CPT-Covariance: Physical Meaning of a New Derivation.

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Usual, proper, orthochronous Lorentz transformations (LT) (*) are known to be interpretable as « rotations » in Minkovski space. If we, for simplicity, confine ourselves only to boosts, then they are « rotations » of an angle φ running (1) continuously from $-45°$ to $+45°$.

Moreover, the proper, *non*orthochronous, collinear boosts may be considered as « rotations » with (1) $+135° < \varphi < +225°$. The former LT's and the latter ones constitute, respectively, two disconnected sets.

In particular, the $\hat{P}\hat{T}$ operation can be reached, when one starts from the identity, only by passing through operations usually considered as nonphysical (3-5).

On the contrary, we can give a physical meaning to those transformations, by considering the theory of special relativity extended also to Superluminal (=faster-than-light) inertial frames and to tachyons (2). Within such a framework—that allowed us to build up a « classical theory » of tachyons (2) without any paradox—the transformations corresponding to $+45° < \varphi < +135°$ (and to $225° < \varphi < 315°$) are just « Superluminal Lorentz transformations » (SLT), *i.e.* transformations between (real) inertial frames with faster-than-light relative velocity (2). From an axiomatic viewpoint, the *extended relativity* follows (2) from the usual postulates, provided that the assumption $|v| < c$ is eliminated. All what precedes is true also in four dimensions (2). All together, the LT's and the SLT's form a new (2) group G of « generalized Lorentz transformations » (GLT).

(*) We are neglecting space-time translations. We understand space-time transformations are represented by 4×4 matrices.
(1) See, *e.g.*, Fig. 9 and 11 of ref. (2). See also ref. (3).
(2) E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209 (1974), and references therein.
(3) V. BERESTETSKY, E. M. LIFSHITZ and P. PITAEVSKY: *Relativistic Quantum Theory* (London, 1971), pag. 34.
(4) J. J. SAKURAY: *Invariance Principles and Elementary Particles* (Princeton, N. J., 1964), pag. 137.
(5) R. F. STREATER and A. S. WIGHTMAN: *PCT, Spin and Statistics, and All That* (New York, N. Y., 1964).

Therefore, starting from the identity operation, we can reach the operation that we shall call (for now) $\hat{P}\hat{T}$, for example by means of a chain of *physical* GLT's expressed in terms of a continuous parameter φ (running, *e.g.*, from 0° to 180° with continuity) (*). This procedure results to be quite interesting, since « extended relativity » reveals itself useful for better understanding *usual* physics (besides tachyons). For example we are going to show the $\hat{P}\hat{T}$ Lorentz transformation results actually to be the $\hat{C}\hat{P}\hat{T}$-operation, so that $\hat{C}\hat{P}\hat{T}$ for us is nothing but a particular *physical* (generalized) Lorentz transformation. As well, we shall clarify the physical meaning of charge conjugation $\hat{C}$ in terms only of pure operations on four-momentum space, the connection particle-antiparticle and so on. Moreover, we derive elsewhere ([2,6]) the « crossing relations » from extended relativity.

In order to make usual special relativity fully consistent, *i.e.* to avoid information transmission into the past, it is necessary to introduce (even in standard relativity) a « *third postulate* », besides tye two usual ones ([2]). That is to say, that « physical signals are transported only by positive-energy objects, *i.e.* by objects that appear with positive energy and travelling forward in time » ([2]).

In fact, if we have in the chronotopical space a signal going backwards in time (that *a priori* is allowed in the usual Minkovski space!), then in the four-momentum space there correspond a signal bearing negative energy ([2]). It is well known that those two paradoxical occurrences, when contemporaneous (as they are), admit a quite orthodox reinterpretation in terms of the *antiparticle* ([2]) with positive energy going forward in time ([7]). This is nothing but the Dirac-Stückelberg-Feynman-Sudarshan

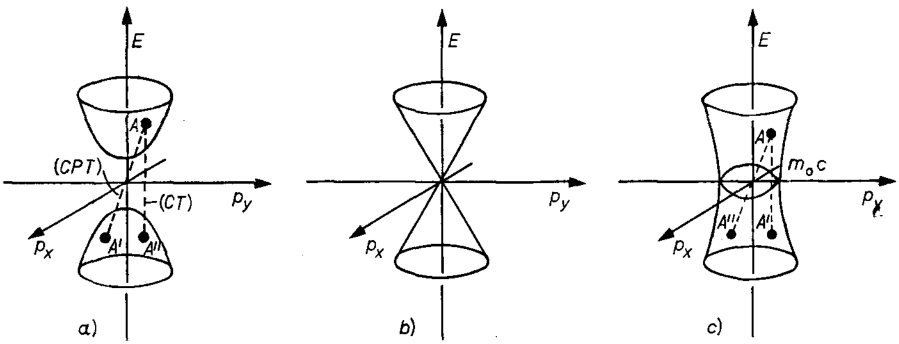

Fig. 1. – Representation of the hypersurfaces $E^2 - \boldsymbol{p}^2 = p^2$ for *a*) bradyons with $p^2 \equiv m_0^2 > 0$ (*timelike* case); *b*) luxons with $p^2 \equiv m_0^2 = 0$ (lightlike case); *c*) tachyons with $p^2 \equiv -m_0^2 < 0$ (*spacelike* case), where m_0 is *always real*. In Fig. 1*a*) the points A' and A'' represent the particle kinematical states obtained by applying the operations $\hat{C}\hat{P}\hat{T}$ and $\hat{C}\hat{T}$, respectively, to the kinematical state A. *In the case when we confine ourselves* to subluminal frames and to usual LT's, then it happens that the « matter » or « antimatter » character is invariant for B's, but relative to the observer for T's. When eliminating previous restriction we may pass from particles to their antiparticles (through GLT's) even in the case of bradyons.

(*) Owing to this fact, we shall say that our generalized (collinear) boosts have been parametrized in a continuous fashion. One has to bear in mind, however, that—in the neighbourhood of the light-cone—it happens that the relative speed of two inertial frames $s(c-\varepsilon_1)$, $S(c+\varepsilon_2)$ cannot go to zero, but runs from c to ∞.

(6) R. MIGNANI and E. RECAMI: preprint PP/392 (Catania, 1973), to be published in *Int. Journ. Theor. Phys.*

(7) See, *e.g.*, ref. (2); O. M. P. BILANIUK, V. K. DESHPANDE and E. C. G. SUDARSHAN: *Am. Journ. Phys.*, **30**, 718 (1962); E. C. G. SUDARSHAN: *Proc. Ind. Acad. Sci.*, **69** (3 A), 133 (1968); in *1968 Proceedings of the VIII Nobel Symposium*, edited by N. SWARTHOLM (New York, N. Y., and Stockolm), pag. 335; in *Symposia in Theoretical Physics and Mathematics*, Vol. **10** (New York, N. Y., 1970), pag. 129.

« reinterpretation principle » ([7,8]). Our « third postulate » (RIP) states that not only we *can* use the « reinterpretation principle », but that we *must* apply it.

The RIP may be immediately checked through its implications. For instance, it easily results to allow interpreting the lower-hyperboloid points in Fig. 1*a*) as the kinematical states of the (free) *antiparticle* (see ref. ([2])) of the free particle whose kine-

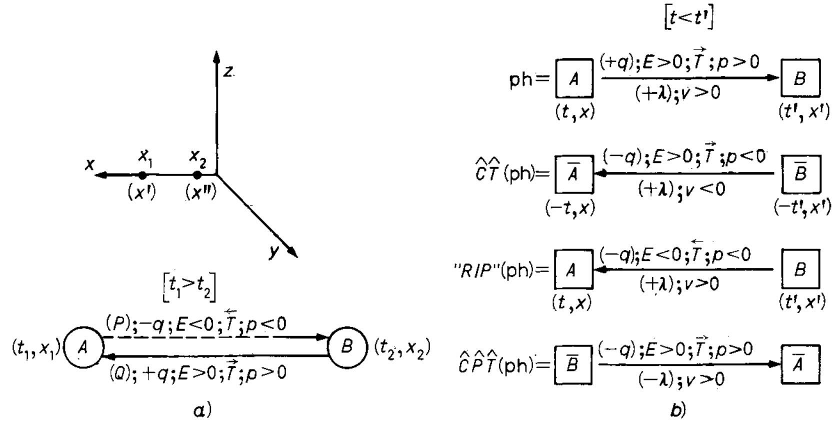

Fig. 2. – *a*) Represents the exchange from A to B of a particle P with *negative* energy (and « charge ») and travelling backwards in time ($t_2 < t_1$). Such a process will appear nothing but the exchange *from B to A* of a (standard) particle Q with *positive* energy (and « charge ») travelling forward in time. Particle Q may be shown to be (maybe except for the helicity) the *antiparticle* of the initial particle: $Q = \overline{P}$. See ref. ([2]). *b*) Shows a certain phenomenon ph, *i.e.* the exchange from emitter A to absorber B of a certain particle, and the transformations on ph (and on A, B) operated *respectively by* $\hat{C}\hat{T}$, *by* the « reinterpretation procedure » (RIP, see the text) used in case *a*), *and by* $\hat{C}\hat{P}\hat{T}$.

matical states are represented by the upper-hyperboloid points. *In fact,* since 1905, from the double sign $E = \pm\sqrt{\boldsymbol{p}^2 + m_0^2}$ existence of an « antiparticle » ([2]) for any m_0-mass particle could have been predicted, provided the RIP had been used. And since antiparticles have been actually discovered (*exactly* with the properties predictable ([2,6]) on the basis of RIP!) the check results to be positive.

Before going on let us explicitly emphasize that from inspection of Fig. 2 it easily ([2]) follows that

$$\text{RIP} \equiv \hat{C}\hat{E}\hat{p}\,, \tag{1}$$

where by $\hat{E}$ and $\hat{p}$ we mean the operations of energy and three-momentum reversal, respectively ([6]).

Let us now pass to consider our (*physical*) GLT's both subluminal and Superluminal (*i.e.*—let us repeat—transformations between actual reference frames with relative speeds $|u| < c$ or $|U| > c$). A quite obvious—but extremely important—remark is in order: *If a* GLT *happens to change the sign of a component of a four-vector* (*e.g.*, four-position), *then it will of course change the sign of the same component of any other four-vector* (*e.g.*, of four-momentum) ([9]). In particular, if we get (among the others) (*) the so-

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([8]) R. P. FEYNMAN: *Phys. Rev.*, **76**, 749, 769 (1949); E. C. G. STÜCKELBERG: *Helv. Phys. Acta*, **14**, 32 L, 588 (1941); P. A. M. DIRAC: *Proc. Roy. Soc.*, A **126**, 160 (1930).

([9]) Compare, *e.g.*, Table II in ref. ([2,6]).

(*) In our framework we *cannot* get only $\hat{T}$ or only $\hat{P}$ by means of a physical GLT, since our GLT's are (all the) « rotations » in Minkowski space, so that always $\det \text{GLT} = +1$.

called time reversal through a (physical) GLT, really we get

$$\hat{T} \to \hat{T}\hat{E}\,.$$

Analogously

$$\hat{P} \to \hat{P}\hat{p}\,.$$

Let us now consider the actual, « physical » GLT $= -\mathbf{1}$ (corresponding to $\beta=0$ but $\varphi = 180°$) [2,10] that operates the *total inversion* (or « strong reflection ») $\hat{P}\hat{T}$; because of what we said, effectively it is

$$\hat{P}\hat{T} \to \hat{P}\hat{p}\hat{T}\hat{E}\,. \tag{2}$$

It is enough now to use the RIP, eq. (1), in order to get

$$\hat{P}\hat{T} \to (\hat{C}\hat{E}\hat{p})\,\hat{P}\hat{p}\hat{T}\hat{E} \equiv \hat{C}\hat{P}\hat{T}; \tag{2'}$$

in other words, the operation

$$-\mathbf{1} = \hat{C}\hat{T}\hat{P} \tag{3}$$

is nothing but one of our physical GLT's.

Therefore, *the mere relativistic covariance implies, as a particular case, the $\hat{C}\hat{P}\hat{T}$-covariance*, and the $\hat{C}\hat{P}\hat{T}$ theorem is a mere product of (extended) relativity that may be derived within the framework of *actual* Lorentz transformations. *That is to say, $\hat{C}\hat{P}\hat{T}$-covariance is a particular case of G-covariance* (*). Notice that in our framework $\hat{C}\hat{P}\hat{T}$ is a *linear* operator [11] as well as *all* GLT's.

We can therefore assert, in extended relativity, that [6] *the true way for doing $\hat{P}\hat{T}$ is doing $\hat{C}\hat{P}\hat{T}$.*

By the way, in the cases when $\hat{T}$-covariance holds by itself, then we get as a corollary that [2,6] *the right way for doing $\hat{P}$ is doing $\hat{C}\hat{P}$*, which is the essential teaching of LEE and YANG [12]. In fact, in the case considered, relativity says that we can « safely » (*i.e.* covariantly) reflect space only if we contemporarily apply $\hat{C}$, so to have particles changed into antiparticles (**).

From what precedes it is evident the *need for new symbols* indicating the sign inversion produced by a GLT in *all* the tetravectors' fourth components, *e.g.*

$$\bar{T} \equiv \hat{T}\hat{E}\,, \tag{4a}$$

and three first components, *e.g.*

$$\bar{P} \equiv \hat{P}\hat{p}\,, \tag{4b}$$

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(10) By the way, this is a subluminal (nonorthochronous!) LT, that we can effectively get through two (physical) SLT's. Compare ref. [1,2].

(*) *I.e.*, of covariance under the new group *G*.

(11) See also E. C. G. SUDARSHAN: *1968 Proceedings of the VIII Nobel Symposium*, edited by N. SWARTHOLM (New York, N. Y., and Stockholm), pag. 335; in *Symposia in Theoretical Physics and Mathematics*, Vol. **10** (New York, N. Y., 1970), p. 129.

(12) T. D. LEE and C. N. YANG: *Phys. Rev.*, **105**, 1671 (1957).

(**) The true operation transforming particles into antiparticles is just the operation [3,4] $\hat{C}\hat{P}\hat{T} = -\mathbf{1}$.

and so on. Besides, we have already understood that

$$\bar{C} \equiv \hat{C} \tag{4c}$$

means the sign inversion of all additive « charges ». It then holds (by using RIP) that

$$\bar{P}\bar{T} = \hat{C}\hat{P}\hat{T}\,. \tag{5}$$

Another interesting point that is important for the physical understanding of « charge conjugation » is the following. From the fact that, in equations like (2′), by applying RIP we get

$$\hat{P}\hat{p}\hat{T}\hat{E} \overset{\text{(RIP)}}{=} \hat{C}\hat{P}\hat{T}$$

we may deduce that (when assuming RIP) one may *always* write

$$\hat{C} \overset{\text{(RIP)}}{=} \hat{p}\hat{E}$$

since $\hat{v}\hat{E} = \hat{p}\hat{E}$, where $\hat{v}$ is the velocity reversal operation (*).

As regards the connection ([2]) between particles and antiparticles—connection that we slightly mentioned before—we would here like to add only what follows ([13]). Let us, for instance, consider the case of an interaction among tachyons, as it appears to inertial frames connected (for simplicity) by x-axis boosts. By means of a suitable transformation ([1,9]) $-\Lambda_s \equiv \text{GLT}(135° < \varphi < 225°)$ of its kinematical effects and of the RIP, we easily get ([2,6]) that the second frame observes the same effects as produced (in the first frame) by the $\hat{C}\hat{P}\hat{T}$-operation. In the particular case when $\text{GLT}(\varphi = 180°) \equiv -\mathbf{1}$, *i.e.* total inversion, one obviously gets the *true* $\hat{C}\hat{P}\hat{T}$, that is to say transition from the tachyonic phenomenon to the $\hat{C}\hat{P}\hat{T}$-ed one (both seen, now, in the same frame, *i.e.* without the effects of any supplementary LT-boost). This result is influenced by the intervention of the RIP, that must enter into action when a tachyon by-pass the *divergent* velocity. In fact, consider the case of a tachyon T and a succession of subluminal frames (all moving collinearly with T, for simplicity). Let s_∞ be (see Fig. 1c)) the frame observing T with divergent velocity ([2]). If a frame moving slower than s_∞ sees T travelling in a certain direction, then any frame moving faster than s_∞ will actually observe ([2,6]) T as an antitachion (*e.g.*, with the opposite electric charge) travelling in the reversed direction ([14]). Therefore, by-passing the frame s_∞, in the above sense, *implies* charge conjugation. In other words, if we observe from our frame s_0 a succession of Superluminal objects (or *vice versa*), for simplicity all moving with positive speeds $U \equiv U_x > c$, when we by-pass the « transcendent » object ([2,6])—corresponding to *infinite* velocity— we get the $\hat{C}$-symmetry.

Thus, when we operate a « rotation » in the four-dimensional space-time aimed to reach, for example, the totally inverted frame $(\hat{P}\hat{T})s_0$, actually we reach the frame $(\hat{C}\hat{P}\hat{T})\,s_0$, as we already saw.

Let us now *go back to bradyons* (usual, slower-than-light particles). We can « physically » apply to them a GLT of the type $-\Lambda_s \equiv \text{GLT}(135° < \varphi < 225°)$ only by successively applying ([1]) two SLT's, *i.e.* only by-passing the *transcendent* (infinite-speed)

(*) The mass obviously changes sign only under $\hat{E}$.

([13]) Compare also ref. ([2,6]) and references therein.

([14]) With respect, of course, to a certain « interaction region ». See ref. ([2]) and references therein.

frame [2] S_∞. It is now easy to understand that, by means [9] of GLT's of the type $-\Lambda_s$, we eventually get that the *second frame* observes (as regards the analysed bradyon) the same effects as produced (*in the first frame*) by a $\hat{C}\hat{T}$-operation [2,6,11] *applied to the analysed bradyon*.

According to our analysis, thus, we can conclude that (since it is $-\mathbf{1} = \hat{C}\hat{P}\hat{T}$ both for *bradyons* and tachyons) under the GLT « strong reflection » the particles in the initial state of an interaction process will be transformed [2] into antiparticles in the final state of the same interaction process and *vice versa* [2]. For instance, the two reactions (even among bradyons!)

$$a + b \to c + d\,, \qquad \bar{c} + \bar{d} \to \bar{a} + \bar{b}\,,$$

are the two different descriptions *of the same phenomenon*, as seen by the two different inertial, physical (*) frames s_0 and $-s_0 = (\hat{C}\hat{P}\hat{T})s_0$, respectively [2,6].

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(*) In the sense that we can go from the first frame to the second one through a continuous chain of reference frames, both subluminal and Superluminal [2].