturity)
  - Dependent on economic cycle (credit and interest rates)

- Our research has 4 aims
  - Aim 1: incorporate interest rate interactions and cycles in risk analysis
  - Aim 2: include credit risk cycles
  - Aim 3: simulate long-term risk scenarios given a bank’s lending strategy
  - Aim 4: set the foundation for long-term balance sheet optimization
2. Simple one-period model

- Assume that we have the initial balance sheet: $E_0$ equity, $D_0$ core deposits, $N_0$ wholesale funding and $L_0$ loans.
- The bank is faced with the decision of taking on new loans $L_{new}$.
- The bank wants to maximize

$$
\max \mathbb{E} U \left[ E_0 + L_0 m_{curr} + L_{new} m_{new} \right]
$$

where

$$
m_{curr} = \frac{I - N_0 f}{L_0} - \lambda
$$

$$
m_{new} = r - f - \lambda
$$

- $r$ – rate on new loans
- $f$ – rate on wholesale funding
- $I$ income on existing loans
- $\lambda$ – charge-off rate (random variable)
2. Simple one-period model

- Approximate solution:

\[ L_{new} = \frac{E_0 \mathbb{E}(m_{new})}{\gamma \mathbb{E}(m_{new}^2)} - L_0 \frac{\mathbb{E}(m_{curr}m_{new})}{\mathbb{E}(m_{new}^2)} \]

- All moments are very easy to evaluate.

- If \( L_0 = 0 \) the formula gives the optimal leverage ratio for the bank

\[ \frac{L_{new}}{E_0} = \frac{\mathbb{E}(m_{new})}{\mathbb{E}(m_{new}^2)} \left( \frac{1}{\gamma} - f \right) - \frac{D_0 f}{E_0} \]

- Higher risk implies lower desired leverage ratio.

- Higher risk aversion implies lower desired leverage ratio.
3. Multiperiod model

- We assume four risk factors: mortgage rate, non-core funding rate, core funding rate and charge-off rate at annual frequency.
3. Multiperiod model

- Interest rate equations: consider the de-averaged processes

\[ r_t^* = r_t - \gamma_r ; \quad f_t^* = f_t - \gamma_f ; \quad d_t^* = d_t - \gamma_d \]

and write

\[ r_{t+1}^* = \phi_{r,r} r_t^* + \phi_{r,f} f_t^* + \phi_{r,d} d_t^* + \phi_{r,m} m_t^r + \sqrt{r_t} \epsilon_{t+1}^{r*} \]

\[ f_{t+1}^* = \phi_{f,r} r_t^* + \phi_{f,f} f_t^* + \phi_{f,d} d_t^* + \phi_{f,m} m_t^f + \sqrt{f_t} \epsilon_{t+1}^{f*} \]

\[ d_{t+1}^* = \phi_{d,r} r_t^* + \phi_{d,f} f_t^* + \phi_{d,d} d_t^* + \phi_{d,m} m_t^d + \sqrt{d_t} \epsilon_{t+1}^{d*} \]

\[ m_t^r = r_t - r_{t-1} ; \quad m_t^f = f_t - f_{t-1} ; \quad m_t^d = d_t - d_{t-1} \]

- Charge-off rate equations: change variables \( y_t = N^{-1}(\lambda_t) \)

and specify

\[ y_{t+1} = c + \phi_y y_t + \phi_m m_t^y + \epsilon_{t+1}^y \]

\[ m_t^y = y_t - y_{t-1} \]
4. Balance sheet equations

- Assume core deposits fixed: the bank will have to finance

\[(1 + n_t - p)L_t - D - E_t\]

in the wholesale funding market.

- Assume that the bank grows loans at a rate \(n_t\) at time \(t\).

- Loan expansion equation:

\[L_{t+1} = L_t(1 + n_t - p - \lambda_{t+1})\]

- Income on loans:

\[I_{t+1} = I_t(1 - p - \lambda_{t+1}) + L_t n_t r_t\]

- Equity accumulation:

\[E_{t+1} = E_t + I_{t+1} - ((1 + n_t - p)L_t - E_t - D)f_t - Dd_t - L_t(c + \lambda_{t+1}).\]
5. Estimation of the model

- Interest rate model can be estimated by OLS:

<table>
<thead>
<tr>
<th>VAR estimation</th>
<th>$\gamma_r$</th>
<th>$r_t^*$</th>
<th>$f_t^*$</th>
<th>$d_t^*$</th>
<th>$m_t$</th>
<th>$R^2$</th>
<th>Std. err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{t+1}^*$</td>
<td>8.92</td>
<td>0.51</td>
<td>-0.17</td>
<td>0.85</td>
<td>0.21</td>
<td>0.95</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(3.94)</td>
<td>(-1.27)</td>
<td>(4.29)</td>
<td>(1.34)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{t+1}^*$</td>
<td>6.01</td>
<td>0.15</td>
<td>-0.19</td>
<td>1.38</td>
<td>0.64</td>
<td>0.97</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>(0.71)</td>
<td>(-1.05)</td>
<td>(4.55)</td>
<td>(5.48)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{t+1}^*$</td>
<td>3.75</td>
<td>0.21</td>
<td>-0.16</td>
<td>0.92</td>
<td>0.47</td>
<td>0.95</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>(1.31)</td>
<td>(-1.30)</td>
<td>(3.35)</td>
<td>(2.69)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Charge-off process also estimated by OLS:

<table>
<thead>
<tr>
<th>Charge-off process estimates</th>
<th>$c$</th>
<th>$\phi_y$</th>
<th>$\phi_m$</th>
<th>Std. err.</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{t+1}$</td>
<td>-0.85</td>
<td>0.70</td>
<td>0.75</td>
<td>0.18</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>(-2.89)</td>
<td>(6.70)</td>
<td>(4.12)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Estimation of the model

- We checked normality of residuals using the Jarque-Bera test

\[ JB = \frac{n}{6} \left( S^2 + \frac{1}{4}K^2 \right) \]

which follows a chi-square distribution with two degrees of freedom (critical value 5.99 at 5% significance):

<table>
<thead>
<tr>
<th>Jarque-Bera tests</th>
<th>( e_t^r )</th>
<th>( e_t^f )</th>
<th>( e_t^d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jarque-Bera test</td>
<td>0.08</td>
<td>1.49</td>
<td>1.83</td>
</tr>
<tr>
<td>Result</td>
<td>not reject</td>
<td>not reject</td>
<td>not reject</td>
</tr>
</tbody>
</table>

- Also checked that lag-one autocorrelation of residuals is very close to zero.
6. Comparison with Vašíček credit model

- Vašíček credit model: loan \( i \) has default probability \( p \) and defaults if \( Z_i < 0 \), where

\[
Z_i = -N^{-1}(p) + \sqrt{\rho} Y + \sqrt{1 - \rho} X_i,
\]

- One can show that for a large portfolio of equal loans the loss is equal to

\[
L = N \left( \frac{N^{-1}(p) - \sqrt{\rho} Y}{\sqrt{1 - \rho}} \right)
\]

- If \( L_t \) are losses at time \( t \), then

\[
N^{-1}(L_t) = \frac{N^{-1}(p)}{\sqrt{1 - \rho}} - \frac{\sqrt{\rho} Y_t}{\sqrt{1 - \rho}}
\]

which by change of variables is the same as

\[
y_t = c + \epsilon_t^y
\]

This idea was used in Kupiec (2009).
6. Comparison with Vašíček credit model

- We test three models.

- Model 1: \( y_t = c + \varepsilon_t^y \) (Vašíček)

- Model 2: \( y_{t+1} = c + \phi_y y_t + \varepsilon_{t+1}^y \) (Kupiec)

- Model 3: \( y_{t+1} = c + \phi_y y_t + \phi_m n_t^y + \varepsilon_{t+1}^y \) (our model)

- All models can be estimated by ordinary least squares.
6. Comparison with Vašíček credit model

- All model residuals pass the normal test.
- Our model has no autocorrelation of residuals.
6. Comparison with Vašíček credit model

- Including momentum is important!

Figure 4: Density of the charge-off rate $\lambda_{t+1}$, conditional on $\lambda_t = 1\%$, using our OLS parameter estimates. Negative momentum assumes that $\lambda_{t-1} = 1.5\%$; no momentum assumes that $\lambda_{t-1} = 1\%$; and positive momentum assumes that $\lambda_{t-1} = 0.5\%$. 
7. Simulation of risk scenarios

- Interest-rate scenarios:
  - Wholesale rate (pink) consistently between core deposit rate (yellow) and mortgage rate (blue)
  - Can achieve very high and very low interest rates: important for risk management.
7. Simulation of risk scenarios

- Charge-off rates:

- Simulated charge-off rates show peaks and autocorrelation (consistent with the data).
7. More simulation results

<table>
<thead>
<tr>
<th>Baseline parameters</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Prepayment rate ( p )</td>
<td>15%</td>
<td>Initial equity ( E_0 )</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Initial mortgage rate ( r_0 )</td>
<td>4.69%</td>
<td>Initial loans ( L_0 )</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>Initial non-core rate ( f_0 )</td>
<td>0.2%</td>
<td>Core deposits ( D )</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>Initial core rate ( d_0 )</td>
<td>0.28%</td>
<td>Growth on loans</td>
<td>16.5%</td>
<td></td>
</tr>
<tr>
<td>Initial charge-off rate ( \lambda_0 )</td>
<td>1.97%</td>
<td>Core deposit growth</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>Costs-to-loans ( c )</td>
<td>1.5%</td>
<td>Number of paths</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>Minimum leverage ratio</td>
<td>5%</td>
<td>Maximum leverage ratio</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>Payout ratio</td>
<td>50%</td>
<td>Number of years</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Discount factor ( \rho )</td>
<td>0.95</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7. More simulation results

### Impact of the funding ratio

<table>
<thead>
<tr>
<th>Funding ratio $(L_0/D_0)$</th>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.55</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>15.64</td>
<td>0.23</td>
</tr>
<tr>
<td>3</td>
<td>13.34</td>
<td>0.48</td>
</tr>
<tr>
<td>5</td>
<td>11.61</td>
<td>0.77</td>
</tr>
</tbody>
</table>

### Impact of the degree of leverage

<table>
<thead>
<tr>
<th>Leverage $(L_0/E_0)$</th>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>13.33</td>
<td>0.01</td>
</tr>
<tr>
<td>10</td>
<td>15.64</td>
<td>0.23</td>
</tr>
<tr>
<td>20</td>
<td>20.31</td>
<td>2.56</td>
</tr>
</tbody>
</table>
7. More simulation results

### Impact of interest rates

<table>
<thead>
<tr>
<th>Initial interest rate shock</th>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>baseline</td>
<td>15.64</td>
<td>0.23</td>
</tr>
<tr>
<td>+5%</td>
<td>30.17</td>
<td>0.35</td>
</tr>
<tr>
<td>+10%</td>
<td>45.63</td>
<td>0.33</td>
</tr>
</tbody>
</table>

### Impact of costs

<table>
<thead>
<tr>
<th>Cost to loans</th>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>21.14</td>
<td>0.10</td>
</tr>
<tr>
<td>1.5%</td>
<td>15.64</td>
<td>0.23</td>
</tr>
<tr>
<td>2%</td>
<td>10.30</td>
<td>0.63</td>
</tr>
</tbody>
</table>
8. Conclusion

- Framework for studying long-term balance sheet management through economic cycles.
- Developed one-period model and estimated the balance-sheet processes for the multiperiod model.
- Model is quite flexible.
- Applications (ongoing research):
  - Assessing effects of individual institutional policies.
  - Long-term risk management.
  - Long-term risk-return optimization.
- Paper 1 available at ssrn (same title as the presentation):