The Stochastic Mixed Capacitated General Routing Problem: formulation and solution approaches

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The Stochastic Mixed Capacitated General Routing Problem: formulation and solution approaches

Outline

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2. Problem description
3. Deterministic equivalent
4. Branch and cut
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The Mixed Capacitated General Routing Problem

A set of closed routes starting and ending at the depot is a feasible solution if:

- Each required edge/arc is serviced by exactly one route;
- The sum of demands of the serviced edges/arcs/vertices in each route does not exceed the vehicle capacity.

Edges/arcs in a route can be either serviced or deadheaded.

**Goal**

Find a feasible solution minimizing the sum of the service cost and the deadheading of the route.
The Mixed Capacitated General Routing Problem

Service activity occurs both at some of the nodes and at some of the arcs and edges of a mixed graph

Applications:

- collection and delivery of goods
- waste collection
- street cleaning
- school-bus routing
- snow removal
The General Routing Problem

<table>
<thead>
<tr>
<th>VRP</th>
<th>Customers/demand concentrated in sites associated with vertices - only required vertices</th>
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<tr>
<td>ARP</td>
<td>Customers/demand evenly distributed along the edges - only required edges</td>
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Challenges

Is one of the more complex combinatorial optimization problems on vehicle routing.
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The General Routing Problem

Literature review

Uncertainty

Motivation

Solid waste collection system

Since the operational plan is updated every five or more periods over a long planning horizon, it becomes crucial to design a set of a priori routes.

Goal:

Optimize the operational plan designing a set of a priori routes with a threshold constraint on the probability of a route failure.
Routing problems under uncertainty

Problem description

Notation

- A mixed graph $G = (V, A, E)$.
- $V = \{1, \ldots, n\}$
- $A = \{(i, j) \subseteq V \times V\}$
- $E = \{(i, j) \subseteq V \times V : i < j\}$
- $L = A \cup E$
- $c_{ij}$
- $V_R \subseteq V$
- $A_R \subseteq A$
- $E_R \subseteq E$
- $m$ vehicles with the same capacity $Q$
The Stochastic Mixed Capacitated General Routing Problem: formulation and solution approaches

Problem description

Decision variables

- $x_{ij}^k$ the binary variable equal to 1 if and only if $(i,j)$ is serviced by vehicle $k$ which travels from vertex $i$ to vertex $j$ and 0 otherwise
- $x_i^k$ the binary variable equal to 1 if $i$ is serviced by $k$ and 0 otherwise $\forall i \in V_R$ and $\forall k = 1, \ldots, m$
- $y_{ij}^k$ the non-negative variable representing the number of deadheading traversals from vertex $i$ to vertex $j$ by $k$
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Problem description

Objective function

Min $f = f_1 + f_2$ \hspace{1cm} (1)

$f_1 = \sum_{k=1}^{m} \sum_{(i,j) \in E_R} c_{ij}(x_{ij}^k + x_{ji}^k) + \sum_{k=1}^{m} \sum_{(i,j) \in A_R} c_{ij}x_{ij}^k$

$f_2 = \sum_{k=1}^{m} \sum_{(i,j) \in E} c_{ij}(y_{ij}^k + y_{ji}^k) + \sum_{k=1}^{m} \sum_{(i,j) \in A} c_{ij}y_{ij}^k$
Three index formulation

The following constraints hold:

\[ \sum_{k=1}^{m} (x_{ij}^k + x_{ji}^k) = 1, \quad (i,j) \in E_R \quad (2a) \]

\[ \sum_{k=1}^{m} x_{ij}^k = 1, \quad (i,j) \in A_R \quad (2b) \]

\[ \sum_{k=1}^{m} x_{i}^k = 1, \quad i \in V_R \quad (2c) \]
Flow constraints

\[ \sum_{j:(i,j) \in \delta^+_AR(i)} x^k_{ij} + \sum_{j:(i,j) \in \delta^+(i)} y^k_{ij} - \sum_{j:(j,i) \in \delta^-AR(i)} x^k_{ji} - \sum_{j:(j,i) \in \delta^-(i)} y^k_{ji} = \sum_{j:(i,j) \in \delta^+_ER(i)} x^k_{ji} + \sum_{j:(i,j) \in \delta(i)} y^k_{ji} - \sum_{j:(i,j) \in \delta^-ER(i)} x^k_{ij} - \sum_{j:(i,j) \in \delta(i)} y^k_{ij} \]

\[ k = 1, \ldots, m, \; i \in V \]
Connectivity constraints

\[
\sum_{(i,j) \in \delta^+_A(S)} x_{ij}^k + \sum_{(j,i) \in \delta^-_A(S)} x_{ji}^k + \sum_{(i,j) \in \delta_E(S)} (x_{ij}^k + x_{ji}^k) + \sum_{(i,j) \in \delta^+(S)} y_{ij}^k + \sum_{(j,i) \in \delta^-(S)} y_{ji}^k + \sum_{(i,j) \in \delta(S)} (y_{ij}^k + y_{ji}^k) \geq \begin{cases} 
2(x_{uv}^k + x_{vu}^k), & (u, v) \in E_R(S), \\
2x_{uv}^k, & (u, v) \in A_R(S), \\
2x_h^k, & h \in S_R,
\end{cases} \\
k = 1, \ldots, m, \ S \subseteq V \setminus \{1\} 
\]
Problem description

Variables definitions

\[ x_{ij}^k \in \{0, 1\}, \quad k = 1, \ldots, m, \ (i, j) \in A_R \cup E_R \] (4a)

\[ y_{ij}^k \in \mathbb{Z}_+, \quad k = 1, \ldots, m, \ (i, j) \in A \cup E \] (4b)

\[ x_i^k \in \{0, 1\}, \quad k = 1, \ldots, m, \ i \in V_R. \] (4c)
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Problem description

Capacity constraints

\[ \{x \in \{0, 1\}^{(|A|+2|E|+|V|) \times m}, \ y \in Z_+^{(|A|+2|E|) \times m} \]

\[ \text{IP}(D(\omega)x^k \leq Q) \geq \alpha, \ k = 1, \ldots, m \]

Here \( D \) is a row vector having the following structure:

\[
D = \begin{pmatrix}
\underbrace{d_{ij}(\omega)}_{(i,j) \in E \cap R} & \underbrace{d_{ij}(\omega)}_{(i,j) \in A \cap R} & \underbrace{d_i(\omega)}_{i \in V} & \underbrace{d_{ij}}_{(i,j) \in E \cap C} & \underbrace{d_{ij}}_{(i,j) \in A \cap C} & \underbrace{d_i}_{i \in V} \\
\end{pmatrix}
\]
Challenges

**Combinatorial problem under probabilistic constraints**

- Separate chance constraints with random left hand side
- Integrality of the variables
- Combinatorial structure
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Deterministic equivalent

\[ \sum_{e \in R_U} \mu_e z_e^k + \sum_{e \in R_C} d_e z_e^k + \Phi^{-1}(\alpha) \sqrt{\sum_{(e,e') \in R_U^2} \theta_{e,e'} z_e^k z_{e'}^k} \leq Q \] (5)

where

\[ z_e^k = x_{ij}^k + x_{ji}^k \text{ if } e = (i,j) \in E_R, \]
\[ z_e^k = x_{ij}^k \text{ if } e = (i,j) \in A_R, \]
\[ z_e^k = x_i^k \text{ if } e = i \in V_R. \] (6)
Deterministic equivalent

\[
\sum_{e \in R} \mu_e z^k_e \leq Q \\
\left[ \Phi^{-1}(\alpha) \right]^2 \left( \sum_{e \in R} \sigma^2_e z^k_e + \sum_{(e,e') \in R^2 \mid e \neq e'} \theta_{e,e'} z^k_e z^k_{e'} \right) \leq \\
\left( Q - \sum_{e \in R} \mu_e z^k_e \right)^2
\]
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Deterministic equivalent

\[
\sum_{e \in R} \mu_e z_e^k \leq Q,
\]
\[
\sum_{e \in R} \left( [\Phi^{-1}(\alpha)]^2 \sigma_e^2 + (2Q - \mu_e) \mu_e \right) z_e^k \leq Q^2 + \sum_{(e, e') \in R^2 | e \neq e'} \left( \mu_e \mu_{e'} - [\Phi^{-1}(\alpha)]^2 \theta_{e, e'} \right) w_{e, e'}^k,
\]
\[
w_{e, e'}^k \leq z_e^k, \quad w_{e, e'}^k \leq z_{e'}^k, \quad w_{e, e'}^k \geq z_e^k + z_{e'}^k - 1, \quad (e, e') \in R^2, \quad e \neq e',
\]
\[
z_e^k = x_{ij}^k + x_{ji}^k \quad \text{if} \quad e = (i, j) \in E_R,
\]
\[
z_e^k = x_{ij}^k \quad \text{if} \quad e = (i, j) \in A_R,
\]
\[
z_e^k = x_i^k \quad \text{if} \quad e = i \in V_R
\]
Capacity cuts separation

(Belenguer and Benavent, 2003) If all the required elements inside $S$ and in the cutset defined by $S$ not include stochastic required elements, then

$$
\sum_{(i,j) \in \delta_L(S)} \eta_{ij} \geq 2 \left[ \frac{D(R(S), \delta_{LR}(S))}{Q} \right] - |\delta_{LR}(S)|, \quad S \subseteq V \setminus \{1\},
$$

(9)

where $R(S)$ is the set of required elements in $S$ and $\delta_{LR}(S)$ are the required links with one endpoint in $S$. 
Probabilistic fractional capacity inequalities

\[ \text{IP} \left( 2 \frac{D(R(S, \omega), \delta_{LR}(S, \omega))}{Q} \right) \leq \sum_{(i,j) \in \delta_L(S)} \eta_{ij} + |\delta_{LR}(S)| \right) \geq \alpha, \]

(10)
Property

For $S \subseteq V \setminus \{1\}$, if the following linear inequalities holds:

$$\sum_{(i,j) \in \delta_L(S)} \eta_{ij} \geq 2 \frac{D(R(S), \delta_{LR}(S))}{Q} - |\delta_{LR}(S)| \quad (11)$$

$$\frac{4}{Q^2} \left[ \Phi^{-1}(\alpha) \right]^2 \left( \sum_{e \in R(S) \cup \delta_{LR}(S)} \sigma_e^2 + \sum_{(e,e') \in (R(S) \cup \delta_{LR}(S))^2 \mid e \neq e'} \theta_{e,e'} \right) \leq$$

$$2 \left( |\delta_{LR}(S)| - 2 \frac{D(R(S), \delta_{LR}(S))}{Q} \right) \sum_{(i,j) \in \delta_L(S)} \eta_{ij} + \quad (12)$$

$$2 \left( 2 \frac{D(R(S), \delta_{LR}(S))}{Q} - |\delta_{LR}(S)| \right)^2$$

then (10) is a valid cut.
Consider all constraints and inequalities, except the connectivity and capacity constraints.

Valid inequalities are included in the root node and in each node of the branch and cut tree.

Branching occurs in priority on variables $x_{ij}^k, x_i^k$ and then on variables $y_{ij}^k$.

An initial upper bound is obtained using an initial partition-first-route-next heuristic.
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Branch and cut

Partition-first-route-next heuristic

Partition

Capacitated concentrator location-based problem with stochastic demands

1. a required element is assigned to itself whether it is a concentrator;

2. each required element is assigned to only one concentrator;

3. the probability that the overall demand of the required elements assigned to a concentrator does not exceed the vehicle capacity should be greater than \( \alpha \)

The goal consists of minimizing the total assignment cost of the required elements to the concentrators.

Route next

Each cluster represents an instance of the MGRP.
A nested large neighborhood search heuristic

Loop until termination criterion met

- Diversification
- Large neighborhood search
- Intensification

Endloop
Diversification

Adaptation of the removal and insertion heuristics presented by Pisinger and Ropke 2007

R1 random remove: selects $p\%$ required elements at random and removes them from the solution

R2 worst remove: removes $p\%$ of the required elements on the basis of the saving achievable

R3 Demand-Oriented: removes $p\%$ of the required elements according to a metric
\[ \Delta_{ij} = \|d_i - d_j\| .\]

I1 random insert: randomly inserts the elements in a feasible route with a random position.

I2 regret insert: inserts the elements with the aim of maximizing the regret associated with each element.
Diversification

- R1+I1
- R2+I2
- R2+I3
Large neighborhood search

1) **merge**: for each couple of routes $r$ and $r'$ in the current solution $s$, an attempt is made in order to merge these routes by servicing all the required elements of $r'$ after the required elements serviced in $r$.

2) **$\lambda$—interchange**: this move is similar to the one defined by Osman 1993 and tries to swap simultaneously a sequence of $\lambda$ consecutives required elements in two different routes. In the computational experiments, we used $\lambda \in \{1, 2, 3, 4, 5\}$.

3) **interchangelInterRoute**: this move is the same introduced by Savelsbergh 1992 and consists of reinserting a single required element at a time in an alternate route.

4) **interchangelInterRouteNewPath**: this move creates a new empty route in which a single required element is inserted.

5) **twoOptPlusInterRoute**: this move corresponds to the 2-opt* move defined by Potvin and Rousseau.
Large neighborhood search

- VND
- $m$-infeasible
- $Q$-infeasible
Route optimization example

Integer programming

\[ m = 2 \text{ and } R = R(h) \cup R(k) \]
How do we select the two or three vehicles whose routes define the neighborhood?

Let $L^{dd}(k)$ be the set of links deadheaded by vehicle $k$ in route $r_k$. We select the couple of vehicles $(k, h)$ with the following rules:

1. $|L^{dd}(k) \cap L^{dd}(h)|$ is maximum.
2. The couple/triplet for which the maximum violation of the capacity constraint is checked.
3. The couple/triplet with the maximal residual capacity.

We stop after evaluating the thirty percent of all the couples.
Dataset

- Randomly generated instances starting from the dataset designed by Bosco et al. 2012
- For each required element $e \in \mathcal{R}$ with demand $d_e$, a random binary number is extracted to decide if $d_e$ is affected by uncertainty or not.
- Then, a random value from the discrete uniform distribution over $\{1, \ldots, 5 \times d_e\}$, is selected as variance of the random variable $d_e(\omega)$.
- Correlation $\rho$ is selected randomly in the interval $[-1, 1]$. 
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Computational experiments

Table: Computational results for small instances.

| Name      | m | V | A | E | VR | AR | ER | VRU | ARU | ERU | UB | CON | PCAP | cost | veh. | time | gap | avg. cost | avg. veh. | best cost | best veh. | avg. gap | avg. time |
|-----------|---|---|---|---|----|----|----|-----|-----|-----|----|-----|------|------|------|-----|------|--------|-------|----------|---------|---------|---------|---------|----------|
| mggbsd15  | 5 | 7 | 32 | 5 | 5 | 12 | 3 | 3 | 7 | 1 | 57 | 3 | 8 | 55 | 4 | 6.19 | 0.00 | 55 | 4 | 55 | 4 | 0.00 | 6.9 |
| mggbsd19  | 4 | 8 | 18 | 2 | 3 | 6 | 1 | 1 | 1 | 0 | 61 | 60 | 17 | 59 | 4 | 10.55 | 0.00 | 59 | 4 | 59 | 4 | 0.00 | 3.32 |
| mggbsd20  | 6 | 11 | 34 | 5 | 5 | 12 | 3 | 1 | 7 | 1 | 127 | 50 | 38 | 116 | 5 | 1,629.39 | 0.00 | 116 | 5 | 116 | 5 | 0.00 | 302.54 |
| mggbsd15  | 5 | 7 | 32 | 5 | 5 | 12 | 3 | 3 | 7 | 1 | 57 | 54 | 15 | 55 | 4 | 3.85 | 0.00 | 55 | 4 | 55 | 4 | 0.00 | 7.43 |
| mggbsd19  | 4 | 8 | 18 | 2 | 3 | 6 | 1 | 1 | 1 | 0 | 61 | 362 | 63 | 59 | 4 | 24.03 | 0.00 | 59 | 4 | 59 | 4 | 0.00 | 6.25 |
| mggbsd20  | 5 | 11 | 34 | 5 | 5 | 12 | 3 | 1 | 7 | 1 | 136 | 684 | 41 | 117 | 5 | 13,790.76 | 0.00 | 117 | 5 | 117 | 5 | 0.00 | 200.42 |
| mggbsd15  | 5 | 7 | 32 | 5 | 5 | 12 | 3 | 3 | 7 | 1 | 61 | 97 | 17 | 55 | 4 | 11.24 | 0.00 | 55 | 4 | 55 | 4 | 0.00 | 19.03 |
| mggbsd19  | 4 | 8 | 18 | 2 | 3 | 6 | 1 | 1 | 1 | 0 | 61 | 659 | 103 | 61 | 4 | 13.63 | 0.00 | 61 | 4 | 61 | 4 | 0.00 | 7.14 |
| mggbsd20  | 6 | 11 | 34 | 5 | 5 | 12 | 3 | 1 | 7 | 1 | 132 | 40 | 1 | 119 | 6 | - | 0.03 | 118 | 6 | 118 | 6 | -0.17 | 184.64 |
| mggbsd15  | 5 | 7 | 32 | 5 | 5 | 12 | 3 | 3 | 7 | 1 | 61 | 165 | 19 | 55 | 5 | 4.76 | 0.00 | 55 | 5 | 55 | 5 | 0.00 | 4.50 |
| mggbsd19  | 4 | 8 | 18 | 2 | 3 | 6 | 1 | 1 | 1 | 0 | 61 | 1063 | 140 | 61 | 4 | 16.44 | 0.00 | 61 | 4 | 61 | 4 | 0.00 | 7.00 |
| mggbsd20  | 6 | 11 | 34 | 5 | 5 | 12 | 3 | 1 | 7 | 1 | 139 | 471 | 11 | 130 | 6 | - | 0.13 | 122 | 6 | 121 | 6 | -5.85 | 302.306 |
Results

Table: Computational results for the mggdbsd dataset with $\alpha = 0.95$.

| Name     | $m$ | $|V|$ | $|A|$ | $|E|$ | $|V_R|$ | $|A_R|$ | $|V_{RU}|$ | $|A_{RU}|$ | $|E_{RU}|$ | UB | CON | Branch-and-cut | PCAP | cost | veh. | time | avg. cost | avg. veh. | best cost | best veh. | avg. gap | avg. time |
|----------|-----|------|------|------|-------|-------|--------|-------|--------|------|-----|-------|-------|-------|-------|-------|-------|----------|----------|----------|----------|---------|----------|
| mggdbsd1 | 10  | 12   | 34   | 5    | 6     | 12    | 3      | 4     | 1      | 443 | 408 | 805   | 443   | 10    | –     | 421.2 | 10   | 414    | 10   | –4.92   | 1,224,427|
| mggdbsd2 | 13  | 12   | 40   | 6    | 6     | 15    | 4      | 3     | 10     | 566 | 391 | 422   | 566   | 13    | –     | 482.6 | 11   | 475    | 11   | -14.73  | 907.76   |
| mggdbsd3 | 11  | 12   | 34   | 7    | 7     | 12    | 3      | 2     | 7      | 456 | 37,302 | 6,915 | 456 | 11    | –     | 443.4 | 11   | 438    | 11   | -2.76   | 326.35   |
| mggdbsd4 | 10  | 11   | 30   | 4    | 4     | 11    | 3      | 8     | 2      | 494 | 11,811 | 2,097 | 494 | 10    | –     | 442   | 8    | 442    | 8    | -10.53  | 497.67   |
| mggdbsd5 | 11  | 13   | 40   | 5    | 5     | 15    | 4      | 3     | 11     | 566 | 17,566 | 3,598 | 547 | 11    | –     | 501.6 | 9    | 495    | 9    | -8.30   | 1,703.24 |
| mggdbsd6 | 10  | 12   | 34   | 5    | 6     | 12    | 3      | 2     | 5      | 473 | 937  | 362   | 473   | 10    | –     | 472.4 | 10   | 470    | 10   | -0.13   | 288.69   |
| mggdbsd7 | 8   | 12   | 34   | 5    | 5     | 12    | 3      | 2     | 6      | 449 | 420  | 362   | 473   | 10    | –     | 374   | 7    | 374    | 7    | -13.63  | 364.31   |
| mggdbsd8 | 17  | 27   | 70   | 11   | 11    | 26    | 8      | 7     | 13     | 530 | 9,726 | 783   | 530  | 11    | –     | 455   | 15   | 448    | 15   | -14.15  | 4,207.79 |
| mggdbsd9 | 14  | 27   | 78   | 7    | 9     | 28    | 9      | 2     | 13     | 430 | 7,435 | 2,547 | 430  | 14    | –     | 396.8 | 14   | 393    | 14   | -7.72   | 6,913.15 |
| mggdbsd10| 6   | 12   | 38   | 6    | 4     | 14    | 4      | 2     | 7      | 309 | 7,825 | 5,196 | 309  | 6     | –     | 291   | 6    | 291    | 6    | -5.83   | 834.79   |
| mggdbsd11| 7   | 22   | 68   | 11   | 8     | 25    | 8      | 4     | 12     | 422 | 14,162| 7,337 | 422  | 7     | –     | 382   | 7    | 380    | 7    | -9.48   | 951.25   |
| mggdbsd12| 9   | 13   | 36   | 5    | 6     | 13    | 3      | 3     | 7      | 610 | 13,644| 6,525 | 610  | 9     | –     | 573.4 | 9    | 563    | 9    | -6.00   | 741.50   |
| mggdbsd13| 8   | 10   | 42   | 7    | 6     | 15    | 5      | 2     | 8      | 455 | 355  | 526   | 455  | 8     | –     | 423   | 8    | 423    | 8    | -7.03   | 1,037.62 |
| mggdbsd14| 6   | 7    | 32   | 5    | 5     | 12    | 3      | 3     | 6      | 112 | 270,485| 37,157 | 108  | 6     | –     | 108.4 | 6    | 108    | 6    | 0.37    | 137.11   |
| mggdbsd15| 5   | 7    | 32   | 5    | 5     | 12    | 3      | 3     | 7      | 61  | 165  | 19    | 55*   | 5     | 4.76  | 55    | 5    | 55     | 5    | 0.00    | 4.50    |
| mggdbsd16| 6   | 12   | 38   | 7    | 5     | 15    | 5      | 3     | 5      | 108 | 148  | 52    | 108  | 8     | –     | 100.4 | 8    | 100    | 8    | -7.04   | 314.18   |
| mggdbsd17| 7   | 8    | 42   | 7    | 5     | 15    | 5      | 3     | 9      | 77  | 1761 | 710   | 71*   | 7     | 926.41| 71    | 7    | 71     | 7    | 0.00    | 230.34   |
| mggdbsd18| 6   | 9    | 54   | 9    | 6     | 20    | 6      | 3     | 11     | 169 | 4    | 4     | 169  | 6     | –     | 149   | 6    | 149    | 6    | -11.83  | 9526.75  |
| mggdbsd19| 4   | 8    | 18   | 2    | 3     | 6     | 1      | 1     | 1      | 61  | 1063 | 140   | 61*   | 4     | 16.44 | 61    | 4    | 61     | 4    | 0.00    | 7.00    |
| mggdbsd20| 6   | 11   | 34   | 5    | 5     | 12    | 3      | 1     | 7      | 139 | 471  | 11    | 130   | 6     | –     | 122.4 | 6    | 121    | 6    | -5.85   | 302.306  |
| mggdbsd21| 10  | 11   | 50   | 8    | 7     | 18    | 6      | 5     | 11     | 172 | 63   | 0     | 168  | 10    | –     | 156.4 | 10   | 156    | 10   | -6.90   | 437.14   |
| mggdbsd22| 11  | 11   | 66   | 11   | 6     | 24    | 8      | 3     | 11     | 181 | 197  | 46    | 176  | 11    | –     | 168.8 | 11   | 168    | 11   | -4.09   | 2,015.75 |
| mggdbsd23| 17  | 11   | 84   | 13   | 8     | 31    | 9      | 4     | 20     | 208 | 267 | 121   | 208  | 17    | –     | 197.8 | 17   | 196    | 17   | -4.90   | 1,090.50 |

Avg. NumB. | 332.70 | 9.30 | 12,562.94 | 297.77 | 8.91 | 295.35 | 8.91 | -6.32 | 4,392.26 |
Results

Table: Computational results for the \textit{mggdbsd} dataset with $\alpha = 0.85$.

| Name     | $m$ | $|V|/|A|/|E|$ | $|VR|/|AR|/|ER|/|VRU|/|ARU|/|ERU|$ | UB | CON | PCAP | cost | veh. | time |
|----------|-----|----------------|----------------|-------|-----|------|------|------|------|------|------|
| mggdbsd1 | 8   | 12 34 5 6     | 12 3 4 4 1    | 434   | 47,556 | 12,944 | 414 | 8   | –    |
| mggdbsd2 | 10  | 12 40 6 6     | 15 4 3 10 1   | 483   | 381    | 576   | 483  | 10  | –    |
| mggdbsd3 | 9   | 12 34 5 7     | 12 3 2 7 2    | 391   | 33,399 | 8,355  | 391  | 9   | –    |
| mggdbsd4 | 8   | 11 30 4 4     | 11 3 0 8 2    | 439   | 184,122 | 29,825 | 432  | 8   | –    |
| mggdbsd5 | 9   | 13 40 6 5     | 15 4 3 11 2   | 503   | 16,376 | 5,055  | 503  | 9   | –    |
| mggdbsd6 | 8   | 12 34 5 6     | 12 3 2 5 2    | 430   | 63,895 | 20,708 | 419  | 10  | –    |
| mggdbsd7 | 9   | 12 34 5 5     | 12 3 2 6 3    | 450   | 402    | 206   | 433  | 9   | –    |
| mggdbsd8 | 13  | 27 70 11 8    | 26 7 8 13 6   | 514   | 19,213 | 3,531  | 453  | 17  | –    |
| mggdbsd9 | 13  | 27 78 72 9    | 29 9 2 13 4   | 399   | 7,534  | 2,313  | 399  | 13  | –    |
| mggdbsd10| 6   | 12 38 6 4     | 14 4 2 7 2    | 319   | 471    | 29    | 302  | 6   | –    |
| mggdbsd11| 6   | 22 68 11 8    | 25 8 4 12 6   | 429   | 1,925  | 438   | 402  | 6   | –    |
| mggdbsd12| 8   | 13 36 5 6     | 13 3 3 7 1    | 610   | 11,089 | 10,848 | 589  | 8   | –    |
| mggdbsd13| 8   | 10 42 7 6     | 15 5 2 8 2    | 456   | 304    | 725   | 455  | 8   | –    |
| mggdbsd14| 11  | 7 32 5 5      | 12 3 3 6 0    | 115   | 8,347  | 4,525  | 107  | 6   | 1,164.20 |
| mggdbsd15| 5   | 7 32 5 5      | 12 3 3 7 1    | 57    | 54     | 15    | 55   | 4   | 3.85 |
| mggdbsd16| 7   | 8 42 7 5      | 15 5 3 5 4    | 112   | 60     | 26    | 112  | 7   | –    |
| mggdbsd17| 6   | 8 42 7 5      | 15 5 3 9 3    | 83    | 1,761  | 710   | 71   | 6   | 926.41 |
| mggdbsd18| 6   | 9 54 9 6      | 20 6 3 11 2   | 170   | 20     | 2     | 170  | 6   | –    |
| mggdbsd19| 4   | 8 18 2 3      | 6 1 1 1 0     | 61    | 362    | 63    | 59*  | 4   | 24.03 |
| mggdbsd20| 5   | 11 34 5 5     | 12 3 1 7 1    | 136   | 684    | 41    | 117* | 5   | 13,790.76 |
| mggdbsd21| 9   | 11 50 8 7     | 18 6 5 11 3   | 171   | 177    | 21    | 165  | 9   | –    |
| mggdbsd22| 10  | 11 66 11 6    | 24 8 3 11 4   | 177   | 235    | 104   | 170  | 10  | –    |
| mggdbsd23| 15  | 11 84 13 8    | 31 9 4 20 5   | 205   | 211    | 101   | 203  | 15  | –    |

Avg. NumB.

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Heuristic

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Conclusions

- We formulate the mixed general routing problem under uncertainty
- We presented an exact solution framework
- We designed a large neighborhood search able to efficiently solve the problem
- We presented the results achieved to show the effectiveness of the method proposed