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Considerations about the Apparent « Superluminal Expansions » Observed in Astrophysics (*).

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Summary. — The orthodox models devised to explain the apparent « superluminal expansions » observed in astrophysics—and here briefly summarized and discussed together with the experimental data—do not seem to be too much successful, especially when confronted with the most recent observations, suggesting complicated expansion patterns, even with possible accelerations. At this point it may be, therefore, of some interest to explore the possible alternative models in which actual Superluminal motions take place. To prepare the ground, we start from a variational principle, introduce the elements of a tachyon mechanics within special relativity, and argue about the expected behaviour of tachyonic objects when interacting (gravitationally, for instance) among themselves or with ordinary matter. We then review and develop the simplest « Superluminal models », paying particular attention to the *observations* which they would give rise to. We conclude that some of them appear to be physically acceptable and are statistically favoured with respect to the orthodox ones.

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1. – Introduction.

The particular—and unreplaceable—role in special relativity (SR) of the light speed, c, in vacuum is due to its *invariance* (namely, to the experimental fact that c does not depend on the velocity of the source), and *not* to its being or not the maximal speed (1).

The subject of tachyons, even if still speculative (2), may deserve some attention for reasons that can be divided into a few categories, some of which we want to mention right now: i) the larger scheme that one tries to build up in order to incorporate spacelike objects in the relativistic theories can allow a better comprehension of many aspects of the ordinary relativistic physics, even if tachyons would not exist in our cosmos as « asymptotically free » objects; ii) Superluminal classical objects can have a role in elementary-particle or quantum interactions; iii) they may have a role even in astrophysics. Let us moreover recall that, in general relativity (GR), spacelike geodesics are « at home », so that tachyons have often been implicit ingredients of GR (3).

In this paper let us fix our attention on the problem of the apparent « superluminal expansions » in astrophysics.

2. – The apparent superluminal expansions.

The theoretical possibility of Superluminal motions in astrophysics has been considered since long (4).

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(1) See, *e.g.*, E. RECAMI and E. MODICA: *Lett. Nuovo Cimento*, **12**, 263 (1975), and references therein.
(2) See, *e.g.*, E. RECAMI, Editor: *Tachyons, Monopoles, and Related Topics* (North-Holland, Amsterdam, 1978); P. SMRZ: *Lett. Nuovo Cimento*, **41**, 327 (1984).
(3) See, *e.g.*, R. W. FULLER and J. A. WHEELER: *Phys. Rev.*, **128**, 919 (1962); R. SACHS and W. WU: *General Relativity for Mathematicians* (Springer, Berlin, 1980).
(4) C. GREGORY: *Nature (London)*, **206**, 702 (1965); *Nature Phys. Sci.*, **239**, 56 (1972); R. MIGNANI and E. RECAMI: *Nuovo Cimento B*, **21**, 210 (1974); *Gen. Rel. Grav.*, **5**, 615 (1974); E. RECAMI: unpublished work (1974) (seminars, private communications, computer work, and pieces of work in collaboration with H. B. NIELSEN, etc.); in *Topics in Theoretical and Experimental Gravitation Physics*, edited by V. DE SABBATA and J. WEBER (Plenum Press, New York, N. Y., 1977), p. 305; in *Tachyons, Monopoles, and Related Topics*, edited by E. RECAMI (North-Holland, Amsterdam, 1978), p. 3; in *Relativity, Quanta and Cosmology in the Development of the Scientific Thought of A. Einstein*, edited by M. PANTALEO and F. DE FINIS (Johnson Rep. Co., New York, N. Y., 1979), Vol. **2**, Chapt. 16, p. 537. The last reference appeared also in Italian, and in Russian (as Chapt. 4 in *Astrofizika, Kvanti i Teorya Otnositelnosti*, edited by F. I. FEDOROVA (Mir, Moscow, 1982), p. 53).

Experimental investigations, started long ago as well ([5]), led at the beginning of the Seventies to the claim that radiointerferometric observations had revealed—at least in the two quasars 3C279, 3C273 and in the Seyfert type-I galaxy 3C120—expansion of small radio components at velocities apparently a few times greater than that of light ([6]). The first claims were followed by extensive collections of data, all obtained by very-long-baseline-interferometry (VLBI) systems with many radio telescopes; reviews of the experimental data can be found in COHEN *et al.*, KELLERMAN, and COHEN and UNWIN ([7]): see also SCHILLIZZI and BRUYN ([7]). The result is, *grosso modo*, that the nucleus of seven strong radiosources (six quasars, 3C273, 3C279, 3C345, 3C179, NRAO-140, BL Lac, and one galaxy, 3C120) consists of two components which appear to recede from each other with Superluminal relative speeds ranging from a few c to a few tens c (cf. ref. ([8])). A result so puzzling that the journal *Nature* even devoted one of its covers (April 2, 1981) to the superluminal expansion exhibited by quasar 3C273. Simplifying it, the experimental situation can be summarized as follows:

i) the Superluminal relative motion of the two components is always a collinear recession;

ii) such a Superluminal « expansion » seems endowed with a roughly constant velocity, which does not depend on the observed wave-length;

iii) the flux density ratio for the two components, F_1/F_2, does depend on the (observed) wave-length and time.

Apparently, those strong radiosources exibit a compact inverted-spectrum

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([5]) H. J. SMITH and D. HOFFLEIT: *Nature (London)*, **198**, 650 (1963); C. A. KNIGHT, D. S. ROBERTSON, A. E. E. ROGERS, I. I. SHAPIRO, A. R. WHITNEY, T. A. CLARK, R. M. GOLDSTEIN, G. E. MARANDINO and N. R. VANDERBERG: *Science*, **172**, 52 (1971).

([6]) A. R. WHITNEY, I. I. SHAPIRO, A. E. E. ROGERS, D. S. ROBERTSON, C. A. KNIGHT, T. A. CLARK, R. M. GOLDSTEIN, G. E. MARANDINO and N. R. VANDERBERG: *Science*, **173**, 225 (1971); M. H. COHEN, W. CANNON, G. H. PURCELL, D. E. SHAFFER, J. J. BRODERICK, K. I. KELLERMAN and D. L. JAUNCEY: *Astrophys. J.*, **170**, 207 (1971); D. B. SHAFFER, M. H. COHEN, D. L. JAUNCEY and K. I. KELLERMAN: *Astrophys. J. Lett.*, **173**, L147 (1972); I. I. SHAPIRO, H. F. INTEREGGER, C. A. KNIGHT, J. J. PUNSKY, D. S. ROBERTSON, A. E. E. ROGERS, A. R. WHITNEY, T. A. CLARK, G. E. MARANDINO and R. M. GOLDSTEIN: *Astrophys. J. Lett.*, **183**, L47 (1973).

([7]) M. H. COHEN, K. I. KELLERMANN, D. B. SHAFFER, R. P. LINFIELD, A. T. MOFFET, J. D. ROMNEY, G. A. SEIELSTAD, I. I. K. PAULINY-TOTH, E. PREUSS, A. WITZEL, R. T. SCHILLIZZI and B. J. GELDZAHLER: *Nature (London)*, **268**, 405 (1977); K. I. KELLERMANN: *Ann. N. Y. Acad. Sci.*, **336**, 1 (1980); M. H. COHEN and S. C. UNWIN: *IAU Symposium* No. 97 (1982), p. 345. See also R. T. SCHILLIZZI and A. G. DE BRUYN: *Nature (London)*, **303**, 26 (1983).

([8]) I. I. K. PAULINY-TOTH, E. PREUSS, A. WITZEL, D. GRAHAM, K. I. KELLERMANN and R. RÖNNÄNG: *Astron. J.*, **86**, 371 (1981).

core component (usually variable) and one extended component which separate from the core with Superluminal velocity. But it is not yet clear whether the compact core is indeed stationary or it too moves. The extended component seems to become weaker with time and more rapidly at high frequencies.

The most recent results, however, seem to show that—at least in quasar 3C345—the situation may be more complex (9). In the same quasar an « extended component » does even appear to accelerate away with time (10).

Many theoretical models were soon devised to explain the apparent Superluminal expansions in an orthodox way (6,11). Reviews of the orthodox models can be found in (12), and in PORCAS (9).

The most successful and, therefore, most popular models resulted to be:

a) *The relativistic jet model*: a relativistically moving stream of plasma is supposed to emanate from the core. The compact core of the « superluminal » sources is identified with the base of the jet and the « moving » component is a shock or plasmon moving down the jet. If the jet points at a small angle α towards the observer, the apparent separation speed becomes Superluminal since the radiation coming from the knot has to travel a shorter distance. Namely, if v is the knot speed with respect to the core, the apparent recession speed $[c = 1]$ will be $w = v \sin\alpha/(1 - v\cos\alpha)$, with $v \geqslant w/(1+w^2)^{\frac{1}{2}}$. The *maximal* probability for a relativistic jet to have the orientation required for producing the apparent Superluminal speed $\bar{w}$—independently of the jet speed v—is $P(\bar{w}) = (1+\bar{w}^2)^{-1} < 1/\bar{w}^2$ (BLANDFORT *et al.* (12), FINKELSTEIN *et al.* (13), CASTELLINO (14)). The relativistic jet models, therefore, for the observed « super-

(9) S. C. UNWIN, M. H. COHEN, J. J. PEARSON, G. A. SEIELSTAD and R. S. SIMON: *Astrophys. J.*, **271**, 536 (1983); A. C. S. READHEAD, O. H. HOUGH, M. S. EWING and J. D. ROMNEY: *Astrophys. J.*, **265**, 107 (1983); J. A. BIRETTA, M. H. COHEN, S. C. UNWIN and I. I. K. PAULINY-TOTH: *Nature (London)*, **306**, 42 (1983); R. PORCAS: *Nature (London)*, **302**, 753 (1983).

(10) R. L. MOORE, A. C. S. READHEAD and L. BAATH: *Nature (London)*, **306**, 44 (1982). See also J. J. PEARSON, S. C. UNWIN, M. H. COHEN, R. P. LINFIELD, A. C. S. READHEAD, G. A. SEIELSTAD, R. S. SIMON and R. C. WALKER: *Nature (London)*, **290**, 365 (1981).

(11) M. J. REES: *Nature (London)*, **211**, 46 (1966); A. CAVALIERE, P. MORRISON and L. SARTORI: *Science*, **173**, 525 (1971); W. A. DENT: *Science*, **175**, 1105 (1972); R. H. SANDERS: *Nature (London)*, **248**, 390 (1974); R. L. EPSTEIN and M. J. GELLER: *Nature (London)*, **265**, 219 (1977).

(12) R. D. BLANDFORT, C. F. MCKEE and M. J. REES: *Nature (London)*, **267**, 211 (1977); P. A. G. SHEUER and A. C. S. READHEAD: *Nature (London)*, **277**, 182 (1979); A. P. MARSCHER and J. S. SCOTT: *Publ. Astron. Soc. Pac.*, **92**, 127 (1980); M. J. OOR and I. W. A. BROWNE: *Mon. Not. R. Astron. Soc.*, **200**, 1067 (1982).

(13) A. M. FINKELSTEIN, V. JA. KREINOVICH and S. N. PANDEY: preprint (Special Astrophysics Observatory, Pulkovo, 1983), unpublished.

(14) A. CASTELLINO: *A theoretical approach to the study of some apparent superluminal expansions in astrophysics*, M.S. Thesis work, supervised by E. RECAMI (University of Catania, Catania, 1984).

luminal» speeds suffer from statistical objections, even if selection effects can play in favour of them (see, *e.g.*, ([15]) and PEARSON *et al.* ([10])).

b) *The «screen» models*: the «superluminal» emissions are triggered by a relativistic signal coming from a central source and «illuminating» a pre-existing screen. For instance, for a spherical screen of radius R illuminated by a concentric spherical relativistic signal, the distant observer would see a circle expanding with speed $w \simeq 2c(R-ct)/(2Rct-c^2t^2)^{\frac{1}{2}}$; such a speed will be superluminal in the time interval $0<t<\frac{1}{2}(2-\sqrt{2})R/c$ only. When the screen is a ring, the observer would see an expanding double source. The defect of such models is that the apparent expansion speed will be $w \geqslant \bar{w}$ (with $\bar{w} \gg 2c$) only for a fraction $c^2/\bar{w}^2$ of the time during which the radiosource exibits its variations. Of course, one can introduce «oriented» screens—or *ad hoc* screens—, but they are statistically unfavoured (BLADFORT *et al.* ([12]), CASTELLINO ([14])).

c) *Other models*: many previous (unsuccessful) models have been abandoned. The gravitational-lens models did never find any observational support, even if a new type of model (where the magnifying lens is just surrounding the source) has been recently suggested by LIAOFU and CHONGMING ([16]).

In conclusion, the orthodox models are not too much successful, especially if the more complicated superluminal expansions (*e.g.*, with acceleration) recently observed will be confirmed.

It may be of some interest, *therefore*, to explore the possible alternative models in which actual Superluminal motions take place (cf., *e.g.*, MIGNANI and RECAMI ([4])).

To prepare the ground, in sect. **3** and **4** we shall develop some tachyon mechanics within SR. Before going on, however, let us immediately put forth the following (simple, but important) remark, valid at least in two dimensions.

Let us consider in SR two bradyonic (= slower than light) bodies A and B that—owing to mutual attraction—for instance *accelerate while approaching each other*. The situation is sketched in fig. 1, where A is chosen as the reference frame $s \equiv (t, x)$ and, for simplicity's sake, only a discrete change of velocity is depicted. From a Superluminal frame they will be described either as two tachyons that *decelerate while approaching* each other (as seen from the frame $S'' \equiv (t'', x'')$), or as two antitachyons ([17,18]) that *accelerate while receding* from

([15]) R. W. PORCAS: *Nature (London)*, **294**, 47 (1981); unsigned: *Sci. News*, **119**, 229 (1981); G. POOLEY: *Nature (London)*, **290**, 363 (1981).

([16]) L. LIAOFU and XU CHONGMING: in *X International Conference on Relativity and Gravitation* (GR 10), edited by B. BERTOTTI, F. DE FELICE and A. PASCOLINI (CNR, Roma, 1984), p. 749.

([17]) See, *e.g.*, E. RECAMI: *Found. Phys.*, **8**, 329 (1978); P. CALDIROLA and E. RECAMI: in *Italian Studies in the Philosophy of Science*, edited by M. DALLA CHIARA (Reidel, Boston, Mass., 1980), p. 249, and references therein. See also E. RECAMI and W.

each other (as seen from the frame $S' \equiv (t', x')$). Therefore, we expect that two tachyons *from the kinematical point of view* will seem to suffer a repulsion, if they attract each other in their own rest frames (and in the other frames in which they are subluminal); we shall, however, see that such a behaviour of tachyons can be still considered—from the more important dynamical and energetical point of view—as due to an *attraction*.

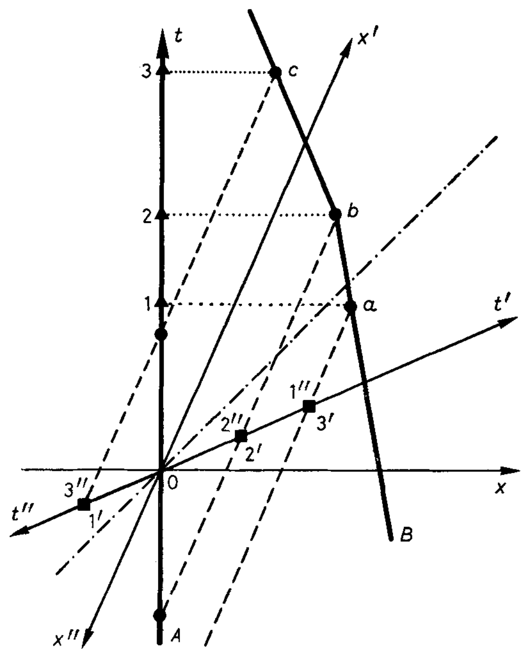

Fig. 1. – Let us consider two bradyonic (= slower than light) objects A and B in two dimensions. Let B accelerate while approaching A, due to a mutual attraction. Then, from a Superluminal frame they will be described i) either as two tachyons that *decelerate while approaching* each other (from the frame $S'' \equiv (t'', x'')$), ii) or as two anti-tachyons [17,18] that *accelerate while receding* from each other (from the frame $S' \equiv \equiv (t', x')$). See also the text.

3. – Some preliminary tachyon mechanics in SR.

3'1. *On the variational principle: a digression.* – Let us first consider the action S for a free object. In the ordinary case it is $S = \alpha\int_a^b ds$; for a free tachyon

RODRIGUES: *Found. Phys.*, **12**, 709 (1982); **13**, 533 (1983); P. PAVŠIČ and E. RECAMI: *Lett. Nuovo Cimento*, **34**, 357 (1982); E. RECAMI: last reference ([4]).

([18]) Cf., *e.g.*, E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209, 398 (1974), hereafter called review I.

let us, tentatively, write

$$S = \alpha \int_a^b |\mathrm{d}s| \, . \tag{1}$$

By analogy with the bradyonic case, we might assume for a free tachyon the Lagrangian ($c = 1$)

$$L = + m_0 \sqrt{V^2 - 1} \qquad (V^2 > 1), \tag{2}$$

and therefore evaluate, in the usual way,

$$\boldsymbol{p} \equiv \frac{\partial L}{\partial \boldsymbol{V}} = + \frac{m_0 \boldsymbol{V}}{\sqrt{V^2 - 1}} \equiv m\boldsymbol{V} \, , \tag{3}$$

which suggests that for tachyons

$$m = \frac{m_0}{\sqrt{V^2 - 1}} \, . \tag{4}$$

If the tachyon is no longer free, we can write as usual

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m_0 \boldsymbol{V}}{\sqrt{V^2 - 1}}\right). \tag{5}$$

By choosing the reference frame, at the considered time instant t, in such a way that $\boldsymbol{V}$ is parallel to the x-axis, *i.e.* $|\boldsymbol{V}| = V_x$, we then get

$$F_x = + m_0 \left[\frac{1}{\sqrt{V^2 - 1}} - \frac{V^2}{\sqrt{(V^2 - 1)^3}}\right] a_x = - \frac{m_0}{(V^2 - 1)^{\frac{3}{2}}} a_x \tag{6a}$$

and

$$F_y = + \frac{m_0}{\sqrt{V^2 - 1}} a_y \, , \qquad F_z = + \frac{m_0}{\sqrt{V^2 - 1}} a_z \, . \tag{6b}$$

The sign in eq. (6*a*) is consistent with the ordinary definition of work $\mathscr{L}$

$$\mathrm{d}\mathscr{L} \equiv + \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{l} \tag{7}$$

and the fact that the total energy of a tachyon increases when its speed *decreases* (as is well known [18]).

Notice, however, that the proportionality constant between force and acceleration does *change sign* when passing from the longitudinal to the transverse components.

The tachyon total energy E, moreover, can still be defined as

$$E \equiv \boldsymbol{p}\cdot\boldsymbol{V} - L = \frac{m_0 c^2}{\sqrt{V^2-1}} \equiv mc^2 , \tag{8}$$

which, together with eq. (4), extends to tachyons the relation $E = mc^2$.

However, the following comments are in order at this point. An ordinary timelike (straight) line can be bent only in a spacelike direction; and it gets shorter. On the contrary, if you take a spacelike line and, keeping two points on it fixed, bend it slightly in between in a spacelike (timelike) direction, the bent line is longer (shorter) than the original straight line (see, *e.g.*, ([19])). For simplicity, let us here skip the generic case when the bending is partly in the timelike and partly in a spacelike direction (even if such a case looks to be the most interesting). Then, the action integral $\int_a^b |\mathrm{d}s|$ of eq. (1) along the straight (spacelike) line is minimal with respect to the « spacelike » bendings and maximal with respect to the « timelike » bendings. *A priori*, one might then choose for a free tachyon, instead of eq. (2), the Lagrangian

$$L = -m_0\sqrt{V^2-1} , \tag{2'}$$

which yields

$$\boldsymbol{p} \equiv \frac{\partial L}{\partial \boldsymbol{V}} = -\frac{m_0 \boldsymbol{V}}{\sqrt{V^2-1}} \equiv -m\boldsymbol{V} . \tag{3'}$$

Equation (3') becomes rather interesting, *e.g.*, when tachyons are substituted for the « virtual particles » as the carriers of the elementary-particle interactions ([20]). In fact, the (classical) exchange of a tachyon endowed with a momentum antiparallel to its velocity would generate an *attractive* interaction.

For nonfree tachyons, from eq. (3') one gets

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m_0 \boldsymbol{V}}{\sqrt{V^2-1}}\right) \tag{5'}$$

and, therefore, when $|\boldsymbol{V}| = V_x$,

$$F_x = +\frac{m_0}{(V^2-1)^{\frac{3}{2}}} a_x , \tag{6'a}$$

$$F_y = -\frac{m_0}{\sqrt{V^2-1}} a_y , \qquad F_z = -\frac{m_0}{\sqrt{V^2-1}} a_z . \tag{6'b}$$

([19]) See, *e.g.*, J. DORLING: *Am. J. Phys.*, **38**, 539 (1970).

([20]) See, *e.g.*, P. CASTORINA and E. RECAMI: *Lett. Nuovo Cimento*, **22**, 195 (1978), and references therein; G. D. MACCARRONE and E. RECAMI: *Nuovo Cimento A*, **57**, 85 (1980).

Due to the sign in eq. ($6'a$), it is now necessary to define the work $\mathscr{L}$ as

$$\mathrm{d}\mathscr{L} \equiv -\boldsymbol{F}\cdot\mathrm{d}\boldsymbol{l}\,, \tag{7'}$$

and analogously the total energy E as

$$E \equiv -(\boldsymbol{p}\cdot\boldsymbol{V} - L) = \frac{m_0 c^2}{\sqrt{V^2-1}} \equiv mc^2\,. \tag{8'}$$

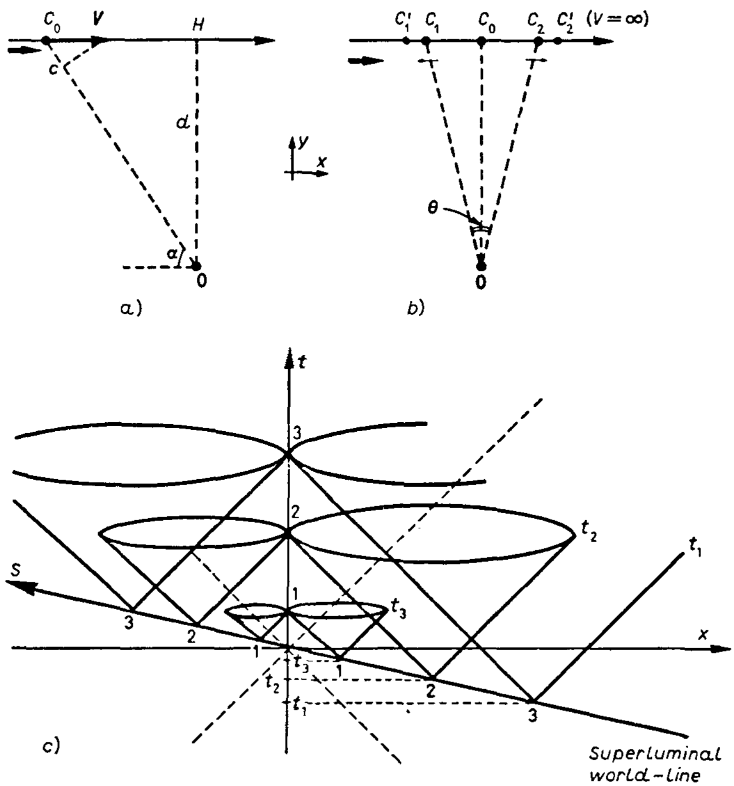

Fig. 2. – A *single* Superluminal object, observed through the radiation emitted by it, will appear as a *couple* of objects receding from each other with Superluminal relative speed. See the text.

3·2. *On radiating tachyons.* – Notice that the previous results in subsect. **3·1** are quite independent of the eventual existence (or not) of Superluminal Lorentz transformations (SLT).

Here, as a further example of results actually independent of the very existence of SLTs, let us report the fact that a tachyon—when seen by means of its electromagnetic emissions (see review I and ([21]))—will appear, in general, as occupying *two positions* at the same time ([4,22]). Let us start by considering a macro-object C emitting spherical electromagnetic waves (fig. 2c)). When we see it travelling at constant Superluminal velocity $\boldsymbol{V}$, because of the distortion due to the large relative speed $|\boldsymbol{V}| > c$, we shall observe the electromagnetic waves to be internally tangent to an enveloping cone Γ having as its axis the motion line of C (([23]), review I); even if this cone has nothing to do with Čerenkov's ([24]). This is analogous to what happens with an airplane moving at a constant supersonic speed in the air. A first observation is the following: as we hear a sonic boom when the sonic contact with the supersonic airplane does start ([25]), so we shall analogously *see* an « optic boom » when we first enter in radiocontact with the body C, *i.e.* when we meet the Γ-cone surface. In fact, when C is seen by us under the angle (fig. 2a))

$$V\cos\alpha = c \qquad (V \equiv |\boldsymbol{V}|)\,, \tag{9}$$

all the radiations emitted by C in a certain time interval around its position C_0 reach us simultaneously. Soon after, we shall receive at the same time the light emitted from suitable *couples of points*, one on the left and one on the right of C_0. We shall thus see the initial body C, at C_0, split in two luminous objects C_1, C_2 which will then be *observed* to recede from each other with the Superluminal « transverse » relative speed W ([26,22]):

$$W = 2b\,\frac{1 + d/bt}{[1 + 2d/bt]^{\frac{1}{2}}}\,, \qquad b \equiv \frac{V}{\sqrt{V^2 - 1}} \qquad (V^2 > 1)\,, \tag{10}$$

where $d \equiv \overline{OH}$, and $t = 0$ is just the time instant when the observer enters in radiocontact with C, or rather sees C at C_0. In the simple case in which C moves with almost infinite speed (fig. 2b)), the apparent *relative* speed of C_1 and C_2 varies in the initial stage as $W \simeq (2cd/t)^{\frac{1}{2}}$, where now $\overline{OH} = \overline{OC_0}$, while $t = 0$ is still the instant at which the observer sees $C_1 \equiv C_2 \equiv C_0$.

We shall come back to this subject in the following. Here let us add the observation that the radiation associated with one of the images of C (namely, the radiation emitted by C while approaching us, in the simple case depicted

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([21]) M. BALDO, G. FONTE and E. RECAMI: *Lett. Nuovo Cimento* (first series), **4**, 241 (1970).
([22]) A. O. BARUT, G. D. MACCARRONE and E. RECAMI: *Nuovo Cimento A*, **71**, 509 (1982). See also Ø. GRØN: *Lett. Nuovo Cimento*, **23**, 97 (1978).
([23]) E. RECAMI and R. MIGNANI: *Lett. Nuovo Cimento*, **4**, 144 (1972).
([24]) R. MIGNANI and E. RECAMI: *Lett. Nuovo Cimento*, **7**, 388 (1973).
([25]) H. BONDI: *Relativity and Common Sense* (Doubleday, New York, N. Y., 1964).
([26]) E. RECAMI, H. B. NIELSEN, H. C. CORBEN, G. D. MACCARRONE and S. GENOVESI: unpublished work (1976).

in fig. 2c)) will be received by us in the *reversed* chronological order; cf. ([27]).

In may be interesting to quote that the circumstance that the image of a tachyon suddenly appears at a certain position C_0 and then splits into two images was already met by BACRY ([28]) and BACRY *et al.* ([28]) while exploiting a group-theoretical definition of the motion of a charged particle in a homogeneous field; definition which was valid for all kinds of particles (bradyons, luxons, tachyons). Analogous solutions, simulating a pair production, have been later on found even in the *subluminal* case by BARUT ([28]), when exploring *nonlinear* evolution equations, and by SALA ([28]), by merely taking account of the finite speed of the light which carries the image of a moving subluminal object. SALA ([28]) did even rediscover—also in subluminal cases—that one of the two images can display a time-reversed evolution.

4. – Some more tachyon mechanics in SR.

While the results in sect. **3** do not depend at all on the eventual existence of SLTs, to go on we need now adopting the following *Assumption*. Namely, let us assume in this section **4** that *such* « transformations » exist in four dimensions (even if at the price of giving up possibly one of the ordinary properties of the Lorentz transformations: see ref. ([29])) *that* carry timelike into spacelike tangent vectors, and *vice versa*. Incidentally they are known to exist in two [(1, 1)] dimensions, as well as in (n, n) dimensions ([29]). Their actual existence in four [(1, 3)] dimensions has been claimed, for instance, by SHAH ([30]) within the « quasi-catastrophes » theory (cf. also SMRZ ([2])). We shall call ([29]) Superluminal Lorentz transformations those « transformations »; let us repeat that, to proceed with, we need nothing but the previous Assumption. The laws of classical physics for tachyons could then be derived just by applying a SLT to the ordinary classical laws of bradyons (cf. ([18,31])).

It is noticeable that tachyon classical physics can then be obtained in terms of purely real quantities. (Notice, moreover, that subsect. **4·1** and **4·2** below do contain improvements with respect, *e.g.*, to review I.)

([27]) R. MIGNANI and E. RECAMI: *Nuovo Cimento A*, **14**, 169 (1973); **16**, 208 (1973); E. RECAMI: in *Topics in Theoretical and Experimental Gravitation Physics*, edited by V. DE SABBATA and J. WEBER (Plenum Press, New York, N. Y., 1977), p. 305.

([28]) H. BACRY: *Phys. Today*, **25**, No. 11, 15 (1972); H. BACRY, PH. COMBE and P. SORBA: preprint 72/449 (Marseille, 1972); A. O. BARUT: *Phys. Lett. A*, **67**, 257 (1978); K. L. SALA: *Phys. Rev. A*, **19**, 2377 (1979).

([29]) See G. D. MACCARRONE and E. RECAMI: *Found. Phys.*, **14**, 367 (1984); G. D MACCARRONE, M. PAVŠIČ and E. RECAMI: *Nuovo Cimento B*, **73**, 91 (1983).

([30]) K. T. SHAH: *Lett. Nuovo Cimento*, **18**, 156 (1977); in *Tachyons, Monopoles, and Related Topics*, edited by E. RECAMI (North-Holland, Amsterdam, 1978), p. 49.

([31]) L. PARKER: *Phys. Rev.*, **188**, 2287 (1969).

4'1. *Tachyon motion equation.* – For example, the fundamental law of bradyon dynamics reads

$$F^\mu = c\,\frac{\mathrm{d}}{\mathrm{d}s}\left(m_0 c\,\frac{\mathrm{d}x^\mu}{\mathrm{d}s}\right) \equiv \frac{\mathrm{d}}{\mathrm{d}\tau_0}\left(m_0\,\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau_0}\right) \qquad (\beta^2 < 1)\,. \tag{11}$$

Notice that eq. (11) in its first form is only Lorentz covariant, while in its second form is G-covariant (cf., *e.g.*, review I).

Even for tachyons, then, we shall have [18]

$$F^\mu = +\frac{\mathrm{d}}{\mathrm{d}\tau_0}(m_0 u^\mu) \equiv +\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau_0} \qquad (\beta^2 > 1)\,, \tag{12}$$

where m_0 is the tachyon (real) rest mass and we defined $p^\mu \equiv m_0 u^\mu$ also for tachyons. Equation (12) is the relativistic Newton law written in G-covariant form: *i.e.* it is expected to hold for $\beta^2 \gtrless 1$. It is essential to recall, however, that u^μ is to be defined as $u^\mu \equiv \mathrm{d}x^\mu/\mathrm{d}\tau_0$ [18,29]. Quantity $\mathrm{d}\tau_0$, where τ_0 is the proper time, is, of course, G-invariant; on the contrary, $\mathrm{d}s = \pm c\,\mathrm{d}\tau_0$ for bradyons, but $\mathrm{d}s = \pm ic\,\mathrm{d}\tau_0$ for tachyons.

Equation (12) agrees with eqs. (5) and (5′) of sect. **3**, where we set $\boldsymbol{F} = \mathrm{d}\boldsymbol{p}/\mathrm{d}t$, and suggests that for tachyons $\mathrm{d}t = \pm\,\mathrm{d}\tau_0/\sqrt{\beta^2-1}$ (see review I), so that in G-covariant form $\mathrm{d}t = \pm\,\mathrm{d}\tau_0(|1-\beta^2|)^{-\frac{1}{2}}$.

For the tachyon case, let us notice the following. If at the considered time instant t we choose the x-axis so that $V \equiv |\boldsymbol{V}| = V_x$, then only the force component F_x will make work. We already mentioned that the total energy of a tachyon decreases when its speed increases, and *vice versa*; it follows that F_x when applied to a tachyon will actually make a positive, elementary work $\mathrm{d}\mathscr{L}$ only if a_x is antiparallel to the elementary displacement $\mathrm{d}x$, *i.e.* if $\operatorname{sign}(a_x) = -\operatorname{sign}(\mathrm{d}x)$. In other words, $\mathrm{d}\mathscr{L}$ in the case of a force $\boldsymbol{F}$ applied to a tachyon *must be defined* (cf. subsect. **3**'1) so that

$$\mathrm{d}\mathscr{L} = -\frac{m_0}{(V^2-1)^{\frac{3}{2}}}\,a_x\,\mathrm{d}x\,, \tag{13}$$

where a_x and $\mathrm{d}x$ possess, of course, their own sign. Equation (13) does agree both with the couple of equations (6a), (7) and with the couple of equations (6′a), (7′).

It is evident that, *with the choice* (review I) represented by eqs. (7) and (2) of subsect. **3**'1, we shall have ($v = v_x$, $V = V_x$)

$$F_x = +\frac{m_0}{(1-v^2)^{\frac{3}{2}}}\,a_x \qquad \text{for bradyons}\,, \tag{14a}$$

$$F_x = -\frac{m_0}{(V^2-1)^{\frac{3}{2}}}\,a_x \qquad \text{for tachyons}\,. \tag{14b}$$

On the contrary, still with the choice (7)-(2), we shall have

$$F_{y,z} = + \frac{m_0}{(|1-\beta^2|)^{\frac{1}{2}}} a_{y,z} \tag{14c}$$

for *both* bradyons and tachyons. Actually, under our hypotheses ($v = v_x$, $V = V_x$), the force transverse components $F_{y,z}$ do not make any work; therefore, one had no reasons *a priori* for expecting any change in eq. (14c) when passing from bradyons to tachyons.

4'2. *Gravitational interactions of tachyons.* – In any gravitational field a bradyon feels the (attractive) gravitational 4-force

$$F^\mu = - m_0 \Gamma^\mu_{\varrho\sigma} \frac{\mathrm{d}x^\varrho}{\mathrm{d}s} \frac{\mathrm{d}x^\sigma}{\mathrm{d}s} \qquad (\beta^2 < 1)\,. \tag{15}$$

In G-covariant form, then, eqs. (15) (see review I, MIGNANI and RECAMI [4,18] and RECAMI [27]) are expected to read

$$F^\mu = - \frac{m_0}{c^2} \Gamma^\mu_{\varrho\sigma} \frac{\mathrm{d}x^\varrho}{\mathrm{d}\tau_0} \frac{\mathrm{d}x^\sigma}{\mathrm{d}\tau_0} \qquad (\beta^2 \gtrless 1)\,, \tag{16}$$

since the Christoffel symbols behave like (third-rank) tensors under *any* linear transformations of the co-ordinates. Equations (16) hold in particular for a tachyon in any gravitational field (both when originated by tachyonic and by bradyonic sources).

Analogously, the equation of motion for both bradyons and tachyons in a gravitational field will still read (review I), in G-covariant form,

$$a^\mu + \Gamma^\mu_{\varrho\sigma} u^\varrho u^\sigma = 0 \qquad (\beta^2 \gtrless 1) \tag{17}$$

with $a^\mu \equiv \mathrm{d}^2 x^\mu / \mathrm{d}\tau_0^2$.

Passing to general relativity, this does agree with the equivalence principle: bradyons, photons and tachyons follow different trajectories in a gravitational field, which depend only on the initial (different) four-velocities and are independent of the masses.

Going back to eqs. (16), we may say that also tachyons are attracted by a gravitational field. However, such an « *attraction* » has to be understood from the energetical and dynamical point of view only.

In fact, if we consider for simplicity a tachyon moving radially with respect to a gravitational source, due to eq. (14b) (*i.e.* due to the couples of equations either (6a)-(7), or (6'a)-(7')) it will *accelerate* when *receding* from the source, and *decelerate* when *approaching* the source. From the kinematical point of view, therefore, we can say that tachyons *seem* to be gravitationally *repelled.*

Analogous results were put forth by VAIDYA, RAYCHAUDHURI, HONIG *et al.* [32], and so on.

In the case of a bradyonic *source*, what precedes agrees with the results obtained within general relativity: see, *e.g.*, [13,33], etc.

4'3. *About Doppler effect.* – In the two-dimensional case, the Doppler-effect formula for a sub- or a Super-luminal source, moving along to x-axis, is (MIGNANI and RECAMI [27])

$$\nu = \nu_0 \frac{\sqrt{|1-u^2|}}{1 \pm u} \qquad (-\infty < u < +\infty), \tag{18a}$$

where the sign $-$ $(+)$ corresponds to approach (recession). The consequences are depicted in figures like fig. 23 of review I. For Superluminal approach, ν happens to be negative, so as explained by our figure 2c). Let us moreover observe that, in the case of recession, *the same* Doppler shift is associated both with $\bar{u} < c$ and with $\bar{U} \equiv c^2/\bar{u} > c$ [27,34].

In the four-dimensional case, if the observer it still located at the origin, eq. (18a) is expected to generalize (see RECAMI and MIGNANI [18,34]) into

$$\nu = \nu_0 \frac{\sqrt{|1-u^2|}}{1 - u\cos\alpha} \qquad (-\infty < u < +\infty), \tag{18b}$$

where $\alpha \equiv \widehat{\boldsymbol{us}}$, vector $\boldsymbol{s}$ being directed from the source to the observer. Let us notice from subsect. **3**'2 (eq. (9)), incidentally, that, when an observer *starts* receiving radiation from a Superluminal pointlike source C (at C_0, *i.e.* in the « optic-boom » situation), the received radiation is infinitely blue-shifted.

5. – The model with a single (Superluminal) source.

The simplest Superluminal model is the one of a single Superluminal source. In fact, we have seen in subsect. **3**'2 (see fig. 2) that a *single* Superluminal source

(32) P. C. VAIDYA: *Curr. Sci. (India)*, **40**, 651 (1971); A. K. RAYCHAUDHURI: *J. Math. Phys. (N. Y.)*, **15**, 256 (1974); E. HONIG, K. LAKE and R. C. ROEDER: *Phys. Rev. D*, **10**, 3155 (1974).

(33) F. SALTZMAN and G. SALTZMAN: *Lett. Nuovo Cimento*, **1**, 859 (1969); C. GREGORY: *Nature Phys. Sci.*, **239**, 56 (1972); R. O. HETTEL and T. M. HELLIWELL: *Nuovo Cimento B*, **13**, 82 (1973); C. P. SUM: *Lett. Nuovo Cimento*, **11**, 459 (1974); J. V. NARLIKAR and E. C. G. SUDARSHAN: *Mon. Not. R. Astron. Soc.*, **175**, 105 (1976); J. V. NARLIKAR and S. V. DHURANDHAR: *Pramāna*, **6**, 388 (1976); R. P. COMER and J. D. LATHROP: *Am. J. Phys.*, **46**, 801 (1978); V. K. MALTSEV: *Teor. Mat. Fiz.*, **47**, 177 (1981); J. CIBOROWSKI: preprint (Institute of Experimental Physics, Warsaw, 1982); C. SHENG-LIN, X. XINGHUA, L. YONGZHEN and D. ZUGAN: preprint (Normal University, Beijing, 1984).

(34) R. MIGNANI and E. RECAMI: *Gen. Rel. Grav.*, **5**, 615 (1974); E. RECAMI: unpublished work (1974) (seminars, private communications, computer work, pieces of work in collaboration with H. B. NIELSEN, etc.).

C will appear as the creation of a pair of sources collinearly receding from each other with relative speed $W > 2c$. This model immediately explains some gross features of the « Superluminal expansions »; *e.g.*, why converging Superluminal motions are never seen, and the high luminosity of the « superluminal » component (possibly due to the *optic-boom* effect mentioned in subsect. **3**.2; see also ref. ([27]) and the last reference ([4])), as well as the oscillations in the received *overall* intensity (perhaps « beats »; cf. RECAMI ([27])). Since, moreover, the Doppler effect will be different for the two images C_1, C_2 of the same source C (subsect. **4**.3), *a priori* the model may even explain why F_1/F_2 does depend on the observed wave-length and on time (see sect. **2**, point iii)).

Such a model for the « superluminal expansions » was, therefore, proposed long ago (see RECAMI, ref. ([4]), MIGNANI and RECAMI ([34]), RECAMI *et al.* ([26]), GRØN ([22]) and BARUT *et al.* ([22])). More details can be found in the M.S. thesis work by CASTELLINO ([14]), where, *e.g.*, the case of an *extended* source C is thoroughly exploited.

5.1. *The model.* – With reference to fig. 2*a*) and subsect. **3**.2, let us first consider the case of an expanding universe (homogeneous isotropic cosmology). If we call $\overline{C_0O} \equiv s = db$, with $b \equiv V/\sqrt{V^2-1}$, the observed angular rate of recession of the two images C_1 and C_2 as a function of time will be (see appendix A)

$$\dot{\theta}(t) \equiv \omega \simeq \frac{2bc}{s} \frac{1+A}{[1+2A]^{\frac{1}{2}}}, \qquad A \equiv \frac{s}{b^2 ct}, \tag{19}$$

provided that s is the « proper distance » between C_0 and O at the epoch of the radiation reception by O, and t is the time at which O receives those images ($t = 0$ being still the instant at which O starts seeing C, *i.e.* receives radiation from C_0). Let us repeat that ω is the separation angular speed of C_1 and C_2, observed by O, in the case of a space-time metric:

$$ds^2 = c^2 dt^2 - R^2(t) \cdot [dr^2 + r^2 d\Omega],$$

where $R = R(t)$ is the (dimensionless) scale factor. Notice that $\dot{\theta}(t) \to \infty$ for $t \to 0$.

If we call t^* and t the emission time and the reception time, respectively, then the observed frequency ν (see subsect. **4**.3 and eq. (18*b*)) and the received radiation intensity I will be given, of course, by (RECAMI ([34]), RECAMI *et al.* ([26]), CASTELLINO ([14]))

$$\nu = \nu_0 \frac{\sqrt{V^2-1}}{|1 - V\cos\alpha|} \frac{R(t^*)}{R(t)}, \qquad I = \frac{(V^2-1)\mathscr{W}_0}{4\pi r^2(1-V\cos\alpha)^2}\left[\frac{R(t^*)}{R(t)}\right]^2, \tag{20}$$

where ν_0 is the intrinsic frequency of emission and $\mathscr{W}_0$ is the emission power of the source in its rest frame. Angle α is the one formed by the source motion line with the source-observer joining line; and r is the source-observer « proper distance » ([35], p. 415) at the reception epoch. The derivation of eqs. (20) is in appendix B.

Let us pass to the case of a *non-pointlike* source C. Let for simplicity C be one-dimensional with size l with respect to the observer O (fig. 2*a*)) and move with speed V in the direction of its own length. Let us call x the co-ordinate of a generic point of the motion line, the value $x = 0$ belonging to H. As in subsect. **3**'2, be $t = 0$ the instant when the observer enters in radiocontact with C.

Once the two (extended) images C_1 and C_2 get fully separated (*i.e.* for $t > l/V$), if the intrinsic spectral distribution $\Sigma(\nu_0)$ of the source C is known, one can evaluate the differential intensities $\mathrm{d}I_1/\mathrm{d}\nu$ and $\mathrm{d}I_2/\mathrm{d}\nu$ observed for the two images (RECAMI *et al.* [26], CASTELLINO [14]). For the moment let us report only that, due to the extension of the moving images, for each emitted frequency ν_0 the *average* observed frequencies (after lengthy calculations) result to be

$$\langle\nu_1\rangle = \frac{2\nu_0}{\sqrt{V^2-1}(1-\alpha_2/\alpha_1)}\,\frac{R(t^*)}{R(t)}\,, \qquad \langle\nu_2\rangle = \frac{\alpha_2}{\alpha_1}\langle\nu_1\rangle\,, \tag{21}$$

quantities α_1, α_2 being the observed angular sizes of the two images, with $\alpha_1 > \alpha_2$. Moreover, $l/d = \frac{1}{2}V^2(\alpha_1 - \alpha_2)$.

5'2. *Corrections due to the curvature.* – Let us consider the corrections due to the curvature of the Universe, which can be important if the observed « expansions » are located very far. Let us consider, therefore, a curved expanding cosmos (closed Friedmann model), where the length element $\mathrm{d}\lambda$ is given by $\mathrm{d}\lambda^2 = \mathrm{d}r^2(1-r^2/a^2)^{-1} + r^2\,\mathrm{d}\Omega$, quantity $a = a(t)$ being the curvature radius of the cosmos. Again, some details can be found in RECAMI *et al.* [26] and CASTELLINO [14]. For instance, the apparent angular speed of separation $\dot\theta(t)$ between the two observed images C_1 and C_2 (cf. eq. (19)) becomes ($h \equiv r/a$, $ct \ll r$)

$$\dot\theta(t) \equiv \omega \simeq b\,\frac{1+bh}{b+h}\left[\frac{2c}{rt}\right]^{\frac{1}{2}}[1-h^2]^{\frac{1}{4}}\,, \tag{22}$$

quantities r and a being the « radial co-ordinate » of C_0 and the Universe radius, respectively, at the present epoch ($r = a\sin(s/a)$, where s is the « proper distance » of C_0; moreover, $a = c/H\sqrt{2q-1}$; $H \equiv$ Hubble constant; $q \equiv$ $\equiv$ deceleration parameter). Further evaluations in the above-quoted literature;

[35] S. WEINBERG: *Gravitation and Cosmology* (J. Wiley, New York, N. Y., 1972), p. 415.

for instance, eq. (22) is obtained by deriving (under the approximation $ct \ll r$) the exact expression

$$\theta(t) = \frac{2b}{r}\,\frac{1+bh}{b+h}\,[(1-h^2)(\beta^2-h^2)\,c^2 t + 2r(\beta^2-1)(1-h^2)^{\frac{1}{2}}\,ct]^{\frac{1}{2}}\,,$$

where $\beta \equiv V/c$ and, as before, $b \equiv \beta/\sqrt{\beta^2-1}$.

5'3. *Comments.* – Equation (19) yields apparent angular velocities of separation two or three orders of magnitude larger than the experimental ones. It is then necessary to make recourse to eq. (22), which includes the corrections due to the Universe curvature; actually, eq. (22) can yield arbitrarily small values of $\dot{\theta}(t)$ provided that $h \to 1$, *i.e.* $r \to a$. To fit the observation data, however, one has to attribuite to the « Superluminal expansions » values of the radial co-ordinate r very close to a. Such huge distances would explain why the possible blue-shifts—often expected from the local motion of the Superluminal source C (cf. the end of subsect. 4'3)—appear masked by the cosmological red-shift. (Notice, incidentally, that a phenomenon as the one here depicted can catch the observer's attention only when the angular separation θ between C_1 and C_2 is small, *i.e.* when C_1 and C_2 are still close to C_0.) But those same large distances make also this model improbable as an explanation of the observed « Superluminal » expansions, at least in the closed models. One could well resort, then, to open Friedmann models. In fact, the present model with a single (Superluminal) source is appealing since it easily explains *a*) the appearance of two images with Superluminal relative speed ($W > 2c$), *b*) the fact that only Superluminal expansions (and not approaches) are observed, *c*) the fact that W is always Superluminal and practically does not depend on ν, *d*) the relative motion collinearity, *e*) the fact that the flux-density ratio *does* depend on ν and t, since the observed flux differential intensities for the two images as a function of time are given by the formulae [14]

$$(23)\qquad \begin{cases} \dfrac{\mathrm{d}I_i}{\mathrm{d}\nu} = \dfrac{V^2-1}{4\pi d^2 VL}\displaystyle\int\limits_{\nu/M_i(t)}^{\nu/m_i(t)} \dfrac{\Sigma(\nu_0)\,\mathrm{d}\nu_0}{\nu_0 F_i} & (i=1,2), \\[2ex] F_{\substack{1\\2}} \equiv \left[V^{-2}\left(\sqrt{V^2-1}\,\dfrac{R(t)}{R(t^*)}\,\dfrac{\nu}{\nu_0} \mp 1\right)^2 - 1\right]^{\frac{1}{2}}, \end{cases}$$

the integration extrema being

$$(24a)\qquad m_{\substack{1\\2}}(t) \equiv K\{VG[VTG']^{-\frac{1}{2}} \pm 1\}\,,$$

$$(24b)\qquad M_{\substack{1\\2}}(t) \equiv K\left\{\frac{V(G-L)}{[VT(G'-2L)+L\,(L-2\sqrt{V^2-1})]^{\frac{1}{2}}} \pm 1\right\},$$

where d is the « proper distance » $\overline{OH}$ at the reception epoch (fig. 2*a*)), $L \equiv l/d$, $T \equiv ct/d$, $K \equiv \sqrt{V^2-1}\,R(t^*)/R(t)$, $G \equiv \sqrt{V^2-1} + VT$ and $G' \equiv \equiv 2G - VT$. All equations (23), (24) become dimensionally correct provided that V/c is substituted for V.

But the present model remains disfavoured since i) the Superluminal expansion seems to regard not the whole quasar or galaxy, but only a « nucleus » of it; ii) at least in one case (3C273) an object was visible there, even before the expansion started; iii) it is incompatible with the acceleration seemingly observed at least in another case (3C345).

Nevertheless, we exploited somewhat the present question since *A*) in general, the above discussion tells us how a single Superluminal cosmic source would appear, *B*) it might still regard *part* of the present-type phenomenology, *C*) and, chiefly, it must be taken into account *for each one* of the Superluminal objects considered in the following models.

6. – The models with more than one radiosources.

Let us, first, recall that black-holes can classically emit (only) tachyonic matter, so that they are expected to be suitable classical sources—and detectors—of tachyons ([36,22]). Notice that, *vice versa*, tachyons can not only enter the horizon of a black-hole, but also come out from a horizon. As is well known, the motion of a spacelike object penetrating the horizon has been already investigated, within GR, in the existing literature.

We also saw in sect. **2** (fig. 1) and in subsect. **4**˙2 that, in a « subluminal » frame, two tachyons may *seem*—as all the precedent authors claimed—to repel each other from the kinematical point of view, due to the novel features of tachyon mechanics (subsect. **4**˙1, eqs. (14*b*), (14*c*)). In reality, they will gravitationally *attract* each other, from the energetical and dynamical points of view (subsect. **4**˙2).

From subsect. **4**˙2 a tachyon is expected to behave *the same way* also in the gravitational field of a bradyonic source. If a central source B (*e.g.* a black-hole) emits, *e.g.*, a Superluminal body T, the object T under the effect of gravity will loose energy and, therefore, accelerate away. If the total energy $E = m_0 c^2/\sqrt{V^2-1}$ of T is larger than the gravitational binding energy $\bar{E}$, it will escape to infinity with *finite* (asymptotically constant) speed. If, on the

([36]) M. PAVŠIČ and E. RECAMI: *Lett. Nuovo Cimento*, **19**, 273 (1977); V. DE SABBATA, M. PAVŠIČ and E. RECAMI: *Lett. Nuovo Cimento*, **19**, 441 (1977); J. V. NARLIKAR and S. V. DHURANDHAR: *Lett. Nuovo Cimento*, **23**, 513 (1978); E. RECAMI: in *Relativity, Quanta and Cosmology*, edited by M. PANTALEO and F. DE FINIS (Johnson, New York, N. Y., 1979), Vol. **2**, p. 537; E. RECAMI and K. T. SHAH: *Lett. Nuovo Cimento*, **24**, 115 (1979).

contrary, $E < \bar{E}$, then T will reach infinite speed (*i.e.* the zero-total-energy state) at a finite distance; afterwards the gravitational field will not be able to subtract any longer energy to T, and T will start going back towards the source B (appearing now—possibly—as an antitachyon $\bar{\mathrm{T}}$ (see ref. ([17,18]))). It should be remembered (*e.g.*, from ref. ([17])) that at infinite speed the motion direction is undefined, in the sense that the transcendent tachyon can be described either as a tachyon T going back or as an antitachyon $\bar{\mathrm{T}}$ going forth, or *vice versa*. (Since at infinite speed a tachyon possesses zero total energy —see subsect. **3·1**—, we *may* regard its total energy as *all* kinetic.)

We shall see, on another accasion, that a tachyon subjected, *e.g.*, to a central attractive elastic force $\boldsymbol{F} = -k\boldsymbol{x}$ can move periodically back and forth with a motion analogous to the harmonic one, reversing its direction at the points at which it has transcendent speed. Let us consider, in general, a tachyon T moving in space-time (fig. 3) along the spacelike curved path AP, so to reach at P the zero-energy state. According to the nature of the force fields acting on T, after P it can proceed along PB (just as expected in the above two cases, with attractive central forces), or along PC, or along PD. In the last case, T would appear to annihilate at P with an antitachyon emitted by D and travelling along the curved world-line DP (see ref. ([17,18,37]); see also DAVIES ([37]), p. 577).

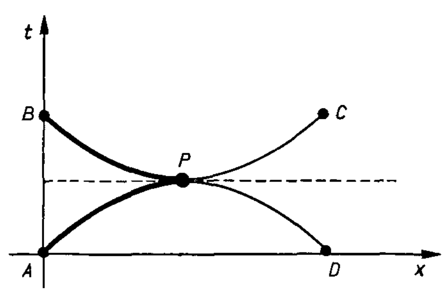

Fig. 3. – See the text.

It is clear that the observed « Superluminal » expansions can be explained i) either by the splitting of a central body into two (oppositely moving) collinear tachyons T_1 and T_2; or by the emission from a central source B of ii) a tachyon T, or iii) a couple of tachyons T_1 and T_2 (in the latter case, T_1 and T_2 can for simplicity's sake be considered as emitted in opposite directions with the same speed). In this respect, it is interesting that NE'EMAN ([38])

([37]) E. RECAMI: *Classical tachyons*, Report INFN/AE-84/8 (Frascati, 1984), to appear in *Riv. Nuovo Cimento*. See also P. C. W. DAVIES: *Nuovo Cimento B*, **25**, 571 (1975), p. 577.

([38]) Y. NE'EMAN: in *High Energy Astrophysics and its Relation to Elementary Particles*, edited by K. BRECHER and G. SETTI (MIT Press, Cambridge, Mass., 1974), p. 405.

regarded quasars—or at least their dense cores—as possible white holes, *i.e.* as possible « lagging cores » of the original expansion.

For simplicity, let us confine ourselves to a flat stationary universe.

6'1. *The case* ii). – In case ii), be O the observer and α the angle between BO and the motion direction of T. Neglecting for the moment the gravitational interactions, the observed *apparent* relative speed between T and B will, of course, be (see fig. 4)

$$W = \frac{V \sin\alpha}{1 - V\cos\alpha} \qquad (V > 1)\,. \tag{25}$$

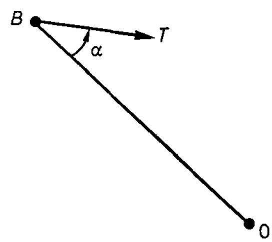

Fig. 4. – A Superluminal object T is emitted by the source B. Point O is the observer's position. See the text.

Let us assume $V > 0$; then $W > 0$ will mean recession of T from B, but $W < 0$ will mean approach. Owing to the cylindrical symmetry of our problem with respect to BO, let us confine ourselves to values $0 < \alpha < 180°$. Let us mention once more that $W \to \infty$ when $\cos\alpha \to 1/V$ (« optic-boom » situation). If the emission angle α of T from B with respect to BO has the value $\alpha = \alpha_b$, with $\cos\alpha_b = 1/V$ $(0 < \alpha_b < 90°$, $b \equiv$ « boom »), tachyon T appears in the optic-boom phase, but the recession speed of T from B would be too high in this case, as we saw in subsect. **3'2**.

Incidentally, to apply the results got in sect. **5** to the Superluminal object T (or T_1 and T_2 in the other cases i), iii)), one has to take account of the fact that the present tachyons are born at a finite time, *i.e.* *do not exist* before their emission from B. It is then immediate to deduce that we shall observe *a*) for $\alpha > \alpha_b$, *i.e.* for $\alpha_b < \alpha < 180°$, the object T recede from B; but *b*) for $0 < \alpha < \alpha_b$, the object T approach B. More precisely, we shall see T receding from B with speed $W > 2$ when

$$\begin{cases} \dfrac{V - \sqrt{5V^2 - 4}}{2(V+1)} < \operatorname{tg}\dfrac{\alpha}{2} < \dfrac{V + \sqrt{5V^2 - 4}}{2(V+1)}\,, \\[2ex] \arccos\dfrac{1}{V} < \alpha < 180°\,. \end{cases} \tag{26}$$

It should be noticed that eq. (25) *can* yield values $W > 2$ whenever $V > 2/\sqrt{5}$: in particular, therefore, *for all possible values* $V > 1$ of V. Due to eqs. (26), the « emission direction » α of T must be however contained inside a certain suitable solid angle: $\alpha_1 < \alpha < \alpha_2$; such a solid angle always including, of course, the optic-boom direction α_b. For instance, for $V \to 1$ we get $0 \leqslant \text{tg}(\alpha/2) < \frac{1}{2}$, $\alpha \geqslant \alpha_b = 0$, wherefrom

$$0 \leqslant \alpha < 53.13° \qquad (V \to 1); \tag{27}$$

in such a case, we shall *never* observe T approaching B. On the contrary, for $V \to \infty$ we get $\frac{1}{2}(1-\sqrt{5}) < \text{tg}(\alpha/2) < \frac{1}{2}(1+\sqrt{5})$, $\alpha_b = 90° \leqslant \alpha < 180°$, wherefrom $-63.44° < \alpha < 116.57°$, $\alpha \geqslant 90°$, that is to say, $90° \leqslant \alpha < 116.57°$. If we add the requirement, *e.g.*, $W < 50$, in order that $2 < W < 50$, we have to exclude in eq. (27)—for $V \to 1$—only the tiny angle $0 < \alpha < 2.29°$, so that in conclusion

$$2.29° < \alpha < 53.13° \qquad (V \to 1)\,.$$

The same requirement $2 < W < 50$ will not affect—on the contrary—the above result $90° \leqslant \alpha < 116.57°$ for the case $V \to \infty$.

Similar calculations were performed also by FINKELSTEIN *et al.* ([13]).

The present case ii) suffers from some difficulties. First, for $\alpha > \alpha_2$ (for instance, for $53 \leqslant \alpha < 180°$ in the case $V \to 1$) we should observe recession speeds with $1 < W < 2$, which is not supported by the data; but this can be understood in terms of the Doppler-shift selective effects (see subsect. **4.3** and BLANDFORT *et al.* ([12])). Second, for $\alpha < \alpha_b$ one should observe also Superluminal approaches; only for $V \simeq 1$ $(V \geqslant 1)$ it is $\alpha_b \simeq 0$ and therefore such Superluminal approaches are *not* predicted.

In conclusion, this model ii) appears acceptable only if the emission mechanism of T from B is such that T has very large kinetic energy, *i.e.* speed $V \gg 1$.

6.2. *The cases* i) *and* iii). – Let us pass now to analyse cases i) and iii), still assuming for simplicity T_1 and T_2 to be emitted with the same speed V in opposite directions. Be α again in the range $[0, 180°]$. In these cases, one would observe faster-than-light recessions for $\alpha > \alpha_b$. When $\alpha < \alpha_b$, on the contrary, we would observe a *single* tachyon $T \equiv T_1$ reaching the position B, passing it, and continuing its motion (as $T \equiv T_2$) beyond B with the same velocity but with a new, different Doppler shift.

One can perform calculations analogous to the ones in subsect. **6.1**; see also ([13]).

In case i), in conclusion, we would *never* observe Superluminal approaches. For $\alpha < \alpha_b$ we would always see only one body at a time (even if $T \equiv T_2$ might result as a feeble radiosource, owing to the red-shift effect): the motion of T would produce a variability in the quasar. For $\alpha > \alpha_b$, as already mentioned, we would see a Superluminal expansion; again, let us recall that the cases with $1 < W < 2$ (expected for large angles α only) could be hidden by the Doppler effect.

Case iii) is not very different from case ii). It becomes « statistically acceptable only if, for some astrophysical reasons, the emitted tachyonic bodies T_1 and T_2 carry very high kinetic energy ($V \gg 1$).

7. – Are « superluminal » expansions Superluminal?

If the emitted tachyonic bodies T (or T_1 and T_2) carry away a lot of kinetic energy ($V \gg 1$), all the models i), ii), iii) may be acceptable from the probabilistic point of view.

Contrariwise, only model i)—and model iii), if B becomes a weak radiosource after the emission of T_1, T_2—remain statistically viable, provided that one considers that the Doppler effect can hide the objects emitted at large angles (say, *e.g.*, between 60° and 180°). On this point, therefore, we do not agree with the conclusions in (13).

In conclusion, the models implying real Superluminal motions investigated in sect. **6** seem to be the most *viable* for explaining the apparent « Superluminal expansions », especially when taking account of the gravitational interactions between B and T, or T_1 and T_2 (or among T_1, T_2, B).

Actually, if we take the gravitational attraction between B and T (subsect. **4**.2) into account—for simplicity, let us confine ourselves to case ii)—, we can easily explain the *accelerations*, probably observed at least for 3C345 and maybe for 3C273 (39).

Some calculations in this direction have been recently performed also by SHENGLIN *et al.* (33) and CAO (40). But those authors did not compare correctly their evaluations with the data, since they overlooked that—because of the finite value of the light speed—the images' apparent velocities do not coincide with the sources' real velocities. The values W_0 calculated by those authors, therefore, have to be corrected by passing to the values $W = W_0 \sin\alpha/(1 - \cos\alpha)$; only the values of W are to be compared with the observation data.

All the calculations, moreover, ought to be corrected for the Universe expansion. However, let us recall (sect. **5**) that in the homogeneous isotropic cosmologies—« conformal » expansions— the angular expansion rates are not expected to be modified by the expansion, at least in the ordinary observational conditions, while the corrections due to the Universe curvature would be appreciable only for *very* distant objects.

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(39) C. SHENGLIN and L. YONGZHEN: in *Proceedings of the III M. Grossmann Meeting on General Relativity*, edited by H. NING (Science Press, Amsterdam, 1983), p. 1319.
(40) SH-L. CAO: preprint (Astronomy Department of the Normal University, Beijing, 1984).

* * *

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APPENDIX A

Let us derive here eq. (19), *i.e.* the *observed* angular rate of recession of the two images C_1 and C_2 as a function of time, in the case of an expanding flat universe (homogeneous isotropic cosmology) corresponding to the space-time metric $\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - R^2(t)\cdot[\mathrm{d}r^2 + r^2(\sin^2\varphi_1\,\mathrm{d}\varphi_2^2 + \mathrm{d}\varphi_1^2)]$, where R is the dimensionless scale factor. With reference to fig. 2*a*), let us take the origin at O and the polar axis parallel to $\boldsymbol{V}$, and consider, *e.g.*, a light-signal emitted by the tachyonic source at the « boom position » C_0 at the time t_0^*. The expansion being conformal, such a light signal propagates in a plane $\varphi_2 = \text{const}$ with $\varphi_1 = \text{const}$, so that its motion is characterized by $\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - R^2(t)\cdot\mathrm{d}r^2 = 0$. If $t = t_0 = 0$ is the instant at which the considered light signal arrives at O, the flight time is given (via variable separation and integration) by

$$c\int\limits_{t_0^*}^{0}\frac{\mathrm{d}t}{R(t)} = \int\limits_{0}^{s^*}\mathrm{d}r = s^*\,,$$

quantity s^* being the « proper distance » between the Superluminal source and the observer at the time t_0^* of the radiation emission. Notice that, in other words, $s^* \equiv r_0^*$ and t_0^* are co-ordinates of the source when it passes through position C_0. At another, generic position C, at the instant t^*, be the source individuated by the space co-ordinates r^*, φ_1. Moreover, be δ the distance of C from C_0, with $|\delta| \ll s^*$ (δ can be both positive and negative, along the motion line of the source); we shall have

$$V\int\limits_{t_0^*}^{t^*}\frac{\mathrm{d}t}{R(t)} = \delta\,. \tag{A.1}$$

Since $|\delta| \ll s^*$, so that $R(t)$ practically does not vary during the time interval $t^* - t_0^*$, we may write $t^* = \delta R(t)/V + t_0^* = t_0^* + \delta/V$, where we set $R(t_0^*) = 1$.

If we call t the time instant at which the observer O receives the radiation emitted (at time t^*) by the tachyonic source at C, then

$$(\text{A.2}) \qquad c\int_{t^*}^{t}\frac{\mathrm{d}t}{R(t)} = c\int_{t_0^*+\delta/V}^{t}\frac{\mathrm{d}t}{R(t)} = r^*\,,$$

and the difference between eq. (A.2) and eq. (A.1) yields the time interval between emission and reception ($s^*\equiv r_0^*$):

$$(\text{A.3}) \qquad r^*-s^*\simeq\frac{ct}{R(0)}-\frac{c\delta}{V} \quad \text{with } r^*=[s^{*2}+\delta^2-2cs^*\delta/V]^{\frac{1}{2}}\,.$$

In Cartesian co-ordinates, the last equation becomes (cf. fig. 2) $bd+ct/R(0)-$ $-x/\beta=[(x-d/\sqrt{\beta^2-1})^2+d^2]^{\frac{1}{2}}$, where $\beta\equiv V/c$, x is the abscissa corresponding to C, and, as before, $b\equiv\beta/\sqrt{\beta^2-1}$. The two solutions x_1, x_2 correspond to the two images received by the observer. It is easy to evaluate that

$$x_2-x_1=2cb^2\left[\frac{t}{R(0)}\left(\frac{t}{R(0)}+\frac{2d}{cb}\right)\right]^{\frac{1}{2}}.$$

Since the angular separation, θ, between the two images is very small in the cases of interest, it will be given by

$$(\text{A.4}) \qquad \theta(t)\simeq\frac{(x_2-x_1)\sin\alpha}{s^*}=\frac{2cb}{s}\left[t^2+\frac{2st}{cb^2}\right]^{\frac{1}{2}},$$

where $s=s^*R(0)/R(t_0^*)=s^*R(0)$ is now the proper distance between source and observer when the latter starts seeing C, *i.e.* receiving radiation from C_0. By deriving eq. (A.4) we finally get eq. (19) of the text, which—for the properties of a flat expanding universe—does essentially coincide *mutatis mutandis* with eq. (10).

Appendix B

Let us derive here also eqs. (20). Notice that the derivations of equations like (21) and (22) are—however—so lengthy that they cannot be reported here; but they are included in the thesis [14].

Let us, then, consider the question of the received radiation intensity in the simple case of a pointlike source and still in a homogeneous isotropic cosmology. Be the reference frame chosen as in appendix A. Let us suppose the Superluminal source to be at time t^* at a generic position C (individuated by the co-ordinates r^* and $\varphi\equiv\varphi_1$) and at time $t^*+\mathrm{d}t^*$ at the position C'.

Moreover, be $d\delta$ and $r^{*\prime}$ the proper distances CC' and $C'O$ at the instant t^*. One has $V\,dt^*/R(t^*) = d\delta$, and $r^{*\prime} = [r^{*2} + d\delta^2 - 2r^*\,d\delta\cos\alpha]^{\frac{1}{2}}$, angle α being the one formed by the source motion line with the source-observer joining line. By expanding the last equation, at the first order one finds

$$r^{*\prime} = r^*\left(1 - \frac{d\delta}{r^*}\cos\alpha\right) \qquad (\alpha \equiv 180^\circ - \varphi) \tag{B.1}$$

If we call t and $t + dt$ the time instants at which the light signals emitted at C and at C', respectively, are received by O, since $R(t)$ practically does not vary during dt^*, we shall have

$$c\int_{t^*}^{t}\frac{dt}{R(t)} = r^*, \qquad c\int_{t^*+dt^*}^{t+dt}\frac{dt}{R(t)} = r^{*\prime},$$

and, by subtraction,

$$dt = \frac{R(t)}{R(t^*)}\,|1-\beta\cos\alpha|\,dt^*, \qquad \beta \equiv \frac{V}{c}. \tag{B.2}$$

In the case of a nonexpanding universe, the relation between observed frequency ν and proper frequency ν_0 is given by eq. (18b) of the text [18,27]. In the case of expansion, we shall have

$$\nu = \nu_0\sqrt{\beta^2-1}\,\frac{dt^*}{dt} = \nu_0\,\frac{\sqrt{\beta^2-1}}{|1-\beta\cos\alpha|}\,\frac{R(t^*)}{R(t)}. \tag{B.3}$$

Let us recall that the absolute-value symbols entering eqs. (B.2) and (B.3) can be omitted: for a discussion of this point see ref. [18] and MIGNANI and RECAMI [4,27,34].

Be, now, $\mathscr{W}_0$ the power emitted by the tachyonic source in its proper reference frame. The radiation intensity, I, observed by O will be inversely proportional to the square both of the source-observer distance r at the epoch of the reception and of the scale factor ratio, so that

$$I = \frac{\mathscr{W}_0(\beta^2-1)}{4\pi r^2(1-\beta\cos\alpha)^2}\left[\frac{R(t^*)}{R(t)}\right]^2. \tag{B.4}$$

Equations (B.3), (B.4) do coincide with eqs. (20) of the text. Let us notice that they hold for each image received by O, when such images come from points C not far from the «boom» position C_0. When $C \to C_0$, then $r \to s$ and α tends to the value represented in fig. 2a) (where $\cos\alpha = 1/\beta$); therefore I should diverge. But such a divergence is only a formal consequence of the fact that the source was artificially supposed to be pointlike; it disappears when dealing with the realistic case of an extended source (see the text and ref. [14]).

● RIASSUNTO

I modelli ortodossi costruiti per spiegare le apparenti « espansioni superluminali » osservate in astrofisica — e qui brevemente riassunti e discussi, insieme coi dati sperimentali relativi — sembra non abbiamo avuto troppo successo, specialmente se posti a confronto con le piú recenti osservazioni, le quali suggeriscono la presenza di meccanismi di espansione complicati, anche con possibili accelerazioni. A questo punto, quindi, può essere di qualche interesse esplorare i possibili modelli alternativi, in cui abbiano luogo effettivi moti Superluminali. Per preparare il terreno partiamo da un principio variazionale, introduciamo le basi di una meccanica dei tachioni nell'ambito della relatività speciale, e individuiamo il comportamento che ci si aspetta da parte dei tachioni quando essi interagiscono (ad esempio *gravitazionalmente*) tra di loro o con la materia ordinaria. Si esaminano, quindi, e si sviluppano i piú semplici modelli « Superluminali », prestando particolare attenzione alle *osservazioni* a cui essi darebbero luogo. Si conclude che alcuni di tali modelli appaiono fisicamente accettabili e statisticamente favoriti rispetto a quelli ortodossi.

Рассмотрение очевидного « сверхсветового расширения » в астрофизике.

Резюме (*). — В этой работе обсуждаются ортодоксальные модели, разработанные для объяснения очевидного « сверхсветового расширения », наблюденного в астрофизике, наряду с экспериментальными данными. Отмечаются недостатки этих моделей. Особенно, когда модели сопоставляются с недавними наблюдениями, предполагающими сложную картину расширения, даже с возможными ускорениями. Следовательно, с этой точки зрения представляет интерес исследовать возможные альтернативные модели, в которых могут иметь место действительные суперсветовые движения. Сначала мы исходим из вариационного принципа, вводим элементы тахионной механики в специальнцю теорию относительности и аргументируем ожидаемое поведение тахионных объектов, взаимодействующих (гравитационным образом) друг с другом и с обыкновенным веществом. Затем развиваем простейшие « суперсветовые модели », обращая особое внимание на наблюдаемые величины, которые получаются в этих моделях. Мы отмечаем, что некоторые из рассмотренных моделей являются физически приемлемыми и статистически предпочтительными относительно ортодоксальных моделей.

(*) *Переведено редакцией.*