More About Lorentz Transformations and Tachyons: Answer to the Comments by Ramachandran, Tagare and Kolaskar.

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In a note (1), Ramachandran et al. have commented about a recent, brief letter by Olkhovsky and one of us (2), regarding Lorentz transformations (LT) and faster-than-light-particles (3-4).

Even if their comments did not appear to us to be very pertinent, we are grateful to them for this occasion, since it allows us to clarify our preliminary letter (2), which remained indeed at the surface of the problems. In particular, we shall show that our previous considerations can be founded much better than in ref. (2).

1. Let us first analyze ref. (1). The whole central body, from eq. (4) to eq. (14), does not at all concern our letter, since it deals with a complex space-time, following a philosophy which was not ours. Namely, considering (6) "the space-time to be complex, with the masses of particles being also complex quantities", is not our starting point, but only a framework in the mind of those authors. In particular, we do not define (6) velocities as in their eq. (4).

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(5) First lines of ref. (1).
(6) Last lines of ref. (1).
Actually, our eqs. (2)—for example—merely group in a compact form the two well-known sets of formulae usually expressing the energy momentum of free (bradyons) (7) and (tachyons) (8), respectively (9-10). By the way, our eqs. (2)—which are always Lorentz covariant for both bradyons and tachyons—have nothing to do with eqs. (13)-(14) of ref. (1), and therefore the quotation of our letter by those authors must be considered a misprint at this point.

The only point of ref. (1) critically interesting and justified is the one forwarded in the second paragraph (it had already been pointed out e.g. in ref. (11)). And we shall discuss it in this answer.

2. — We were essentially interested in the magnitude of the velocity $v'$ relative to a (superluminal) reference frame $S$, for objects appearing either bradyons (B) or tachyons (T) in a (subluminal) frame $s$. In fact, briefly speaking, our problem was considering if a bradyon (tachyon) will appear as a tachyon (bradyon) to a superluminal observer, or not. Therefore, we turned first our attention only to the velocity composition law in the form reported, e.g., by Terletsky (12):

$$
\frac{c^2 - (v')^2}{c^2} = \frac{(c^2 - v^2)(c^2 - u^2)}{(c^2 - u \cdot v)^2},
$$

where $u$ is the relative velocity of two (inertial) subluminal frames $s, s'$, and $v, v'$ are the velocities of the considered object with respect to those frames. In ref. (11) the extensibility of eq. (1) to both B’s and T’s (with $s, s'$ subluminal) was already realized. Our following step was assuming that eq. (1) holds also for $u > c$, i.e. also when going from a subluminal $s$ to a superluminal $S$. We assumed as well the validity of eq. (1) for both the frames being superluminal, under the hypothesis that only the relative velocity $u$ meant in this context.

Equation (1), in its wider interpretation, can be a good extrapolation of the usual one, since it obviously coincides with that equation, in the usual velocity domain. Besides, the continuity condition at the boundary are well satisfied since—for a couple of (inertial frames) either $s$-type or $S$-type—when $u \to c^2$, one has $v' \to c$ for any value of $v$.

But we must first overcome the following difficulty, connected with using eq. (1). Accepting this equation for $u > c$ means assuming some kind of generalized LT also for $u > c$.

The trivial generalization of the LT, although consistent with eq. (1), does not work. In fact, let us consider one (free) B and two inertial frames $s$ and $S$, respectively sub

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and superluminal, with identical space orientation and moving for simplicity along the $x$-direction with constant velocity $u$. Because of the asymmetry of the usual velocity transformation formulae for the $B$ velocity component along the frame relative velocity $u$ and in the plane normal to $u$, respectively,

$$v'_z = \frac{v_z - u}{1 - uv_z/c^2}, \quad v'_{\perp u, z} = \frac{v_{\perp u, z} \sqrt{1 - (u/c)^2}}{1 - uv_z/c^2}, \quad [u = u_\perp > c],$$

we shall meet imaginary $\perp$ transversal $\perp$ velocities $v'_{\perp u, z}$ when passing from $s$ to $S$.

Let us explicitly notice since now that this asymmetry is due to the asymmetry between the number of the spatial and temporal dimensions in the chronotopical space $(3, 1)$. In a fictitious bidimensional chronotope $(1, 1)$, where space-time dimensional symmetry holds, any such difficulty is not met. Therefore, one has to be careful in avoiding reasonings based on a bidimensional space-time: on the contrary, that misleading procedure has been till now followed by almost all the authors $(12-14)$ who approached the problem.

In particular, the transformation group proposed by Parker $(12)$ (i.e. his $\perp$ extended Lorentz group in one spatial dimension $\perp$) has been shown to have no standard (real) analogous in the pseudo-Euclidean four-dimensional space-time $(15)$. By the way, also the assertions (*) forwarded by Mariwalla $(14)$ have been postulated or worked out only in the case of one-dimensional space.

Therefore, we must seek for different generalized Lorentz transformations (GLT). At this purpose we have to bear in mind that Gorini $(16)$ showed that in four dimensions no real GLT exist, which connect $(17)$ frames $s$ to frames $S$ (i.e. for $u > c$).

As a guide in our research for GLT let us choose the following observation. The almost universal starting point for studying $T$'s—since the first paper by Sudharshan and coworkers $(4)$ is generalizing the relativistic equation $m = \mu_0 \sqrt{1 - \gamma^2}$ also for $\beta^2 > 1$ under the convention that $\mu_0 = \mu_0 \cdot \theta(1 - \beta^2) + im_0 \cdot \theta(\beta^2 - 1)$, i.e. assuming for $u > c$:

$$m = \frac{im_0}{\sqrt{1 - \beta^2}}, \quad \left[ m_0 \text{ real}, \quad \beta = \frac{u}{c}, \quad u > c \right].$$

This important generalization has been commonly accepted in an uncritical way. Now, let us temporarily assume that superluminal frames $S$ do exist with standard properties. We shall show in Sect. 3 that physical frames $S$ must regard our $T$'s as $B$'s. Therefore, we shall have $S$-frames in which our $T$'s are actually at rest (behaving as tachyons with regard to us, but as $B$'s with regard to $S$, in particular to their rest systems). Of course, the proper mass of tachyons will be real, $m_0$ (as well as the proper lengths


(*) Since in the first part of ref. $(14)$ superluminal frames were not yet considered, its eqs. (6) are not even understandable. In any case, they look to be wrong, and not easily compatible with the eqs. (1) of the same paper. Moreover, the second part of that note is tautological, since it assumes tachyons having always $\beta > c$ with respect to both $B$'s and $T$'s, and afterwards it finds out of course—in eq. (8)—that the relative velocity between two $T$'s is greater than $c$. In the following of the text, we shall criticize that hypothesis.

$(17)$ V. Gorini: private communication (1971).

and the proper times: see the following). Let us explicitly repeat that the proper mass of T's is not imaginary, in this framework \(^{(12)}\). Now, when passing from the rest system of a T to a subluminal system, we get that its relativistic mass transforms according to eq. (3) if we assume, for the energy the general transformation law

\[
m = \frac{m_0}{\sqrt{1 - \beta^2}} \quad \left[ m_0 \text{ always real, } \beta = \frac{u}{c}, \quad u \geq c \right],
\]

valid both for \(u < c\) and for \(u > c\). In fact, for \(u > c\) eq. (4) reads

\[
m' = \frac{m_0}{\sqrt{1 - \beta^2}} = \frac{m_0}{\sqrt{\beta^2 - 1}} = \frac{m_0}{-i \sqrt{1 - \beta^2}} = \frac{im_0}{\sqrt{1 - \beta^2}},
\]

thus yielding eq. (3). In this way we gave a meaning to the (real) rest mass of T's, evidencing the ultimate consequences of the assumption of eq. (3). Incidentally, the imaginary unit entering eq. (3) appears not because of a proper mass supposedly imaginary, but because of the modulus symbol in the GLT, eq. (4).

In order to show that the previous procedure is not merely a formal trick, let us consider the transformations \(L\) between a frame \(s\) and a frame \(S\) moving with velocity \(u\), \([u > c]\) along the \(x\)-direction (for simplicity) under the conditions that \(a\) \(L\) be linear; \(b\) the speed of light be \(c\) in both \(s\) and \(S\); \(c\) 4-dimensional space-time be homogeneous, and 3-dimensional space be isotropic. Then, partially following ref. \((19)\), one may show that transformations \(L\) must conserve the norm of fourvectors, except for a sign; \(i.e.\) the \(L\)'s must be such that, \(e.g.,\)

\[
(5a) \quad c^2 t'^2 - x'^2 = \pm (c^2 t^2 - x^2) \quad \text{for } u < c,
\]

\[
(5b) \quad c^2 t'^2 - x'^2 = - (c^2 t^2 - x^2) \quad \text{for } u > c,
\]

for instance, with the pseudo-Euclidean metric \((+ - - - -)\). In eq. (5b) we chose the sign \(\pm\), instead of the sign \(\pm\), because \(a\) we want (among the others) to get eq. (4), \(i.e.\) to save the validity of eq. (3); \(b\) this choice is in accord with our \(\phi\) phylosophy \(\sigma\), as we shall soon see in the 4-momentum space. Besides, this choice, in the bidimensional case, coincides with Parker's. If we want to assign standard meaning to frames \(S\), the previous one seems to be the only possible choice.

In fact, condition (5) is satisfied by the usual LT for \(u < c\) and by the transformations \(^{(1)}\)

\[
x' = \frac{x - ut}{\sqrt{1 - \beta^2}} , \quad t' = \frac{t - ux/c^2}{\sqrt{1 - \beta^2}} , \quad y' = iy , \quad z' = iz \quad [\beta^2 > 1]
\]

for \(u > c\). We underline that precisely the modulus \(|1 - \beta^2|\) enters eqs. (6).

\(^{(1)}\) By the way, the GLT for both \(u > c\) and \(u < c\) may read, \(e.g.,\)

\[
x' = \frac{x - ut}{\sqrt{1 - \beta^2}} ; \quad t' = \frac{t - ux/c^2}{\sqrt{1 - \beta^2}} , \quad y' = (-1)^{\tau} y , \quad z' = (-1)^{\tau} z \quad [u > c],
\]

where a * tachyonic index * (function of the only relative velocity \(u\)) has been formally introduced; \(\tau = 0\) for B's and \(\tau = \frac{1}{2}\) for T's.
These transformations (6) may well be chosen as generalized Lorentz transformations for \( u > c \), since they i) form a group together with the usual LT, ii) are unimodular (with determinant +1), iii) are "pseudounitary", iv) satisfy our fundamental equation (1) and of course eq. (3).

Why imaginary units appear \((16)\) in \( y', z' \) will become clear later on.

In our terminology, \textit{pseudo unitary} means satisfying condition (5b); notice that here "unitary" does not have the common significance, since the chronotope is not a Hilbert space (it does not even have a positive-definite metric, and its norm is not a Hermitian form). In this framework usual LT will be also valid between frames \( S \).

If we write condition (5b) in the four-momentum space

\[
E^2 - c^2 p^2 = -(E'^2 - c^2 p'^2), \quad [u > c],
\]

we see immediately that our pseudounitary GLT operate (for \( u > c \)) a symmetry with respect to the "light-cone" \( p^2 = 0 \), i.e. transform two-sheeted hyperboloids \( p^2 = m_0^2 > 0 \) into single-sheeted hyperboloids \( p^2 = -m_0^2 < 0 \). This fact accords with our philosophy, as exploited, e.g., in Sect. 4.

As regards velocities, from eqs. (6) the usual LT will follow for \( u < c \). In the case \( u > c \), from eqs. (6) we shall get the velocity GLT (*)

\[
\begin{align*}
v'_x &= \frac{v_x - u}{1 - uv_x/c^2} \quad [u > c], \\
v'_y &= \frac{iv_y}{1 - uv_y/c^2} \\
v'_z &= \frac{-iv_z \cdot \sqrt{1 - \beta^2}}{1 - uv_z/c^2} = \frac{-v_z \cdot \sqrt{1 - \beta^2}}{1 - uv_z/c^2}.
\end{align*}
\]

Transformations (7) are consistent with eq. (4) and may be verified to satisfy eq. (1). Therefore, our eq. (1) is not only LT covariant \((u < v)\), but also GLT covariant \((u > c)\); briefly, we shall say that it is "G covariant". Our Table in ref. (2) has a quite general validity, contrarily to what claimed by Ramachandran et al. (1).

Why imaginary units formally enter \( v', y', z' \) (as well as \( y', z' \)) will be clear soon.

It is \textit{first} interesting to observe that our GLT, eq. (6), in the bidimensional case reduce \((**)\) to the transformations \((19)\) by Parker, who possibly did not realize that eqs. (4)-(5) of ref. (12) may be written in the simple form of our first two eqs. (6). Let us explicitly point out that our philosophy too coincides with the Parker's one \((19)\). The fact that our eqs. (6) generalize the Parker transformations to the four-dimensional chronotope is not in contrast with Gorini's theorem \((18)\), since we introduced GLT not real but \textit{complex}.

The circumstance that the GLT for \( u > c \) are complex—precisely they are just the usual LT, for \( u < c \), multiplied by the imaginary unit \( i \)—reflects in the formal writing of eqs. (6) and (7). When going from one frame to another, both either subluminal or superluminal, we have \textit{no} change in the metric \((i.e.\text{ the way that the two frames})\)

\[\text{(*) This well-known fact was, e.g., reported by W. Pauli: Relativitätstheorie, in Enc. der Math. Wissen., 5, Chapt. 19 (Leipzig, 1921).}\]
\[\text{(**) From Parker's transformations one gets the velocity transformation law in two dimensions:}\]
\[v = \frac{a(v'+c) - b(v'-c)}{a(v'+c) + b(v'-c)}, \quad a = \sqrt{\frac{u + c}{u - c}}, \quad b = a^{-1}[u > c],\]
\[\text{which of course coincides with the first of our eqs. (7).}\]
\[\text{(***) In the bidimensional case, Parker's transformations are the unique ones satisfying the bidimensional eq. (5b); this may indicate that our GLT too are the unique ones satisfying eq. (5b).}\]
possess for describing the Minkovsky space-time): namely, two \( s - s' \) (or \( S - S' \)) frames are connected by LT, which satisfy eq. (5a):

\[
e^2 t'^2 - x'^2 = c^2 t^2 - x^2.
\]

In that case, if we would use a Euclidean metric \((++++)\), we should write the timelike components in a real form, and the spacelike components in an imaginary form, and this for both frames.

When going, on the contrary, from a \( s \) to a \( S \) frame (or from \( S \) to \( s \)), we have a change of metric \((---++)\); for instance, we go from a frame, \( s \), describing the Minkovsky space-time with the metric \((++++)\), to a frame, \( S \), describing it with the metric \((---++)\). This evenience is represented by the fact that our GLT (for \( u > c \)) satisfy eq. (5b).

\[
e^2 t'^2 - x'^2 = -(e^2 t^2 - x^2).
\]

This same evenience is represented as well by saying that the 3 spacelike co-ordinates will go into only 1, and that the timelikc co-ordinate will go into 3 (and vice versa):

\[
(3, 1) \rightarrow (1, 3)
\]

\([u > C]\),

The other, previous considerations may be easily modified for this case. What is important is that it must be symbolically:

\[
(8) \quad \text{GLT} = i \cdot \text{LT},
\]

and this is what we have in eqs. (6) and (7). Of course, the imaginary (or real) units merely record how the second observer is supposed to use his own observations for building up four-vector squares. They do not mean, obviously, that the second frame will really observe imaginary or real quantities! (The observed quantities, briefly speaking, will be given by eqs. (6)-(7) except for an imaginary unit.) Incidentally, let us add that the components of velocity behave essentially as the 3 spacelike components of the four-dimensional velocity in this respect.

In our philosophy, when passing from a \( s \) to a \( S \) frame, \( B \)'s go into \( T \)'s and vice versa, whilst \( \mathit{luxons} \) \((4)\) transform into luxons. Since this question of symmetry is essential for attributing a standard meaning to superluminal reference frames, we want to discuss it furtherty.

3. - The preliminary point to be answered when considering frames \( S \) is: may the \( S \) be \( \mathit{physical} \) frames, or do they have to be necessarily regarded as \( \mathit{unphysical} \)?

We would like to emphasize that necessary condition for a standard interpretations of the frames \( S \) is that the above-depicted \( \mathit{kinematical} \ symmetry \) does hold. In fact, let us consider a subluminal frame \( s \) and two objects \( P, Q \) travelling parallel, side by side, with the same velocity \( u \) with regard to \( s \). Let us name \( w \) the velocity of \( Q \) as seen by \( P \), or vice versa. Of course, when \( u < c \), we have \( w = 0 \). But we know \((19)\) that, for \( u = c \), we should have \( w = c \) (see the Figure). Many possibilities of extrapolation for \( u > c \) exist. \( E.g., \), either \( w \) equals \( c \) also for \( u > c \) [case I]; or \( w \) increases as \( u \) increases [case II]; or \( w \) decreases [case III]; or \( w = 0 \) for \( u > c \), as for \( u < c \) [case IV], and so on.

\((14)\) Cf., \( e.g., \) V. Fock: \( \mathit{The Theory of Space, Time and Gravitation} \) (Oxford, 1964).
The kinematically symmetrical case IV is the only one compatible with \( S \) to be physical, and with the existence of (superluminal) rest systems for tachyons. Otherwise, a superluminal observer would see every object «around him» (which, according to us, is «joint» with him, side-by-side) moving off from himself; even his instruments and his own body! Refusing «case IV» would mean to extend the unphysical properties of luminal frames «—having the singular velocity \( c \)—to all the superluminal ones.

On the contrary, the situation considered in the bidimensional chronotope by MARIWALLA \((14)\) is the asymmetrical one of case II. In fact, eq. (8) of ref. \((14)\) may be written

\[
\frac{c^2 - (r')^2}{c^2} = \frac{(c^2 - v^2)(u^2 - c^2)}{(u + v)^2} \quad [u > c],
\]

which is quite different from eq. (1), since conserves the bradyonic (tachyonic) characters as absolute ones. To repeat ourselves, this case II is incompatible with the existence of \( S \)-frames.

In conclusion, the starting philosophies may be:

i) The \( S \) are unphysical: In this case, solution as eq. (9) may a priori be acceptable, as well as any GLT.

ii) The \( S \) are assumed to be physical. This is our own philosophy and Parker’s one. In this case, only «case IV» of the Figure is acceptable; and one has to rule out relations as eq. (9). One has then to accept our eq. (1), which we know to be \( G \)-covariant in four dimensions.

Incidentally, let us mention that the double sign which may be taken by the «invariant square velocity» \( c^2 \) (whose existence follows directly from the principle of relativity \((20)\)) does not matter in all our considerations. In fact, only the first one of the two possibilities: \( c \) real and \( c \) pure imaginary, seems to be acceptable, because of experimental evidence (the second case would not yield any real singular velocity, like the speed of light).

4. — Before concluding the present note we wish to underline that our «kinematical symmetry» between the «bradyonic world» and the «tachyonic world» \((2)\) does not
mean at all that this symmetry may be extended to the dynamics, when it is described by one and the same observer. Moreover, due to the special role played by the speed of light, B’s and T’s are expected to show (to a certain observer O) a different behaviour as regards the exchanging of photons. Let us notice that “tachyon” (or bradyon) signifies simply an object with velocity \( v > c \) (or \( v < c \)), as seen by O, since we are not entitled to attribute absolute meaning to this character (besides, in our philosophy, this character—for the same object—does change when pass in from a sub to a superluminal frame, or vice versa).

Actually, let us examine a subluminal observer s, and the Minkowsky space-time as seen by him. From the only study of the relative position of the world lines of B’s and T’s, and the light-cones associated with one or more events of their history, we can get all what follows. (Only notice that—in a 3-dimensional space-time, for simplicity—the light-cones springing from a bradyonic world-line are strictly inside the other, since the locus of the vertices passes inside the cones; this is not true for the light-cones springing from a tachyonic worldline. In this second case, their enveloping surface is constituted by two planes: the “retarded light-cones” occupy entirely one dihedron with angle larger than \( 90^\circ \)).

A) Case of a subluminal receiver B. The detector B may receive radiosignals (RS) from both bradyonic and tachyonic emitters.

B) Case of a superluminal receiver T.

1) The detector T will receive RS from a bradyon only until a certain instant (i.e. T will no more receive any RS from a bradyon since a certain instant). Precisely, T will first receive the radiotrasmision in the same sequence of emission, and afterwards in the reverse order.

2) T will receive RS from another tachyon t, moving along a parallel path (with respect to a subluminal frame!), in the following way:

   a) If \( v(T) = v(t) \), then T and t will be either always, or never in radio contact.

   b) If \( v(T) > v(t) \), then T will receive (retarded) RS until a certain instant; afterwards the contact will break; afterwards T will start receiving the advanced RS.

   c) If \( v(T) < v(t) \), then previous sequence will be reversed. For example, T will start receiving (retarded) RS only since a certain instant.

3) T will receive, from a second, unparallel tachyon t (retarded) RS either never, or until a certain instant, or since a certain instant, according to their positions and velocities (relative to the subluminal frame!). In general, the sequence of emission will result highly mixed up at the reception.

It is very important to notice that the conclusions above are drawn on the basis of the s-frame observations, and they are \( a \) priori quite independent of the transformations between frames s and S. In particular, they themselves \( a \) priori favour neither the “kinemtical asymmetry”, nor the “symmetry”.

For instance, WIMMEL (21) claimed that two objects \( P, Q \), travelling side by side with the same velocity, along parallel paths, may decide if they are both B’s or T’s, since the two T’s could not exchange photons each other (cf. case B.2.a). But the whole description is valid only with respect to a (particular) subluminal observer s. \( A \) priori,

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(21) H. K. WIMMEL: private communication.
we do not know (if, \( P, Q \) are \textit{\`{t}achyons} \) for \( s \)) what each tachyon will \textit{\`{s}ee}, since \textit{a priori} we would not know which description will be developed by any superluminal observer \( S \) (if they physically exist). In Parker's and our own philosophy, \textit{\`{t}achyon \( P, \) \textit{\`{e}g.}, will see \textit{\`{t}achyon \( \) \( Q \) at rest (i.e. as a bradyon); moreover, we \textit{\`{b}elieve} \( \) that the superluminal frames \( S \) will picture the \textit{\`{M}inkovskye space} \) essentially in the same manner as the frames \( s \). This is supported, and required, by our Sect. 2. Therefore, according to us, \textit{to every observer} \( O \) the impossibility of exchanging photons will appear for any couple of side-by-side objects \( P, Q \) that travel at a velocity \( u > c \) \textit{with respect to him himself}. Let us explicitly notice that \textit{only physical laws}—and not \textit{descriptions} of phenomena (*)—are required by the principle of relativity to be covariant (\textit{\`{a}}). Therefore, the same \( P, Q \) \textit{may appear} as exchanging photons each other to an observer having (relative) velocity \( u < c \), and unable to exchange photons each other to an observer having (relative) speed \( u > c \). At last, in our philosophy, the assertions \textit{\`{t}wo bradyons (for a certain observer!) can always exchange photons each other}, and \textit{\`{t}wo tachyons (for a certain observer!) cannot always exchange photons each other} could be \textit{physical laws}, as well as the constancy of light speed, since they appear valid to any observer (\( s \) or \( S \)). At this point, even the words \textit{\`{s}ubluminal} \( \) and \textit{\`{s}uperluminal} \( \) would not have an absolute meaning, but only a relative one.

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Unfortunately, the co-author of the previous letter, V. S. Olkhovsky, could not participate in the present work because of the enormous postal delays in communicating between Catania and Kiev.

(*) In general relativity we have an analogous situation, \textit{\`{e}g.}, for the gravitational collapse. In the proper frame of the collapsing star, the Schwarzschild radius will be reached in a finite time; while this time will appear as infinite to other observers. The more suitable framework for dealing with such our problems may be \textit{\`{n}ot} the special relativity. \textit{\`{S}ee}, \textit{\`{e}g.}, L. Fantappi\`{e}: \textit{\`{R}endic. Lineri,} 17, 158 (Roma, 1954).

(\textit{\`{s}}) \textit{\`{S}ee}, \textit{\`{e}g.}, M. Baldo, G. Fonte and E. Recami: \textit{\`{L}ett. Nuovo Cimento,} 4, 241 (1970), and references therein.