About Lorentz Transformations and Tachyons.

V. S. Olkhovsky
Kiev State University - Kiev

E. Recami (*)
Institute of Theoretical Physics, Ukrainian Academy of Sciences - Kiev
Istituto di Fisica Teorica dell'Università - Catania
Centro Siciliano di Fisica Nucleare e di Struttura della Materia - Catania
Istituto Nazionale di Fisica Nucleare - Sezione di Catania

(ricevuto il 24 Novembre 1970)

It is known that from the (special) relativity principle the existence follows of an invariant velocity \( c \); but it does not follow—\textit{a priori}—any restriction on the relative velocities of the reference-frames and/or of the objects considered (1).

Thus, from that principle (i.e. practically from the Lorentz transformations) the usual velocity addition theorem can be \textit{univocally} derived; and it must hold, \textit{e.g.} for all kinds of \textit{particles} (bradyons (2), luxons, tachyons), independently of their real or unreal proper mass.

Therefore, if \( u \) is the relative velocity of two \textit{inertial} frames \textit{(subluminal and/or superluminal)}, we may write (2)

\[
\frac{c^2 - (v')^2}{c^2} = \frac{(c^2 - v^2)(c^2 - u^2)}{(c^2 - u \cdot v)^2}
\]

where \( v, v' \) are the velocities of any considered \textit{particle} with respect to the two frames of reference.

The form (1) of the velocity addition law is particularly transparent. We can notice immediately what sketched in Table I.

(1) On leave of absence from the Istituto di Fisica dell'Università di Catania, under an exchange program supported by the Istituto Nazionale di Fisica Nucleare, Sezione di Torino, and The Ukrainian Academy of Sciences, Kiev.

(2) See, \textit{e.g.}, V. S. Olkhovsky and E. Recami: \textit{About the Possibility of Faster-than-Light Particles in Special Relativity}, in Visnyk K. D. Universitetu (Seria Fisichna), n. II (Kiev, 1970) (in Ukrainian).


(4) Our considerations are in disagreement, partially at least, with K. H. Mariwalla: \textit{Am. Journ. Phys.}, 37, 1281 (1969), and private communication.
To fix our ideas, we may suppose the «first» frame to be subluminal. This assumption limits by no means the generality, as itself has no absolute (but only relative) meaning. There is formally a complete «indistinguishability» between the two «sets» of sub- and super-luminal frames (or a «duality»). We might say that, in special relativity, the velocity \( u = c \) divides the world of the reference frames, and of the objects \((*)\) \((4,5)\), in two complementary and «symmetrical» parts. But, in the previous assumption, we shall call physical the «subluminal» frames and unphysical the «superluminal» ones.

In particular, from the Table it is self-evident that according to superluminal frames our tachyons appear as bradyons, and vice-versa \((*)\). Besides, photons (if they have actually proper mass zero \((6)\)) will always appear as having velocity \( c \).

From what precedes, it follows that for every particle a couple of quantities may correspond to each property of it, as manifesting itself with regard to frames of the two «worlds» respectively. Therefore, complex numbers may be formally associated to particle properties. In particular, every particle may be given a couple of «proper masses», one for the first «world» and one for the second «world».

Namely, each particle possesses formally a complex proper mass \((**\))

\[
M = m + i\mu ,
\]

and the following (real!) observables can be written, for frames of both the «worlds»,

\[ u < c \quad \begin{array}{c} v \triangleleft c \rightarrow v' \triangleleft c \\ v = c \rightarrow v' = c \\ v > c \rightarrow v' > c \end{array} \]

\[ u = c \quad v \triangleleft c \rightarrow v' = c \]

\[ u > c \quad \begin{array}{c} v \triangleleft c \rightarrow v' > c \\ v = c \rightarrow v' = c \\ v > c \rightarrow v' < c \end{array} \]
as $\beta = u/c$:

$$
\begin{align*}
E &= \text{Re} \frac{(m + i\mu)u}{\sqrt{1 - \beta^2}}, \\
p &= \text{Re} \frac{(m + i\mu)u}{\sqrt{1 - \beta^2}}, \\
F &= \text{Re} \frac{d}{dt} \left[ \frac{(m + i\mu)u}{\sqrt{1 - \beta^2}} \right],
\end{align*}
$$

where obviously the real proper mass will be considered for subluminal particles (their velocities relative to the frame considered being $u < c$), and the pure imaginary one for superluminal particles ($u > c$). The relations (2) simply express, in compact form, equations well known in relativity (7) (*).

Analogously, if $u$ is the relative velocity of two collinear inertial frames, we may write, e.g., the Lorentz transformations $\beta = u/c \geq 1$

$$
\begin{align*}
x' &= \text{Re} \frac{(x + i\xi) - u(t + iv)}{\sqrt{1 - \beta^2}}, \\
y' &= y, z' = z,
\end{align*}
$$

$$
\begin{align*}
t' &= \text{Re} \frac{(t + iv) - u(x - i\xi)/c^2}{\sqrt{1 - \beta^2}}, \\
\Delta t &= \text{Re} \left[ (\Delta t_0 - i\cdot \Delta \lambda_0) \cdot \sqrt{1 - \beta^2} \right], \\
\Delta t &= \text{Re} \left[ (\Delta t_0 + i\cdot \Delta \lambda)/\sqrt{1 - \beta^2} \right],
\end{align*}
$$

where the symbols are self-evident. And if, besides, $v$ is the velocity of the considered particle with regard to the nonprimed (first) frame:

$$
\begin{align*}
dt' &= \text{Re} \frac{(dt + i\cdot dr)(c^2 - u \cdot v)}{c \sqrt{c^2 - v^2}}, \\
E' &= \text{Re} \frac{(E + i\cdot E_0)(c^2 - u \cdot v)}{c \sqrt{c^2 - v^2}}.
\end{align*}
$$

As regards eqs. (4), see again also the first footnote of this work. From relations (4) (**), it is immediate to notice the conditions under which a tachyon appears as an antitachyon (in the sense clarified in ref. (2) and references therein).

At last, we want recall that ROLNICK (*) raised a paradox, trying to show that tachyons could not interact with usual matter. But that paradox has been destroyed


(*) In this work we do not consider complex position, for relativistic particles, as in ref. (4). Generalizing eqs. (2) for complex velocities $u$, it would be perhaps possible to relate actually global complex proper masses with relativistic particles.

(**) Considered for the particular case when the two frames of references are both of bradyonic type.

by Sudarshan (10); and in any case it ought to take into due account the interesting
observations put forward long time ago by Wheeler and Feynman (11) (when resolv-
ing an analogous paradox arisen in the context of advanced electromagnetic effects),
i.e. that discontinuous forces cannot exist in (macro)-physics.

***

The authors are grateful to Prof. V. P. Shelest for the kind interest, and to Profs.
A. Agodi, A. S. Davydov, P. K. Kobuzhkin, A. G. Sitenko and E. C. G. Sudarshan
for very useful discussions.

One of them (E.R.) thanks also Profs. A. Agodi, N. N. Bogolubov, Yu. L. Mente-
kovsky, V. P. Shelest, M. Verde, G. V. Wataghin, who allowed or helped his leave
of absence from Italy to Kiev, and acknowledges the very kind hospitality received at
the Institute of Theoretical Physics, Ukrainian Academy of Sciences, Kiev (*).

(*) E. C. G. Sudarshan: private communication.
(*) The present work appeared first as preprint ITF/70, Kiev (Sept. 21st, 1970).