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On the Shape of Tachyons (*).

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Summary. — We study some aspects of the experimental behaviour of tachyons, in particular by finding out their « apparent » shape. A Superluminal particle, which in its own rest frame is spherical or ellipsoidal (and with an infinite lifetime), would « appear » to a laboratory frame as occupying the whole region of space bound by a double cone and a two-sheeted hyperboloid. Such a structure (the tachyon « shape ») rigidly travels with the speed of the tachyon. However, if the Superluminal particle has a finite lifetime *in its rest frame*, then in the laboratory frame it gets a *finite* space extension. As a by-product, we are able to interpret physically the imaginary units entering—as is well known—the transverse co-ordinates in the Superluminal Lorentz transformations. The various particular or limiting cases of the tachyon shape are thoroughly considered. Finally, some brief considerations concerning possible experiments to look for tachyons are added.

1. – Introduction.

Tachyons (or spacelike states) are already known to exist as *internal states*. Can they also exist as asymptotically free states? Here we shall address ourselves to this latter possibility.

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In connection with the role of tachyons as intermediate states or exchanged objects, let us recall the following. If we consider a tachyon T emitted by body A and absorbed by body B, it is well known that suitable (subluminal) observers exist seeing T to have infinite speed, *i.e.* deeming A, B to be connected by a simultaneous, symmetric interaction (1). Other observers, moreover, exist which describe that process as the exchange of an antitachyon $\overline{T}$ emitted by B and absorbed by A. Therefore, tachyons are actually quite fit to be the carriers of two-body *mutual* interactions (1,2). Let us add that in the first of ref. (1) we showed that—when describing the elementary processes happening at A and at B during the tachyon exchange in the A and B rest frame, respectively—one meets all the *four* kinematical possibilities: *a*) « intrinsic emission » at A and « intrinsic absorption » at B, *b*) « intrinsic absorption » at A and « intrinsic emission » at B, *c*) « intrinsic emission » both at A and at B, *d*) « intrinsic absorption » both at A and at B, where the last two situations are kinematically possible only in the case of tachyon exchange.

Let us also recall that it appeared convenient always to consider (in each frame) the tachyons together with their own source and detector.

For instance, from the classical point of view, typical tachyon sources are expected to be the black holes, in the sense that only tachyons can be clas-

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(1) G. D. MACCARRONE and E. RECAMI: *Nuovo Cimento A*, **57**, 85 (1980); E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209, 398 (1974), and references therein; E. RECAMI, Editor: *Tachyons, Monopoles, and Related Topics* (Amsterdam, 1978), and references therein; E. RECAMI: Chapt. 18 in *Centenario di Einstein: Astrofisica e Cosmologia, Gravitazione, Quanti e Relatività nello sviluppo del pensiero scientifico di A. Einstein*, edited by M. PANTALEO (Firenze, 1979), p. 1021; E. RECAMI: Chapt. 16 in *Albert Einstein 1879-1979: Relativity, Quanta and Cosmology in the Development of the Scientific Thought of A. Einstein*, edited by F. DE FINIS, Vol. **2** (New York, N. Y., 1979), p. 537.

(2) See, *e.g.*, P. CASTORINA and E. RECAMI: *Lett. Nuovo Cimento*, **22**, 195 (1978), and references (especially ref. (1,4,5)) therein; R. MIGNANI and E. RECAMI: *Phys. Lett. B*, **65**, 148 (1976); E. RECAMI: Chapt. 18 in *Centenario di Einstein: Astrofisica e Cosmologia, Gravitazione, Quanti e Relatività nello sviluppo del pensiero scientifico di A. Einstein*, edited by M. PANTALEO (Firenze, 1979), p. 1071 and following; E. RECAMI: Chapt. 16 in *Relativity, Quanta and Cosmology in the Development of the Scientific Thought of A. Einstein*, edited by F. DE FINIS, Vol. **2** (New York, N. Y., 1979), p. 575 and following; E. RECAMI: in *Tachyons, Monopoles, and Related Topics*, edited by E. RECAMI (Amsterdam, 1978), p. 17; P. CASTORINA and E. RECAMI: *Lett. Nuovo Cimento*, **22**, 195 (1978); M. PAVŠIČ and E. RECAMI: *Nuovo Cimento A*, **36**, 171 (1976), particularly footnotes (17,21,32); E. RECAMI, R. MIGNANI and G. ZIINO: in *Recent Developments in Relativistic Quantum Field Theory and its Application*, edited by W. KARWOWSKI, Vol. **2** (Wroclaw, 1976), p. 269; E. RECAMI and R. MIGNANI: *Phys. Lett. B*, **62**, 41 (1976), p. 43; R. MIGNANI and E. RECAMI: *Phys. Lett. B*, **65**, 149 (1976), footnotes at p. 149; *Nuovo Cimento A*, **30**, 533 (1975), sect. **4**, p. 538; M. BALDO and E. RECAMI: *Lett. Nuovo Cimento*, **2**, 643 (1969), p. 646; V. S. OLKHOVSKY and E. RECAMI: *Nuovo Cimento A*, **63**, 814 (1969), p. 122; E. RECAMI: *G. Fis.*, **10**, 195 (1969), p. 203; *Possible ... and comments on tachyons, virtual particles, resonances*, Report IFUM-088/S.M., University of Milan (August 1968), p. 4.

sically emitted by black holes. By the « reinterpretation procedure » ([3]) of *extended relativity* ([1,4]) it then follows that black holes must also be suitable tachyon absorbers. As a consequence, tachyonic matter should be possibly exchanged between black holes ([5,6])—where we mean *a priori* both gravitational black holes and « strong black holes » (= hadrons).

However, in this paper we want to deal—as already mentioned—with the problem how *free* tachyons would look like and how they are expected to behave experimentally.

That such a problem does deserve a careful investigation is suggested even by simple, *preliminary* considerations of the kind of the two following ones:

i) Free bradyons always admit a particular class of subluminal reference frames (the rest frames) wherefrom they appear—in Minkowski space—as « points » in space extended in time along a line. On the contrary, free tachyons always admit a particular class of subluminal (with respect to us) reference frames wherefrom they appear with divergent speed ($V = \infty$), *i.e.* as « points » in time extended in space along a line ([1]). Considerations of this kind correspond to the fact that the little groups of the timelike and spacelike representations of the Poincaré group are SO_3 and $SO_{2,1}$, respectively ([7]).

ii) When tachyons are seen by us by means of their electromagnetic emissions ([8]), they will generally appear as occupying *two positions* at the same

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([3]) See, *e.g.*, E. Recami: *Found. Phys.*, **8**, 329 (1978), and references therein; E. Recami and R. Mignani: *Riv. Nuovo Cimento*, **4**, 209, 398 (1974), and references therein; P. Caldirola and E. Recami: in *Italian Studies in the Philosophy of Science*, edited by M. Dalla Chiara (Boston, Mass., 1980), p. 249; E. Recami and W. A. Rodrigues: *Found. Phys.*, **12**, 709 (1982); E. Recami: in *Annuario '73, Enciclopedia EST-Mondadori* (Milano, 1973), p. 85. See also D. M. Bilaniuk, V. K. Deshpande and E. C. G. Sudarshan: *Am. J. Phys.*, **30**, 718 (1962).

([4]) G. D. Maccarrone and E. Recami: *Lett. Nuovo Cimento*, **34**, 251 (1982); E. Recami and G. D. Maccarrone: *Lett. Nuovo Cimento*, **28**, 151 (1980); P. Caldirola, G. D. Maccarrone and E. Recami: *Lett. Nuovo Cimento*, **29**, 241 (1980); G. D. Maccarrone and E. Recami: to be submitted to *Nuovo Cimento*; G. D. Maccarrone, M. Pavšič and E. Recami: *Nuovo Cimento B* (to appear).

([5]) See, *e.g.*, E. Recami: in *Tachyons, Monopoles, and Related Topics*, edited by E. Recami (Amsterdam, 1978), p. 16; V. De Sabbata, M. Pavšič and E. Recami: *Lett. Nuovo Cimento*, **19**, 441 (1977); E. Recami and K. T. Shah: *Lett. Nuovo Cimento*, **24**, 115 (1979).

([6]) This point might *a priori* be interesting even with respect to some astrophysical observations; cf. M. H. Cohen, K. I. Kellermann, D. B. Shaffer, R. P. Linfield A. T. Moffet, J. D. Romney, G. A. Seielstad, I. I. K. Pauliny-Toth, E. Preuss, A. Witzel, R. T. Schillizzi and B. J. Geldzahler: *Nature (London)*, **268**, 405 (1977).

([7]) A. O. Barut: in *Tachyons, Monopoles, and Related Topics*, edited by E. Recami (Amsterdam, 1978), p. 148.

([8]) Cf., *e.g.*, R. Mignani and E. Recami: *Nuovo Cimento A*, **30**, 533 (1975); *Phys. Lett. B*, **62**, 41 (1976); E. Recami: in *Tachyons, Monopoles, and Related Topics*, edited by E. Recami (Amsterdam, 1978), p. 3; E. Recami and R. Mignani: in *The Uncertainty Principle and Foundations of Quantum Mechanics*, edited by C. Price and S. S. Chissick (London, 1977), p. 321.

time. Let us start by considering a macro-object C (emitting spherical electromagnetic waves). When we see it travelling with Superluminal, constant velocity $\boldsymbol{V}$, because of the *distortion* due to the large relative speed $|\boldsymbol{V}| > c$, we shall observe the electromagnetic waves to be internally tangent to an enveloping (double) cone Γ having as axis the motion line of body C (this cone has nothing to do with Čerenkov's, see, *e.g.*, ref. [9]). This is analogous to what happens with an airplane moving at a constant, supersonic speed in the air. A first observation is the following one. As we hear a sonic boom when we meet the initial sound contact with the supersonic airplane, so we

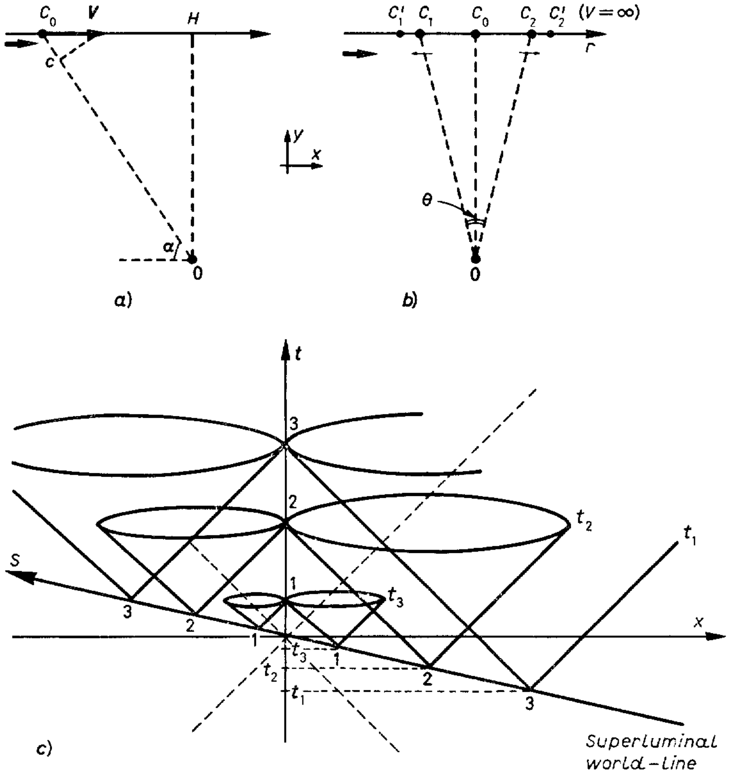

Fig. 1. – *a*) When a source C of electromagnetic radiation approaches at constant Superluminal speed V, an «optical boom» will be *seen* by any observer O as soon as he enters in radiocontact with C, *i.e.* for $V\cos\alpha = c$. The latter condition is equivalent to saying that O lies on the «retarded» half of the double cone Γ (see the text at the end of sect. 2) *enveloping* the spherical light waves emitted by C; *b*) the same case, when $V\to\infty$. From *a*), *b*) we can notice that, after the «optic boom», the tachyon source will appear to occupy simultaneously *two* positions. See the text. *c*) Representation of the same situation as in *a*), but in Minkowski space-time. Again we can notice that O will receive light simultaneously from two positions.

(9) R. MIGNANI and R. RECAMI: *Lett. Nuovo Cimento*, 7, 388 (1973).

shall analogously *see* an optic boom when we first enter in radiocontact with body C, *i.e.* when we meet the Γ-cone surface. In fact, when C is seen by us under the angle α such that (see fig. 1*a*))

$$V \cos \alpha = c\,, \tag{1}$$

all the radiations emitted by C in a certain interval around its position C_0 reach us simultaneously. Soon after the initial optic (or radio) contact with the emitting body C, we shall simultaneously receive the light emitted from suitable *couples of points*, one on the left and one on the right of C_0, respectively. We shall thus see the initial body, at C_0, to split in two luminous objects C_1, C_2 receding from each other with the Superluminal (relative) speed U:

$$U = 2b^2 \frac{1 + d/bt}{\sqrt{1 + 2d/bt}}, \qquad b \equiv \frac{V}{\sqrt{V^2 - 1}}, \qquad c = 1 \qquad (V^2 > 1)\,, \tag{1'}$$

where $d \equiv \overline{OH}$, and $t = 0$ is just the time instant when the observer enters in radiocontact with C, or rather sees C at C_0. In the simple case in which C moves with almost infinite speed along r (see fig. 1*b*)), the apparent relative speed of C_1 and C_2 varies in the initial stage as $U \simeq (2dc/t)^{\frac{1}{2}}$, where now $\overline{OH} = \overline{OC_0}$ while $t = 0$ is still the instant in which the observer sees $C_1 \equiv C_2 \equiv C_0$ [6,10]; cf. also fig. 1*c*).

2. – On tachyon shape.

Let us then investigate what shape a Superluminal particle would show to us. Let us first recall that special relativity has been generalized by extending the principle of relativity also to Superluminal reference frames [1,4]; the fundamental requirement of such an «extended relativity» is that the Superluminal Lorentz transformations (SLT) change timelike quantities into spacelike quantities, so that under any SLT the quadratic form is invariant except for its sign [1,11].

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[10] It is interesting enough that a similar behaviour, associated with solitonic solutions of nonlinear equations, was already considered from the mathematical viewpoint in A. O. BARUT: *Phys. Lett. A*, **67**, 257 (1978). See also Ø. GRØN: *Lett. Nuovo Cimento*, **23**, 97 (1978).

[11] E. RECAMI and R. MIGNANI: *Lett. Nuovo Cimento*, **4**, 144 (1972); L. PARKER: *Phys. Rev.*, **188**, 2287 (1969). See also V. S. OLKHOVSKY and E. RECAMI: *Vis. Kiiv. Univ. Ser. Fiz. (Kiev)*, **11**, 58 (1970); preprint ITF/70, Ukrainian Ac. Sc. (Kiev, 1970); E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209, 398 (1974), and references therein; H. C. CORBEN: *Nuovo Cimento A*, **29**, 415 (1975); *Int. J. Theor. Phys.*, **15**, 703 (1976).

It follows in particular that, if we consider a particle P_T which is *a tachyon* with respect to the Superluminal frames, to us it will behave as an ordinary particle (*bradyon*). Let us initially assume such a particle P to be spherical (in particular pointlike) when at rest:

$$0 \leqslant x^2 + y^2 + z^2 \leqslant r \quad \text{(at rest)} . \tag{2}$$

In the frame in which P moves with subluminal speed $v \equiv \beta c$ along x ($P \equiv P_B$), the equation of its *world-tube* becomes (with the metric $(+ - - -)$, and in natural units)

$$0 \leqslant \frac{(x - vt)^2}{1 - v^2} + y^2 + z^2 \leqslant r^2 \quad (v^2 < 1) , \tag{3}$$

which in Lorentz-invariant form reads (cf. fig. 2)

$$0 \leqslant \frac{[(x_\mu - c_\mu) u^\mu]^2}{u_\mu u^\mu} - (x_\mu - c_\mu)(x^\mu - c^\mu) \leqslant r^2 \quad (v^2 < 1) , \tag{4}$$

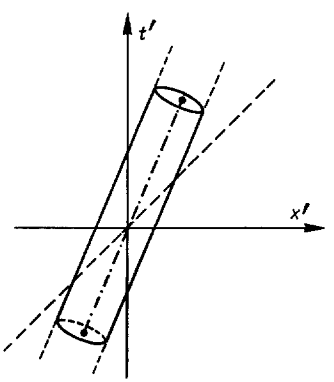

Fig. 2. – The « world-tube » of an ordinary (bradyonic) particle $P \equiv P_B$, assumed to be spherical—or ellipsoidal—in its rest frame. For simplicity, particle P is assumed to move along the x-axis and the world-line of the centre C of P to pass through the space-time origin, so that $C \equiv O$ for $t = 0$. Notice, however, that eqs. (4), (6), (7), (9) of the text have been written down for the most general case.

where $x_\mu \equiv (t, x, y, z)$, the co-ordinates c_μ refer to the *centre* C of P_B, and [1,4] the four-velocity u_μ is defined as $u_\mu \equiv dx_\mu/d\tau_0$ (see appendix A). Equation (4) reduces to eq. (3) in the special case in which the world-line of C passes through the space-time origin, and, moreover,

$$x_\mu \equiv (t, x, y, z) , \qquad c_\mu \equiv (t, vt, 0, 0) . \tag{4'}$$

In the more general case in which P_B has in its own rest frame an ellipsoidal shape with semi-axes $x_0 \equiv r$, y_0, z_0, then eq. (3) becomes

$$0 \leqslant \frac{(x-vt)^2}{x_0^2(1-v^2)} + \frac{y^2}{y_0^2} + \frac{z^2}{z_0^2} \leqslant 1 \qquad (v^2 < 1,\ x_0 \equiv r) \tag{5}$$

and, in Lorentz-invariant form, instead of eq. (4) we get

$$0 \leqslant \frac{[\Delta X_\mu u^\mu]^2}{u_\mu u^\mu} - \Delta X_\mu \Delta X^\mu \leqslant 1 \qquad (v^2 < 1) \tag{6}$$

with $\Delta X_\mu \equiv X_\mu - c_\mu$, $X_\mu \equiv (t, x/x_0, y/y_0, z/z_0)$. Equation (6) reduces to eq. (5) if

$$\Delta X_\mu \equiv X_\mu - c_\mu\,, \qquad X_\mu \equiv (t, x/x_0, y/y_0, z/z_0)\,, \qquad c_\mu \equiv (t, vt/x_0, 0, 0)\,. \tag{6'}$$

We have now to consider the same object P endowed, however, with Superluminal speed V along x, *i.e.* a tachyonic particle, $P \equiv P_T$, under the condition that it is a sphere (or an ellipsoid) when seen in its rest frame. In order to get the shape that P assumes with respect to us when it is faster than light, we have merely to apply a SLT to eq. (4), or to eq. (6). Actually, the only characteristic we need to know abort the SLT's is that they *invert* the quadratic-form sign [1,4]. Once we borrow this information from extended relativity, we are able to state that eq. (4) transforms—for a *tachyon* P_T moving with speed $V \equiv \beta c$—into

$$0 \leqslant (x_\mu - c_\mu)(x^\mu - x^\mu) - \frac{[(x_\mu - c_\mu)u^\mu]^2}{u_\mu u^\mu} \leqslant r^2 \qquad (V^2 > 1)\,, \tag{7}$$

where for a motion along x we have

$$x_\mu \equiv (t, x, y, z)\,, \qquad c_\mu \equiv (x/V, x, 0, 0)\,. \tag{7'}$$

Again, the quantities c_μ are the co-ordinates of the *centre* C of P_T. Equations (7′) still imply that the world-line of C passes through the space-time origin. Therefore, the *shape of tachyon* P_T is given by (cf. fig. 3*a*) and *b*))

$$0 \geqslant -\frac{(x-Vt)^2}{V^2-1} + y^2 + z^2 \geqslant -r^2 \qquad (V^2 > 1)\,. \tag{8}$$

One can immediately notice that (if the world-tube of P_B was supposed to be unlimited, *i.e.* if P_B was supposed to be indefinitely extended in time) the tachyon P_T appears as occupying the whole space bound by the double, unlimited cone $y^2 + z^2 = (x - Vt)^2/(V^2 - 1)$ and the (two-sheeted, rotation) hyper-

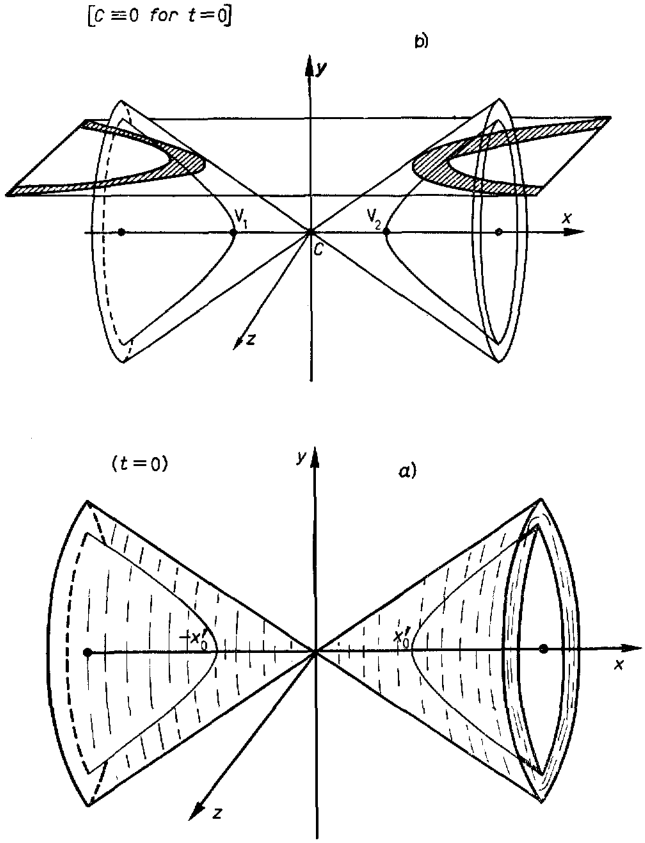

Fig. 3. – *a*) Shape of a particle P, which in its rest frame is intrinsically spherical or ellipsoidal, when seen from a Superluminal frame. Namely, when this particle ($P \equiv P_T$) moves with relative Superluminal speed V, it appears to be spread over the whole spatial region delimited by a double cone and by a two-sheeted hyperboloid asymptotic to the cone. The whole structure moves, of course, with the speed V of P_T. If P is intrinsically spherical, then the cone semi-angle is $\operatorname{tg}\alpha = (V^2-1)^{-\frac{1}{2}}$. Notice that P_T is infinitely extended in space *only* if P_B was supposed to be infinitely extended in time: see the following. *b*) The same structure already depicted, for $t=0$, in *a*). We cut it with a plane $\mathscr{Q}$ parellel to the motion line x, and in particular orthogonal to the y-axis. The intersection is a (twofold) «hyperbolic annulus», delimited by the (four) branches of two hyperbolae. Such intersections travel rigidly with the same speed and direction of P_T. The intersections with planes $\mathscr{P}$ orthogonal, on the contrary, to the motion line are shown in fig. 6.

boloid $y^2 + z^2 = (x - Vt)^2/(V^2-1) - r^2$, where the latter is asymptotic to the former. Notice that the cone semi-angle α is (fig. 3*a*))

$$\operatorname{tg}\alpha = 1/\sqrt{V^2-1}\,. \tag{8'}$$

More generally, for the ellipsoidal case, eq. (6) transforms *in the case of* *tachyons* into

$$0 \geqslant \frac{[\Delta X_\mu u^\mu]^2}{u_\mu u^\mu} - \Delta X_\mu \Delta X^\mu \geqslant -1 \qquad (V^2 > 1)\,, \tag{9}$$

where, for (Superluminal) motion along x, if $C \equiv O$ for $t = 0$,

$$\Delta X_\mu \equiv X_\mu - c_\mu\,, \qquad X_\mu \equiv (t/x_0, x, y/y_0, z/z_0)\,, \qquad c_\mu \equiv (x/Vx_0, x, 0, 0)\,. \tag{9'}$$

We conclude that the *shape of tachyon* P_T is given by

$$0 \geqslant -\frac{(x - Vt)^2}{x_0^2(V^2 - 1)} + \frac{y^2}{y_0^2} + \frac{z^2}{z_0^2} \geqslant -1 \qquad (V^2 > 1)\,. \tag{10}$$

It follows that *(if* P_B *was supposed to be indefinitely extended in time) a generic tachyon* P_T *will appear to be spread over the whole space confined between the double, unlimited cone* $\mathscr{C}$

$$\frac{y^2}{y_0^2} + \frac{z^2}{z_0^2} = \frac{(x - Vt)^2}{x_0^2(V^2 - 1)} \qquad (V^2 > 1) \tag{11a}$$

and the two-sheeted hyperboloid $\mathscr{H}$

$$\frac{y^2}{y_0^2} + \frac{z^2}{z_0^2} = \frac{(x - Vt)^2}{x_0^2(V^2 - 1)} - 1 \qquad (V^2 > 1)\,. \tag{11b}$$

The hyperboloid $\mathscr{H}$ is asymptotic to the cone $\mathscr{C}$ (cf. fig. 3*a*), *b*)). For $t = 0$, the cone vertex C coincides with the space origin O and the co-ordinates of the vertices V_1, V_2 of $\mathscr{H}$ are

$$V_{1,2} = \mp r\sqrt{V^2 - 1} \qquad (V^2 > 1)\,. \tag{12}$$

Equation (12) needs, of course, to be suitably modified if $C \neq O$ when $t = 0$; if the co-ordinate x_C of C is $x_C \equiv \varrho$ for $t = 0$, then, as time elapses,

$$V_{1,2} = \varrho + Vt \mp r\sqrt{V^2 - 1}\,. \tag{12'}$$

The whole « structure » $\mathscr{C} + \mathscr{H}$ moves rigidly, with the speed V of tachyon P_T, along x. We can also notice that the central point $(x - vt)^2/[x_0^2(1 - v^2)] + y^2/y_0^2 + z^2/z_0^2 = 0$ of the bradyonic ellipsoid (cf. eqs. (5)) goes into the cone (11*a*): *i.e.* the centre of the bradyon ellipsoid (internal boundary, in a sense) goes into $\mathscr{C}$, that *might* be regarded as the « external boundary » of the tachyon. *Vice versa,* the external ellipsoidal surface of the bradyon goes—under the considered SLT—into the hyperboloid (11*b*), that *might* be regarded as the « internal boundary » of the tachyon.

Also the equipotential surfaces associated with the electrostatic field of a charged tachyon will, of course, appear in the shape of two-sheeted hyperboloids (fig. 4).

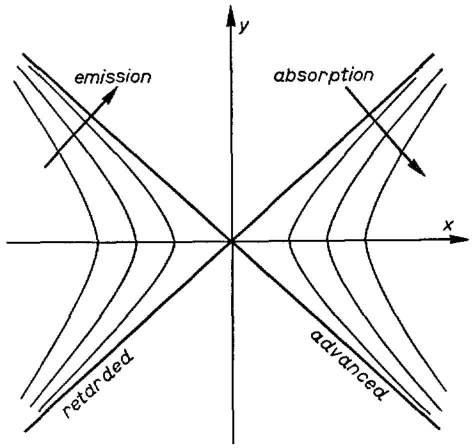

Fig. 4. – The equipotential surfaces of the electrostatic field of a charged particle P assume the form of two-sheeted hyperboloids when the particle (P_T) travels with Superluminal, constant speed V (*e.g.*, along x). Such is the result of the physical « distortion » due to the very high relative speed. The asymptotic cone has nothing to do—of course—with Čerenkov's, since *no* actual radiation energy is emitted by P_T during its inertial (Superluminal) motion. In a sense, one might say that the apparent *emission* associated to the « retarded » cone is exactly counterbalanced by the apparent *absorption* associated to the « advanced » cone. The asymptotic cone, of course, is the one associated to the shape of P_T (*i.e.* is the same as in fig. 3).

The problem is quite different, however, when P_B is emitting electromagnetic waves, *i.e.* when the emitted spherical waves propagate with the light speed c. In such a case, the waves will appear as spherical also to Superluminal observers (4), even if *enveloped* now by a (double) cone Γ in partial analogy with what is known to happen for a supersonic sound source in the air (*).

But let us go back to the above-mentioned fact that also the equipotential surfaces associated with the electrostatic field of a (charged) tachyon P_T will appear to us shaped like the (hyperboloidal) surface of particle P_T itself. In other words, the equipotential surfaces of the electrostatic field of a (charged) tachyon—moving with *constant* velocity V—will be two-sheeted hyperboloids just asymptotic to the double cone $\mathscr{C}$ (4). We thus reobtain previous results (9,12)

(*) The semi-angle α of cone Γ is still given by $\operatorname{tg}\alpha = (V^2 - 1)^{-\frac{1}{2}}$.

(12) See, *e.g.*, C. C. CHIANG: private communication; V. A. GLADKIKH: *Fizika* (*Is. Tomsk Univ.*), No. 6, p. 69 (1978); No. 6, p. 130 (1978); No. 12, p. 52 (1978); YA. P. TERLETSKY: in *Tachyons, Monopoles and Related Topics*, edited by E. RECAMI (Amster-

which showed that i) the cone $\mathscr{C}$ has nothing to do—of course—with Čerenkov's, and ii) it is associated with *no* radiation emission, as required by the static character of the initial field and by the tachyon inertial motion (since, in a sense, the apparent emission of the «retarded» cone is compensated by the apparent absorption of the «advanced» cone). See fig. 4. If P_B is pointlike and charged, then the charge of P_T *a priori* will be suitably distributed over the whole cone $\mathscr{C}$; an interesting mathematical problem would be to find out explicitly those nonradiating solutions of the Maxwell equations for tachyons (in Mignani-Recami's [8] or in Corben's [11] form) corresponding to such a charge distribution. But we are confining ourselves here to the tachyon shape problem.

3. – Tachyon localizability in time and space: various possible cases.

Let us explicitly notice that, if the «world-tube» of P_B was *not* supposed to be unlimited in time but, on the contrary, has a finite lifetime, then also the space-time extension of tachyon P_T gets constrained. For instance, let us start by considering the case in which the particle P in its rest frame i) is spherical and with its centre situated at the space origin, ii) is created at time $\bar{t}'_1$ and iii) is absorbed at time $\bar{t}'_2$. Then, when P_T is endowed with speed V along x, we shall see (instead of the whole structure in fig. 3, for $-\infty < x < +\infty$) only that part of $\mathscr{C} + \mathscr{H}$ *confined* between the 2-dimensional planes

$$(13) \qquad x = \bar{t}'_1\sqrt{1-v^2} + vt \equiv x_1(t)\,, \qquad x = \bar{t}'_2\sqrt{1-v^2} + vt \equiv x_2(t) \qquad \left(v = \frac{1}{V}\right),$$

such a couples of planes $x = x_1$, $x = x_2$ shifting, however, in space along the tachyon direction *with the «dual» (subluminal) speed* $v = 1/V$. Since the tachyon travels on the contrary with the (Superluminal) speed V, even the tachyon finite shape—*i.e.* the *portion* of $\mathscr{C} + \mathscr{H}$ confined between that couple of planes—changes in time. Chosen any fixed plane $x = \bar{x}$, the considered (*finite*) tachyon will be crossing it during the *finite* time interval

$$(14) \qquad t_1(\bar{x}) \equiv \bar{x}V - \bar{t}'_2\sqrt{V^2-1} \leqslant t \leqslant \bar{x}V - \bar{t}'_1\sqrt{V^2-1} \equiv t_2(\bar{x}) \qquad (x = \bar{x})\,,$$

dam, 1978), p. 47; H. C. CORBEN: *Lett. Nuovo Cimento*, **11**, 533 (1974); *Nuovo Cimento A*, **29**, 415 (1975). See also N. FLEURY, J. LEITE-LOPES and G. OBERLECHNER: *Acta Phys. Austriaca*, **38**, 113 (1973); J. R. GOTT III: *Nuovo Cimento B*, **22**, 49 (1974). For the different case of the spherical electromagnetic waves actually emitted by a source, cf. fig. 1 in E. RECAMI and G. D. MACCARRONE: *Lett. Nuovo Cimento*, **28**, 151 (1980).

i.e. for a time duration Δt independent of $\bar{x}$:

$$\Delta t = (\bar{t}'_2 - \bar{t}'_1)\sqrt{V^2-1} \equiv \overline{\Delta t}'\sqrt{V^2-1} \qquad (x = \text{const})\,. \tag{14 bis}$$

Cf. appendix B.

In other words, we expect the tachyon P_T to be a double unlimited structure $\mathscr{C}+\mathscr{H}$, infinitely extended in space, *only* when the corresponding bradyon P_B exists for $-\infty < t' < +\infty$, *i.e.* is infinitely extended in time. On the contrary, *if the lifetime of* P_B *is finite, the space extension of* P_T *is finite too.*

To go on, let us first restrict ourselves—for simplicity—to the case of a pointlike P_B. Then one finds that the vertex C of cone $\mathscr{C}$ will be visible (*i.e.* the actually existing portion of $\mathscr{C}$ will contain C) in the range $\bar{x}_1 \leqslant x \leqslant \bar{x}_2$, where $\bar{x}_1$, $\bar{x}_2$ are the fixed positions

$$\bar{x}_1 \equiv \bar{t}'_1 V/\sqrt{V^2-1}\,, \qquad \bar{x}_2 \equiv \bar{t}'_2 V/\sqrt{V^2-1}\,, \tag{15}$$

corresponding to the time range $\bar{t}_1 \leqslant t \leqslant \bar{t}_2$, where $\bar{t}_1$, $\bar{t}_2$ are the fixed time instants

$$\bar{t}_1 \equiv \bar{t}'_1/\sqrt{V^2-1}\,, \qquad \bar{t}_2 \equiv \bar{t}'_2/\sqrt{V^2-1} \qquad (x' = y' = z' = 0)\,. \tag{16}$$

Notice from eqs. (13) that also the *mobile* space interval

$$\Delta x \equiv x_2(t) - x_1(t) = \overline{\Delta t}'\sqrt{1-v^2} \qquad (v \equiv 1/V) \tag{13 bis}$$

is actually independent of time; it is always smaller than the fixed space interval given by eqs. (15)

$$\overline{\Delta x} \equiv \bar{x}_2 - \bar{x}_1 = \overline{\Delta t}'/\sqrt{1-v^2} \qquad (v \equiv 1/V)\,, \tag{15 bis}$$

that is to say

$$\overline{\Delta x} \geqslant \Delta x\,,$$

the equality sign holding only when the tachyon P_T has divergent speed $V = \infty$. (On the contrary, the time duration Δt in eq. (14 bis) can be smaller or larger than the time duration $\overline{\Delta t}$ given by eqs. (16)

$$\overline{\Delta t} \equiv \bar{t}_2 - \bar{t}_1 = \overline{\Delta t}'/\sqrt{V^2-1}\,, \tag{16 bis}$$

depending on whether $V \lessgtr \sqrt{2}$.) As depicted in fig. 5*a*), in the fixed space range $\bar{x}_1 \leqslant x \leqslant \bar{x}_2$ we have

i) during the time interval $\bar{t}_1$ to $\bar{t}_2$, a finite double cone (with the x-axis as symmetry axis and bound by the two planes $x = \bar{x}_1$ and $x = \bar{x}_2$) moving with speed V, so that its vertex C moves from $\bar{x}_1$ to $\bar{x}_2$;

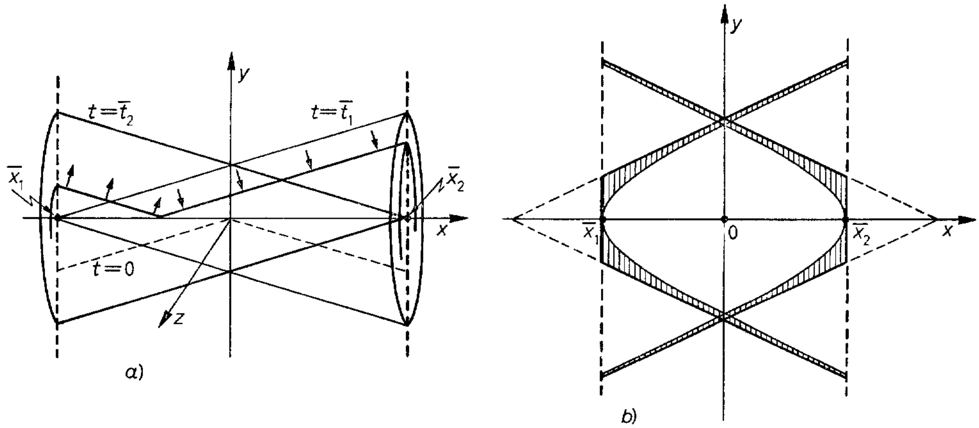

Fig. 5. – *a*) If a particle P_B has a *finite* time extension in its rest frame (*e.g.*, is created at time $\bar{t}'_1$ and absorbed at time $\bar{t}'_2$), then P_T will appear to possess a *finite* space extension. Namely, instead of the whole structure in fig. 3, we obtain that P_T will consist only in that part of $\mathscr{C}+\mathscr{H}$ *confined* between the spatial, 2-dimensional planes $x=x_1(t)$ and $x=x_2(t)$ given by eqs. (13) in the text; such a couple of limiting planes travelling rigidly in space along the tachyon direction *with the dual* (*subluminal*) *speed* $v=1/V$. This figure refers to the case of a *pointlike* P_B. It shows the (fixed) range $\bar{x}_1 \leqslant x \leqslant \bar{x}_2$ within which the cone vertex C is visible, *i.e.* in which the actually existing portion of $\mathscr{C}$ happens to contain C. Of course, we shall *not* see the whole pattern at the same time, due to limitations (13). *b*) Here the same picture is shown as in *a*), for a particle P intrinsically spherical (not pointlike). See the text.

ii) at time $\bar{t}_1$, a finite *single* cone with vertex at $\bar{x}_1$ and base on the plane $x=\bar{x}_2$;

iii) at time $\bar{t}_2$, a finite *single* cone with base on the plane $x=\bar{x}_1$ and vertex at $\bar{x}_2$,

even if we shall *not* see simultaneously the whole pattern, because of eqs. (13). (We spent some time on this description, expressed in eqs. (15)-(16 bis), also since for various reasons it *may* be convenient to assume the (finite) tachyon P_T to exist only within the global time interval $\bar{t}_1 \leqslant t \leqslant \bar{t}_2$ given by eqs. (16).)

If P_B is *not* pointlike, such a description has to be suitably modified (we still assume that particle P_B is istantaneously born at time $\bar{t}'_1$ and instantaneously disappears at time $\bar{t}'_2$ *in its rest frame*). When $r \neq 0$, the vertex C of $\mathscr{C}$ has to be substituted ([4]) by the vertices V_1, V_2 of $\mathscr{H}$; and eqs. (16) become

$$(16') \qquad \bar{t}_1 = (\bar{t}'_1 - Vr)/\sqrt{V^2-1}\,, \qquad \bar{t}_2 = (\bar{t}'_2 + Vr)/\sqrt{V^2-1}\,.$$

In correspondence with these time instants, for the vertices of $\mathscr{H}$ it is $V_1 = \bar{x}_1$

for $t = \bar{t}_1$ and $V_2 = \bar{x}_2$ for $t = \bar{t}_2$, with (*) (see fig. 5*b*))

$$(15') \qquad \bar{x}_1 = (\bar{t}_1' V - r)/\sqrt{V^2 - 1}\,, \qquad \bar{x}_2 = (\bar{t}_2' V + r)/\sqrt{V^2 - 1}\,;$$

while the cone sections with both the planes $x = \bar{x}_1$ and $x = \bar{x}_2$ have radius $r_{ext} = r$. In correspondence with the same time instants $\bar{t}_1, \bar{t}_2$, the co-ordinate of the cone vertex C is

$$(15'') \qquad C_1 = \bar{t}_1 V\,, \quad C_2 = \bar{t}_2 V\,,$$

respectively; which in particular yields that now $C_1 < \bar{x}_1$, $C_2 > \bar{x}_2$. Figure 5*b*) depicts the general pattern in the fixed space range $\bar{x}_1 \leqslant x \leqslant \bar{x}_2$ (for time ranging in the interval $\bar{t}_1 \leqslant t \leqslant \bar{t}_2$) of the «finite» tachyon P_T, when endowed with speed V along the positive x-axis and under the conditions specified in eqs. (13). (Again, it *may* be convenient for various reasons to assume the global time extension of tachyon P_T to be confined within the range $\bar{t}_1 \leqslant t \leqslant \bar{t}_2$.)

Let us now go back to the case in which (P_B having bee nassumed to be infinitely extended in time) tachyon P_T results to be infinitely extended in space (and in time). Let us also assume P to be *spherical* in its rest frame.

In any frame f' in which P is subluminal (P_B), its shape at a certain time instant $\bar{t}'$ corresponds to the intersection of the P_B world-tube with the hyperplane $t' = \bar{t}'$. According to the extended principle of relativity, a way for investigating the shape of P when Superluminally moving—*e.g.* along the positive x-axis—with respect to f' (P_T) consists in *a*) finding out the shape of the Superluminal particle P_T with respect to a Superluminal frame (*e.g.* the frame f_∞ which travels with divergent speed $V = +\infty$ along x with respect to f'); this means cutting the P world-tube with hyperplanes $x = \bar{x}$; and then *b*) transforming the result back to frame f'. This agrees with the formal considerations expounded in ref. [13] in connection with the localization of spacelike objects. There it was concluded that the space in which tachyons can *a priori* be localized is any hypersurface Σ orthogonal to a spacelike line; for example, perpendicular to the vector $(0, x, 0, 0)$: in this case we reduce ourselves to the above-mentioned hyperplanes $x = \bar{x}$. Any hypersurface Σ has, of course, two spacelike and one timelike orthogonal basis vectors.

Let us analyse what *we* shall see in any such space Σ when observing a tachyon P_T. To us a hyperplane $x = \bar{x}$, *e.g.*, is nothing but the «world-space» described as time elapses by the 2-dimensional space plane parallel to (y, z)

(*) For simplicity's sake, we disregard the double sign entering the generalized Lorentz transformations [1,4].

[13] A. O. BARUT: in *Tachyons, Monopoles, and Related Topics*, edited by E. RECAMI (Amsterdam, 1978), p. 143.

at $x = \bar{x}$, so that we have to investigate the evolution in time of the intersection of tachyon P_T with a given spatial plane $\mathscr{P}$ parallel to (y, z). Inserting $x = \bar{x}$ in eq. (8), we get

$$\frac{(\bar{x} - Vt)^2}{V^2 - 1} \geqslant y^2 + z^2 \geqslant \frac{(\bar{x} - Vt)^2}{V^2 - 1} - r^2 \qquad (x = \bar{x}\,,\ V^2 > 1)\,,$$

which means that, in the plane $\mathscr{P}$, tachyon P_T occupies the circular ring $\mathscr{R}$:

$$\begin{cases} 0 > y^2 + z^2 - \dfrac{(\bar{x} - Vt)^2}{V^2 - 1} & (17a) \\ y^2 + z^2 - \dfrac{(\bar{x} - Vt)^2}{V^2 - 1} > -r^2 & (17b) \end{cases} \qquad (V^2 > 1\,,\ \bar{x} = \text{const})\,,$$

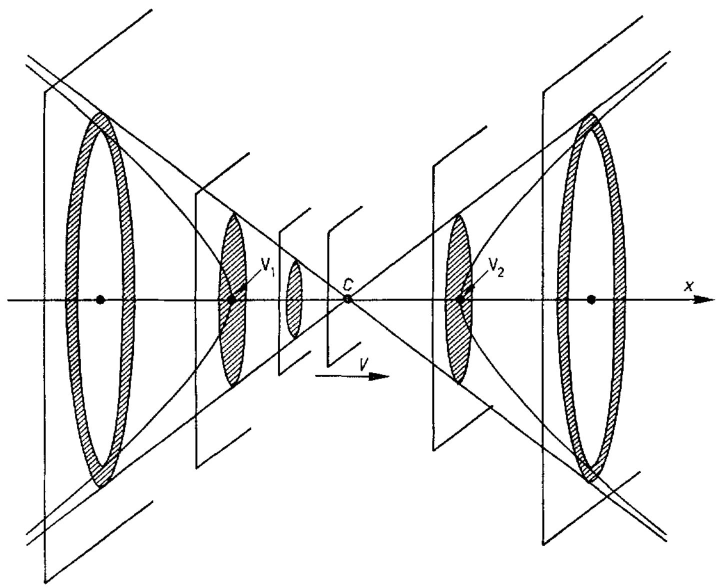

Fig. 6. – Here we show the intersections of tachyon P_T with (2-dimensional, spatial) planes $\mathscr{P}$ orthogonal to the tachyon motion line, the x-axis, in the same case considered in fig. 3. For simplicity, we assume P to be spherical in its rest frame, and $C \equiv O$ for $t = 0$. Such intersections evolve in time, so that the same pattern reproduces on a second plane—shifted by Δx—after the time $\Delta t = \Delta x/V$. On each plane, as time elapses, the intersection is a circular ring which, for negative times, goes on shrinking till it reduces to a circle and then to a point (for $t = 0$); afterwards, such a point becomes again a circle and then a circular ring that goes on broadening. If P_B is supposed to have a finite lifetime, then also the above pattern exists and evolves for a *finite* time on each plane $\mathscr{P}$. Shape and time evolution of those intersections are, of course, relevant to any possible experiments (the main problem still open being: what ordinary « material » would reveal the intersection with a tachyon?).

where $C \equiv O$ for $t = 0$, and where eqs. (17*a*), (17*b*) individuate the *external circumference* (= cone intersection), with radius

$$r_{\text{ext}} = |Vt - \bar{x}|/\sqrt{V^2 - 1}\,, \tag{18a}$$

and the *internal circumference* (= hyperboloid intersection), with radius

$$r_{\text{int}} = \sqrt{r_{\text{ext}}^2 - r^2}\,, \tag{18b}$$

respectively, that vary with time. Of course, if P_B is pointlike, then P_T corresponds only to the cone $\mathscr{C}$ and—on the plane $\mathscr{P}$—we obtain only the circumference in eq. (17*a*). On the contrary, if P_B is an ellipsoid, then the circular ring $\mathscr{R}$ becomes an « elliptical ring ».

The circular ring $\mathscr{R}$ appears, on each plane $\mathscr{P}$, first to move inwardly (until it reduces to a point) and then outwardly (cf. fig. 6), in such a way that the external radius r_{ext} varies with constant speed

$$|\dot{r}_{\text{ext}}| = \frac{V}{\sqrt{V^2 - 1}} \qquad (V^2 > 1)\,, \tag{19a}$$

while the internal radius r_{int} possesses the speed (varying with time)

$$\dot{r}_{\text{int}} = \dot{r}_{\text{ext}}\left[1 - \left(\frac{r}{r_{\text{ext}}}\right)^2\right]^{-\frac{1}{2}} \qquad (x = \bar{x} = \text{const}\,,\; V^2 > 1)\,. \tag{19b}$$

In the simple case $\bar{x} = 0$, we have, as time elapses,

$$\text{for } t = -\infty \quad \Rightarrow \begin{cases} r_{\text{ext}} = \infty\,, & r_{\text{int}} = \infty\,, \\ \dot{r}_{\text{ext}} = \dfrac{-V}{\sqrt{V^2 - 1}}\,, & \dot{r}_{\text{int}} = \dot{r}_{\text{ext}}\,; \end{cases} \tag{20a}$$

$$\text{for } t = -\frac{r\sqrt{V^2 - 1}}{V} \Rightarrow \begin{cases} r_{\text{ext}} = r\,, & r_{\text{int}} = 0\,, \\ \dot{r}_{\text{ext}} = \dfrac{-V}{\sqrt{V^2 - 1}}\,, & \dot{r}_{\text{int}} = \infty\,; \end{cases} \tag{20b}$$

$$\text{for } t = 0^{\mp} \quad \Rightarrow \begin{cases} r_{\text{ext}} = 0\,, \\ \dot{r}_{\text{ext}} = \mp \dfrac{V}{\sqrt{V^2 - 1}}\,; \end{cases} \tag{20c}$$

$$(20d)\qquad \text{for } t=+\frac{r\sqrt{V^2-1}}{V} \Rightarrow \begin{cases} r_{\text{ext}}=r\,, & r_{\text{int}}=0\,,\\ \dot r_{\text{ext}}=\dfrac{+V}{\sqrt{V^2-1}}\,, & \dot r_{\text{int}}=\infty\,; \end{cases}$$

$$(20e)\qquad \text{for } t=+\infty \Rightarrow \begin{cases} r_{\text{ext}}=\infty\,, & r_{\text{int}}=\infty\,,\\ \dot r_{\text{ext}}=\dfrac{+V}{\sqrt{V^2-1}}\,, & \dot r_{\text{int}}=\dot r_{\text{ext}}\,. \end{cases}$$

If $\bar{x}\neq 0$, everything results to be shifted by the time interval $\bar{x}/V$. Notice, moreover, that *from* the time Δt_0 during which the circular ring $\mathscr{R}$ reduces to a circle (*i.e.* during which the internal circumference is absent), one can infer the intrinsic diameter $2r$ of our tachyon P_T:

$$2r=\Delta t_0\frac{V}{\sqrt{V^2-1}}\,;$$

and that the above equations slightly simplify by the substitution $\sqrt{V^2-1}/V\equiv$ $\equiv\sqrt{1-v^2}$, with $v\equiv 1/V$.

In summary, when both $\bar{t}'_1, \bar{t}'_2\to\infty$ and $r\neq 0$, we have

$$(21)\qquad -\infty<t<+\infty\,,\qquad -\infty<x<+\infty\,,$$

$$(22)\qquad -\infty<x_C<+\infty\,,$$

where eq. (22) refers to the position of the cone vertex C.

In the limiting case in which $r\to 0$, the tachyon shape—depicted in fig. 3, 6—reduces only to the mere cone $\mathscr{C}$.

As to the tachyon speed, we have for instance ($\bar{x}=0$, and $C\equiv O$ for $t=0$)

$$(23a)\qquad \text{if } V=c\sqrt{2} \Rightarrow \begin{cases} r_{\text{ext}}=Vt\,, & r_{\text{int}}=\sqrt{V^2t^2-r^2}\,;\\ |\dot r_{\text{ext}}|=V\,, & |\dot r_{\text{int}}|=V\dfrac{r_{\text{ext}}}{r_{\text{int}}}\,,\\ \mathrm{V}_{1,2}=\mp r\,, & \alpha=45^\circ\,; \end{cases}$$

$$(23b)\qquad \text{if } V\to c^+ \Rightarrow \begin{cases} r_{\text{ext}}\to\infty\,, & r_{\text{int}}\to\infty\,,\\ |\dot r_{\text{ext}}|\to\infty\,, & |\dot r_{\text{int}}|\to\infty\,,\\ \mathrm{V}_{1,2}\to 0\,, & \alpha\to 90^\circ\,, \end{cases}$$

where also the positions V_1, V_2 of the hyperboloid vertices (cf. eqs. (12)) and the value of the cone semi-angle α (cf. eq. (8′)) are given.

On the contrary, when P_B is finite in time, that is to say $\bar{t}'_1 < t < \bar{t}'_2$ with $\bar{t}'_1$, $\bar{t}'_2$ finite, then eqs. (14), (14 bis) tell us that the above-seen pattern on each plane $\mathscr{P}$ will last a finite time $\Delta t = \overline{\Delta t}' \sqrt{V^2 - 1}$. In other words, the intersection of the (finite) tachyon P_T with any plane $x = \bar{x}$ (see fig. 6) will be existing only in the finite time interval $t_1(\bar{x}) < t < t_2(\bar{x})$: see eq. (14).

4. – The infinite-speed cases.

The case $V \to \infty$, when the cone semi-angle $\alpha \to 0$, requires a detailed analysis (since, *e.g.*, a cone with $\alpha \to 0$ whose vertex C goes to infinity will appear as a cilinder). Still we are assuming $C \equiv O$ for $t = 0$.

First, let us consider $\bar{t}'_1$, $\bar{t}'_2$ to be *finite*, as well as r.

It is essential to recall (cf. eqs. (13 bis), (15 bis)) that, when $V \to \infty$, the mobile space interval Δx does coincide with the fixed space interval $\overline{\Delta x}$, so that the *mobile* space interval (within which the finite tachyon P_T is actually confined, as time elapses) becomes *fixed*. And, instead of eqs. (13), (14), we can make recourse to eqs. (15), (16). Or, more generally, since $\Delta x = \overline{\Delta x}$ also for $r \neq 0$ when $V = \infty$, instead of eqs. (13), (14) we can make recourse to eqs. (15′), (16′).

Then, from eqs. (16′), (15′) and (15″), we get for $V \to \infty$, $\alpha \to 0$,

$$\bar{t}_1 \equiv -\frac{r}{c} \leqslant t \leqslant +\frac{r}{c} \equiv \bar{t}_2 \qquad (V \to \infty\,,\ \alpha \to 0)\,, \tag{24}$$

$$\bar{x}_1 \equiv c\bar{t}'_1 \leqslant x \leqslant c\bar{t}'_2 \equiv \bar{x}_2 \qquad (V \to \infty\,,\ \alpha \to 0)\,, \tag{25}$$

$$-\infty < x_C < +\infty\,; \qquad |x_C| = \infty \text{ for } t \neq 0 \qquad (V \to \infty\,,\ \alpha \to 0)\,, \tag{26}$$

respectively. The shape of the tachyon P_T in this situation is illustrated in fig. 7*a*). The tachyon would appear as a solid cylinder, finite in space and time,

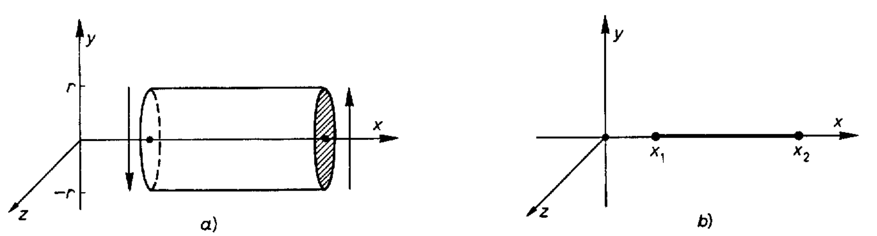

Fig. 7. – *a*) A particle P (which is spherical, with $r \neq 0$, and exists for a *finite* time in its rest frame) appears, when travelling along x with *divergent speed* $V \to \infty$, as a solid cylinder *finite both in space and time*. The cylinder radius shrinks with constant speed c, reducing from the initial value r to zero (in which case the cylinder becomes a segment), and then increases again—with the same speed—to the value r; *b*) under the previous hypotheses, in the particular case in which P_B is *pointlike* in space, tachyon P_T appears as a mere segment on the x-axis, existing only for an instant (*i.e.* as an *instantaneous segment*). See also fig. 8.

with varying radius (when $C \equiv O$ for $t = 0$):

$$|r_{\text{ext}}| \to ct \qquad (V \to \infty),$$

whose speed is

$$|\dot{r}_{\text{ext}}| \to c^+ \qquad (V \to \infty).$$

Initially, as well as at the end, the cylinder radius is just $|r_{\text{ext}}| = r$, and at $t = 0$ it is $r_{\text{ext}} = 0$. For instance, the (cylinder-generating) straight segment, which in particular is at $y = +r$ for $t = \bar{t}_1$, will be at $y = 0$ for $t = 0$ and at $y = -r$ for $t = \bar{t}_2$. In order to understand the present results, one ought intuitively to think that, since $V \to \infty$, during the infinitesimal time interval $\delta t = [-\varepsilon, \varepsilon]$, the vertex co-ordinate x_C travels from $x_C \to -\infty$ to $x_C \to +\infty$, so that for all times outside the infinitesimal interval δt the cone-vertex C lies at infinity:

$$x_C\left(-\frac{r}{c} \leqslant t < 0\right) = -\infty, \qquad x_C\left(0 < t \leqslant \frac{r}{c}\right) = +\infty.$$

Notice that r_{int} exists only at the initial and final time instants $\bar{t}_1$, $\bar{t}_2$ when $r_{\text{int}} = 0$ (for $\bar{t}_1 < t < \bar{t}_2$ it becomes imaginary). In any case, the tachyon P_T must be thought to occupy at each time $\bar{t}_1 \leqslant t \leqslant \bar{t}_2$ the whole interior of the cylinder existing at that time.

In the limiting case $r = 0$, we merely get (for $V \to \infty$, $\alpha \to 0$ and for finite $\bar{t}'_1$, $\bar{t}'_2$) the linear segment $\bar{x}_1 \mapsto \bar{x}_2$, existing only at time $t = 0$ (under our hypotheses): see fig. 7*b*). In fact, eqs. (15), (16) yield

(24′) $$\bar{t}_1 = 0, \qquad \bar{t}_2 = 0,$$

(25′) $$\bar{x}_1 = c\bar{t}'_1, \qquad \bar{x}_2 = c\bar{t}'_2,$$

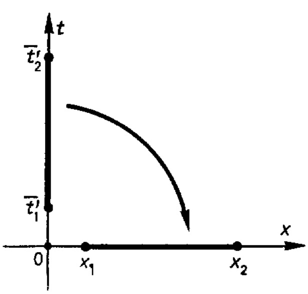

Fig. 8. – The same case as in fig. 7*b*). Namely, here it is schematically shown that, under a transcendent Lorentz boost along x [1,4], a pointlike particle P_B characterized in its rest frame by the finite lifetime $\Delta t'$ *transforms* into the «instantaneous segment» $\Delta x = c\,\Delta t'$ lying on the x-axis. In other words, a bradyon P_B at rest (living for a *finite* time, and *pointlike in space*) is transformed by a transcendent Lorentz boost into an infinite-speed tachyon P_T (extended over a *finite* segment, and *pointlike in time*).

as can be verified also by starting from a pointlike object, existing (at $x' = y' = z' = 0$) in the time interval $\bar{t}'_1$ to $\bar{t}'_2$, and then applying a « transcendent Lorentz transformation » [1,4]. Cf. also fig. 8. This means that, under our hypotheses, the infinite-speed tachyon appears to us as an *instantaneous* line segment confined between $\bar{x}_1$ and $\bar{x}_2$ (in agreement with the heuristical considerations at point i) in sect. **1**).

Before going on, let us recall that in relativity the pointlike case appears to be meaningful only as a limiting case [14]; such a consideration does actually inspire our present analysis.

As second case, we have to consider not only $V \to \infty$, but also both $\bar{t}_1, \bar{t}_2 \to \infty$ (with $r \neq 0$). Let us for simplicity assume $\bar{t}'_2 = -\bar{t}'_1 \equiv \bar{t}'$; then, when $\bar{t}' \to \infty$, let us call

$$\xi \equiv \frac{1}{c} \lim_{V, \bar{t}' \to \infty} \frac{V}{\bar{t}'}.$$

We get

$$-\frac{1 + \xi r/c}{\xi} \leqslant t \leqslant + \frac{1 + \xi r/c}{\xi}, \tag{27}$$

$$-\infty < x < + \infty, \tag{28}$$

$$-\infty < x_c < + \infty, \tag{29}$$

where actually eqs. (28), (29) do *not* depend on V. With regard to eq. (27), we have to distinguish the following subcases:

$$\text{if } \xi = 0 \quad \Rightarrow -\infty < t < \infty; \tag{27a}$$

$$\text{if } \xi = \text{finite} \Rightarrow -\bar{\Delta} \leqslant t \leqslant \bar{\Delta} \qquad \left(\bar{\Delta} > \frac{r}{c}\right); \tag{27b}$$

$$\text{if } \xi = \pm \infty \Rightarrow -\frac{r}{c} \leqslant t \leqslant + \frac{r}{c}. \tag{27c}$$

In the limiting case for $r \to 0$, one has that (when V, $t' \to \infty$, $r \to 0$):

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[14] See, *e.g.*, J. M. JAUCH: *Foundations of Quantum Mechanics* (London, 1968); A. J. KÁLNAY: in *Tachyons, Monopoles, and Related Topics*, edited by E. RECAMI (Amsterdam, 1978), p. 53, and references therein; *Phys. Rev. D*, **7**, 1707 (1973); J. C. GALLARDO, A. J. KÁLNAY, B. A. STEC and B. P. TOLEDO: *Nuovo Cimento A*, **48**, 1008 (1967); A. J. KÁLNAY and B. P. TOLEDO: *Nuovo Cimento A*, **48**, 997 (1967); V. S. OLKHOVSKY and E. RECAMI: *Lett. Nuovo Cimento*, **4**, 1165 (1970); E. RECAMI: in *Quarks and the Nucleus* (*Progress in Particle and Nuclear Physics*, Vol. **8**), edited by D. WILKINSON (Oxford, 1982), p. 401; P. CALDIROLA, M. PAVŠIČ and E. RECAMI: *Nuovo Cimento B*, **48**, 205 (1978); *Phys. Lett. A*, **66**, 9 (1978); *Lett. Nuovo Cimento*, **24**, 565 (1979); P. CALDIROLA: *Riv. Nuovo Cimento*, **2**, No. 13 (1979).

a) Equations (27*a*), (28), (29), for $\xi = 0$, yield that the tachyon P_T appears as a nonstatic cylinder infinitely extended both in space and time with

$$r_{ext} \equiv r_{int} = ct\,, \qquad |\dot{r}_{ext}| \equiv |\dot{r}_{int}| = c^+\,. \tag{30}$$

For instance, $r_{ext} \equiv r_{int} \to -\infty$ for $t \to -\infty$, $r_{ext} \equiv r_{int} = 0$ for $t = 0$, $r_{ext} \equiv r_{int} \to +\infty$ for $t \to +\infty$. See fig. 9*a*).

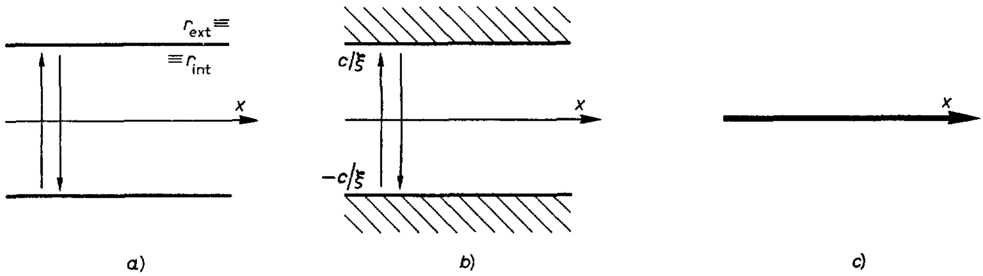

Fig. 9. – When the lifetime of P_B diverges ($\Delta t' \to \infty$) together with the speed of tachyon P_T along x ($V \to \infty$), the tachyon shape depends on the value $c\xi$ of the limit of $V/\Delta t'$ for V, $\Delta t' \to \infty$. For instance, in the case in which furtherly $r \to 0$, we observe (if $C \equiv O$ when $t = 0$), *a*) for $\xi = 0$, a nonstatic cylinder having the x-axis as symmetry axis and infinitely extended both in space and time. The cylinder radius varies for negative times from $-\infty$ to zero (at $t = 0$), and from zero to $+\infty$ for positive times; *b*) for $\xi \neq 0$ and finite, a nonstatic cylinder similar to the previous one (*i.e.* infinitely extended in space), but existing only for a finite time. The cylinder radius contracts (with constant speed c) from the finite value c/ξ to zero (at $t = 0$) and afterwards increases from zero again to c/ξ; *c*) for $\xi = \pm\infty$, a mere *straight line*, infinitely extended along the x-axis but existing only for a time instant.

b) Equations (27*b*), (28), (29), for ξ *finite*, yield that tachyon P_T appears as a cylinder infinitely extended in space but existing only for a *finite time*

$$-\frac{1}{\xi} \leqslant t \leqslant \frac{1}{\xi}\,; \tag{31}$$

for instance, if $\xi = 1\ \mathrm{s}^{-1}$, one gets (in seconds)

$$-1 \leqslant t \leqslant 1\,.$$

Also this cylinder is nonstatic, in the sense that

$$r_{ext} \equiv r_{int} = ct \quad (\text{with } -1 \leqslant t \leqslant 1)\,, \qquad |\dot{r}_{ext}| \equiv |\dot{r}_{int}| = c\,,$$

so that (see fig. 9*b*))

$$-c \leqslant r_{int} \equiv r_{ext} \leqslant +c\,. \tag{32}$$

c) Equations (27*c*), (28), (29), for $\xi = \pm \infty$, yield that tachyon P_T appears as a straight line, infinitely extended along the x-axis, but existing only for a time instant (*i.e.* as an object *pointlike in time*)

$$0 \leqslant t \leqslant 0 \,. \tag{33}$$

Moreover,

$$r_{ext} \equiv r_{int} = 0 \,. \tag{34}$$

See fig. 9*c*).

In connection with fig. 7, 9, let us add that charged tachyons appear to attract each other when they possess equal charges; and, *vice versa*, they appear to repell each other when endowed with opposite charges; this is *actually* reminiscent of the behaviour of ordinary electric-current wires. (Let us recall, in fact, that approach motion transforms into departure motion—and *vice versa* —under a Superluminal Lorentz transformation, as can be got from direct inspection in Minkowski space-time.)

5. – Further remarks.

Let us consider again the generic case of a tachyon P_T moving with Superluminal speed V along x, assuming P_B to be infinitely extended in time (sect. **2**). However, instead of cutting the (moving) structure $\mathscr{C} + \mathscr{H}$, depicted for $t = 0$ in fig. 3*a*), with planes $\mathscr{P}$ orthogonal to the motion line, let us now consider its intersections with space planes $\mathscr{Q}$ *parallel* to the motion line (*e.g.*, orthogonal to y, or to z). Then instead of the pattern in fig. 6 we get the pattern already shown in fig. 3*b*). Namely, the general shape of the intersection $\mathscr{I}$ of tachyon P_T with a plane $\mathscr{Q}$ parallel to the tachyon motion direction is a (twofold) « hyperbolic annulus »; in the sense that the intersection $\mathscr{I}$ results to be the portion of $\mathscr{Q}$ delimited by the branches of two hyperbolas (such a portion consisting, of course, of two disconnected parts $\mathscr{I}_1$ and $\mathscr{I}_2$). On any such plane $\mathscr{Q}$, as the tachyon moves, the intersections $\mathscr{I}_1$ and $\mathscr{I}_2$ move rigidly with the same speed and direction of P_T. See fig. 3*b*). In the limiting case ($r \to 0$) of a pointlike P_B, when the P_T shape degenerates into the cone $\mathscr{C}$, the intersections of P_T with planes orthogonal to y or to z are shown in fig. 3*a*), *b*), respectively, of ref. ([15]). In relation to the present remark, and in particular to those fig. 3 ([15]), we like to recall here the following.

As a by-product of all what derived above on the shape of tachyons, we are now able to geometrico-physically interpret—at least in some relevant cases—the role and meaning of the imaginary units entering, as is well known,

([15]) E. RECAMI and G. D. MACCARRONE: *Lett. Nuovo Cimento*, **28**, 151 (1980).

the Superluminal Lorentz transformations for the *transverse* space co-ordinates. Such a point has been exploited in ref. ([4]).

This paper constitutes, moreover, a first step in the direction of suggesting —eventually—« sensible » experiments with the object of detecting tachyons. We consider any such experiment to be sensible only when based on a definite, developed *theoretical framework* for tachyons (even if not yet complete in its background and/or in its applications). A preliminary problem still to be carefully investigated is finding out how a (charged) tachyon interacts electromagnetically with an ordinary-matter electron, by using—again—Maxwell equations for tachyons either in Mignani-Recami's ([8]) or in Corben's ([11]) form.

Here let us merely stress once more how unconventional the behaviour of tachyons can be—even if tachyons and bradyons are particles « relativistically dual »—because of the physical « distortion » due to the very high relative speed. This is just shown by the present paper. To add something more, we should like to submit the following consideration, taken out from ref. ([16]). It is known that, in a gravitational field, tachyons are subjected to a repulsive force, since ([1])

$$F^\mu = \pm\, m_0 \Gamma^\mu_{\varrho\sigma} \frac{\mathrm{d}x^\varrho}{\mathrm{d}s}\frac{\mathrm{d}x^\sigma}{\mathrm{d}s} \qquad (V^2 \gtrless 1)\,,$$

such that they absorb (emit) gravitons, while in the same situations bradyons would emit (absorb) gravitons. Let us assume that an analogous behaviour holds also for the electromagnetic field, *i.e.* when a charged tachyon interacts with ordinary matter. In the case of interaction between a charged ordinary particle and matter, energy is released by the particle to the medium. For instance, in a bubble chamber one encounters along the track of high-energy particles a series of *overheated* spots. Contrariwise, along the track of a tachyon, we may expect to encounter a series of *underheated* spots, with all its consequences ...

We shall deal with these questions in another paper.

* * *

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([16]) E. Recami, K. T. Shah and W. Yourgrau: *On a possible experiment concerning tachyons,* unpublished preprint (August 1978).

APPENDIX A

Let us first remind that in extended relativity [1,3,4] any Superluminal Lorentz transformation (SLT) does invert the sign of the quadratic forms, so that four-vector products like $x_\mu x^\mu$ or $p_\mu p^\mu$ behave as « pseudoscalars » under the SLTs, and in particular under the discrete « Superluminal operation » $\mathscr{S}: x_\mu \to i x_\mu$ (where $\mathscr{S}$ represents the « transcendent » SLT: see, *e.g.*, ref. [4]). The SLTs, together with the subluminal Lorentz transformations, have been shown [4] to form a new group G.

Moreover, let us recall that quantities so as $\bar{u}_\mu \equiv \mathrm{d}x_\mu/\mathrm{d}s$ are Lorentz four-vectors, but are *not* G-four-vectors, due to the fact that $\mathrm{d}s \to \pm i\mathrm{d}s$ under any SLT. So that, in order to get a G-four-vector, we defined [1] the four-velocity as follows:

$$u_\mu \equiv \mathrm{d}x_\mu/\mathrm{d}\tau_0\,, \tag{A.1}$$

$\mathrm{d}\tau_0$ being the (G-invariant) proper-time element. From eq. (A.1) we got [1,4], *e.g.*, that $p_\mu = m_0 u_\mu$ also for tachyons. In the bradyonic case $u_\mu \equiv \bar{u}_\mu$; but in the tachyonic case it is $u_\mu = \pm i\bar{u}_\mu$. For further details, cf. ref. [4].

APPENDIX B

Let us consider the subluminal particle $\mathrm{P_B}$, intrinsically spherical and subjected to the conditions set at the beginning of sect. **3**. That is to say, $\mathrm{P_B}$ is suddenly created at time $\bar{t}'_1$ in its rest frame, and is suddenly absorbed at time $\bar{t}'_2$ in its rest frame. In Minkowski space-time, therefore, the world-tube of $\mathrm{P_B}$ is to be confined between two suitable hyperplanes. The generic equation of any such hyperplane (cfr. sect. **2**) is $(x_\mu - c_\mu)u^\mu = \text{const}$. In the particular case in which the world-line of C passes through the space-time origin, one may simply write $x_\mu u^\mu = \text{const}$. In any case, since the four-vector products are scalars under subluminal Lorentz transformations, the equation of the $\mathrm{P_B}$ world-tube, *i.e.* $0 \leqslant (x_\mu u^\mu)^2/u_\mu u^\mu - x_\mu x^\mu \leqslant r^2$, must be associated with the further constraint $\bar{t}'_1 \leqslant x_\mu u^\mu \leqslant \bar{t}'_2$. Passing to the Superluminal Lorentz transformation case, the four-vector products are still invariant (except for the sign, since they behave as pseudoscalars under SLTs [1,4]); the essential point is that in the tachyon case u_μ is spacelike and no longer timelike, so that the limiting hypersurfaces are no longer spacelike, but are referred to 2 spatial and 1 temporal basis vectors. In the tachyonic case, therefore, one has to associate, with the world-tube *transformed* equation, the additional constraint

$$-\bar{t}'_2\sqrt{V^2-1} + xV \leqslant t \leqslant -\bar{t}'_1\sqrt{V^2-1} + xV\,,$$

which yields as well

$$\bar{t}'_1\sqrt{V^2-1}/V + t/V \leqslant x \leqslant \bar{t}'_2\sqrt{V^2-1}/V + t/V\,.$$

Notice that, in the case in which $y = z = 0$, the last relation becomes

$$\bar{t}'_1 V/\sqrt{V^2-1} \leqslant x \leqslant \bar{t}'_2 V/\sqrt{V^2-1}\,.$$

● RIASSUNTO

Si studiano alcuni aspetti del comportamento dei tachioni, in particolare trovando quale « apparirebbe » la loro forma. Una particella Superluminale, che sia sferica o ellissoidale (e con vita di durata finita) nel proprio riferimento a riposo, a un osservatore nel laboratorio sembrerebbe occupare l'intera regione di spazio limitata da un doppio cono e da un iperboloide a due falde. Tale struttura (la « forma » del tachione) viaggerà rigidamente con la velocità del tachione. Si noti però che, se la particella Superluminale ha una vita finita (*nel suo riferimento a riposo*), allora nel laboratorio essa risulta avere un'estensione spaziale *finita*. Come conseguenza della precedente analisi, siamo in grado d'interpretare fisicamente le unità immaginarie che entrano — come noto — nelle coordinate trasversali per azione delle trasformazioni di Lorentz Superluminali. Si esaminano dettagliatamente i vari casi particolari o casi limite della forma dei tachioni. Infine, si aggiungono alcune considerazioni circa eventuali esperimenti atti alla ricerca effettiva dei tachioni.

О форме тахионов.

Резюме (*). — Мы исследуем некоторые аспекты экспериментального поведения тахионов, в частности, посредством нахождения их кажущейся формы. Суперлюминальная частица, которая в своей собственной системе соординат является сферической или эллипсоидальной (и с бесконечным временем жизни), в лабораторной системе координат представляется занимающей всю область пространства, ограниченную двойным конусом и гиперболоидом с двумя слоями. Такая структура (« форма » тахиона) движется со скоростью тахиона. Однако, ески суперлюминальная частица имеет конечное время жизни в своей собственной системе координат, то в лабораторной системе координат эта частица занимает конечное пространство. Как вспомогательный результат, мы можем физически интерпретировать мнимые единицы, входящие, как известно, в попереченые координаты в суперлюминальных преобразованиях Лоренца. Подробно исследуются различные частные и предельные случаи формы тахиона. В заключение, проводится обсуждение возможных экспериментов по наблюдению тахионов.

(*) *Переведено редакцией.*