Integrated planning of operations and spare parts logistics under uncertainty in the supply chain of maintenance service providers

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Research motivation

- 3rd party maintenance service providers (3PMP)
- Aircraft, automobiles, medical equipment, etc.
- High system availability is essential
- Local repair is technically impossible
  - Lack of appropriate equipment or skills

Supply Chain Sales & Operations Planning (S&OP)

New perspective in the industry
• If an equipment fails, we assume that the failure is caused by the failure of one or more of its subassemblies
• The equipment (e.g., aircraft engine) is dismantled and inspected
• The defective component(s) are identified
• The required spare parts are supplied from in-house inventory or are purchased from external suppliers
• The defective elements are replaced/repaired and get reassembled
3PMP overview

- A well-known problem in this industry:
  - To choose optimal spare part stock levels, such that target delivery dates are attained at minimum inventory investment
- Other decisions:
  - Resource (budget/machinery/labor) allocation
  - Delivery dates (sales decisions (available to promise))
Spare part logistics networks

- Multi-echelon networks
- Differences from other materials supply chains:
  - Large number and diversity of spare parts
  - Sporadic item failure rates (demand) and repair times (extremely difficult to forecast)
  - Very high prices of some individual parts
  - Financially remarkable effect of spare part shortage
- Taking a conservative strategy, where high amounts of demand for different elements are predicted, might guarantee a high service level
  - A costly solution, particularly for expensive parts
  - It might also lead to the obsolescence of spare parts over time
3PMP challenges

- Multi-indenture structure of the equipment
- Large number of unreliable components
  - Diversity of required spare parts
- Different types of components might randomly fail to perform at different points of time
- The demand is hard to forecast
  - Hard to set a robust service lead time
  - High penalties for the late delivery of equipment (e.g., from airlines)
Literature review

- Inventory modeling of spare parts logistics systems
  - Review of spare parts inventory models [Kennedya et al., 2002]

- Integrated design and inventory management of service parts logistics
  - Candas and Kutanoglu (2007) introduced an optimization model that reflects the interdependency between spare part logistics network design and inventory stocking decisions, (stock levels and their corresponding fill rates)
  - Kutanoglu and Lohiya (2008) proposed an optimization-based model for the integrated inventory and transportation for a single echelon, multi facility service parts logistics system
Literature review

• The majority of contributions in the literature on spare parts management are focused on inventory modeling.
• Many contributions in the literature are mostly relied on basic theories of inventory control and/or queuing networks.
• Less attention is paid on the spare part inventory control in the supply chain context.
• No contribution addresses the spare part logistics problem from a 3rd party maintenance provider viewpoint.
• Mainly focused on the spare part inventory management in an in-house maintenance context:
  – The manufacturer seeks the optimal spare part order quantity and interval in order to maximize system availability or to minimize system down time over a given period of time.
• 3PMP is faced with strict due dates for the delivery of repaired equipment (sale decisions).
Outline

• Deterministic model for planning of operations and spare part logistics in the framework of supply chain S & OP planning
  – The number of maintenance jobs that can be completed to deliver at each period
  – The amount of spare parts that must be purchased from external suppliers
  – Objective: to minimize the cost of spare parts procurement and inventory in addition to the delivery delay penalty
• Modeling the uncertain spare part demand
• Reformulating the deterministic model into a multi-stage stochastic program with recourse
• Discussion on the problem complexity
• Some solutions to reduce the complexity
Problem assumptions

- Single-product, multi-period setting
- Positive lead time for the spare part supply
- Capacitated spare part supply
- Single spare part supplier (per component)
- Limited budget
- Limited resource (labor/machine) capacity
- Known (average) failure probability of components
- Deterministic spare part supply lead time
Problem formulation (deterministic context)

- Maintenance demand in each period (at product level):
  - The number of received maintenance orders (or the forecasted ones) with the associated due dates

- The number of repaired items that the company can promise to deliver in each period can be interpreted as the sale amount in a supply chain context

- Any delay in delivering the equipment can be interpreted as a shortage (backorder) variable
  - The repaired item will be delivered in the upcoming periods
Problem formulation (deterministic context)

• To link the sale decisions to the spare part procurement plan, the product maintenance demand must be translated into the demand for components (spare parts)
  • Disassembly tree (reverse BOM)
  • Average failure probability of each component
  • Component (spare part) demand:
    – By multiplying the product demand by average failure probability of each component

• Example:
  • The maintenance demand= 100 units
  • 1 unit of component c with the average failure probability equal to 0.4
  • Component demand= 0.4*100=40 units.
Problem formulation (deterministic context)

- Decision variables:
  - Number of orders (maintenance jobs) that can be completed to deliver at each period
  - Number of delayed maintenance orders
  - The amount of spare parts that must be purchased from external suppliers
  - Spare part inventory
Problem formulation (deterministic context)

- **Objective:** to minimize the cost of spare parts procurement, inventory, and delivery delay penalty
- **Number of delayed orders (repaired items) is identified based on the amount of components shortage**
- **The component with the highest amount of shortage will identify the total number of delayed orders (shortage amount at the equipment level)**
  - All components, associated to the equipment, must be repaired and replaced before the reassembly
  - No need to consider a penalty cost for the shortage of each individual component
  - The delivery delay (shortage amount) in each period depends only on the component with the highest amount of shortage
Problem formulation (deterministic context)

• Example:
  – In a given period, 2 units of component 1, 4 units of component 2, and 6 units of component 3 are missing, the company will face the delivery delay for 6 repaired items (instead of 2+4+6=12 items).

• Instead of including the number of items (equipment) that can be repaired in each period, we consider the number of different components that can be replaced as the decision variable.

• The delay (shortage), however, is calculated at the equipment level.

• Linking the spare part logistics decisions with the equipment delivery (sale) decisions in such supply chains:
  • The industry is faced with due dates at equipment (product) level.

(New approach in the literature of spare parts logistics)
S&OP model in the 3PMP supply chain

\[ \text{Min} \quad \sum_{t \in T} \sum_{c \in C} p_c Q_{ct}^{\text{proc}} \quad + \sum_{t \in T} \sum_{c \in C} h_c I_{ct} \quad + b \sum_{t \in T} \theta_t \]

\[ \text{St.} \]

\[ \text{Spare part procurement cost} \quad \text{Spare part inventory cost} \quad \text{Product delivery penalty} \]

Product due dates (sale constraint)

\[ Q_{ct}^{\text{del}} - B_{c,t-1}^{\text{del}} + B_{ct}^{\text{del}} = d_{ct} \]

\[ \forall t \in T, c \in C, \]

Spare part inventory balance

\[ Q_{c,(t-l_{c}^{\text{proc}})}^{\text{proc}} + I_{c,t-1} - I_{c,t} = Q_{ct}^{\text{del}} \]

\[ \forall t \in \{1, \ldots, l_{c}^{\text{proc}}\}, c \in C, \]

Spare part supply capacity

\[ Q_{ct}^{\text{proc}} \leq M_{ct}, \]

\[ \forall c \in C, t \in T, \]

Spare part supply budget limit

\[ \sum_{c \in C} p_c Q_{ct}^{\text{proc}} \leq L_t, \]

\[ \forall t \in \{1, \ldots, l_{c}^{\text{proc}}\}, \]
S&OP model in the 3PMP supply chain

Resource capacity

$$\alpha_c Q^{del}_{ct} \leq E_{ct}, \quad \forall t \in T, c \in C,$$

Spare part safety stock

$$I_{ct} \geq SS_c, \quad \forall t \in T, c \in C,$$

Product delivery delay

$$\theta_t \geq B^{del}_{ct}, \quad \forall c \in C, t \in T,$$

Component to product link

$$\theta_t: \text{ auxiliary variable representing the maximum amount of shortage among all components in period } t$$

$$Q^{proc}_{ct}, I_{ct}, Q^{del}_{ct}, B^{del}_{ct}, \theta_t \geq 0, \quad \forall t \in T, c \in C.$$
Model complexity

• Given a product with \( |C| \) components and a planning horizon, including \( T \) periods:
  – Number of decision variables: \( (4|C| + 1) \times T \)
  – Number of constraints: \( (6|C| + 1) \times T \)

• Large-scale model for products including thousands of components

• Can be solved in Polynomial time in the deterministic context
Uncertainty

• We assume that the monthly demand for maintenance services can be forecasted with high precision based on historical data
  – Periodic or preventive maintenance constitute the majority of demand

• The nature of service and the type of spare part that might be required is highly unpredictable

• Forecasting the component (spare part) demand is a challenge
  – Which components are defective in a product at a given point of time?

• Uncertain lead times and capacities of spare part suppliers

This study is focused on uncertain spare part demand
Modeling the uncertain spare part demand

\( d_t \): the number of products waiting for maintenance services at the 3PMP in period \( t \)

- The demand for each component (spare part) can then take a value between zero and \( d_t \)
- Each possibility can be considered as a scenario for \( d_{ct} \)
- The probability distribution of such scenarios follows a binomial distribution with \( d_t \) and \( r_{ct} \)

\( r_{ct} \): failure probability of component \( c \).
  - Can be estimated based on historical data by dividing the number of times a particular component (\( c \)) has been categorized as defective to the total number of products received maintenance services over a given period of time
  - Provided by the customer
Modeling the uncertain spare part demand

• The probability distribution of spare part (component) demand

\[
\Pr\{d_{ct} = n\} = \binom{d_t}{n} (r_{ct})^n (1 - r_{ct})^{d_t-n}
\]

• Example: 10 products are waiting for maintenance services in a given month, possible scenarios for a given component vary between zero and 10

• Given that the failure probability for that component is equal to 0.6

\[
\Pr\{d_{ct} = 0\} = \binom{10}{0} (0.6)^0 (0.4)^{10},
\]

\[
\Pr\{d_{ct} = 1\} = \binom{10}{1} (0.6)^1 (0.4)^9,
\]

\[
\Pr\{d_{ct} = 10\} = \binom{10}{10} (0.6)^{10} (0.4)^0.
\]
Modeling the uncertain spare part demand

- Total number of scenarios for each spare part demand over a $T$-period planning horizon grows exponentially and would be equal to:

$$\left(1 + d_t\right)^T \quad 11^4 = 14,641$$

- In the case where the failure probability distribution of different components in a product are independent (a realistic assumption in many cases), the total number of demand scenarios over the set of spare parts ($C$) would be equal to

$$\left(\left(1 + d_{ct}\right)^T\right)^C = \left(1 + d_{ct}\right)^{CT} \quad 11^{16} = 4.5 \times 10^{16}$$

- Number of possible scenarios for the spare parts demand over a multi-period planning horizon is humongous and grows exponentially by the number of unreliable components in the product as well as the number of periods in the planning horizon
Modeling the uncertain spare part demand

- It is not reasonable to assume a stationary behavior for the spare part demand over time
  - Equivalent to detecting the same type of defective components in the products during the planning horizon
- Spare part demand is featured as a dynamic stochastic process
A Multi-stage stochastic programming formulation

• The spare par procurement decision ($Q_{ct}^{\text{proc}}$) is the main decision variable
  – Must be taken before the actual demand for different spare parts are known
  – The procurement amount of different components (spare parts) in a given period should be the same for all demand scenarios for the period that those spare parts will be delivered (first-stage decision)

• The quantity of spare parts (components) that can be delivered to the re-assembly unit, as well as the inventory and shortage of components along with the total delay are the state variables
  • They can be determined once the product is inspected and the number of defective components are identified (recourse decisions)
Multi-stage stochastic programming model  
(compact formulation)

\[
\text{Min } \sum_{n \in \text{Tree}} \sum_{t=1, \ldots, l_c} p(n) \sum_{c \in C} p_c Q_{ct}^{\text{proc}}(n) + \sum_{n \in \text{Tree}} p(n) \sum_{t \in T} \sum_{c \in C} h_c I_{ct}(n) + b \sum_{t \in T} \theta_t(n)
\]

\[
\text{St.}
\]

\[
Q_{ct}^{\text{del}}(n) - B_{c,t-1}^{\text{del}}(a(n)) + B_{ct}^{\text{del}}(n) = d_{ct}(n)
\]
\[\forall t \in T, c \in C, n \in \text{Tree},\]

\[
Q_{c,(t-1)}^{\text{proc}}(a'(n)) + I_{c,t-1}(a(n)) - I_{ct}(n) = Q_{ct}^{\text{del}}(n)
\]
\[\forall t \in \{1, \ldots, l_c^{\text{proc}}\}, c \in C, n \in \text{Tree},\]

\[
Q_{ct}^{\text{proc}}(n) \leq M_{ct},
\]
\[\forall c \in C, t \in T, n \in \text{Tree},\]

\[
\alpha_c Q_{ct}^{\text{del}}(n) \leq E_{ct},
\]
\[\forall t \in T, c \in C, n \in \text{Tree},\]

\[
\sum_{c \in C} p_c Q_{ct}^{\text{proc}}(n) \leq L_t,
\]
\[\forall t \in \{1, \ldots, l_c^{\text{proc}}\}, n \in \text{Tree},\]

\[
I_{ct}(n) \geq SS_c,
\]
\[\forall t \in T, c \in C, n \in \text{Tree},\]

\[
\theta_t(n) \geq B_{ct}^{\text{del}}(n),
\]
\[\forall c \in C, t \in T, n \in \text{Tree},\]
Multi-stage stochastic programming model
(Split-variable formulation)

\[\begin{align*}
\text{Min} & \quad \sum_{s \in S} p(s) \sum_{c \in C} p_c Q^\text{proc}_{ct}(s) + \sum_{s \in S} p(s) \left( \sum_{t \in T} \sum_{c \in C} h_c I_{ct}(s) + b \sum_{t \in T} \theta_t(s) \right) \\
\text{St.} & \\
Q^\text{del}_{ct}(s) - B^\text{del}_{c,t-1}(s) + B^\text{del}_{ct}(s) &= d_{ct}(s) \quad \forall t \in T, c \in C, s \in S, \\
Q^\text{proc}_{c,(t-l^\text{proc}_c)}(s) + I_{c,t-1}(s) - I_{c,t}(s) &= Q^\text{del}_{ct}(s) \quad \forall t \in \{1, \ldots, l^\text{proc}_c\}, c \in C, s \in S, \\
I_{ct}(s) &\geq S_{sc}, \quad \forall t \in T, c \in C, s \in S, \\
Q^\text{proc}_{ct}(s) &\leq M_{ct}, \quad \forall c \in C, t \in T, s \in S, \\
\alpha_c Q^\text{del}_{ct}(s) &\leq E_{ct}, \quad \forall t \in T, c \in C, s \in S, \\
\sum_{c \in C} p_c Q^\text{proc}_{ct}(s) &\leq L_t, \quad \forall t \in \{1, \ldots, l^\text{proc}_c\}, s \in S, \\
\theta_t(s) &\geq B^\text{del}_{ct}(s), \quad \forall c \in C, t \in T, s \in S, \\
Q^\text{proc}_{ct}(s) &= Q^\text{proc}_{ct}(s'), \quad \forall c \in C, t \in T, s \in S, s' \in B_{s,t}. 
\end{align*}\]
Case study

- Planning horizon: 4 periods (month or season)
- Maintenance demand per period: 10 products
- Number of components in the product: 4
- Failure probability of each component: 0.7
- The spare part supply lead time: 1 period for all components
- Initial inventory of components: 7 units
- Spare part safety stock: 1 unit
- Spare part supply capacity: 9 units
- Product inventory cost: 5% of the price
- 11 scenarios (i.e., 0, 1, 2, ..., 10) for each spare part demand
Demand scenarios

• To reduce the size of scenario tree:
  – Clustering scenarios into 3 groups with the mean values equal to 2, 7, and 10.
    • Scenarios 0-4 belong to cluster 1, scenarios 5-9 belong to cluster 2 and scenario 10 belongs to the third cluster
    • The probability of each cluster is then calculated as the some of the probabilities of scenarios in each cluster
    • Example: the probability of cluster 1 is calculated as \(0+0+0+0.01+0.04=0.05\)
  • For the sake of simplicity, we assume that the demand for all components is correlated (not a realistic assumption)
  • 5-stage scenario tree
    – 3 nodes in period 1 (stage 2), 9 nodes in stage 3, 27 nodes in stage 4, and 81 nodes in the last stage.
### Experimental results

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Worst scenario</th>
<th>Best scenario</th>
<th>Average scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective function</td>
<td>19,062</td>
<td>1,160</td>
<td>7,112</td>
</tr>
</tbody>
</table>

- MSP model solution: 7,270
- Discussion:
  - A robust optimization method (worst-case scenario) results in an expensive solution
    - Highest possible order quantity of spare parts
    - Worst case scenario has a small probability (0.028 in this example)
  - Multi-stage stochastic model: less conservative order quantities
Observations

- When the probability of failure is high and we cluster scenarios into 3 groups, the middle cluster has a high probability while the other extreme clusters have a small probability of occurrence.

- As we proceed in the planning horizon, the probability of demand scenarios far from the average demand (middle cluster) gets close to zero:
  - After the first 2 stages (periods), the only acting scenario would be the average demand with a high probability, while the probability of majority of other scenarios is equal to zero.
  - The probabilities of the extreme clusters are very small and the multiplication of such probabilities over a number of stages results a value close to zero.
Observations

• In this case, it would be more reasonable to consider a scenario tree only for the first two stages, while considering average demand for the rest of periods (stages)

• Reduced scenario tree
  – Example: 9 nodes instead of 31

• Zero-probability nodes can be observed for any value of failure probability

• The above observation highlights the importance of pre-processing the scenario tree before formulating the MSP model
Discussion on reducing problem complexity

• To decompose the original model into a number of manageable-size sub-models
• *No budget constraint:*
  – Decomposing the deterministic model for each component by relaxing the delivery delay constraint
  – Revising component sub-problem by adding a term in the objective function to minimize the component backorder cost
  – Reformulating each component sub-problem based on the sub-scenario tree
  – Calculating the number of products with late delivery in each node of the master scenario tree as the maximum amount of backorder over all components in each node
S&OP model in the 3PMP supply chain

\[
\begin{align*}
\text{Min} & \quad \sum_{t \in T} \sum_{c \in C} p_c Q_{ct}^{\text{proc}} + \sum_{t \in T} \sum_{c \in C} h_c I_{ct} + b \sum_{t \in T} \theta_t \\
\text{St.} & \quad Q_{ct}^{\text{del}} - B_{c,t-1}^{\text{del}} + B_{ct}^{\text{del}} = d_{ct} \quad \forall t \in T, c \in C, \\
& \quad Q_{c,(t-l_{c}^{\text{proc}})}^{\text{proc}} + I_{c,t-1} - I_{c,t} = Q_{ct}^{\text{del}} \quad \forall t \in \{1, \ldots, l_{c}^{\text{proc}}\}, c \in C, \\
& \quad I_{ct} \geq SS_c, \quad \forall t \in T, c \in C, \\
& \quad Q_{ct}^{\text{proc}} \leq M_{ct}, \quad \forall c \in C, t \in T, \\
& \quad \alpha_c Q_{ct}^{\text{del}} \leq E_{ct}, \quad \forall t \in T, c \in C, \\
& \quad \theta_t \geq B_{ct}^{\text{del}}, \quad \forall c \in C, t \in T, \\
& \quad \sum_{c \in C} p_c Q_{ct}^{\text{proc}} \leq L_t, \quad \forall t \in \{1, \ldots, l_{c}^{\text{proc}}\},
\end{align*}
\]
Component sub-problem

\[ Z(c) = \text{Min} \sum_{t \in T \setminus 1, \ldots, l_c^{\text{proc}}} p_c Q_{ct}^{\text{proc}} + \sum_{t \in T} (h_c I_{ct} + bB_{ct}^{\text{del}}) \]

\[ \text{St.} \]

\[ Q_{ct}^{\text{del}} - B_{c,t-1}^{\text{del}} + B_{ct}^{\text{del}} = d_{ct} \quad \forall t \in T, c \in C, \]

\[ Q_{c,(t-l_c^{\text{proc}})}^{\text{proc}} + I_{c,t-1} - I_{c,t} = Q_{ct}^{\text{del}} \quad \forall t \in 1, \ldots, l_c^{\text{proc}}, c \in C, \]

\[ Q_{ct}^{\text{proc}} \leq M_{ct}, \quad \forall c \in C, t \in T, \]

\[ \alpha_c Q_{ct}^{\text{del}} \leq E_{ct}, \quad \forall t \in T, c \in C, \]

\[ I_{ct} \geq SS_c, \quad \forall t \in T, c \in C, \]
Discussion on reducing problem complexity

**Budget constraint**

- The model cannot be decomposed for each component due to the linking constraint (budget constraint)
- A Lagrangian relaxation approach can be proposed
  - By relaxing the budget constraint and solving component sub-problem
  - Feasibility check of the decomposed solutions at each node of the master scenario tree:
    - At each node of the master scenario tree, calculate the left-side of budget constraint and compare it to the available budget in the corresponding period
    - For the nodes, where this constraint is violated, penalty terms can be added to the objective function of component sub-problems
Key findings on reducing the problem complexity

• Scenario tree pre-processing
• Component decomposition method
  – Lagrangian method
• Scenario reduction methods
• To reduce the degree of uncertainty in the spare part demand and lead time, the company can adopt a collaboration mechanism with competitors, customers and suppliers
  • Collaboration with customers in terms of information sharing on condition monitoring of equipment (VMI perspective)
  • An inventory pooling strategy with competitors
  • Information sharing with suppliers
Thank you!

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