IL NUOVO CIMENTO Vol. 30 A, N. 4 21 Dicembre 1975

Complex Electromagnetic Four-Potential and the Cabibbo-Ferrari Relation for Magnetic Monopoles (*).

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(ricevuto il 12 Agosto 1975)

Summary. — Within « extended relativity » we generalize the Maxwell equations in terms of four-potentials for both ordinary and faster-than-light charges. We succeed in giving a physical meaning to the complex electromagnetic four-potential (and to complex electromagnetic four-current, tensor and field), and in giving a new interpretation to the Cabibbo-Ferrari relation for magnetic monopoles.

1. – We showed (by using our « extended relativity » (1), generalized for faster-than-light inertial frames and for tachyons) that magnetic monopoles are probably expected to exist only as Superluminal (= faster than light) objects (2-5).

In fact (6) let $F_{\mu\nu}$ be the usual electromagnetic tensor and let us define its

(*) Supported in part by CNR, and by INFN, Sezione di Catania and Sezione di Roma.

(**) On leave of absence from Istituto di Fisica dell'Università, Roma.

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(1) E. Recami and R. Mignani: *Riv. Nuovo Cimento*, **4**, 209, 398 (1974).

(2) R. Mignani and E. Recami: *Lett. Nuovo Cimento*, **9**, 367 (1974).

(3) E. Recami and R. Mignani: *Lett. Nuovo Cimento*, **9**, 479 (1974).

(4) R. Mignani and E. Recami: *Lett. Nuovo Cimento*, **11**, 417 (1974).

(5) E. Recami and R. Mignani: in the 1976 Volume in honour of Prof. W. Heisenberg, to appear.

(6) We use the metric $(+ - - -)$ by writing, however, the generic vector as $x_\mu \equiv (ct, ix, iy, iz)$, *i.e.* $x_0 \equiv ct$, $x_1 \equiv ix$, $x_2 \equiv iy$, $x_3 \equiv iz$. We thus obtain $g_{\mu\nu} = \delta_{\mu\nu}$, and no distinction between covariant and contravariant components. Summation is understood over the repeated indices. Natural units (numerically $c = 1$) will be used. For the electromagnetic quantities we always use rationalized Gaussian units, *i.e.* Heaviside-Lorentz units.

« dual » as follows ($\mu, \nu, \varrho, \sigma = 0, 1, 2, 3$):

$$F^*_{\mu\nu} \equiv \frac{i}{2}\,\varepsilon_{\mu\nu\varrho\sigma} F_{\varrho\sigma}\,, \qquad (F^*_{\mu\nu})^* = -F_{\mu\nu}\,, \tag{1}$$

where $\varepsilon_{\mu\nu\varrho\sigma}$ is the *real*, completely antisymmetric Ricci tensor (normalized so that $\varepsilon_{1234} = 1$). Notice that our present « dual » tensor (7) is simply the *opposite* in sign to that of Cabibbo-Ferrari (8), in agreement (9) with our previous ref. (4). Let us confine ourselves for simplicity to requiring covariance only under the usual group of proper, orthochronous Lorentz transformations, and moreover define the new tensor (10)

$$T_{\mu\nu} \equiv F_{\mu\nu} - iF^*_{\mu\nu}\,, \tag{2}$$

where $T_{\mu\nu}$ is invariant under the exchanges (1) $\boldsymbol{E} \to i\boldsymbol{H}$, $\boldsymbol{H} \to -i\boldsymbol{E}$ and where

$$T^*_{\mu\nu} = iT_{\mu\nu}\,. \tag{2'}$$

Then, the *generalized* Maxwell equations, valid when both subluminal (= slower than light) and Superluminal electric charges are present, were shown (1) to read

$$\begin{cases} \partial_\nu T_{\mu\nu} = j_\mu(s) - ij_\mu(S)\,, \\ T^*_{\mu\nu} = iT_{\mu\nu}\,, \end{cases} \qquad v^2 \lessgtr c^2, \tag{3}$$

where $s \equiv$ subluminal and $S \equiv$ Superluminal. Equations (3) are equivalent to the following self-clear equations (1-5):

$$(v^2 \lessgtr c^2) \begin{cases} \operatorname{div} \boldsymbol{D} = \varrho(s)\,, & \operatorname{div} \boldsymbol{B} = -\varrho(S)\,, \\ \operatorname{rot} \boldsymbol{E} = \boldsymbol{j}(S) - \partial\boldsymbol{B}/\partial t\,, & \operatorname{rot} \boldsymbol{H} = \boldsymbol{j}(s) + \partial\boldsymbol{D}/\partial t\,. \end{cases} \tag{4}$$

Therefore, *in extended relativity* only one (electromagnetic) charge is explicitly expected to exist, which behaves as electric when subluminal, and as magnetic when Superluminal. In addition, one gets that the magnetic strength is expected to appear (1)

$$g = -e\,. \tag{5}$$

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(7) Notice that the present definition (1) of « duality » is however different from the one adopted in ref. (1), since there we chose $\tilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\varrho\sigma} F_{\varrho\sigma} = -iF^*_{\mu\nu}$.

(8) N. CABIBBO and E. FERRARI: *Nuovo Cimento*, **23**, 1147 (1962).

(9) A misprinted sign in eqs. (5) and (8) in ref. (4) should however be corrected.

(10) Incidentally, under a « Superluminal Lorentz transformation » L, the quantity $T_{\mu\nu}$ will no longer be (1) a standard tensor, since one would obtain (1-5) $T_{\mu\nu} \to T'_{\mu\nu} = -iL_{\mu\varrho} L_{\nu\sigma} T_{\varrho\sigma}$. Analogously for $F_{\mu\nu}$, for $A_{\mu/\nu}$, etc.

In particular, an « electrically » charged tachyon in an electromagnetic field will probably experience the « Lorentz force » [1-5]

$$F = -e(H - V \wedge E)\,, \qquad v^2 > c^2. \tag{6}$$

2. – All that precedes was directly derived by us from the electromagnetic fields [1-3] without introducing four-potentials. In this work we want to extend, for Superluminal charges, the Maxwell equations also *in terms of electromagnetic four-potentials.*

When in presence of *only a subluminal* electric current $j_\mu(s)$, Maxwell equations (in terms of the usual four-potential vector A_μ) are known to become

$$\begin{cases} \Box A_\mu = j_\mu(s)\,, \\ \partial_\mu A_\mu = 0\,, \end{cases} \qquad v^2 < c^2, \tag{7}$$

where the standard « Lorentz gauge » has been explicitly added. As usual, it will be $F_{\mu\nu} = A_{\nu/\mu} - A_{\mu/\nu}$.

Let us now suppose to be in the presence [11] of only Superluminal « electric » charges. Owing to the principle of relativity [1]—and to the « duality principle » [1]—those charges are represented by a standard (subluminal) four-current $j'_\mu(s)$ with reference to any Superluminal inertial frame S. With respect to S it then holds that

$$\Box' A'_\mu = j'_\mu(s)\,, \qquad \partial'_\mu A'_\mu = 0\,, \qquad v'^2 < c^2\,, \tag{8}$$

with $F'_{\mu\nu} = A'_{\nu/\mu} - A'_{\mu/\nu}$. Therefore, we shall observe—in our frame s—a Superluminal four-current $j_\mu(S)$ obeying the *transformed* laws of eqs. (8) under the suitable « Superluminal Lorentz transformation » [1] L. In other words, according to the « rule of tachyonization » [1], let us apply L to eqs. (8), noticing that $A_\mu \to A'_\mu = -iL_{\mu\varrho}A_\varrho$ (cf. footnote [10]). One immediately obtains

$$\begin{cases} \Box(iA_\mu) = j_\mu(S)\,, \\ \partial_\mu(iA_\mu) = 0\,, \end{cases} \Rightarrow \begin{cases} \Box A_\mu = -ij_\mu(S)\,, \\ \partial_\mu A_\mu = 0\,, \end{cases} \qquad v^2 > c^2. \tag{8'}$$

In conclusion, when in presence of *both* sub- *and* Superluminal four-currents, we need to introduce a *complex four-potential* $\bar{A}_\mu$ and the *complex four-current*

(11) Let us call s our own frame of reference.

$\bar{J}_\mu \equiv j_\mu(s) - ij_\mu(S)$, so as to be able to write in total

$$(9) \qquad \begin{cases} \Box \bar{A}_\mu = \bar{J}_\mu , \\ \partial_\mu \bar{A}_\mu = 0 , \end{cases} \qquad v^2 \gtrless c^2 .$$

Equations (9) *extend* eqs. (7) for the case of *both slower and faster-than-light currents*. If we now call

$$(10) \qquad A_\mu \equiv \mathrm{Re}\, \bar{A} , \qquad B_\mu \equiv - \mathrm{Im}\, \bar{A} ,$$

then we can write the *generalized* equations, for the *extended* electromagnetic four-potential, as

$$(9') \qquad \begin{cases} \Box(A_\mu - iB_\mu) = j_\mu(s) - ij_\mu(S) , \\ \partial_\mu(A_\mu - iB_\mu) = 0 , \end{cases} \qquad v^2 \gtrless c^2 ,$$

holding when both bradyonic and tachyonic charges are considered. Equations (9′) of course are equivalent to

$$(9'') \qquad \begin{cases} \Box A_\mu = j_\mu(s) , \quad \partial_\mu A_\mu = 0 , & v^2 < c^2 , \\ \Box B_\mu = j_\mu(S) , \quad \partial_\mu B_\mu = 0 , & v^2 > c^2 . \end{cases}$$

By comparing our eqs. (9″) with eqs. (8) in ref. (8), or with eqs. (77) in ref. (12), one can realize that our four-potential B_μ is to be *identified* with the four-potential B_μ introduced by CABIBBO and FERRARI for magnetic monopoles in the « Dirac additive term » (8,12). Within extended relativity we thus give a new physical meaning to the Cabibbo-Ferrari four-potential B_μ. Namely, we have

$$(10') \qquad B_\mu = - L^{-1}_{\mu\varrho} A'_\varrho .$$

From eqs. (9″) one can again verify that a Superluminal « electric » charge behaves as a magnetic monopole with opposite (magnetic) charge (13).

At this point it is important to observe that we of course do *not* need any condition of the type $eg = n\hbar$, since « consistency requirements » of the kind of eq. (13) in ref. (8) do *not* come in, when extended relativity is used. In fact, our four-vector B_μ calls into the game nothing but A'_μ, and for A'_μ (in the Super-

(12) E. AMALDI: in *Old and New Problems in Elementary Particles*, edited by G. PUPPI (New York, N .Y., 1968), p. 1.

(13) See, *e.g.*, ref. (8), p. 1149 and ref. (12), p. 53. For a contrary belief, see H. C. CORBEN: *Nuovo Cimento*, **29** A, 415 (1975); and then R. MIGNANI and E. RECAMI: *Lett. Nuovo Cimento*, **13**, 589 (1975).

luminal frames S) eqs. (9)-(12) of ref. (8) do hold, as they also hold for A_μ in the subluminal frames s.

Now, let us remember that the upper eqs. (9″) are equivalent to $\partial_\nu F_{\mu\nu}=j_\mu(s)$. Moreover, the equations (8) $\Box B_\mu = -g_\mu$, $\partial_\mu B_\mu = 0$ would be equivalent (8,12) to $\partial_\nu F^*_{\mu\nu} = -g_\mu$; so that we expect the lower eqs. (9″) to be *equivalent* to

$$\partial_\nu F^*_{\mu\nu} = +j_\mu(S)\,;$$

we want to set forth explicitly the conditions under which that equivalence holds.

In other words, we want to find out the conditions under which our generalized equations (9) and (9′) are equivalent to the extended Maxwell equations (3):

$$\left\{\begin{aligned}\Box\bar{A}_\mu &= \bar{J}_\mu\\ \partial_\mu\bar{A}_\mu &= 0\end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned}\partial_\nu T_{\mu\nu} &= \bar{J}_\mu\\ T^*_{\mu\nu} &= iT_{\mu\nu}\end{aligned}\right\}, \qquad v^2 \gtrless c^2\,, \tag{11}$$

that is to say

$$\left\{\begin{aligned}&\Box(A_\mu - iB_\mu) = j_\mu(s) - ij(S)\\ &\partial_\mu(A_\mu - iB_\mu) = 0\end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned}&\partial_\nu(F_{\mu\nu} - iF^*_{\mu\nu}) = j_\mu(s) - ij_\mu(S)\\ &T^*_{\mu\nu} = iT_{\mu\nu}\end{aligned}\right\}. \tag{11′}$$

We may write the identity

$$\Box\bar{A}_\mu = -\partial^2_\nu\bar{A}_\mu + (\partial_\mu\partial_\nu\bar{A}_\nu + \varepsilon_{\mu\nu\varrho\sigma}\partial_\nu\partial_\varrho\bar{A}_\sigma)\,, \tag{12}$$

where we added (in brackets) a *null* part (14). We can immediately derive from eq. (12) that *relations* (11) *and* (11′) *hold if* we set ($v^2 \gtrless c^2$)

$$T_{\mu\nu} \equiv \bar{A}_{\nu/\mu} - \bar{A}_{\mu/\nu} + \varepsilon_{\mu\nu\varrho\sigma}\bar{A}_{\sigma/\varrho}\,. \tag{13}$$

By explicitating eq. (13), one may obtain the above condition in the form of one of the following conditions:

$$F_{\mu\nu} \equiv A_{\nu/\mu} - A_{\mu/\nu} - i\varepsilon_{\mu\nu\varrho\sigma}B_{\sigma/\varrho}\,, \tag{14a}$$

$$F^*_{\mu\nu} \equiv B_{\nu/\mu} - B_{\mu/\nu} + i\varepsilon_{\mu\nu\varrho\sigma}A_{\sigma/\varrho}\,. \tag{14b}$$

Of course, in connection with eqs. (7) and (8), the terms $B_{\sigma/\varrho}$ *there* were zero. Equation (14*b*) has been written because of the identity (15)

$$B_{\nu/\mu} - B_{\mu/\nu} + i\varepsilon_{\mu\nu\varrho\sigma}A_{\sigma/\varrho} = \frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}\{A_{\beta/\alpha} - A_{\alpha/\beta} - i\varepsilon_{\alpha\beta\gamma\delta}B_{\delta/\gamma}\} \equiv F^*_{\mu\nu}\,. \tag{15}$$

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(14) For example, the last term in eq. (12) vanishes, being the contraction of a symmetric tensor with an antisymmetric tensor.

(15) See, *e.g.*, B. Finzi and M. Pastori: *Calcolo tensoriale e applicazioni* (Bologna, 1961), p. 157.

3. – We have therefore derived the Cabibbo-Ferrari relation ([8]) as our condition (14*a*), giving it a new physical interpretation.

Let us emphasize that, by using the complex four-current, the *extended* Maxwell equations—when both bradyonic and tachyonic charges are taken into account—assume the form

$$(3') \qquad \partial_\nu T_{\mu\nu} = \bar{J}_\mu\,, \qquad T^*_{\mu\nu} = iT_{\mu\nu}\,, \qquad v^2 \gtrless c^2.$$

We have, moreover, shown the connection *between* the duality here defined in eq. (1) *and* the « dual correspondence » ([1]) bradyons $\leftrightarrow$ tachyons.

Finally, it is evident the essential role of complex quantities in formulating extended relativity. For example, the importance of the *complex* « vector » $\boldsymbol{F} \equiv i(\boldsymbol{E} + i\boldsymbol{H})$ in physics was previously realized also by MAJORANA ([16]) and by WEINBERG ([17]). Under duality (1) we have $\boldsymbol{F} \to i\boldsymbol{F}$. In terms of that complex « vector », our « *complex* electromagnetic tensor » $T_{\mu\nu}$ reads

$$(T_{\mu\nu}) = \begin{pmatrix} 0 & F_x & F_y & F_z \\ -F_x & 0 & F_z & -F_y \\ -F_y & -F_z & 0 & F_x \\ -F_z & F_y & -F_x & 0 \end{pmatrix}.$$

4. – We cannot refrain from mentioning the *possible* role ([1]) of faster-than-light objects in hadron structure. It has long since been known that existence of spacelike components ([18]) seems a natural and perhaps unavoidable feature of *interacting* fields ([19]). It has indeed been proved ([20]) *e.g.* that, if the Fourier transform of a local field *vanishes* in a domain of spacelike vectors in momentum space, then the field is a generalized *free* field. Moreover, since usual « virtual particles » generally bear *negative* squared four-momentum, it was suggested ([21]) in 1968 that virtual particles be actually considered as tachyons. Following

([16]) See R. MIGNANI, E. RECAMI and M. BALDO: *Lett. Nuovo Cimento*, **11**, 568 (1974).
([17]) S. WEINBERG: in *1964 Lectures on Particles and Field Theory*, edited by S. DESER and K. W. FORD (Englewood Cliffs, N. J., 1965), p. 405.
([18]) Cf., *e.g.*, E. MAJORANA: *Nuovo Cimento*, **9**, 335 (1932).
([19]) D. TZ. STOYANOV and I. T. TODOROV: *Journ. Math. Phys.*, **9**, 2146 (1968) and references therein; A. O. BARUT and I. H. DURU: ICTP, preprint IC/72/116 (Trieste, 1972).
([20]) G. F. DELL'ANTONIO: *Journ. Math. Phys.*, **2**, 572 (1972); O. W. GREENBERG: *Journ. Math. Phys.*, **3**, 859 (1962).
([21]) E. RECAMI: Report IFUM-088/S.M. (Milano, 1968); *Giornale di Fisica*, **10**, 195 (1969); V. S. OLKHOVSKY and E. RECAMI: *Nuovo Cimento*, **63** A, 814 (1969).

that line, results have been put forth *e.g.* in ref. ([22]). In particular, since infinite-speed tachyons bear ([1]) zero energy but finite momentum, such tachyons may be useful for interpreting « pomeron exchange reactions », or diffractive scatterings ([1]) and elastic scatterings ([23]).

What we want to emphasize here is the following. We have shown that the « duality » between bradyons and tachyons is essentially the duality between electric and magnetic charges (see also ref. ([24])). And TZE ([25]) seems to have shown, following NAMBU, that a structural identity can be established between electrodynamics and dual strong interactions (exemplified by dual-resonance models). Possibly, a link is thus found between tachyons and hadron structure.

Furthermore, PARISI ([26]) has suggested to identify magnetic monopoles with quarks; BARUT ([26]) underlined the connections between electromagnetic ([27]) and dual ([28]) « strings »; and much other work along similar ideas may be found in the recent literature ([29,1]). For example, some results have been obtained by considering quarks as quantized loops ([30]).

In our opinion, all these considerations—and many other ones—indicate that quarks (made up of partons ([1])) may well be quantized loops of Superluminal charges, *i.e.* of faster-than-light leptons (*electrons*); and partons may be nothing but such tachyonic leptons.

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The authors are grateful to Prof. L. FONDA for grants and his interest in this work. They are also grateful to Prof. A. SALAM, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, and to Prof. A. AGODI and Dr. L. R. BALDINI for kind interest.

([22]) E. C. G. SUDARSMAN: *Ark. Phys.*, **39** (40), 585 (1969); A. M. GLEESON, M. G. GUNDZIK, E. C. G. SUDARSHAN and A. PAGNAMENTA: *Phys. Rev. D*, **6**, 807 (1972); E. VAN DER SPUY: *Phys. Rev. D*, **7**, 1106 (1973).

([23]) R. MIGNANI and E. RECAMI: ICTP, Internal Report IC/75/82 (Trieste, 1975).

([24]) See also D. WEINGARTEN: *Ann. of Phys.*, **76**, 510 (1973).

([25]) H. C. TZE: *Nuovo Cimento*, **22** A, 507 (1974). See also second reference in ([26]).

([26]) G. PARISI: *Phys. Rev. D*, **11**, 970 (1975); A. O. BARUT: ICTP, preprint IC/73/34 (Trieste, 1973).

([27]) P. A. M. DIRAC: *Phys. Rev.*, **74**, 817 (1948).

([28]) See, *e.g.*, L. SUSSKIND: *Nuovo Cimento*, **69** A, 457 (1970).

([29]) See, *e.g.*, H. B. NIELSEN and P. OLESEN: *Nucl. Phys.*, **61** B, 45 (1973); P. OLESEN: preprint NBI-HE-74-12 (Copenhagen, 1974); T. SAWADA: *Nucl. Phys.*, **71** B, 82 (1974); T. YONEDA: *Prog. Theor. Phys.*, **51**, 1907 (1974); M. KOBAYASHI: *Prog. Theor. Phys.*, **51**, 1636 (1974); M. ADEMOLLO, A. D'ADDA, R. D'AURIA, E. NAPOLITANO, S. SCIUTO, P. DI VECCHIA, F. GLIOZZI, R. MUSTO and F. NICODEMI: *Nuovo Cimento*, **21** A, 77 (1974).

([30]) M. JEHLE: *Phys. Rev. D*, **3**, 306 (1971); **6**, 441 (1972).

● RIASSUNTO

Nell'ambito della « relatività estesa », si generalizzano le equazioni di Maxwell per cariche sia ordinarie sia più veloci della luce, *partendo dai tetra-potenziali* elettromagnetici. Si riesce a dare significato fisico a un tetra-potenziale complesso (e a una tetracorrente, un tensore, e un campo *complessi*), e a dare una nuova interpretazione della relazione di Cabibbo e Ferrari per i monopoli magnetici.

Комплексный электромагнитный четырехмерный потенциал и соотношение Кабиббо-Феррари для магнитных монополей.

Резюме (*). — В рамках « протяженной относительности » мы обобщаем уравнения Максвелла в терминах четырехмерных потенциалов для случая обычных зарядов и для случая зарядов, движущихся быстрее скорости света. В этой работе мы обсуждаем физический смысл комплексного электромагнитного четырехмерного потенциала. Мы предлагаем новую интерпретацию соотношения Кабиббо-Феррари для магнитных монополей.

(*) *Переведено редакцией.*