Flow Balancing with Uncertain Demand for Package Sorting Facilities

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Flow Balancing with Uncertain Demand for Package Sorting Facilities
Novoa, Jarrah, Morton

Agenda

Problem Description

Literature Review

Deterministic and Stochastic Modeling Approaches

Alternative Stochastic Formulations
  - Non-linear
  - Quadratic
  - Linear
  - Second Order Cone

Solution Techniques
  - Challenge of solving the Deterministic MIP (FBP2-D)
  - Performance Comparison for SMA1
  - Performance Comparison for SMA2
  - Binary Search

Results Analysis

Final Remarks and Future Research

References
Problem Description
General Characteristics and Considerations

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Problem Description

General Characteristics and Considerations

- Package carriers such as DHL, FedEx and UPS
- Sophisticated automated sorting systems in their regional hubs with very similar characteristics
- Efficiently process inbound packages and sort them to their downline destinations
- Packages arrival to hub → up to 20,000 packages/hour - up to 150 destinations → high level sortation according to destination to secondary workcenters → package segregation and transportation to outbound loading doors.
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Four 4-hour shifts (or “sorts”) are utilized for package sorting.
Problem Description
General Characteristics and Considerations

- Four 4-hour shifts (or “sorts”) are utilized for package sorting
  - Sort 1 (Day 2:00 – 6:00 pm). Typically the largest volume - wide cross-section of destinations
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  - **Sort 1** (Day 2:00 – 6:00 pm). Typically the largest volume - wide cross-section of destinations
  - **Sort 2** (Twilight 7:00 – 11:00 pm). Most volume for local and regional destinations
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- Important considerations when assigning destinations to secondary workcenters
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  - Each destination has to be assigned to one-and-only one secondary workcenter
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Important considerations when assigning destinations to secondary workcenters

- Each destination has to be assigned to one-and-only one secondary workcenter
- Assignment of destinations is constrained by the number of doors available in each of the workcenters
- Need to balance the usage of the secondary sorters and secondary workcenters
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Target throughputs (in packages per hour) for each of the secondary sorters are set → assignment of destinations should result in throughputs close as possible to the stipulated ones
Problem Description
General Characteristics and Considerations

Planning of the assignment of destinations to secondary workcenters has been done using average daily flow per hour for each destination and sort. Fails to take into account the daily fluctuation in package flows that is observed in practice. We discuss stochastic programming modeling and solution approaches for addressing this problem while explicitly incorporating stochasticity in the package flows. → Minimize deviations from the pre-established target throughputs for the secondary sorters.
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- We discuss stochastic programming modeling and solution approaches for addressing this problem while explicitly incorporating stochasticity in the package flows → Minimize deviations from the pre-established target throughputs for the secondary sorters.
Literature Review
Related Assignment and Balancing Problems

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Related Assignment and Balancing Problems

Generalized Assignment Problem (Ross and Soland, 1975)

\[
\begin{align*}
\min & \quad \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \\
\text{s.t.} & \quad \sum_{j \in J} x_{ij} = 1 \quad \text{for all } i \in I \\
& \quad \sum_{i \in I} r_{ij} x_{ij} \leq b_j \quad \text{for all } j \in J \\
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- Martello et.al [11], Duin et.al [6] and Punnen et.al [4]: workload balancing problem among agents minimizing the difference between the maximum and the minimum workload considering at most one task per agent
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- Similar workforce balancing problems (Pentico et.al [3])
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s.t. \( \sum_{j \in J} x_{ij} = 1 \) for all \( i \in I \)

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- Similar workforce balancing problems (Pentico et.al [3])
- Similar line balancing problems (Becker et.al[2])
- Load balancing in project assignment (Liang et.al [10])
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Sources of Uncertainty

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- Alternative Stochastic Formulations
- Solution Techniques
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- **Amount of resource available** \( (b_j) \): Set of two-stage stochastic programming formulations (Totkas et.al [9])
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- Amount of resource available \((b_j)\): Set of two-stage stochastic programming formulations (Totkas et.al [9])
- Amount of resource required \((r_{ij})\): Chance-constrained deterministic equivalent for a machine loading problem (Resh [5])
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- Amount of resource available \((b_j)\): Set of two-stage stochastic programming formulations (Totkas et.al [9])
- Amount of resource required \((r_{ij})\): Chance-constrained deterministic equivalent for a machine loading problem (Resh [5])
- Other applications: Stochastic line balancing (Ağpak et.al [1]), Stochastic supplier allocation (Bilsel et.al[3])

References
Modeling Approaches
Stochastic Flow Balancing Problem FBP1

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Set $I$ of destinations, to be assigned uniquely to a set $J$ of secondary workcenters during each one on the four elements of the set $S$ of sorts. Set $G$ contains all pairs of destinations to be assigned to the same secondary zone.
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Stochastic Flow Balancing Problem FBP1

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\begin{align*}
\min & \quad \max_{j \in J, s \in S} \left( \sum_{i \in I} \xi_{is} x_{ij} - P_{js} \sum_{i \in I} \xi_{is} \right) \\
\text{s.t.} & \quad \sum_{j \in J} x_{ij} = 1, \quad \text{for all } i \in I \\
& \quad \sum_{i \in I} \frac{\xi_{is}}{T_j} x_{ij} \leq C_j, \quad \text{for all } j \in J, s \in S \\
& \quad x_{ij} = x_{kj}, \quad \text{for all } j \in J, (i, k) \in G \\
& \quad x_{ij} \in \{0, 1\}, \quad \text{for all } i \in I, j \in J
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\(\text{(FBP1 - 1)}\), \(\text{(FBP1 - 2)}\), \(\text{(FBP1 - 3)}\), \(\text{(FBP1 - 4)}\), \(\text{(FBP1 - 5)}\)
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$\xi_{is}$: the number of packages with destination $i$ arriving during sort $s$. 

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\end{align*}
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- $\xi_{is}$: the number of packages with destination $i$ arriving during sort $s$
- $P_{js}$: target throughput proportion for the $j$ secondary zone during sort $s$
Modeling Approaches
Stochastic Flow Balancing Problem FBP1

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- $P_{js}$: target throughput proportion for the $j$ secondary zone during sort $s$
- $C_j$: number of doors available in the $j$ secondary zone
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- $\xi_{is}$: the number of packages with destination $i$ arriving during sort $s$
- $P_{js}$: target throughput proportion for the $j$ secondary zone during sort $s$
- $C_j$: number of doors available in the $j$ secondary zone
- $T_j$: capacity per door at the $j$ secondary zone (packages/hour)
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\text{s.t.} & \quad \sum_{j \in J} x_{ij} = 1, \quad \text{for all } i \in I \quad \text{(FBP1 - 2)} \\
& \quad \sum_{i \in I} \frac{\xi_{is}}{T_j} x_{ij} \leq C_j, \quad \text{for all } j \in J, s \in S \quad \text{(FBP1 - 3)} \\
& \quad x_{ij} = x_{kj}, \quad \text{for all } j \in J, (i, k) \in G \quad \text{(FBP1 - 4)} \\
& \quad x_{ij} \in \{0, 1\}, \quad \text{for all } i \in I, j \in J \quad \text{(FBP1 - 5)}
\end{align*}
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- $P_{js}$: target throughput proportion for the $j$ secondary zone during sort $s$
- $C_j$: number of doors available in the $j$ secondary zone
- $T_j$: capacity per door at the $j$ secondary zone (packages/hour)
- $x_{ij} = 1$, if destination $i$ is assigned to secondary zone $j$ and 0, otherwise.
Modeling Approaches
Stochastic Flow Balancing Problem Reformulation FBP2

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\min & \quad Z \\
\text{s.t.} & \quad \sum_{j \in J} x_{ij} = 1, & \text{for all } i \in I \\
& \quad \sum_{i \in I} \xi_{is} x_{ij} - p_{js} \leq Z, & \text{for all } j \in J, s \in S \\
& \quad p_{js} - \sum_{i \in I} \xi_{is} x_{ij} \leq Z, & \text{for all } j \in J, s \in S \\
& \quad \sum_{i \in I} \xi_{is} x_{ij} \leq c_j, & \text{for all } j \in J, s \in S \\
& \quad x_{ij} = x_{kj}, & \text{for all } j \in J, (i, k) \in G \\
& \quad x_{ij} \in \{0, 1\}, & \text{for all } i \in I, j \in J \\
\end{align*}
\]

\(FBP2 - 1\)

\(FBP2 - 2\)

\(FBP2 - 3\)

\(FBP2 - 4\)

\(FBP2 - 5\)

\(FBP2 - 6\)

\(FBP2 - 7\)
Modeling Approaches
Stochastic Flow Balancing Problem Reformulation FBP2

\[
\begin{align*}
\min & \quad Z \\
\text{s.t.} & \quad \sum_{j \in J} x_{ij} = 1, \quad \text{for all } i \in I \\
& \quad \sum_{i \in I} \xi_{is} x_{ij} - P_{js} \leq Z, \quad \text{for all } j \in J, s \in S \\
& \quad P_{js} - \sum_{i \in I} \frac{\xi_{is} x_{ij}}{\sum_{i \in I} \xi_{is}} \leq Z, \quad \text{for all } j \in J, s \in S \\
& \quad \sum_{i \in I} \frac{\xi_{is}}{T_j} x_{ij} \leq C_j, \quad \text{for all } j \in J, s \in S \\
& \quad x_{ij} = x_{kj}, \quad \text{for all } j \in J, (i, k) \in G \\
& \quad x_{ij} \in \{0, 1\}, \quad \text{for all } i \in I, j \in J
\end{align*}
\]

\(Z\): imbalance measure \(\rightarrow\) largest difference between the actual and target proportions for all the secondary workcenters and sorts (FBP2-3 and FBP2-4)
**Modeling Approaches**

Stochastic Flow Balancing Problem Reformulation FBP2

\[
\begin{align*}
\text{min} & \quad Z & \quad (FBP2 - 1) \\
\text{s.t.} & \quad \sum_{j \in J} x_{ij} = 1, \quad \text{for all } i \in I & \quad (FBP2 - 2) \\
& \quad \sum_{i \in I} \frac{\xi_{is} x_{ij}}{\sum_{i \in I} \xi_{is}} - P_{js} \leq Z, \quad \text{for all } j \in J, s \in S & \quad (FBP2 - 3) \\
& \quad P_{js} - \sum_{i \in I} \frac{\xi_{is} x_{ij}}{\sum_{i \in I} \xi_{is}} \leq Z, \quad \text{for all } j \in J, s \in S & \quad (FBP2 - 4) \\
& \quad \sum_{i \in I} \frac{\xi_{is}}{T_j} x_{ij} \leq C_j, \quad \text{for all } j \in J, s \in S & \quad (FBP2 - 5) \\
& \quad x_{ij} = x_{kj}, \quad \text{for all } j \in J, (i, k) \in G & \quad (FBP2 - 6) \\
& \quad x_{ij} \in \{0, 1\}, \quad \text{for all } i \in I, j \in J & \quad (FBP2 - 7)
\end{align*}
\]

- $Z$: imbalance measure $\rightarrow$ largest difference between the actual and target proportions for all the secondary workcenters and sorts (FBP2-3 and FBP2-4)

- $\xi_{is}$ random nature: one or more capacity constraints could be violated despite being feasible when the $\xi_{is}$ assume their expected values (FBP2-D: model considering expected values)
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Stochasticity Inclusion

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Stochasticity Inclusion

\[ \xi_{is} \rightarrow N \left( \mu_{is}, \sigma_{is}^2 \right) \]
Modeling Approaches
Stochasticity Inclusion

\[ \xi_{is} \rightarrow N \left( \mu_{is}, \sigma_{is}^2 \right) \]

Assumption: \( \xi_s = (\xi_{1s}, \ldots, \xi_{|I|s})^T \rightarrow |I|-\text{variate normal distribution for all } s \in S \)
Modeling Approaches
Stochasticity Inclusion

\[ \xi_{is} \rightarrow N \left( \mu_{is}, \sigma_{is}^2 \right) \]

Assumption: \( \xi_s = (\xi_1s, \ldots, \xi_{|I|s})^T \rightarrow \text{|I|-variate normal distribution for all } s \in S \)

Let \( \mu_s = (\mu_1s, \ldots, \mu_{|I|s})^T \) for all \( s \in S \),
\( \xi_{is} \rightarrow N \left( \mu_{is}, \sigma_{is}^2 \right) \)

Assumption: \( \xi_s = (\xi_{1s}, \ldots, \xi_{|I|s})^T \rightarrow |I|-\text{variate normal distribution for all } s \in S \)

Let \( \mu_s = (\mu_{1s}, \ldots, \mu_{|I|s})^T \) for all \( s \in S \),

\[ x_j = (x_{1j}, \ldots, x_{|I|j})^T \text{ for all } j \in J, \]
Modeling Approaches
Stochasticity Inclusion

\[ \xi_{is} \rightarrow N \left( \mu_{is}, \sigma^2_{is} \right) \]

Assumption: \( \xi_s = (\xi_{1s}, \ldots, \xi_{|I|s})^T \rightarrow |I|-\text{variate normal distribution for all } s \in S \)

Let \( \mu_s = (\mu_{1s}, \ldots, \mu_{|I|s})^T \) for all \( s \in S \),
\[ x_j = (x_{1j}, \ldots, x_{|I|j})^T \text{ for all } j \in J \],
\[ \Sigma_s = \text{E}[ (\xi_s - \mu_s)(\xi_s - \mu_s)^T ] \text{ for all } s \in S \text{ (Variance-Covariance Matrix)} \]
Modeling Approaches
Stochasticity Inclusion

\[ \xi_{is} \to N \left( \mu_{is}, \sigma_{is}^2 \right) \]

Assumption: \( \xi_s = (\xi_{1s}, \ldots, \xi_{|I|s})^T \to |I|-\text{variate normal distribution for all } s \in S \)

Let \( \mu_s = (\mu_{1s}, \ldots, \mu_{|I|s})^T \) for all \( s \in S \),
\( x_j = (x_{1j}, \ldots, x_{|I|j})^T \) for all \( j \in J \),
\[ \Sigma_s = E[(\xi_s - \mu_s)(\xi_s - \mu_s)^T] \] for all \( s \in S \) (Variance-Covariance Matrix)

We are interested in guaranteeing that the doors capacities are not exceeded with a high probability level \( p \):

\[ P(\xi_s^T x_j \leq C_j T_j) \geq p \quad \text{for all } j \in J, s \in S. \]
Modeling Approaches  
Stochasticity Inclusion

\[ \xi_{is} \rightarrow N \left( \mu_{is}, \sigma_{is}^2 \right) \]

Assumption: \( \xi_s = (\xi_{1s}, \ldots, \xi_{|I|s})^T \rightarrow |I|\)-variate normal distribution for all \( s \in S \)

Let \( \mu_s = (\mu_{1s}, \ldots, \mu_{|I|s})^T \) for all \( s \in S \),

\[ x_j = (x_{1j}, \ldots, x_{|I|j})^T \quad \text{for all} \quad j \in J, \]

\[ \Sigma_s = \text{E}[(\xi_s - \mu_s)(\xi_s - \mu_s)^T] \quad \text{for all} \quad s \in S \quad \text{(Variance-Covariance Matrix)} \]

We are interested in guaranteeing that the doors capacities are not exceeded with a high probability level \( p \):

\[ P(\xi_s^T x_j \leq C_j T_j) \geq p \quad \text{for all} \quad j \in J, s \in S. \]

Each one of the individual probabilistic constraints can be written as:

\[
F \left( \frac{C_j T_j - \mu_s^T x_j}{\sqrt{x_j^T \Sigma_s x_j}} \right) \geq p \quad \Rightarrow \quad F^{-1}(p) \sqrt{x_j^T \Sigma_s x_j} + \mu_s^T x_j \leq C_j T_j
\]
Modeling Approaches
Convexity

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Convexity

For probability levels greater than 0.5, the left hand side expression is a convex function, generating a convex set:

\[ F^{-1}(p) \sqrt{x_j^\top \Sigma_s x_j + \mu_s^\top x_j C_j T_j} \]

Intersection of convex sets is convex → feasible set of individual chance constraints is convex (for probability levels > 0.5)
For probability levels greater than 0.5, the left hand side expression is a convex function, generating a convex set.
For probability levels greater than 0.5, the left hand side expression is a convex function, generating a convex set

\[ F^{-1}(p) \sqrt{x_j^T \Sigma_s x_j + \mu_s^T x_j} \]

\[ C_j T_j \]
For probability levels greater than 0.5, the left hand side expression is a convex function, generating a convex set:

\[
F^{-1}(p) \sqrt{x_j^T \Sigma_s x_j + \mu_s^T x_j}
\]

Intersection of convex sets is convex \( \rightarrow \) feasible set of individual chance constraints is convex (for probability levels > 0.5)
Modeling Approaches
Stochastic Objective Function - SMA1

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How to deal with the stochasticity implied in the linearization of the objective function?
How to deal with the stochasticity implied in the linearization of the objective function?

1. Threshold for the coefficients of variability → work with Expected Values
Modeling Approaches
Stochastic Objective Function - SMA1

How to deal with the stochasticity implied in the linearization of the objective function?

1. Threshold for the coefficients of variability → work with Expected Values
2. Introduce new probabilistic constraints
Modeling Approaches
Stochastic Objective Function - SMA1

How to deal with the stochasticity implied in the linearization of the objective function?

1. Threshold for the coefficients of variability → work with Expected Values
2. Introduce new probabilistic constraints

Following Stochastic Modeling Approach 1 (SMA1):

\[
\begin{align*}
\min & \quad Z \\
\text{s.t.} & \quad \sum_{j \in J} x_{ij} = 1, \quad \text{for all } i \in I \\
& \quad \left( \sum_{i \in I} \mu_{is} x_{ij} / \sum_{i \in I} \mu_{is} \right) - P_{js} \leq Z, \quad \text{for all } j \in J, s \in S \\
& \quad P_{js} - \left( \sum_{i \in I} \mu_{is} x_{ij} / \sum_{i \in I} \mu_{is} \right) \leq Z, \quad \text{for all } j \in J, s \in S \\
& \quad (F^{-1}(\rho)) \sqrt{x_j^T \Sigma_s x_j} \leq (C_j T_j - \mu_s T x_j), \quad \text{for all } j \in J, s \in S \\
& \quad x_{ij} = x_{kj}, \quad \text{for all } j \in J, (i, k) \in G \\
& \quad x_{ij} \in \{0, 1\} \quad \text{for all } i \in I, j \in J 
\end{align*}
\]
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Stochastic Objective Function - SMA2

Following Stochastic Modeling Approach 2 (SMA2):

\[ P \{ \sum_{i \in I} \xi_{is} (x_{ij} - \mu_{is}) \sum_{i \in I} \mu_{is} \leq Z, \geq q, \text{for all } j \in J, s \in S \} \geq q, \text{for all } j \in J, s \in S \} \]

Letting \( y_{js} = (x_{1j} - P_{js} \sum_{i \in I} \mu_{is}, \ldots, x_{|I|j} - P_{js} \sum_{i \in I} \mu_{is}) ^{\top} \)

Now, the deterministic equivalent can be written as:

\[ F \left( Z - \mu_{s} ^{\top} y_{js} \sqrt{y_{js} ^{\top} \Sigma_{s} y_{js}} \right) \geq q + \frac{1}{2} \iff \left( Z - \mu_{s} ^{\top} y_{js} \right) \geq F^{-1}(q + \frac{1}{2}) \sqrt{y_{js} ^{\top} \Sigma_{s} y_{js}}, \text{for all } j \in J, s \in S \]
Modeling Approaches
Stochastic Objective Function - SMA2

Following Stochastic Modeling Approach 2 (SMA2):
Modeling Approaches
Stochastic Objective Function - SMA2

Following Stochastic Modeling Approach 2 (SMA2):

\[
P \left\{ \left( \sum_{i \in I} \xi_{is} \left( \frac{x_{ij}}{\sum_{i \in I} \mu_{is}} \right) \right) - \left( \frac{P_{js}}{\sum_{i \in I} \mu_{is}} \right) \sum_{i \in I} \xi_{is} \right) \leq Z \right\} \geq q, \text{ for all } j \in J, s \in S
\]

(1)

\[
P \left\{ -Z \leq \left( \sum_{i \in I} \xi_{is} \left( \frac{x_{ij} - P_{js}}{\sum_{i \in I} \mu_{is}} \right) \right) \leq Z \right\} \geq q, \text{ for all } j \in J, s \in S
\]

(2)
Modeling Approaches
Stochastic Objective Function - SMA2

Following Stochastic Modeling Approach 2 (SMA2):

\[
P \left\{ \left( \sum_{i \in I} \xi_{is} \left( \frac{x_{ij}}{\sum_{i \in I} \mu_{is}} \right) \right) - \left( \frac{P_{js}}{\sum_{i \in I} \mu_{is}} \right) \sum_{i \in I} \xi_{is} \right) \leq Z \mid \geq q, \text{ for all } j \in J, s \in S \right\}
\]

(1)

\[
= P \left\{ -Z \leq \left( \sum_{i \in I} \xi_{is} \left( \frac{x_{ij} - P_{js}}{\sum_{i \in I} \mu_{is}} \right) \right) \leq Z \mid \geq q, \text{ for all } j \in J, s \in S \right\}
\]

(2)

Letting \( y_{js} = \left( \frac{x_{1j} - P_{js}}{\sum_{i \in I} \mu_{is}}, \ldots, \frac{x_{|I|j} - P_{js}}{\sum_{i \in I} \mu_{is}} \right)^T \)
Modeling Approaches
Stochastic Objective Function - SMA2

Following **Stochastic Modeling Approach 2 (SMA2)**:

\[
P \left\{ \left( \sum_{i \in I} \xi_{is} \left( \frac{x_{ij}}{\sum_{i \in I} \mu_{is}} \right) - \left( \frac{P_{js}}{\sum_{i \in I} \mu_{is}} \right) \sum_{i \in I} \xi_{is} \right) \leq Z \right\} \geq q, \quad \text{for all } j \in J, s \in S \tag{1}\]

\[
= P \left\{ -Z \leq \left( \sum_{i \in I} \xi_{is} \left( \frac{x_{ij} - P_{js}}{\sum_{i \in I} \mu_{is}} \right) \right) \leq Z \right\} \geq q, \quad \text{for all } j \in J, s \in S \tag{2}\]

Letting \( y_{js} = \left( \frac{x_{1j} - P_{js}}{\sum_{i \in I} \mu_{is}}, \ldots, \frac{x_{|I|j} - P_{js}}{\sum_{i \in I} \mu_{is}} \right)^T \)

Now, the deterministic equivalent can be written as:

\[
F \left( \frac{Z - \mu_s^T y_{js}}{\sqrt{y_{js}^T \Sigma_s y_{js}}} \right) \geq \frac{q + 1}{2} \iff (Z - \mu_s^T y_{js}) \geq F^{-1} \left( \frac{q + 1}{2} \right) \sqrt{y_{js}^T \Sigma_s y_{js}}, \quad \text{for all } j \in J, s \in S \tag{3}\]
Modeling Approaches
Stochastic Objective Function SMA2

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Following **Stochastic Modeling Approach 2 (SMA2):**
Modeling Approaches
Stochastic Objective Function SMA2

Following Stochastic Modeling Approach 2 (SMA2):

\[
\begin{align*}
\min & \quad Z \\
\text{s.t.} & \quad \sum_{j \in J} x_{ij} = 1, & \text{for all } i \in I \quad (SMA2 - 1) \\
& \quad F^{-1}\left(\frac{q + 1}{2}\right) \sqrt{y_{js}^T \Sigma_s y_{js}} \leq (Z - \mu_s^T y_{js}), & \text{for all } j \in J, s \in S \quad (SMA2 - 2) \\
& \quad (F^{-1}(p)) \sqrt{x_j^T \Sigma_s x_j} \leq (C_j T_j - \mu_s^T x_j), & \text{for all } j \in J, s \in S \quad (SMA2 - 3) \\
& \quad x_{ij} = x_{kj}, & \text{for all } j \in J, (i, k) \in G \quad (SMA2 - 4) \\
& \quad x_{ij} \in \{0, 1\} & \text{for all } i \in I, j \in J \quad (SMA2 - 5) \\
\end{align*}
\]
Alternative Stochastic Formulations

Non-linear Formulation

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SMA1 and SMA2 resulting formulations (after expressing the originally stated probabilistic constraints as their corresponding deterministic equivalents) are non-linear mixed integer convex problems.
SMA1 and SMA2 resulting formulations (after expressing the originally stated probabilistic constraints as their corresponding deterministic equivalents) are non-linear mixed integer convex problems.

Computationally challenging to solve, especially as the dimensionality of the problem increases.
Alternative Stochastic Formulations
Non-linear Formulation

- SMA1 and SMA2 resulting formulations (after expressing the originally stated probabilistic constraints as their corresponding deterministic equivalents) are non-linear mixed integer convex problems.
- Computationally challenging to solve, especially as the dimensionality of the problem increases.
- Convex problems.
Alternative Stochastic Formulations

Quadratic Formulation

Considering that $p \geq 0.5$, then $C_j^T_j \geq \mu_s^\top x_j$ and we can write SMA1-5, (which is the same as SMA2-4) as:

$$\left(F - 1 \left( \frac{p}{2} \right) \right) \left( x_j^\top \Sigma_s x_j \right) \leq \left( C_j^T_j - \mu_s^\top x_j \right)^2,$$

for all $j \in J$, $s \in S$. (4)

include set of constraints $C_j^T_j \geq \mu_s^\top x_j$ → eliminate cases in which the inequality can hold when squared with $C_j^T_j < \mu_s^\top x_j$ → contradiction with the specified probability level

By the same token, we can write SMA2-3 as:

$$F \left( Z - \mu_s^\top y_{js} \sqrt{y_{js}^\top \Sigma_s y_{js}} \right) \left( \geq q + \frac{1}{2} \right) = \Rightarrow \left( Z - \mu_s^\top y_{js} \right)^2 \geq F - 1 \left( q + \frac{1}{2} \right)^2 \left( y_{js}^\top \Sigma_s y_{js} \right),$$

for all $j \in J$, $s \in S$. (5)

given that $Z \geq \mu_s^\top y_{js}$, as $q \geq 0.5$.5.
Considering that \( p \geq 0.5 \), then \( C_j T_j \geq \mu_s^T x_j \) and we can write SMA1-5, (which is the same as SMA2-4) as:
Alternative Stochastic Formulations

Quadratic Formulation

Considering that \( p \geq 0.5 \), then \( C_j T_j \geq \mu_s^T x_j \) and we can write SMA1-5, (which is the same as SMA2-4) as:

\[
(F^{-1}(p))^2(x_j^T\Sigma_s x_j) \leq (C_j T_j - \mu_s^T x_j)^2, \quad \text{for all } j \in J, s \in S
\]  

(4)
Alternative Stochastic Formulations

Quadratic Formulation

Considering that \( p \geq 0.5 \), then \( C_j T_j \geq \mu_s^T x_j \) and we can write SMA1-5, (which is the same as SMA2-4) as:

\[
(F^{-1}(p))^2(x_j^T \Sigma_s x_j) \leq (C_j T_j - \mu_s^T x_j)^2, \quad \text{for all } j \in J, s \in S
\] (4)

include set of constraints \( C_j T_j \geq \mu_s^T x_j \)
Alternative Stochastic Formulations

Quadratic Formulation

Considering that $p \geq 0.5$, then $C_j T_j \geq \mu_s^T x_j$ and we can write SMA1-5, (which is the same as SMA2-4) as:

$$(F^{-1}(p))^2(x_j^T \Sigma_s x_j) \leq (C_j T_j - \mu_s^T x_j)^2, \quad \text{for all } j \in J, s \in S$$

(4)

include set of constraints $C_j T_j \geq \mu_s^T x_j$

$\rightarrow$ eliminate cases in which the inequality can hold when squared with $C_j T_j < \mu_s^T x_j$

$\rightarrow$ contradiction with the specified probability level
Considering that $p \geq 0.5$, then $C_j T_j \geq \mu_s^T x_j$ and we can write SMA1-5, (which is the same as SMA2-4) as:

$$(F^{-1}(p))^2(x_j^T \Sigma_s x_j) \leq (C_j T_j - \mu_s^T x_j)^2, \text{ for all } j \in J, s \in S$$

(4)

include set of constraints $C_j T_j \geq \mu_s^T x_j$

$\rightarrow$ eliminate cases in which the inequality can hold when squared with $C_j T_j < \mu_s^T x_j$

$\rightarrow$ contradiction with the specified probability level

By the same token, we can write SMA2-3 as:
Alternative Stochastic Formulations
Quadratic Formulation

Considering that \( p \geq 0.5 \), then \( C_j T_j \geq \mu_s^T x_j \) and we can write SMA1-5, (which is the same as SMA2-4) as:

\[
(F^{-1}(p))^2(x_j^T \Sigma_s x_j) \leq (C_j T_j - \mu_s^T x_j)^2, \quad \text{for all } j \in J, s \in S
\]  

(4)

include set of constraints \( C_j T_j \geq \mu_s^T x_j \)

\( \rightarrow \) eliminate cases in which the inequality can hold when squared with \( C_j T_j < \mu_s^T x_j \)

\( \rightarrow \) contradiction with the specified probability level

By the same token, we can write SMA2-3 as:

\[
F\left(\frac{Z - \mu_s^T y_{js}}{\sqrt{y_{js}^T \Sigma_s y_{js}}}\right) \geq \frac{q + 1}{2} \implies (Z - \mu_s^T y_{js})^2 \geq F^{-1}\left(\frac{q + 1}{2}\right)^2 (y_{js}^T \Sigma_s y_{js}), \quad \text{for all } j \in J, s \in S
\]  

(5)

given that \( Z \geq \mu_s^T y_{js} \), as \( q \geq 0.5 \).
Alternative Stochastic Formulations

Quadratic Formulation - Approach 1 - SMAQ1

\[
\begin{align*}
\text{SMAQ1} & \\
\min & \quad Z \\
\text{s.t.} & \quad \sum_{j \in J} x_{ij} = 1, \quad \text{for all } i \in I \\
& \quad \left( \sum_{i \in I} \mu_i s x_{ij} / \sum_{i \in I} \mu_i s \right) - P_{js} \leq Z, \quad \text{for all } j \in J, s \in S \\
& \quad P_{js} - \left( \sum_{i \in I} \mu_i s x_{ij} / \sum_{i \in I} \mu_i s \right) \leq Z, \quad \text{for all } j \in J, s \in S \\
& \quad (F^{-1}(p))^2 (x_j^T \Sigma_s x_j) \leq (C_j T_j - \mu_s T x_j)^2, \quad \text{for all } j \in J, s \in S \\
& \quad \mu_s T x_j \leq C_j T_j, \quad \text{for all } j \in J, s \in S \\
& \quad x_{ij} = x_{kj}, \quad \text{for all } j \in J, (i, k) \in G \\
& \quad x_{ij} \in \{0, 1\}, \quad \text{for all } i \in I, j \in J
\end{align*}
\]
Alternative Stochastic Formulations

Quadratic Formulation - Approach 2 - SMAQ2

SMAQ2

\[ \min \quad Z \]
\[ \text{s.t.} \quad \sum_{j \in J} x_{ij} = 1, \quad \text{for all } i \in I \]  
\[ F^{-1} \left( \frac{q + 1}{2} \right)^2 (y_{js}^T \Sigma_s y_{js}) \leq (Z - \mu_s^T y_{js})^2, \quad \text{for all } j \in J, s \in S \]  
\[ (F^{-1}(p))^2 (x_j^T \Sigma_s x_j) \leq (C_j T_j - \mu_s^T x_j)^2, \quad \text{for all } j \in J, s \in S \]  
\[ \mu_s^T y_{js} \leq Z, \quad \text{for all } j \in J, s \in S \]  
\[ \mu_s^T x_j \leq C_j T_j, \quad \text{for all } j \in J, s \in S \]  
\[ x_{ij} = x_{kj}, \quad \text{for all } j \in J, (i, k) \in G \]  
\[ x_{ij} \in \{0, 1\}, \quad \text{for all } i \in I, j \in J \]  
\[ Z \geq 0, \quad \text{for all } j \in J, s \in S \]
Alternative Stochastic Formulations
Quadratic Formulation - Convexity
Re-writing SMAQ2-4 by expanding the binomial expression at the right hand side as

$$(F^{-1}(p))^2(x_j^T \Sigma_s x_j) \leq (C_j T_j)^2 - (2C_j T_j) \mu_s^T x_j + x_j^T (\mu_s \mu_s^T) x_j, \text{ for all } j \in J, s \in S$$  \hspace{1cm} (6)
Re-writing SMAQ2-4 by expanding the binomial expression at the right hand side as

\[(F^{-1}(p))^2 x_j^T \Sigma_s x_j \leq (C_j T_j)^2 - (2C_j T_j) \mu_s^T x_j + x_j^T (\mu_s \mu_s^T) x_j, \text{ for all } j \in J, s \in S \]  

(6)

we obtain

\[x_j^T ((F^{-1}(p))^2 \Sigma_s - \mu_s \mu_s^T) x_j + (2C_j T_j) \mu_s^T x_j \leq (C_j T_j)^2, \text{ for all } j \in J, s \in S \]  

(7)
Alternative Stochastic Formulations

Quadratic Formulation - Convexity

Re-writing SMAQ2-4 by expanding the binomial expression at the right hand side as

\[(F^{-1}(p))^2 (x_j^T \Sigma_s x_j) \leq (C_j T_j)^2 - (2C_j T_j) \mu_s^T x_j + x_j^T (\mu_s \mu_s^T) x_j, \text{ for all } j \in J, s \in S \] (6)

we obtain

\[x_j^T ((F^{-1}(p))^2 \Sigma_s - \mu_s \mu_s^T) x_j) + (2C_j T_j) \mu_s^T x_j \leq (C_j T_j)^2, \text{ for all } j \in J, s \in S \] (7)
Alternative Stochastic Formulations
Quadratic Formulation - Convexity

Re-writing SMAQ2-4 by expanding the binomial expression at the right hand side as

\[(F^{-1}(p))^2(x_j^T \Sigma_s x_j) \leq (C_j T_j)^2 - (2C_j T_j) \mu_s^T x_j + x_j^T (\mu_s \mu_s^T) x_j, \text{ for all } j \in J, s \in S \]  

(6)

we obtain

\[x_j^T ((F^{-1}(p))^2 \Sigma_s - \mu_s \mu_s^T) x_j + (2C_j T_j) \mu_s^T x_j \leq (C_j T_j)^2, \text{ for all } j \in J, s \in S \]  

(7)

where \(((F^{-1}(p))^2 \Sigma_s - \mu_s \mu_s^T)\) could be indefinite, generating a non-convex set as illustrated in the figure.
Alternative Stochastic Formulations

Linearized Formulation

\[ \begin{align*}
\text{Problem Description} & \\
\text{Literature Review} & \\
\text{Deterministic and Stochastic Modeling Approaches} & \\
\text{Alternative Stochastic Formulations} & \\
\text{Non-linear Quadratic} & \\
\text{Linear} & \\
\text{Second Order Cone} & \\
\text{Solution Techniques} & \\
\text{Results Analysis} & \\
\text{Final Remarks and Future Research} & \\
\text{References} & 
\end{align*} \]
Alternative Stochastic Formulations

Linearized Formulation

Let $n = |I|$

$$x_j^T (\mu_s \mu_s^T) x_j = x_{1j}^2 \mu_{1s}^2 + x_{1j} x_{2j} \mu_{1s} \mu_{2s} + x_{1j} x_{3j} \mu_{1s} \mu_{3s} + \ldots + x_{nj}^2 \mu_{ns}^2$$
Alternative Stochastic Formulations

Linearized Formulation

Let $n = |I|$

\[ x_j^T (\mu_s \mu_s^T) x_j = x_{1j}^2 \mu_{1s}^2 + x_{1j} x_{2j} \mu_{1s} \mu_{2s} + x_{1j} x_{3j} \mu_{1s} \mu_{3s} + \ldots + x_{nj}^2 \mu_{ns}^2 \]

- $x_{ij}^2$ terms are equivalent to $x_{ij}$ as variables are binary
Alternative Stochastic Formulations
Linearized Formulation

Let \( n = |I| \)

\[
x_j^T (\mu_s \mu_s^T) x_j = x_{1j}^2 \mu_{1s}^2 + x_{1j} x_{2j} \mu_{1s} \mu_{2s} + x_{1j} x_{3j} \mu_{1s} \mu_{3s} + \ldots + x_{nj}^2 \mu_{ns}^2
\]

- \( x_{ij}^2 \) terms are equivalent to \( x_{ij} \) as variables are binary
- cross terms are linearized by replacing each product with a positive continuous variable \( w_{ikj} = x_{ij} x_{kj} \) and adding the following set of constraints (for \( i = 1, \ldots, n - 1; k = i + 1, \ldots, n \)):

\[
x_{ij} + x_{kj} - w_{ikj} \leq 1 \tag{8}
\]
\[
w_{ikj} \leq x_{ij} \tag{9}
\]
\[
w_{ikj} \leq x_{kj} \tag{10}
\]
Alternative Stochastic Formulations

Linearized Formulation

Let \( n = |I| \)

\[
x_j^T (\mu_s \mu_s^T) x_j = x_{1j}^2 \mu_{1s}^2 + x_{1j}x_{2j} \mu_{1s} \mu_{2s} + x_{1j}x_{3j} \mu_{1s} \mu_{3s} + \ldots + x_{nj}^2 \mu_{ns}^2
\]

- \( x_{ij}^2 \) terms are equivalent to \( x_{ij} \) as variables are binary
- cross terms are linearized by replacing each product with a positive continuous variable \( w_{ikj} = x_{ij} x_{kj} \) and adding the following set of constraints (for \( i = 1, \ldots, n - 1; k = i + 1, \ldots, n \)):

\[
x_{ij} + x_{kj} - w_{ikj} \leq 1 \quad (8)
\]

\[
w_{ikj} \leq x_{ij} \quad (9)
\]

\[
w_{ikj} \leq x_{kj} \quad (10)
\]
Alternative Stochastic Formulations

Linearized Formulation

Let $n = |I|$

$$x_j^T(\mu_s \mu_s^T)x_j = x_{1j}^2 \mu_s^2 + x_{2j}x_{1s}\mu_s \mu_{2s} + x_{1j}x_{3j}\mu_s \mu_{3s} + \ldots + x_{nj}^2 \mu_{ns}^2$$

- $x_{ij}^2$ terms are equivalent to $x_{ij}$ as variables are binary
- Cross terms are linearized by replacing each product with a positive continuous variable $w_{ikj} = x_{ij}x_{kj}$ and adding the following set of constraints (for $i = 1, \ldots, n - 1; k = i + 1, \ldots, n$):
  $$x_{ij} + x_{kj} - w_{ikj} \leq 1 \quad (8)$$
  $$w_{ikj} \leq x_{ij} \quad (9)$$
  $$w_{ikj} \leq x_{kj} \quad (10)$$

Then $(C_j^T T_j)^2 - (2C_j^T T_j)\mu_s^T x_j + x_j^T(\mu_s \mu_s^T)x_j$ can be linearly formulated as:
Alternative Stochastic Formulations
Linearized Formulation

Let \( n = |I| \)

\[ x_j^T (\mu_s \mu_s^T)x_j = x_{1j}^2 \mu_{1s}^2 + x_{1j} x_{2j} \mu_{1s} \mu_{2s} + x_{1j} x_{3j} \mu_{1s} \mu_{3s} + \ldots + x_{nj}^2 \mu_{ns}^2 \]

- \( x_{ij}^2 \) terms are equivalent to \( x_{ij} \) as variables are binary
- cross terms are linearized by replacing each product with a positive continuous variable \( w_{ikj} = x_{ij} x_{kj} \) and adding the following set of constraints (for \( i = 1, \ldots, n - 1; k = i + 1, \ldots, n \)):
  
  \[
  \begin{align*}
  x_{ij} + x_{kj} - w_{ikj} & \leq 1 \\
  w_{ikj} & \leq x_{ij} \\
  w_{ikj} & \leq x_{kj}
  \end{align*}
  \]

Then \( (C_j T_j)^2 - (2C_j T_j) \mu_s^T x_j + x_j^T (\mu_s \mu_s^T)x_j \) can be linearly formulated as:

\[
(C_j T_j)^2 - (2C_j T_j) \sum_{i=1}^{n} x_{ij} \mu_{is} + \sum_{i=1}^{n} x_{ij} \mu_{is}^2 + 2 \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} \mu_{is} \mu_{ks} w_{ikj}, \quad \text{for all } j \in J, s \in S
\]
Alternative Stochastic Formulations

Linearized Formulation

Let \( n = |I| \)

\[
x_j^T (\mu_s \mu_s^T) x_j = x_{11}^2 \mu_{11}^2 + x_{12} x_{21} \mu_{11} \mu_{21} + x_{13} x_{31} \mu_{11} \mu_{31} + \ldots + x_{nn}^2 \mu_{nn}^2
\]

- \( x_{ij}^2 \) terms are equivalent to \( x_{ij} \) as variables are binary
- cross terms are linearized by replacing each product with a positive continuous variable \( w_{ikj} = x_{ij} x_{kj} \) and adding the following set of constraints (for \( i = 1, \ldots, n - 1; k = i + 1, \ldots, n \)):
  \[
  \begin{align*}
  x_{ij} + x_{kj} - w_{ikj} & \leq 1 \\
  w_{ikj} & \leq x_{ij} \\
  w_{ikj} & \leq x_{kj}
  \end{align*}
  \]

Then \((C_j T_j)^2 - (2 C_j T_j) \mu_s^T x_j + x_j^T (\mu_s \mu_s^T) x_j\) can be linearly formulated as:

\[
(C_j T_j)^2 - (2 C_j T_j) \sum_{i=1}^{n} x_{ij} \mu_{is} + \sum_{i=1}^{n} x_{ij} \mu_{is}^2 + 2 \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} \mu_{is} \mu_{ks} w_{ikj}, \quad \text{for all } j \in J, s \in S (11)
\]

Likewise, \( x_j^T \Sigma_s x_j = \sum_{i=1}^{n} \sum_{k=1}^{n} x_{ij} x_{kj} \text{Cov}_{iks} \) can be written as:

\[
\sum_{i=1}^{n} \sum_{k=1}^{n} x_{ij} x_{kj} \text{Cov}_{iks} \]
Alternative Stochastic Formulations

Linearized Formulation

Let $n=|I|$

$$x_j^T (\mu_s \mu_s^T) x_j = x_{1j}^2 \mu_{1s}^2 + x_{2j} x_{1j} \mu_{1s} \mu_{2s} + x_{3j} x_{1j} \mu_{1s} \mu_{3s} + \ldots + x_{nj}^2 \mu_{ns}^2$$

- $x_{ij}^2$ terms are equivalent to $x_{ij}$ as variables are binary
- cross terms are linearized by replacing each product with a positive continuous variable $w_{ikj} = x_{ij} x_{kj}$ and adding the following set of constraints (for $i = 1, \ldots, n - 1; k = i + 1, \ldots, n$):

$$x_{ij} + x_{kj} - w_{ikj} \leq 1 \quad (8)$$

$$w_{ikj} \leq x_{ij} \quad (9)$$

$$w_{ikj} \leq x_{kj} \quad (10)$$

Then $(C_j T_j)^2 - (2C_j T_j) \mu_s^T x_j + x_j^T (\mu_s \mu_s^T) x_j$ can be linearly formulated as:

$$(C_j T_j)^2 - (2C_j T_j) \sum_{i=1}^{n} x_{ij} \mu_{is} + \sum_{i=1}^{n} x_{ij}^2 \mu_{is}^2 + 2 \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} \mu_{is} \mu_{ks} w_{ikj}, \text{ for all } j \in J, s \in S \quad (11)$$

Likewise, $x_j^T \Sigma_s x_j = \sum_{i=1}^{n} \sum_{k=1}^{n} x_{ij} x_{kj} \text{ Cov}_{iks}$ can be written as:

$$\sum_{i=1}^{n} x_{ij} \text{ Cov}_{iis} + 2 \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} w_{ikj} \text{ Cov}_{iks}, \text{ for all } j \in J, s \in S \quad (12)$$
Alternative Stochastic Formulations
Linearized Formulation - Approach 1 - SMAL1

Flow Balancing with Uncertain Demand for Package Sorting Facilities
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Linearized Formulation - Approach 1 - SMAL1

\[
\begin{align*}
\text{min} & \quad Z \\
\text{s.t.} & \quad \sum_{j \in J} x_{ij} = 1, \quad \text{for all } i \in I \\
& \quad \left( \sum_{i \in I} \mu_{is} x_{ij} / \sum_{i \in I} \mu_{is} \right) - P_{js} \leq Z, \quad \text{for all } j \in J, s \in S \\
& \quad P_{js} - \left( \sum_{i \in I} \mu_{is} x_{ij} / \sum_{i \in I} \mu_{is} \right) \leq Z, \quad \text{for all } j \in J, s \in S \\
& \quad (F^{-1}(p))^2 \left( \sum_{i=1}^{n} x_{ij} \text{Cov}_{is} + 2 \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} w_{ikj} \text{Cov}_{ks} \right) \leq (C_j T_j)^2 - (2C_j T_j) \sum_{i=1}^{n} x_{ij} \mu_{is} + \sum_{i=1}^{n} x_{ij} \mu_{is}^2 + 2 \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} \mu_{is} \mu_{ks} w_{ikj}, \quad \text{for all } j \in J, s \in S \\
& \quad \mu_{s}^T x_j \leq C_j T_j, \quad \text{for all } j \in J, (i, k) \in G \\
& \quad x_{ij} = x_{kj}, \quad \text{for all } j \in J, (i, k) \in G \\
& \quad x_{ij} + x_{kj} - w_{ikj} \leq 1, \quad \text{for } i = 1, \ldots, n-1; k > i \\
& \quad w_{ikj} \leq x_{ij}, \quad \text{for } i = 1, \ldots, n-1; k > i \\
& \quad w_{ikj} \leq x_{kj}, \quad \text{for } i = 1, \ldots, n-1; k > i \\
& \quad x_{ij} \in \{0, 1\}, \quad \text{for all } i \in I, j \in J \\
& \quad w_{ikj} \geq 0, \quad \text{for all } i \in I, k \in K, j \in J
\end{align*}
\]
Alternative Stochastic Formulations
Linearized Formulation - Approach 2 - SMAL2

Now, SMAL2 has basically the same formulation as SMAL1 but expressing the balancing constraints (SMAL2-3) as (SMAL2-3):

\[
F - 1 \left( q + \frac{1}{2} \right) \sum_{i=1}^{n} \left( x_{ij} - 2x_{ij}P_{js} + P_{js}^2 \right) \left( C_{ovj} \sum_{i \in I} \mu_{is} \right)^2 \leq Z_2 - \frac{2}{n} \sum_{i=1}^{n} \sum_{k=i+1}^{n} \left( w_{ikj} - P_{js}(x_{ij} + x_{kj}) + P_{js}^2 \right) \left( \mu_{is} \sum_{i \in I} \mu_{is} \right)^2, \quad \text{for all } j \in J, s \in S
\]
Now, SMAL2 has basically the same formulation as SMAL1 but expressing the balancing constraints (SMA2-3) as (SMAL2-3):
Alternative Stochastic Formulations
Linearized Formulation - Approach 2 - SMAL2

Now, SMAL2 has basically the same formulation as SMAL1 but expressing the balancing constraints (SMA2-3) as (SMAL2-3):

\[
F^{-1} \left( \frac{q+1}{2} \right)^2 \left( \sum_{i=1}^{n} \left( x_{ij} - 2x_{ij}P_{js} + p_{js}^2 \right) \right) \left( \frac{\text{Cov}_{iis}}{\left( \sum_{i \in I} \mu_{is} \right)^2} \right) + \\
2 \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} \left( w_{ikj} - P_{js}(x_{ij} + x_{kj}) + p_{js}^2 \right) \left( \frac{\text{Cov}_{iks}}{\left( \sum_{i \in I} \mu_{is} \right)^2} \right) \leq \\
Z^2 - 2Z \sum_{i=1}^{n} \left( \frac{x_{ij} - P_{js}}{\sum_{i \in I} \mu_{is}} \right) \mu_{is} + \sum_{i=1}^{n} \left( x_{ij} - 2x_{ij}P_{js} + p_{js}^2 \right) \left( \frac{\mu_{is}}{\sum_{i \in I} \mu_{is}} \right)^2 + \\
2 \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} \left( w_{ikj} - P_{js}(x_{ij} + x_{kj}) + p_{js}^2 \right) \left( \frac{\mu_{is}\mu_{ks}}{\left( \sum_{i \in I} \mu_{is} \right)^2} \right), \text{ for all } j \in J, s \in S
\]
Alternative Stochastic Formulations

SOCP Formulation

The original chance constraint deterministic equivalent (SMA1-5, SMA2-4):

\[ F - 1(p) \sqrt{x_j^\top \Sigma_s x_j + \mu_s^\top x_j} \leq C_j^T, \quad \text{for all } j \in J, \quad s \in S \] (13)

can be written as a Second Order Cone constraint:

\[ F - 1(p) |\| L_s x_j^\top |\|_2 + \mu_s^\top x_j \leq C_j^T, \quad \text{for all } j \in J, \quad s \in S \] (14)

where \( L_s \) is the lower triangular matrix resulting from the Cholesky factorization \( \Sigma_s = L_s L_s^\top \).

Then, we can define \( \nu_{js} = L_s^\top x_j \) and \( \omega_{js} = (C_j^T - \mu_s^\top x_j) \). So,

\[ (F - 1(p)) 2(x_j^\top \Sigma_s x_j) \leq (C_j^T - \mu_s^\top x_j)^2, \quad \text{for all } j \in J, \quad s \in S \] (15)

can be written as a second order cone constraint, preserving convexity:

\[ (F - 1(p)) 2(\nu_{js})^\top \nu_{js} \leq \omega_{js}^2, \quad \text{for all } j \in J, \quad s \in S \] (16)
The original chance constraint deterministic equivalent (SMA1-5, SMA2-4):

\[ F^{-1}(p) \sqrt{x_j^T\Sigma_s x_j + \mu_s^T x_j} \leq C_j T_j, \quad \text{for all } j \in J, s \in S \]  

\[ (13) \]
Alternative Stochastic Formulations

SOCP Formulation

The original chance constraint deterministic equivalent (SMA1-5, SMA2-4):

\[ F^{-1}(p) \sqrt{x_j^T \Sigma_s x_j + \mu_s^T x_j} \leq C_j T_j, \quad \text{for all } j \in J, s \in S \quad (13) \]

can be written as a Second Order Cone constraint:

\[ F^{-1}(p)\|L_s^T x_j\|_2 + \mu_s^T x_j \leq C_j T_j, \quad \text{for all } j \in J, s \in S \quad (14) \]
Alternative Stochastic Formulations

SOCP Formulation

The original chance constraint deterministic equivalent (SMA1-5, SMA2-4):

\[
F^{-1}(p) \sqrt{x_j^T \Sigma_s x_j + \mu_s^T x_j} \leq C_j T_j, \quad \text{for all } j \in J, s \in S
\]  

(13)
can be written as a Second Order Cone constraint:

\[
F^{-1}(p) \| L_s^T x_j \|_2 + \mu_s^T x_j \leq C_j T_j, \quad \text{for all } j \in J, s \in S
\]  

(14)
where \( L_s \) is the lower triangular matrix resulting from the Cholesky factorization \( \Sigma_s = L_s L_s^T \).

Then, we can define \( \nu_{js} = L_s^T x_j \) and \( w_{js} = (C_j T_j - \mu_s^T x_j) \). So,
Alternative Stochastic Formulations

SOCP Formulation

The original chance constraint deterministic equivalent (SMA1-5, SMA2-4):

\[ F^{-1}(p) \sqrt{x_j^T \Sigma_s x_j + \mu_s^T x_j} \leq C_j T_j, \quad \text{for all } j \in J, s \in S \]  \hspace{1cm} (13)

can be written as a Second Order Cone constraint:

\[ F^{-1}(p) \| L_s^T x_j \|_2 + \mu_s^T x_j \leq C_j T_j, \quad \text{for all } j \in J, s \in S \]  \hspace{1cm} (14)

where \( L_s \) is the lower triangular matrix resulting from the Cholesky factorization (\( \Sigma_s = L_s L_s^T \)).

Then, we can define \( \nu_{js} = L_s^T x_j \) and \( w_{js} = (C_j T_j - \mu_s^T x_j) \). So,

\[ (F^{-1}(p))^2(x_j^T \Sigma_s x_j) \leq (C_j T_j - \mu_s^T x_j)^2, \quad \text{for all } j \in J, s \in S \]  \hspace{1cm} (15)
Alternative Stochastic Formulations

SOCP Formulation

The original chance constraint deterministic equivalent (SMA1-5, SMA2-4):

$$F^{-1}(p) \sqrt{x_j^T \Sigma_s x_j + \mu_s^T x_j} \leq C_j T_j, \text{ for all } j \in J, s \in S$$  \hspace{1cm} (13)

can be written as a Second Order Cone constraint:

$$F^{-1}(p) \|L_s^T x_j\|_2 + \mu_s^T x_j \leq C_j T_j, \text{ for all } j \in J, s \in S$$  \hspace{1cm} (14)

where $L_s$ is the lower triangular matrix resulting from the Cholesky factorization ($\Sigma_s = L_s L_s^T$).

Then, we can define $\nu_{js} = L_s^T x_j$ and $w_{js} = (C_j T_j - \mu_s^T x_j)$. So,

$$(F^{-1}(p))^2 (x_j^T \Sigma_s x_j) \leq (C_j T_j - \mu_s^T x_j)^2, \text{ for all } j \in J, s \in S$$  \hspace{1cm} (15)

can be written as a second order cone constraint, preserving convexity

$$(F^{-1}(p))^2 (\nu_{js}^T \nu_{js}) \leq \omega_{js}^2, \text{ for all } j \in J, s \in S.$$  \hspace{1cm} (16)
**Alternative Stochastic Formulations**

**SOCP Formulation**

The original chance constraint deterministic equivalent (SMA1-5, SMA2-4):

\[ F^{-1}(p) \sqrt{x_j ^T \Sigma_s x_j + \mu_s ^T x_j} \leq C_j T_j, \text{ for all } j \in J, s \in S \]  \hspace{1cm} (13)

can be written as a Second Order Cone constraint:

\[ F^{-1}(p) \|L_s x_j\|_2 + \mu_s ^T x_j \leq C_j T_j, \text{ for all } j \in J, s \in S \]  \hspace{1cm} (14)

where \( L_s \) is the lower triangular matrix resulting from the Cholesky factorization (\( \Sigma_s = L_s L_s ^T \)).

Then, we can define \( \nu_{js} = L_s ^T x_j \) and \( w_{js} = (C_j T_j - \mu_s ^T x_j) \). So,

\[ (F^{-1}(p))^2(x_j ^T \Sigma_s x_j) \leq (C_j T_j - \mu_s ^T x_j)^2, \text{ for all } j \in J, s \in S \]  \hspace{1cm} (15)

can be written as a second order cone constraint, preserving convexity

\[ (F^{-1}(p))^2(\nu_{js} ^T \nu_{js}) \leq \omega_{js}^2, \text{ for all } j \in J, s \in S. \]  \hspace{1cm} (16)
Alternative Stochastic Formulations

SOCP Formulation - Approach 1 - SMAC1

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Linear

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Alternative Stochastic Formulations
SOCP Formulation - Approach 1 - SMAC1

**SMAC1**

\[
\begin{align*}
\text{min} & \quad Z \\
\text{s.t.} & \quad \sum_{j \in J} x_{ij} = 1, \quad \text{for all } i \in I \\
& \quad \left( \sum_{i \in I} \mu_i s x_{ij} / \sum_{i \in I} \mu_i s \right) - P_{js} \leq Z, \quad \text{for all } j \in J, s \in S \\
& \quad P_{js} - \left( \sum_{i \in I} \mu_i s x_{ij} / \sum_{i \in I} \mu_i s \right) \leq Z, \quad \text{for all } j \in J, s \in S \\
& \quad (F^{-1}(p))^2 (\nu_{js}^T \nu_{js}) \leq \omega_{js}^2, \quad \text{for all } j \in J, s \in S \\
& \quad \nu_{js} = L_s^T x_j, \quad \text{for all } j \in J, s \in S \\
& \quad \omega_{js} = (C_j T_j - \mu_s^T x_j), \quad \text{for all } j \in J, s \in S \\
& \quad \omega_{js} \geq 0, \quad \text{for all } j \in J, s \in S \\
& \quad x_{ij} = x_{kj}, \quad \text{for all } j \in J, (i, k) \in G \\
& \quad x_{ij} \in \{0, 1\}, \quad \text{for all } i \in I, j \in J
\end{align*}
\]
Alternative Stochastic Formulations

SOCP Formulation - Approach 2 - SMAC2

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Quadratic

Linear

Second Order Cone

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Let $\tau_{js} = L_s^T y_{js}$ and $\zeta_{js} = (Z - \mu_s^T y_{js})$ and express SMA2-3 as

$$F^{-1} \left( \frac{q + 1}{2} \right)^2 (\tau_{js}^T \tau_{js}) \leq \zeta_{js}^2,$$

for all $j \in J$, $s \in S$. (17)

**SMAC2**

$$\min \quad Z$$

s.t. $$\sum_{j \in J} x_{ij} = 1,$$

for all $i \in I$ (SMAC2 - 1)

$$F^{-1} \left( \frac{q + 1}{2} \right)^2 (\tau_{js}^T \tau_{js}) \leq \zeta_{js}^2,$$

for all $j \in J$, $s \in S$ (SMAC2 - 2)

$$F^{-1}(p)^2(\nu_{js}^T \nu_{js}) \leq \omega_{js}^2,$$

for all $j \in J$, $s \in S$ (SMAC2 - 3)

$$\tau_{js} = L_s^T y_{js},$$

for all $j \in J$, $s \in S$ (SMAC2 - 4)

$$\nu_{js} = L_s^T x_j,$$

for all $j \in J$, $s \in S$ (SMAC2 - 5)

$$\zeta_{js} = (Z - \mu_s^T y_{js}),$$

for all $j \in J$, $s \in S$ (SMAC2 - 6)

$$\omega_{js} = (C_j^T x_j - \mu_s^T x_j),$$

for all $j \in J$, $s \in S$ (SMAC2 - 7)

$$\zeta_{js} \geq 0,$$

for all $j \in J$, $s \in S$ (SMAC2 - 8)

$$\omega_{js} \geq 0,$$

for all $j \in J$, $s \in S$ (SMAC2 - 9)

$$x_{ij} = x_{kj},$$

for all $j \in J$, $(i, k) \in G$ (SMAC2 - 10)

$$x_{ij} \in \{0, 1\},$$

for all $i \in I$, $j \in J$ (SMAC2 - 11)
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Test data obtained for the Chicago sorting facility of a major package carrier
Flow information for each one of the 104 destinations served by the facility over 24 days
AMPL for model formulations
−→ ILOG CPLEX 12.4 and/or KNITRO 8.0
−→ Runs on PC - Intel Core i7-2620 M CPU 2.7 GHz and 6 GB of memory

Deterministic MIP Challenge (FBP2-D)

Liang et.al [10] show that this type of balancing problems is strongly NP-hard by demonstrating that it is a special case of the known 3-partition problem
−→ CPLEX is able to find an integer solution in 0.01 seconds; however, the search for better solutions continues indefinitely
−→ After one hour and 7,490,000 nodes explored
−→ Best integer solution encountered is $Z = 0.06\%$
−→ Once the level of imbalance ($Z$) is fixed at some pre-specified value (e.g. 0.1%), it is relatively easy to find a feasible solution for FBP2-D (if it exists) that satisfies the pre-specified value for $Z$
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<table>
<thead>
<tr>
<th>SMAQ1 (CPLEX)</th>
<th>SMAL1 (CPLEX)</th>
<th>SMAC1 (CPLEX)</th>
<th>SMA1 (KNITRO)</th>
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<tbody>
<tr>
<td>0.27</td>
<td>1.76</td>
<td>1.44</td>
<td>27.68</td>
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<tr>
<td>0.22</td>
<td>1.53</td>
<td>0.34</td>
<td>17.93</td>
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<td>0.25</td>
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<td>17.99</td>
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<td>0.22</td>
<td>213.61</td>
<td>0.52</td>
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<td>269.06</td>
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<td>0.41</td>
<td>346.93</td>
<td>0.76</td>
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</tr>
<tr>
<td>4.29</td>
<td>—</td>
<td>6.29</td>
<td>—</td>
</tr>
<tr>
<td>12.03</td>
<td>—</td>
<td>20.37</td>
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### Solution Techniques

#### Performance Comparison for SMA1

<table>
<thead>
<tr>
<th>SMA1 - SMAQ1 - SMC1 - SMAL1 solving times (in seconds) given Z</th>
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</thead>
<tbody>
<tr>
<td>SMA1 (KNITRO)</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>50.00</td>
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<tr>
<td>25.00</td>
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<td>10.00</td>
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<td>5.00</td>
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<td>0.50</td>
</tr>
<tr>
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<tr>
<td>0.05</td>
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### Solution Techniques

#### Performance Comparison for SMA1

<table>
<thead>
<tr>
<th>$Z$ (%)</th>
<th>SMAQ1 (CPLEX)</th>
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<tr>
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<td>0.27</td>
<td>1.76</td>
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<td>20.37</td>
<td>—</td>
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<tr>
<td>0.05</td>
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</table>
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Performance Comparison for SMA2
Solution Techniques
Performance Comparison for SMA2

SMA2 - SMAQ2 - SMC2 - SMAL2 solving times (in seconds) given $Z$
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#### Performance Comparison for SMA2

<table>
<thead>
<tr>
<th>Z (%)</th>
<th>SMAQ2 (CPLEX)</th>
<th>SMAL2 (CPLEX)</th>
<th>SMAC2 (CPLEX)</th>
<th>SMA2 (KNITRO)</th>
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<tr>
<td>10.00</td>
<td>7.91</td>
<td>348.77</td>
<td>1.01</td>
<td>44.06</td>
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<td>9.00</td>
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<td>264.81</td>
<td>1.00</td>
<td>40.17</td>
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<tr>
<td>5.00</td>
<td>—</td>
<td>212.61</td>
<td>1.22</td>
<td>68.19</td>
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<tr>
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<td>1325.29</td>
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Binary Search

**Initialization:** Let $L=0$ and $U=1$ be the initial Lower and Upper bounds respectively on $Z$; Let $h=0$ the initialization of the iteration counter; Let $Z=0.1$ the initialization of the $Z$ imbalance measure; Let $\rho=0.1$ the initialization of percentage difference between two feasible $Z$ values; Let $T=900$ the time limit (in seconds) for integer feasible solution search for a given $Z$; Let $t$ the time until feasible solution is found for a given $Z$ (if it exists);

while $(U-Z) > \rho U$ do
  Let $h=h+1$;
  Solve for $Z$;
  if Feasible Integer Solution is found then
    Let $U=Z$;
  end
  else if $t>T$ or $Z$ is infeasible then
    Let $L=Z$;
  end
  Let $Z=\frac{U-L}{2} + L$
end

**Results Analysis**

<table>
<thead>
<tr>
<th>SMA1</th>
<th>SMA2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum $Z$ (%)</td>
<td>0.029</td>
</tr>
<tr>
<td>Time (sec)</td>
<td>1022</td>
</tr>
<tr>
<td>Iterations</td>
<td>12</td>
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```

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<tr>
<th></th>
<th>FBP2-D</th>
<th>SMAC1</th>
<th>SMAC2</th>
</tr>
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<tbody>
<tr>
<td>Minimum Z (%)</td>
<td>0.029</td>
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<td>Time (sec)</td>
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Stochastic Modeling Approach 1 (SMA1) - v.s. Deterministic (FBP2-D)

▶ FBP2-D → binary search algorithm → $Z = 0.029\%$

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Stochastic Modeling Approach 1 (SMA1) - v.s. Deterministic (FBP2-D)
- FBP2-D → binary search algorithm → $Z = 0.029\%$
- Fix this solution in our SMAC1 and test for feasibility → solution for FBP2-D is feasible for SMAC1 → capacity is not exceeded with this assignment
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Stochastic Modeling Approaches (SMA1, SMA2) - v.s. Deterministic (FBP2-D)

**Stochastic Modeling Approach 1 (SMA1) - v.s. Deterministic (FBP2-D)**
- FBP2-D $\rightarrow$ binary search algorithm $\rightarrow Z=0.029\%$
- Fix this solution in our SMAC1 and test for feasibility $\rightarrow$ solution for FBP2-D is feasible for SMAC1 $\rightarrow$ capacity is not exceeded with this assignment

**Stochastic Modeling Approach 2 (SMA2) - v.s. Deterministic (FBP2-D)**
- FBP2-D $\rightarrow$ binary search algorithm $\rightarrow Z=0.029\%$
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Stochastic Modeling Approach 1 (SMA1) - v.s. Deterministic (FBP2-D)
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- Fix this solution in our SMAC1 and test for feasibility $\rightarrow$ solution for FBP2-D is feasible for SMAC1 $\rightarrow$ capacity is not exceeded with this assignment

Stochastic Modeling Approach 2 (SMA2) - v.s. Deterministic (FBP2-D)
- FBP2-D $\rightarrow$ binary search algorithm $\rightarrow Z=0.029\%$
- Fix this solution in our SMAC2 and test for feasibility $\rightarrow$ solution for FBP2-D is infeasible for SMAC2 $\rightarrow$ imbalance measure not achievable
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Stochastic Modeling Approaches (SMA1, SMA2) - v.s. Deterministic (FBP2-D)

Stochastic Modeling Approach 1 (SMA1) - v.s. Deterministic (FBP2-D)

- FBP2-D → binary search algorithm → $Z=0.029\%$
- Fix this solution in our SMAC1 and test for feasibility → solution for FBP2-D is feasible for SMAC1 → capacity is not exceeded with this assignment

Stochastic Modeling Approach 2 (SMA2) - v.s. Deterministic (FBP2-D)

- FBP2-D → binary search algorithm → $Z=0.029\%$
- Fix this solution in our SMAC2 and test for feasibility → solution for FBP2-D is infeasible for SMAC2 → imbalance measure not achievable
- Lowest imbalance measure value achievable by SMAC2 → $Z = 2.583\%$ as opposed to 0.029\%
Final Remarks and Future Research

We presented two general stochastic modeling approaches and proposed and evaluated the performance of four alternative formulations for solving the problem of assigning package destinations to secondary sorters, while balancing the workload accounting for variability in the package arrivals.

The alternative formulations included three non-linear approaches (quadratic, second order cone programming and nonlinear non-quadratic) and a linearized version of the original non-linear stochastic approaches formulations.

When comparing the formulations based on their computational performance, we observed that the second order cone programming formulation displayed the most robust performance.

We proposed a binary search algorithm which iterates through the imbalance measure space ($Z$) and implements the second order cone programming formulation at each iteration.

We were able to find better and faster solutions compared to those obtained when trying to solve the problem directly without iterating over $Z$.

We noticed that in case of not considering the stochastic nature of the arrivals, we would incur in a non-planned imbalance that would probably impact logistically in the operation.
Final Remarks and Future Research

Final Remarks

We presented two general stochastic modeling approaches and proposed and evaluated the performance of four alternative formulations for solving the problem of assigning package destinations to secondary sorters, while balancing the workload accounting for variability in the package arrivals.

The alternative formulations included three non-linear approaches (quadratic, second order cone programming and nonlinear non-quadratic) and a linearized version of the original non-linear stochastic approaches formulations.

When comparing the formulations based on their computational performance, we observed that the second order cone programming formulation displayed the most robust performance.

We proposed a binary search algorithm which iterates through the imbalance measure space ($Z$) and implements the second order cone programming formulation at each iteration.

We were able to find better and faster solutions compared to those obtained when trying to solve the problem directly without iterating over $Z$.

We noticed that in case of not considering the stochastic nature of the arrivals, we would incur in a non-planned imbalance that would probably impact logistically in the operation.
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- It is important to validate empirically the normality assumption, although we rely on the Central limit Theorem, as we consider the sum of a considerable amount of random variables.
- Fine tune the proposed search algorithm in order to make it more robust, by fixing variables based on reduced gradients.
- Investigate the impact of considering the door requirements as discrete random variables, when rounding up the number of doors required by a destination for a given sort.
- A combination of chance-constrained programming and scenario generation can be implemented for solving the problem when considering this issue.
- In order to solve the problem efficiently we intend to implement a lagrangian heuristic, so faster solutions can be found when considering scenario generation.
- Performance validation tool. A MonteCarlo simulation can be used to compare the facility performance based on recommended solutions from various formulations.
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