A Derivation of « Crossing Relations » and its Physical Meaning (*).

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Summary. — In this paper the « crossing relations » of the usual elementary-particle (high-energy) physics are derived on the basis of Lorentz covariance, generalized also to Superluminal inertial frames. In this framework the « analiticity » postulate is unnecessary, and is better substituted by the G-covariance requirement (*i.e.* covariance under the new group G of generalized Lorentz transformations, both subluminal and Superluminal). Moreover, new « crossing type » relations are predicted on the basis of mere « extended relativity ». They may well serve as a test for relativistic covariance of « force fields » like strong interactions and, particularly, weak interactions, and possible new « interaction fields » (which *a priori* are not relativistically covariant).

1. – Introduction.

Let us confine ourselves to the framework of standard special relativity, *i.e.* assume the following postulates:

i) Principle of relativity (PR): Physical laws of mechanics and electromagnetism are covariant under transitions between two inertial frames. The vacuum is also covariant.

(*) To speed up publication, the authors of this paper have agreed to not receive the proofs for correction.

ii) Space is isotropic and space-time homogeneous.

It is well known ([1,2]) that from postulates i) and ii) the existence of an invariant speed follows, which is recognized through experience to be the light speed.

Now we want introductorily to show that a *third postulate* is necessary ([2]), in usual relativity, to avoid the possibility of information being sent into the past. Let us first state the « third postulate » that we shall call the *Dirac-Stückelberg-Feynman-Sudarshan principle* ([2]) (RIP) as follows:

iii) Third postulate: For any observer, physical signals are actually transported only by positive-energy objects (*i.e.* by the objects that appear to the observer as carrying positive energy and going forward in time) ([3]).

The « third postulate » may seem *a priori* obvious and superfluous. On the contrary, it is necessary for solving the following two basic problems:

a) *In Minkovski space.* Let us consider the chronotopous as referred to a certain observer. What prevents, in the relativity framework, a (chronologically subsequent) event $A_2(\boldsymbol{x}, t_2)$ from affecting a (chronologically preceding) event $A_1(\boldsymbol{y}, t_1)$, with $t_1 < t_2$, through a signal sent back (into the past) by A_2 to A_1? ([4]).

b) *In four-momentum space.* See Fig. 1*a*), *i.e.* the free-particle hyperboloid $[c = 1]$

$$E = \pm \sqrt{\boldsymbol{p}^2 + m_0^2}\,; \tag{1}$$

how to interpret the lower hyperboloid points corresponding to the negative sign in eq. (1)?

([1]) W. v. IGNATOWSKY: *Phys. Zeits.*, **21**, 972 (1910); P. FRANK and H. ROTHE: *Ann. der Phys.*, **34**, 825 (1911); E. HAHN: *Arch. Math. Phys.*, **21**, 1 (1913); F. SEVERI: in *Cinquant'anni di relatività*, edited by M. PANTALEO (Firenze, 1955); M. CAMENZIND: *General Relat. Gravit.*, **1**, 71 (1970); V. BERZI and V. GORINI: *Journ. Math. Phys.*, **10**, 1518 (1969); V. GORINI and A. ZECCA: *Journ. Math. Phys.*, **11**, 2226 (1970); A. AGODI: unpublished (1973).

([2]) E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209 (1974); R. MIGNANI and E. RECAMI: *Int. Journ. Theor. Phys.* (to appear).

([3]) The « third postulate » is known also as « reinterpretation principle » (RIP). See ref. ([2]), and P. A. M. DIRAC: *Proc. Roy. Soc.*, A **126**, 360 (1930); E. C. G. STÜCKELBERG: *Helv. Phys. Acta*, **14**, 32L, 588 (1941); R. P. FEYNMAN: *Phys. Rev.*, **76**, 749, 769 (1949); O. M. P. BILANIUK, V. K. DESHPANDE and E. C. G. SUDARSHAN: *Amer. Journ. Phys.*, **30**, 718 (1962); S. WEINBERG: *Gravitation and Cosmology* (New York, N. Y., 1972).

([4]) Notice that, in relativity, all events of the Minkowski space should be *a priori* considered as « always existing ». Cf., *e.g.*, G. ARCIDIACONO: *Relatività e cosmologia* (Roma, 1973).

At this point it is important to remember that we have shown ([2]) that, if a particle travels backwards in time, then it carries also negative energy, and *vice versa* ([2]). It is easy to see that those two paradoxical occurrences allow an orthodox *reinterpretation* ([3]) when they are (as they do) simultaneous. In

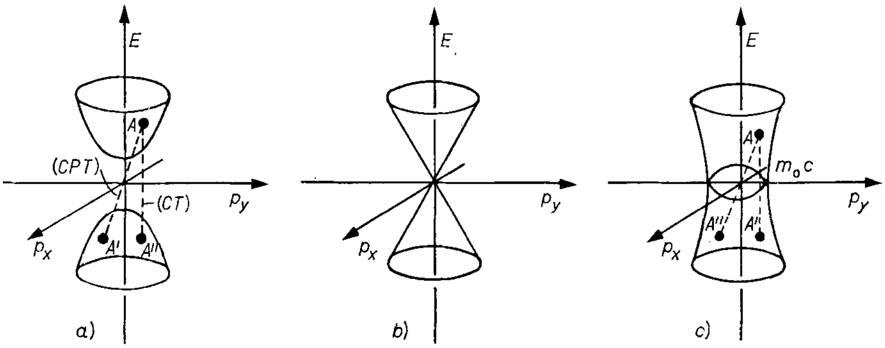

Fig. 1. – Representation of the hypersurfaces $E^2 - \boldsymbol{p}^2 = p^2$ for *a*) bradyons, with $p^2 \equiv m_0^2 > 0$ (*timelike* case); *b*) luxons, with $p^2 \equiv m_0^2 = 0$ (*lightlike* case); *c*) tachyons, with $p^2 \equiv -m_0^2 < 0$ (*spacelike* case), where m_0 is *always real*. In *a*) the points A' and A'' represent the particle kinematical states obtained by applying the operations $\hat{C}\hat{P}\hat{T}$ and $\hat{C}\hat{T}$, respectively, to the kinematical state A. *In the case when we confine ourselves* to subluminal frames and to usual LT's, then it happens that the « matter » or « antimatter » character is invariant for B's, but relative to the observer for T's. When eliminating the previous restriction, we may pass from particles to their antiparticles (through GLT's) even in the case of bradyons.

fact ([3]), let us suppose (see Fig. 2*a*)) that a particle P, with negative energy (and, *e.g.*, « charge » $-e$) ([5]) and travelling backwards in time, is emitted by A at time t_1 and absorbed by B at time $t_2 < t_1$. Therefore, at time t_1 the object A « loses » negative energy and « charge » $-e$, *i.e.* *gains* energy and « charge » $+e$; and at time $t_2 < t_1$ the object B « gains » negative energy and « charge » $-e$, *i.e.* *loses* energy and « charge » $+e$. Such a physical phenomenon will of course appear to be nothing but the exchange *from B to A* of a (standard) particle Q with positive energy (and « charge » $+e$) and travelling forward in time.

Notice that *a priori* we can adopt either the first description or the reinterpreted one through the « RIP ». The third postulate, on the contrary, tells us that we *must* use « RIP », *i.e.* that we *must* adopt the reinterpreted description (and forget about the first one).

Moreover, we have seen that Q has the opposite « charge » with respect to P; this means that the third postulate (RIP) operates, among others, a charge

([5]) In this work the word « charge » is used in its *widest* meaning to indicate any possible « additive charge ». Cf. ref. ([2]).

conjugation $\hat{C}$. A closer inspection of « RIP » (see Fig. 2*b*) and ref. (2)) tells us that actually

$$\text{(2)} \qquad \text{« RIP »} = \hat{C}\hat{E}\hat{p}\,,$$

where by $\hat{E}$ and $\hat{p}$ we mean the operations of energy and three-momentum reversal, respectively (2). Notice that, in our terminology, $\hat{C}$ means conjuga-

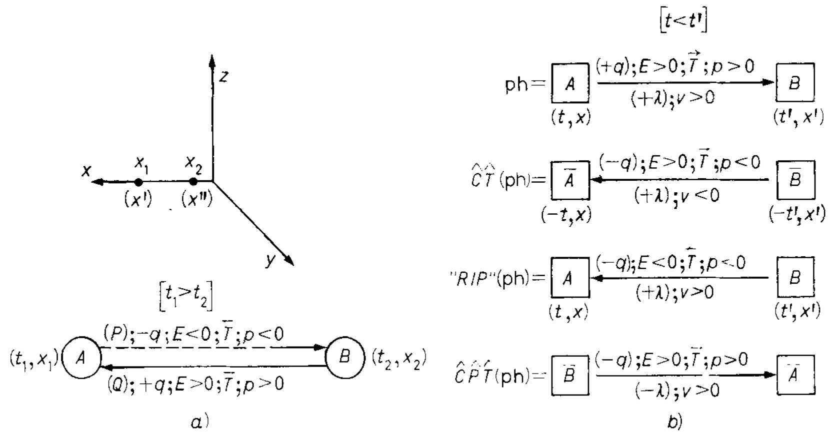

Fig. 2. – *a*) represents the exchange from A to B of a particle P with *negative* energy (and « charge ») and travelling backwards in time ($t_2 < t_1$). Such a process will appear to be nothing but the exchange *from B to A* of a (standard) particle Q with *positive* energy (and « charge »), travelling forward in time. Particle Q may be shown to be (maybe except for the helicity) the *antiparticle* of the initial particle: $Q = \bar{P}$. See the text. *b*) shows a certain phenomenon ph, *i.e.* the exchange from emitter A to absorber B of a certain particle, and the transformations on ph (and on A, B) operated *respectively by* $\hat{C}\hat{T}$, *by* the « reinterpretation procedure » (RIP: see the text) used in case *a*) *and by* $\hat{C}\hat{P}\hat{T}$.

tion of *all* (additive) charges (*e.g.* also of magnetic charge, if it exists) (6). It is possible to show *rigorously* (2) that Q will appear exactly as the *antiparticle* $\bar{P}$ of P in the usual sense (2).

Therefore, if we go back to our two previous problems, we shall have them solved as follows:

a) *In Minkovski space*: the particle P possibly going (backwards in time and with negative energy) from A_2 to A_1 will be nothing but the exchange

(6) By the way, let us mention that in ref. (2) we have shown that, when assuming « RIP », it is always possible to write the relation

$$\text{(3)} \qquad \hat{C}^{(\text{RIP})}\hat{v}\hat{E} = \hat{p}\hat{E}\,,$$

which is very interesting for the physical understanding of « charge conjugation ».

from A_1 *to* A_2 of its antiparticle $\bar{P}$ (with positive energy and going forward in time).

b) In four-momentum space: as the upper hyperboloid points correspond to the kinematical states of a free particle P, as seen by all the inertial frames, so the lower hyperboloid points will represent all the possible (free) kinematical states of its *antiparticle* $\bar{P}$.

It is quite interesting the fact that, once the notion of *particle* (as usually done) is introduced in special relativity, then the concept of *antiparticle* follows merely from special relativity itself [2]. Precisely, since 1905, on the basis of the double sign entering eq. (1), the existence—for any particle—of its antiparticle could have been expected, provided that the third postulate had been used.

2. – A note on extended relativity: tachyons and Superluminal frames.

In the first Section we essentially introduced for special relativity a third postulate, the « RIP ».

We want to stress that its validity has been already checked by us, since its application allowed us, in the previous Section, to give a *correct* meaning to the lower hyperboloid, *i.e.* to predict the existence of antiparticles (prediction verified by experience).

Let us now pass to consider, besides the usual $(|v| < c)$ particles (bradyons) and luxons (photons, neutrinos: see Fig. 1*b*)), also faster-than-light objects (tachyons) [2]. A « classical theory » of tachyons has been recently put forth by us [2]. We have shown [2] that, given a tachyon t, a class of (usual) Lorentz transformations exists for which—roughly speaking—t is transformed into the antitachyon [2] $\bar{\text{t}}$. The meaning of such a sentence, however, vanishes (*e.g.* because of the possible exchange of emission and absorption processes: cf. Fig. 2*a*)) if we cannot refer our particle to some *interaction* (space-time) *regions* [7]. For example, when a tachyon overcomes the divergent velocity (see Fig. 1*c*)), it passes from being *e.g.* a tachyon t *entering* (a certain interaction region) to being an antitachyon $\bar{\text{t}}$ *outgoing* (from that interaction region) [2]. In conclusion, the third postulate will be completed by precising that «under suitable "Lorentz tranformations", when *e.g.* the rôles of emission and absorption happen to be interchanged, *any negative-energy object in the initial « state » corresponds physically to its positive-energy antiobject in the final « state »*, and *vice versa* », in the above sense.

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[7] The particle *source* and *detectors* must as well be taken always into due consideration. See ref. [2] and A. AGODI: *Lezioni di fisica teorica* (Catania, 1973), unpublished.

If we deal with tachyons (and antitachyons), then the « Lorentz transformations » we are speaking of can be just the usual proper, orthochronous Lorentz transformations (LT).

However, if we deal with usual particles (bradyons and antibradyons), the suitable Lorentz transformations of the third postulate's last form must be chosen among the « Superluminal Lorentz transformations » (SLT) ([8]). We must, then, refer ourselves to the theory of special relativity, generalized to faster-than-light inertial frames. Such an « extended relativity », with its « generalized Lorentz transformations » (GLT), has been by us built up and recently put forth ([2]). It comes down exactly from the same postulates i), ii) and iii) of Sect. **1**, provided that we do not confine ourselves arbitrarily to slower-than-light speeds ([2]).

The extended relativity may be found in ref. ([2]). Let us report here only its features more relevant to us:

i) The SLT's result ([2]) to have the form SLT $= \pm i\Lambda_>$, where, if the matrices $\Lambda_< = \Lambda_<(\beta^2 < 1)$ represent the usual ([9]) LT's, then $\Lambda_> = \Lambda_>(\beta^2 > 1)$ are (complex) matrices *formally* identical to the $\Lambda_<$'s, but corresponding to values $|\beta| > 1$. It is easy to verify *e.g.* that the product of two SLT's yields a subluminal LT.

ii) While subluminal LT's do not change (as usual) the four-vector type, on the contrary the SLT's transform timelike vectors into spacelike vectors, and *vice versa* (even if they preserve the four-vector magnitudes—except for the sign) ([2]).

iii) The extended velocity composition law bears for instance the consequences listed in Tables I and II, which will be useful to us in the following.

TABLE I. – *Consequences of the extended velocity composition law for velocity magnitudes.*

$u^2 < c^2$	$v^2 < c^2 \Rightarrow v'^2 < c^2$
	$v^2 = c^2 \Rightarrow v'^2 = c^2$
	$v^2 > c^2 \Rightarrow v'^2 > c^2$
$u^2 = c^2$	$v^2 \gtreqless c^2 \Rightarrow v'^2 = c^2$
$u^2 > c^2$	$v^2 < c^2 \Rightarrow v'^2 > c^2$
	$v^2 = c^2 \Rightarrow v'^2 = c^2$
	$v^2 > c^2 \Rightarrow v'^2 < c^2$

([8]) « Superluminal » means faster-than-light, and « subluminal » slower-than-light. Cf., *e.g.*, ref. ([2]) and references therein.

([9]) However, we are not considering space-time translations. Cf., *e.g.*, ref. ([2]).

TABLE II. – *Effect of GLT's on the sign of various four-vector components of an observed object, in the case of collinear motion along the x-axis.* Both subluminal ($u \equiv u_x$) and Superluminal ($U \equiv U_x$) relative velocities are considered. Analogously, both bradyons (having velocities $\boldsymbol{v}$, relative to the *first*, unprimed frame) and tachyons (having velocities $\boldsymbol{V}$, relative to the unprimed frame) are as well considered. For simplicity, *only* the cases $+\Lambda_<(\beta^2 < c^2)$ and $-i\Lambda_>(\beta^2 > c^2)$ are considered, as well as the *only* cases $v > 0$ and $V > 0$. Notice explicitly that only x-components of $\boldsymbol{v}$ or $\boldsymbol{V}$ are effective, in this context. For compactness' sake, it is assumed $V_x > c$.

Relative velocity	$\lvert u_x\rvert \equiv \lvert u\rvert < c$, $V_x > c$		$\lvert U_x\rvert \equiv \lvert U\rvert > c$				$\lvert u_x\rvert \equiv \lvert u\rvert < c$, $0 < v_x < c$	
			$0 < v_x < c$		$V_x > c$			
	$u > 0$	$u < 0$	$U > 0$	$U < 0$	$U > 0$	$U < 0$	$u > 0$	$u < 0$
$\dfrac{\text{sign}\, x'}{\text{sign}\, x}$	$+$	$+$	$+$	$-$	$\pm$ for $U \gtrless V_x$	$-$	$\pm$ for $u \lessgtr v_x$	$+$
$\dfrac{\text{sign}\, t'}{\text{sign}\, t}$	$\pm$ for $c^2/V_x \gtrless u$	$+$	$\pm$ for $U \gtrless c^2/v_x$	$-$	$+$	$-$	$+$	$+$
$\dfrac{\text{sign}\, p'_x}{\text{sign}\, p_x}$	$+$	$+$	$+$	$-$	$\pm$ for $U \gtrless V_x$	$-$	$\pm$ for $u \lessgtr v_x$	$+$
$\dfrac{\text{sign}\, E'}{\text{sign}\, E}$	$\pm$ for $c^2/V_x \gtrless u$	$+$	$\pm$ for $U \gtrless c^2/v_x$	$-$	$+$	$-$	$+$	$+$
$\dfrac{\text{sign}\, v'_x}{\text{sign}\, v_x}$	$\pm$ for $c^2/V_x \gtrless u$	$+$	$\pm$ for $U \gtrless c^2/v_x$	$-$	$\pm$ for $U \gtrless V_x$	$-$	$\pm$ for $u \lessgtr v_x$	$+$
$\dfrac{\text{sign}\, j'_x}{\text{sign}\, j_x}$	$+$	$+$	$+$	$-$	$\pm$ for $U \gtrless V_x$	$-$	$\pm$ for $u \lessgtr v_x$	$+$
$\dfrac{\text{sign}\, \varrho'}{\text{sign}\, \varrho}$	$\pm$ for $c^2/V_x \gtrless u$	$+$	$\pm$ for $U \gtrless c^2/v_x$	$-$	$+$	$-$	$+$	$+$

3. – Deriving crossing relations.

When the same interaction process p originates different descriptions [2] from different observers, *i.e.* appears as different scattering processes $d_1(\text{p})$ and $d_2(\text{p})$ in different frames r_1 and r_2, then the PR requires that $d_1(\text{p})$ and $d_2(\text{p})$ be ruled by the same dynamical law [2]. Therefore, because of relativistic covariance, *processes d_1 and d_2 present the same scattering amplitude* $A = A(s, t, ...)$, provided that *the physical meanings* (and possibly the signs) of the invariant Mandelstam variables $s, t, ...$ [10] are accordingly changed [10]. Notice that GLT's and « RIP » do automatically save the validity of the usual conservation laws as well [2].

[10] See, *e.g.*, R. HAGEDORN: *Relativistic Kinematics* (New York, N. Y., 1963); G. CHEW: *S-Matrix Theory of Strong Interactions* (New York, N. Y., 1962); P. ROMAN: *Introduction to Quantum Field Theory* (New York, N. Y., 1969).

Now, let us consider subluminal and Superluminal boosts along the x-direction. We shall first consider only tachyons (T) having $V_x > c$. It is then easy to observe (*e.g.* from Table II) that (2):

1) A subluminal boost, $L = \pm \Lambda_<$, applied to an interaction among T's (and/or luxons) allows a transition (11) from a certain process p either to p itself or to i) any scattering p′ obtained by $\hat{C}\hat{T}$-ing (2,12) one or more particles, ii) any scattering p″ obtained by $\hat{P}$-ing no, one or more particles *and* $\hat{C}\hat{P}\hat{T}$-ing all the remaining particles, *provided that* processes p′ and p″ are kinematically allowed (or, better, satisfy the conservation laws of energy, momentum, angular momentum and all « charges »). Scatterings p″ are nothing but the $\hat{C}\hat{P}\hat{T}$-ed ones of scatterings p′. Besides:

2) A Superluminal boost, $L = \pm i\Lambda_>$, applied to an interaction among B's (and/or luxons), allows a transition from a certain process p either to p itself or to i) any scattering p′ obtained by $\hat{C}\hat{T}$-ing (2,12) one or more B's with $v_x > 0$ *and* $\hat{C}\hat{P}\hat{T}$-ing all B's with $v_x \leqslant 0$ or *vice versa*, ii) any scattering p″ obtained by either $\hat{P}$-ing or $\hat{C}\hat{P}\hat{T}$-ing all B's with $v_x > 0$ (and leaving unaffected all B's with $v_x \leqslant 0$) or *vice versa*, *provided that* processes p′ and p″ satisfy the conservation laws of energy, momentum, angular momentum and all « charges ». Moreover, the Superluminal boost will transform B's into T's (*i.e.* will change $s, t, \ldots$ into $-s, -t, \ldots$). As before, scatterings p″ are the $\hat{C}\hat{P}\hat{T}$-ed ones of scatterings p′.

It is noticeable—as we have said—that, by using SLT's, we may get results *holding for usual particles* (bradyons) (10). In fact, if two processes among B's (*e.g.* an interaction and the *crossed* one (13)) are different reactions p_1, p_2 as seen by us (frame s_0), but they are seen as the *same* interaction (2) $d_S = d_1 \equiv d_2$ (among T's) by two different superluminal observers, S_1, S_2 (cf. ref. (2)), *then* we may conclude the following. We may get the scattering amplitude (14) of p_1, *i.e.* $A(p_1)$, by applying the $\mathrm{SLT}(S_1 \to s_0) \equiv L_1$ to the amplitude $A_1(d_1)$ found by S_1 when observing scattering p_1 (2)

$$A(p_1) = L_1[A_1(d_1)] \; ; \tag{4}$$

conversely, we may get the scattering amplitude of p_2, *i.e.* $A(p_2)$, by applying the $\mathrm{SLT}(S_2 \to s_0) \equiv L_2$ to the amplitude (2) $A_2(d_2)$ found by S_2 when observ-

(11) M. Baldo, G. Fonte and E. Recami: *Lett. Nuovo Cimento*, **4**, 241 (1970).
(12) See, *e.g.*, E. C. G. Sudarshan: *Proc. Indian Acad. Sci.*, **69** (3A), 133 (1969); in *1968 Proceedings of the VIII Nobel Symposium* (New York, N. Y., 1970).
(13) See also D. A. Atkinson: preprint (Tekn. Technology Univ., 1973); A. Yaccarini: *Can. Journ. Phys.*, **51**, 1304 (1973); and to appear.
(14) Notice that a scattering amplitude may be introduced even in classical (macrophysical, nonquantum) theory. Cf. ref. (2).

ing scattering p_2

$$A(p_2) = L_2[A_2(d_2)] \,. \tag{5}$$

But, since by hypothesis

$$A_1(d_1) = A_2(d_2) = A(d_S) \,, \tag{6}$$

it follows that

$$A(p_1) = A(p_2) \tag{7}$$

for all reactions among B's satisfying the initial hypothesis.

Then, if we limit ourselves, for simplicity, to four-body processes, it follows (2) that *extended relativity requires the scattering amplitude* $A(s, t, \ldots)$ *to be given by the same function of the kinematical variables for the following reactions*:

1) the process

$$A(\vec{p}_A, q_A, \lambda_A) + B(\vec{p}_B, q_B, \lambda_B) \to C(\vec{p}_C, q_C, \lambda_C) + D(\vec{p}_D, q_D, \lambda_D) \,, \tag{8}$$

where $\vec{p}$, q, λ are trimomentum, « charge » and helicity (2), respectively;

2) *and* the totally $\hat{C}\hat{P}\hat{T}$-ed one (2);

3) *and the crossed processes* like (2)

$$A + \bar{C}(\vec{p}_{\bar{C}}, -q_C, -\lambda_C) \to \bar{B}(\vec{p}_{\bar{B}}, -q_B, -\lambda_B) + D \,; \tag{9}$$

4) *and* the partly $\hat{C}\hat{P}\hat{T}$-ed and partly $\hat{C}\hat{T}$-ed precesses (2) like

$$\bar{C}(-\vec{p}_{\bar{C}}, -q_C, +\lambda_C) + \bar{D}(\vec{p}_{\bar{D}}, -q_D, -\lambda_D) \to \\ \to \bar{A}(\vec{p}_{\bar{A}}, -q_A, -\lambda_A) + \bar{B}(-\vec{p}_{\bar{B}}, -q_B, +\lambda_B) ; \tag{10}$$

notice explicitly that in eqs. (10) (and following ones) the arrows attached *e.g.* to the trimomenta $\vec{p}_{\bar{C}}$ and $\vec{p}_{\bar{B}}$, appearing inside brackets, *only* record the versus of the transformed (2) trimomenta with respect to the original ones $\vec{p}_C$, $\vec{p}_B$;

5) the « decay processes » of the type (2)

$$\bar{C}(\vec{p}_{\bar{C}}, -q_C, -\lambda_C) \to \bar{A}(-\vec{p}_{\bar{A}}, -q_A, +\lambda_A) + \bar{B}(\vec{p}_{\bar{B}}, -q_B, -\lambda_B) + D \,, \tag{11}$$

when kinematically allowed;

6) the « formation processes » of the type (2)

$$A + \bar{C}(\vec{p}_{\bar{C}}, -q_C, -\lambda_C) + \bar{D}(-\vec{p}_{\bar{D}}, -q_D, +\lambda_D) \to \bar{B}(\vec{p}_{\bar{B}}, -q_B, -\lambda_B) \,, \tag{12}$$

when allowed.

Of course, the kinematical variables [13] $s, t, \ldots$ will have for the different processes the different meanings and values pertaining to them for the new processes. In particular, notice that

$$(\text{SLT})\,s = -s\,, \qquad (\text{SLT})\,t = -t\,. \tag{13}$$

We conclude that:

1) We have derived *crossing relations* [10], even for B's, from mere extended relativity.

2) New « *crossing type* » relations are required by PR: such relations may well serve as a test for relativistic covariance of « force fields » like « strong interactions » and particularly « weak interactions » or, possibly, new « interaction fields » (which *a priori* are not relativistically covariant).

3) Extended relativity itself requires that the same function $A(s, t, \ldots)$ gives the scattering amplitudes of different processes (as channels $s, t, u, \ldots$ of a four-bradyon reaction) in correspondence to their physical domains of $s, t, \ldots$. Therefore, in this framework « *analiticity* [10] *is unnecessary, and is better substituted by the G-covariance requirement* (*i.e.* covariance under the new group G of GLT's, both subluminal and Superluminal) [2].

To further clarify the physical meaning of our procedure, let us lastly observe the following. In a two-body to two-body scattering between elementary particles (let us consider it as the reaction s-channel) the square t of the transferred four-momentum is well known [15] to be generally *negative* [10,16,17]:

$$t = p^2 < 0\,. \tag{14}$$

This quantity is moreover known [15] *both* to become positive *and* to change its meaning (*e.g.* from « square momentum transfer » to « square total energy ») when passing from the s-channel to the t-channel.

This accords to the above-seen fact that a SLT may transform a reaction (*among bradyons*) into the *crossed* one (*among tachyons*).

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(15) V. BERESTESKY, E. M. LIFSHITZ and P. PITAEVSKY: *Relativistic Quantum Theory* (London, 1971), p. 34. See also E. C. G. SUDARSHAN: *Proc. Indian Acad. Sci.*, **67** (5A), 284 (1968).

(16) I. FERRETTI and M. VERDE: *Atti Accad. Sci. Torino* (1966), p. 318; F. T. HADJIOANNOU: *Nuovo Cimento*, **44** A, 185 (1966).

(17) Since 1968, within the framework of « peripheral models » (*e.g.* the one-particle exchange model), the possibility that usual « *virtual particles* » be actually considered as tachyons has been suggested: see quotations in ref. (2).

These points ([18]) help clarifying why recourse to two Superluminal Lorentz transformations (*e.g.* one SLT changing channel *s* for B's into channel *t* for T's and one SLT changing *only* T's into B's, without affecting the channel) is needed to prove *crossing relations* among *bradyons.*

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([18]) Since GLT's may change also the observed *channel*, in order to avoid confusion it is better to consider the action of GLT's on *physical quantities* as the « observed total entering four-momentum » (or the « observed transferred four-momentum »), rather than on their *formal* expressions according to a certain observer (*i.e.* relative to a certain channel). In fact, such expressions for the above-mentioned *physical* quantities may well change when changing observer (*i.e.* when the observed channel changes). See also ref. ([2]).

● RIASSUNTO

In questo articolo si derivano la « relazioni d'incrocio » dell'usuale fisica delle particelle elementari (ad alta energia), sulla sola base della covarianza relativistica, estesa anche a sistemi di riferimento Super-luminali. In un tale contesto il postulato di « analiticità » non risulta necessario, e viene anzi sostituito dalla richiesta di G-covarianza (cioè di covarianza sotto il nuovo gruppo G delle trasformazioni « di Lorentz » generalizzate). Si predicono inoltre nuove relazioni del tipo di quelle d'incrocio, sulla base sempre della « relatività estesa ». Esse potrebbero servire come test della covarianza relativistica di « campi di forza » come quelli delle interazioni forti e, specialmente, delle interazioni deboli, e per altri eventuali nuovi « campi d'interazione » (i quali *a priori* non sono relativisticamente covarianti).

Вывод « перекрестных соотношений » и их физический смысл.

Резюме (*). — В этой статье на основе Лорентц-ковариантности, обобщенной также на случай сверхсветовых инерциальных систем отсчета, выводятся « перекрестные соотношения » для физики элементарных частиц при высоких энергиях. В этом подходе постулат « аналитичности » не является необходимым, и он заменяется требованием G-ковариантности (т.е. ковариантности относительно новой G группы обобщенных преобразований Лорентца, как субсветовых так и сверхсветовых). Кроме того, новые « перекрестные соотношения » предсказываются на основе « протяженной относительности ». Они также могут служить проверкой релятивистской ковариантности « силовых полей », подобных сильным взаимодействиям и, в частности, слабым взаимодействиям, и возможных новых « полей взаимодействия » (которые, априори, не являются релятивистски ковариантными).

(*) *Переведено редакцией.*