

Cherenkov radiation has nothing to do with X-shaped Localized Waves (Comments on “Cherenkov-Vavilov Formulation of X-Waves”) ^(†)

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Abstract — *The Localized Waves (LW) are nondiffracting (“soliton-like”) solutions to the wave equations, and are known to exist with subluminal, luminal and superluminal peak-velocities V . For mathematical, and experimental, reasons, the ones that called more attention are the “X-shaped” superluminal waves. Such waves are associated with a cone, so that some authors –let us confine ourselves to electromagnetism– have been tempted to look for links between them and the Cherenkov radiation: A good example of such attempts is represented by the article by Walker and Kuperman recently appeared in PRL 99, no.244802, of Dec.2007. However, the X-shaped waves belong to a very different realm: For instance, they exist even in the vacuum, independently of any media, as localized non-diffracting pulses propagating rigidly with a peak-velocity $V > c$, as verified in a number of papers [cf., e.g., the refs. in the book Localized Waves (J.Wiley; Jan.2008)]. We deem it necessary to clarify the whole question on the basis of a rigorous formalism, in part original, and of clear physical considerations. In particular we clarify, by explicit calculations based on Maxwell equations only, that, at variance with what assumed by some authors: (i) the “X-waves” exist in all space, and in particular inside both the front and the rear part of their double cone (that has nothing to do with Cherenkov’s); (ii) they have not been found heuristically, via ad hoc assumptions, but by*

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use of strict mathematical (or experimental) procedures; (iii) the ideal X-waves (as well as plane waves) are actually endowed with infinite energy, but finite-energy X-waves can be easily constructed (even without space-time truncations): And at the end of this Ms., by following an original technique, we construct exact finite-energy solutions (totally free from backward-traveling waves); (iv) the large majority of the researchers working in this area do not aim at using the X-waves for “superluminal transmission of information”, but are interested in the circumstance that they are localized waves, endowed with a self-reconstruction property, and in their important practical applications (in part already realized, since 1992); (v) an actual attempt at comparing Cherenkov radiation and X-waves would lead one to consider the very different situation of the (X-shaped, too) field generated by a superluminal point-charge, a question actually exploited in previous papers, appeared e.g. in 2004 in Phys.Rev.E [Recami, Zamboni-Rached & Dartora, PRE 69, no.027602]: We show here explicitly that in this case the point-charge would not lose energy in the vacuum, and that the field generated by it would not need to be continuously feeded by incoming side-waves (as it is indeed the case for an ideal, ordinary X-wave).

PACS nos.: 41.60.Bq; 03.50.De; 03.30.+p; 41.20.Jb; 04.30.Db; 42.25.-p; 42.25.Fx; 47.35.Rs.

Keywords: Cherenkov radiation; Localized waves; X-shaped waves; Superluminal pulses; Maxwell equations; Special Relativity; Lorentz transformations; Superluminal point-charge; Wave equations; Bessel beams.

The Localized Waves (LW) are nondiffracting (“soliton-like”) solutions to the wave equations, and are known to exist with subluminal, luminal and superluminal peak-velocities V . For mathematical, and experimental, reasons, the ones that called more attention are the “X-shaped” superluminal waves. Such waves are associated with a cone, so that some authors —let us confine ourselves to electromagnetism— have been tempted to look for links between them and the Cherenkov radiation: A good example of such attempts is represented by the article by Walker and Kuperman, recently appeared[1]. However, the X-shaped waves belong to a very different realm: For instance, they exist even in the vacuum, independently of any media, as localized non-diffracting pulses propagating

rigidly with a peak-velocity $V > c$, as verified in a number of papers (cf., e.g., the refs. in the book *Localized Waves*[2]). We deem it necessary to clarify all the question on the basis of a rigorous formalism, in part original, and of clear physical considerations. In particular we clarify, by explicit calculations based on Maxwell equations only, that, at variance with what assumed by some authors: (i) the “X-waves” exist in all space, and in particular inside both the front and the rear part of their double cone (that has nothing to do with Cherenkov’s); (ii) they have not been found heuristically, via *ad hoc* assumptions, but by use of strict mathematical (or experimental) procedures; (iii) the ideal X-waves (as well as plane waves) are actually endowed with infinite energy, but finite-energy X-waves can be easily constructed (even without space-time truncations): And at the end of this work, by following an original technique, we construct *exact* finite-energy solutions (totally free from backward-traveling waves); (iv) the large majority of the researchers working in this area do not aim at using the X-waves for “superluminal transmission of information”, but are interested in the circumstance that they are localized waves, endowed with a self-reconstruction property, and in their important practical applications (in part already realized, since 1992); (v) an actual attempt at comparing Cherenkov radiation and X-waves would lead one to consider the very different situation of the (X-shaped, too) field generated by a superluminal point-charge, a question actually exploited in previous papers, appeared e.g. in 2004 in Phys.Rev.E [3]: We show here explicitly that in this case the point-charge would not lose energy in the vacuum, and that the field generated by it would not need to be continuously feeded by incoming side-waves (as it is indeed the case for an ideal, ordinary X-wave).

As already said, we wish to reply to attempts, like the one in Ref.[1], at setting forth “Cherenkov-Vavilov formulations of the X-shaped Localized Waves” (in the following we shall write only “Cherenkov” for brevity’s sake). The classical problem of the Cherenkov radiation[4] from a point-charge traveling in a medium with speed v such that $c_n < v < c$, where c_n and c are the speed of light in the medium and in the vacuum, respectively, is normally investigated in correct terms. Also in [1] it is presented in a mathematically correct way; sometimes, however, the language used in such a context is ambiguous: For example, in [1] the speed c_n in the medium is just called c ; furthermore, the point-charge associated with the Cherenkov radiation is called “superluminal”, despite the fact that its speed is smaller than the light-speed in the vacuum. In the existing theoretical and experimental literature on Localized Waves (see again, e.g., Ref.[2]) and in particular on X-shaped waves[5, 6], the word superluminal is reserved to group-speeds actually larger than the speed of light in the vacuum. These ambiguities create difficulties, especially

whenever a comparison is made of the Cherenkov radiation with X-shaped waves. Let us repeat that, in reality, the latter belong to a very different realm; and exist even in vacuum as localized non-diffracting pulses, propagating rigidly with superluminal (in our language) peak-velocities, $V > c$, independently of any media. Let us try to clarify the whole subject, and several unjustified implications in papers like [1], by using a rigorous formalism and clear physical considerations.

Our specific considerations and comments are as follows:

1) — As already mentioned, the papers addressing the Cherenkov effect do incorporate sometimes[1] an ambiguous terminology (see above), about which the readers must be warned. Except for the ambiguous notation, the analysis of the ordinary scalar-valued Cherenkov radiation is normally correct —cf., e.g., the first part of [1]— as it can be explicitly checked; anyway, the relevant results are well-known[4]. Except for the ambiguous notation, the analysis of the ordinary scalar-valued Cherenkov radiation in the first part of [1] is correct, as it can be explicitly checked; however, the relevant results seem to be already known[4]. It can be pointed out, incidentally, that even the vector-valued electromagnetic fields generated by a *really* superluminal point-charge, endowed with speed $V > c$, have already been considered and published in [3] using a procedure totally different, of course, from the one followed e.g. in [1].

2) — The main equivocal aspect of works like [1] is that such authors attempt at using the ordinary Cherenkov radiation to draw conclusions about superluminal Localized Waves (LW) known as X-shaped waves (or simply X-waves). The latter, as we said, are nondiffractive solutions to the homogeneous wave equations and propagate rigidly with superluminal *peak-velocities*. They have been predicted long ago[7, 8], have been mathematically constructed[5], and finite-energy versions of these wave-packets have been experimentally produced[6] in the vacuum, independently of any media. We are going to show the drawbacks of the path followed in articles as [1] by using explicit calculations.

To be clear and self-contained, we initially address the (simpler) scalar case, in which a field ψ is governed by the inhomogeneous wave equation

$$\left(\nabla^2 - \frac{1}{c_n^2} \frac{\partial^2}{\partial t^2} \right) \psi(\mathbf{r}, t) = -\frac{4\pi}{c_n} j(\mathbf{r}, t) \quad (1)$$

with $j = qv \delta(\rho) \delta(z - vt)/(2\pi\rho)$. Here c_n is the speed of light in the considered medium

and $j(\mathbf{r}, t)$ is the generating source, assumed to be pointlike and moving along the positive z -axis with subluminal speed $c_n < v < c$, while ρ denotes the cylindrical radial coordinate.

A Green's function for Eq.(1) is given explicitly as

$$G(\mathbf{r}, t, \mathbf{r}', t') = l G^+(\mathbf{r}, t, \mathbf{r}', t') + (1 - l) G^-(\mathbf{r}, t, \mathbf{r}', t') \quad (2)$$

where

$$G^\pm(\mathbf{r}, t, \mathbf{r}', t') = \frac{\delta(t' - (t \mp R/c_n))}{c_n R} \quad (3)$$

and $R \equiv \sqrt{(z - z')^2 + \rho^2 + \rho'^2 - 2\rho\rho' \cos(\theta - \theta')}$.

Quantities G^+ and G^- are the retarded and advanced Green's functions, respectively.

When considering only the retarded Green's function ($l = 1$), the solution to the wave equation can be expressed as

$$\psi(\mathbf{r}, t) = \int dx'^3 \int dt' G^+(\mathbf{r}, t, \mathbf{r}', t') j(\mathbf{r}', t'). \quad (4)$$

We can use the expression for G^+ given in Eq.(3) for a direct calculation of the wave function $\psi(\mathbf{r}, t)$. However, for reasons that will be made clear in the sequel, we shall follow the procedure adopted in papers like [1]. Specifically, we shall determine the Fourier transform of G^+ in the variables z and t , and subsequently the transform of ψ in the same variables. For $c_n < v < c$, we obtain:

$$\psi(\rho, \zeta) = \frac{1}{2\pi c_n} \left[\left(\int_{-\infty}^0 d\omega (-i\pi) q H_0^2(\rho \gamma_n^{-1} |\omega|/v) e^{i\omega\zeta/v} \right) + \left(\int_0^{\infty} d\omega (i\pi) q H_0^1(\rho \gamma_n^{-1} \omega/v) e^{i\omega\zeta/v} \right) \right]$$

where $\zeta \equiv z - vt$, and $H_0^{1,2}$ are the zero-order Hankel functions of first and second kind. Using, next, the relation $H_0^2(x) = -H_0^1(-x)$ for $x \geq 0$, the last equation is reduced to Eq.(7) of Ref.[1], viz.,

$$\psi(\rho, \zeta) = \frac{1}{2c_n} \int_{-\infty}^{\infty} d\omega i q H_0^1(\rho \gamma_n^{-1} \omega/V) e^{i\omega\zeta/V} \quad (5)$$

from which one determines the well-known expression for the Cherenkov radiation

$$\psi(\rho, z, t) = \begin{cases} \frac{2q\beta_n}{\sqrt{\zeta^2 - \gamma_n^{-2}\rho^2}} & \text{for } \zeta < -\gamma_n^{-1}\rho \\ 0 & \text{elsewhere,} \end{cases} \quad (6)$$

where it should be noted that $\beta_n \equiv v/c_n$ and $\gamma_n \equiv 1/\sqrt{v^2/c_n^2 - 1}$. The radiation exists only inside the rear part $\zeta = -\gamma_n^{-1}\rho$ of the Cherenkov cone.

As to the vectorial case, physically more significant, let us only mention that it can be constructed by adopting the Lorentz gauge, and by considering the vector potential $\mathbf{A} = A_z \hat{\mathbf{e}}_z$, with $A_z \equiv \psi$ and the current density $\mathbf{j} = j_z \hat{\mathbf{e}}_z$, with $j_z \equiv j$.

2a) – Some authors, as the ones of Ref.[1], have been induced at looking for a connection between the Cherenkov emission and X-shaped waves. The simple X-waves to which many authors refer to are solutions to the homogeneous scalar wave equation: They are wave functions of the type $X(\rho, z, t) = X(\rho, \zeta)$, with $\zeta \equiv z - Vt$ and $c_n < V < \infty$, and can be obtained by suitable superpositions of axially symmetric Bessel beams, propagating in the positive z -direction with the same phase-velocity (cf., e.g., Refs.[5]); precisely,

$$\psi_X(\rho, \zeta) = \int_0^\infty d\omega S(\omega) J_0\left(\rho \frac{\omega}{V} \sqrt{V^2/c^2 - 1}\right) e^{i\omega/V\zeta} \quad (7)$$

where $J_0(\cdot)$ is an ordinary zero-order Bessel function and $S(\omega)$ is the temporal frequency spectrum. Incidentally, let us stress that Eq.(7) represents a particular case of the superluminal waves, which in their turn are just a particular case of the (subluminal, luminal or superluminal) Localized Waves. For the specific spectrum $S(\omega) = \exp[-a\omega]$, with a a positive constant, one obtains the zero-order (classic) X-wave:

$$X \equiv X(\rho, \zeta) = \frac{V}{\sqrt{(aV - i\zeta)^2 + (V^2/c^2 - 1)\rho^2}}. \quad (8)$$

Authors like the ones in Ref.[1] are led to attempt a comparison between the Cherenkov effect and the zero-order X-wave by the apparent mathematical similarity of equations (5) and (7) [which correspond, for instance, to equations (7) and (13) of Ref.[1].] To be more specific: (i) we have seen that for the inhomogeneous wave-equation (1), a Cherenkov solution of the type given in Eq.(6) exists only inside the cone rear part $\zeta = -\gamma^{-1}\rho$. To obtain a solution existing only inside the cone forward part $\zeta = \gamma^{-1}\rho$, one should make use of the advanced Green's function G^- : This —to go on with our example— induced the authors in [1] to state that the forward part of the X-wave is non-causal; (ii) Another point that misled Walker and Kuperman[1] is that, if one puts $S(\omega) = i$ into the ψ_X -wave (7), that is, if one sets $a = 0$ in Eq.(8) and multiplies it by i , one obtains

$$\tilde{X}(\rho, \zeta) = \frac{V}{\sqrt{\zeta^2 - (V^2/c^2 - 1)\rho^2}}, \quad (9)$$

which is mathematically identical, apart from a constant, to the Cherenkov solution in Eq.(6)], with a real part existing this time inside both the rear cone $\zeta = -\gamma^{-1}\rho$ and the forward cone $\zeta = \gamma^{-1}\rho$. The statement in papers like [1] that the advanced part of the zero-order X-wave [cf. Eq.(8)] is non-causal is due to an illicit extrapolation. Being a solutions to the homogeneous wave equation, the X-wave cannot admit singularities. In contrast, the solution given in Eq.(9), which was obtained from the Bessel beam superposition (7) using a *constant* spectrum $S(\omega)$, does have singularities and cannot be considered a solution to the homogeneous wave equation. Actually, the real part of the solution (9) can be an acceptable solution for the inhomogeneous case only: Indeed, we shall show later on that it represents the field of a point-charge traveling with speed $V > c$ when using as a Green's function the expression $G = G^+/2 + G^-/2$, half retarded and half advanced (which means, again, that it refers to an inhomogeneous problem).

It should be pointed out that the Bessel beam synthesis in Eq.(7) is a particular case of more general spectral representations leading not only to infinite but also to finite energy superluminal LWs[2, 5]. Not less important, even a finite-energy close replica of the the classic X-wave (8) (endowed, incidentally, also with a finite field-depth) can be generated in a *causal* manner by means of a “dynamic” finite aperture (antennas, holographic or optical elements, etc.). For instance, it is enough an array of circular elements excited according to the function $X(\rho, z = 0, t)$, given by equations (7) or (8) on the aperture plane located at $z = 0$. The emitted finite-energy X-wave can be calculated by means of the Rayleigh-Sommerfeld (II) formula [9]

$$\psi_{\text{RS(II)}}(\rho, z, t) = \int_0^{2\pi} d\phi' \int_0^{D/2} d\rho' \rho' \frac{1}{2\pi R} \left\{ [X] \frac{(z - z')}{R^2} + [\partial_{c_n t'} X] \frac{(z - z')}{R} \right\}. \quad (10)$$

The quantities inside the square brackets are evaluated at the retarded time $c_n t' = c_n t - R$. The distance $R = \sqrt{(z - z')^2 + \rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi')}$ is now the separation between source and observation points, and the aperture diameter is denoted by D . The depth of field of the particular solution given in Eq.(10) is known[10] to be $Z = R\gamma_n$.

What stated above has been theoretically (even via numerical simulations...) and experimentally verified, as published in a large number of well-known papers: see, for example, Refs.[5, 6, 11, 10, 12, 13, 2].

2b) – We agree with authors as Walker and Kuperman[1] that the superluminal spot of any X-wave is fed by the waves coming from the elements of the aperture, and that these waves carry energy with at most luminal ($V = c_n$) speed. In such cases, the

X-wave *intensity peaks* at two different locations are not causally correlated. But such a correct claim, contained also in [1], is well known and firmly accepted since the nineties by practically all the scholars working in the area of LWs (cf., e.g., the references in [14] and [13], as well as Refs.[2, 5, 10, 11]): The claim in [1] that efforts in the area of X-waves are aimed at transmitting information superluminally is not correct. The large majority of the experts in such a subject are interested in X-waves due to their spatio-temporal localization, unidirectionality, “soliton-like”, and self-reconstruction properties in the near-to-far zone. Such properties bear interesting consequences, from theoretical and experimental points of view in all sectors of physics in which a role is played by a wave equation (including —*mutatis mutandis*— elementary particle physics, and even gravitation).

3) — The statement, contained in papers like [1], that the (ideal) zero-order X-wave in Eq.(7) has infinite energy (as well as the plane-waves) does not convey new information. The fact that such a solution needs to be fed for an infinite time has been known since the start of LW theory. In any case, as mentioned earlier, such a problem can be overcome either by using (cf. Eq.(10)) apertures finite in space and time, i.e., by truncating the X-wave, or by constructing exact, analytical finite-energy solutions[12, 11]. For reasons of space, we shall show *only briefly*, but in an original rigorous way, how closed-form solutions of the latter type can be actually constructed, without any recourse to the backward-traveling waves that trouble the ordinary approaches: See point **5)** below.

Here, for the moment, let us recall that X-waves endowed by themselves with finite energy even without truncation have been easily constructed in the past by use of diffraction integrals: See, for instance, the approximate solution in Eqs.(2.31),(2.32) of Ref.[15], that is, the “SMPS pulse”, which is depicted in Figs.1. A finite-energy X-wave gets deformed while propagating: and Fig.1(b) shows the pulse in Fig.1(a) after it has travelled 50 km.

Let us emphasize, as well, that the formulations leading to LWs, and to X-waves, are not chosen *ad hoc*, as believed in papers as [1], but are to be based on proper choices of the spectra (which imply a specific space-time coupling[12, 11, 16]) and of the Bessel functions (in order to avoid singularities both at $\rho = 0$ and at $\rho = \infty$): In contrast, the choice suggested, e.g., in Eq.(15) of [1] presents singularities. To be clearer, let us observe that a general solution to the scalar homogeneous wave-equation in free space can be written [when eliminating evanescent waves] in the form

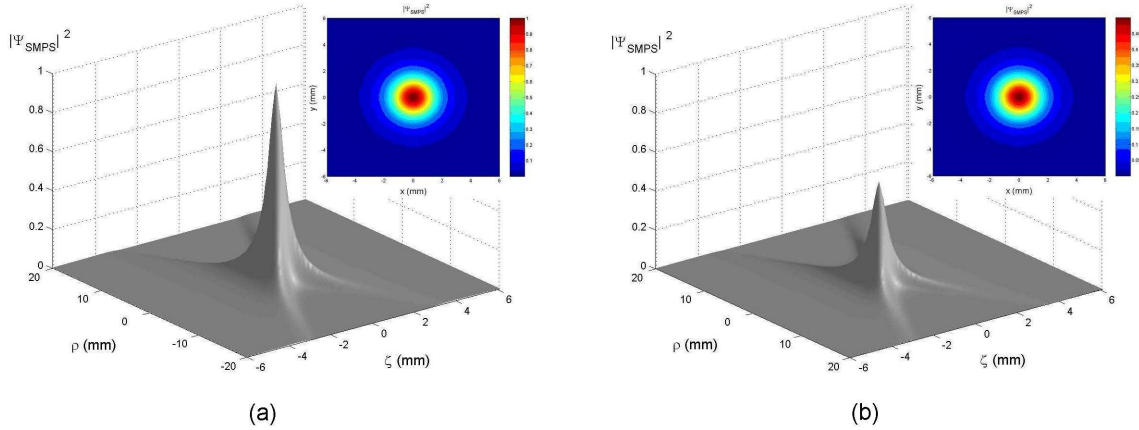


Figure 1: Example of an X-type LW endowed with finite energy (even without truncation), and that consequently gets deformed while propagating: (b) represents the pulse depicted in (a) after it has traveled 50 km. See Ref.[15], and the text.

$$\psi(\rho, \phi, z, t) = \sum_{\nu=-\infty}^{\infty} \left[\int_0^{\infty} d\omega \int_{-\omega/c}^{\omega/c} dk_z A_{\nu}(k_z, \omega) J_{\nu} \left(\rho \sqrt{\frac{\omega^2}{c^2} - k_z^2} \right) e^{ik_z z} e^{-i\omega t} e^{i\nu\phi} \right] \quad (11)$$

by considering positive angular frequencies ω only. For obtaining ideal LWs propagating along the positive z -direction with peak-velocity V (that can assume a priori any value $0 \leq V \leq \infty$), the spectra A_{ν} *must* have the form:[12, 2, 16]

$$A_{\nu}(k_z, \omega) = \sum_{\mu=-\infty}^{\infty} S_{\nu\mu}(\omega) \delta[\omega - (Vk_z + b_{\mu})] \quad (12)$$

where $b_{\mu} = 2\pi\mu V/\Delta z_0$, and it can be easily shown[12, 16] that solution (11) possesses the important property $\psi(\rho, \phi, z, t) = \psi(\rho, \phi, z + \Delta z_0, t + \Delta z_0/V)$, where Δz_0 is a chosen space-interval along z . The last two equations already show that LWs are not to be found by “ad hoc” assumptions: Equation (12) does explicitly show that —as mentioned above— the ideal LWs exist only in correspondence with linear relations between ω and k_z , that is, with *specific* space-time couplings. By assuming in particular $A_0(k_z, \omega) \equiv A_{\nu}(k_z, \omega) = \delta_{\nu 0} S(\omega) \Theta(k_z) \delta[\omega - (Vk_z + \alpha_0)]$, where the Heaviside function Θ does eliminate, as desired, any backward components, and α_0 is a constant, we can construct an infinite number of subluminal, luminal, or superluminal Localized Solutions, with axial symmetry, and in closed form. For instance, the X-shaped waves represented in Eq.(7), and used for the sake of comparison also in Ref.[1], correspond to the particular value

$\alpha_0 = 0$. [To get finite-energy solutions one has to abandon the strict request of a linear relation between ω and k_z , and impose instead that the spectral functions $A(k_z, \omega)$ possess non negligible values only *in the vicinity* of a straight line of the mentioned type[12]: This is briefly exploited under point **5**) below].

4) — We have already established that the analogy attempted in works like [1] between the Cherenkov radiation and the X-wave solution is not justified even in material media. In the case of vacuum, the aforementioned analogy should have rather led one to consider the field generated by a *really* superluminal point-charge: A problem that was investigated in [3], and refs. therein. In such a situation, the point-charge superluminally traveling in the vacuum is not expected to radiate due to physical reasons published long ago[17, 8, 18]. Such a charge does not radiate in its rest-frame[8, 18] and, consequently, does not radiate also according to observers for whom it is superluminal[17]. We shall establish this result below, by explicit calculations based on Maxwell equations only.

Let us consider the wave equation (1) in vacuum ($c_n \rightarrow c$) with a superluminally moving ($v \rightarrow V > c$) point charge source. Use of the Green's function $G = (G^+ + G^-)/2$ (cf., e.g., Refs.[19, 20]) yields the following integral representation for the solution:

$$\psi(\rho, \zeta, t) = \frac{q}{c} \int_0^\infty d\omega N_0\left(\rho \frac{\omega}{V} \gamma^{-1}\right) \cos\left(\frac{\omega}{V} \zeta\right), \quad (13)$$

where N_0 is the zero-order Neumann function, and, now, β_n and γ_n have been replaced with $\beta \equiv V/c$ and $\gamma^{-1} \equiv \sqrt{V^2/c^2 - 1}$. The integration can be carried out explicitly, and yields, for the field generated by a point-charge traveling superluminally in the vacuum, the expression

$$\psi(\rho, z, t) = \begin{cases} q\beta [\zeta^2 - \rho^2 \gamma^{-2}]^{-1/2} & \text{when } 0 < \rho \gamma^{-1} < |\zeta| \\ 0 & \text{elsewhere .} \end{cases} \quad (14)$$

This solution is different from zero inside the rear *and* front parts of the unlimited double cone[3, 14] generated by the rotation around the z -axis of the straight lines $\rho = \pm \gamma \zeta$, in agreement with the predictions of the “extended” (or, rather, “non-restricted”) theory of Special Relativity[8, 18]. The expression in Eq.(14) is precisely equivalent to the solution given by Eq.(8) in our Ref.[3]; except for a constant that was wrong therein.

Going on to the (physically more suited) vectorial formalism, adopting the Lorentz gauge, and choosing a current density $\mathbf{j} = j_z \hat{\mathbf{e}}_z$; $j_z \equiv j$, a scalar electric potential

$\phi = c\psi/V$ and a vector magnetic potential $A \equiv \psi\hat{e}_z$ (cf. Fig.2 in Ref.[3], which refers to a negative point-charge), one obtains, in analogy to [3], the electric and magnetic fields

$$\mathbf{E}(\rho, \zeta) = -q\gamma^{-2}Y(\rho\hat{e}_\rho + \zeta\hat{e}_z); \quad \mathbf{B} = -q\beta\gamma^{-2}Y\rho\hat{e}_\theta, \quad (15)$$

in Gaussian units; where

$$Y \equiv [\zeta^2 - \rho^2\gamma^{-2}]^{-3/2}$$

inside the double cone (i.e., for $0 < \rho\gamma^{-1} < |\zeta|$), while $Y = 0$ outside it (that is, \mathbf{E} and \mathbf{B} are zero outside it). The corresponding Poynting vector is given by

$$\mathbf{S} = \frac{c}{4\pi}q^2\beta\gamma^{-4}Y^2\rho(\rho\hat{e}_z - \zeta\hat{e}_\rho). \quad (16)$$

The total flux, through any closed surface containing the point-charge at the considered instant of time, is equal to zero. Thus, a point-charge traveling at a constant superluminal speed in vacuum does not radiate energy. This fact is depicted and explained in an intuitive way in Fig.4 of Ref.[3], which originally appeared in Refs.[7, 8] and, for clarity, is reproduced in this article as Fig.2.

We wish to emphasize that in the present case the field needs not be fed, at variance with the case of the ordinary X-waves. Once more, one can see that the analogy exploited in papers like [1] between the Cherenkov effect and the X-waves completely breaks down in the case of the vacuum.

Eventually, most authors do not address many of the other interesting physical points. For example, no mention appears in [1] of the fact that a superluminal charge is expected to behave as a magnetic pole, in the sense fully clarified in Refs.[21, 7, 8]. One can see even from Eqs.(15) that $\mathbf{E} \rightarrow 0$, and one is left with a pure magnetic field, in the limit $V \rightarrow \infty$.

5) — *At last*, as anticipated in point **3)** above, finite-energy solutions can be obtained in closed form without any recourse to the backward-traveling waves that trouble the usual approaches (even if the intervention of such components has been already minimized in Refs.[12, 11], at the cost —however— of going on to frequency spectra with a very large bandwidth). In fact, when confining ourselves to superluminal LWs with axial symmetry, let us put in Eq.(11) $A_\nu(k_z, \omega) = \delta_{\nu 0} A(k_z, \omega)$, and adopt the “unidirectional decomposition”

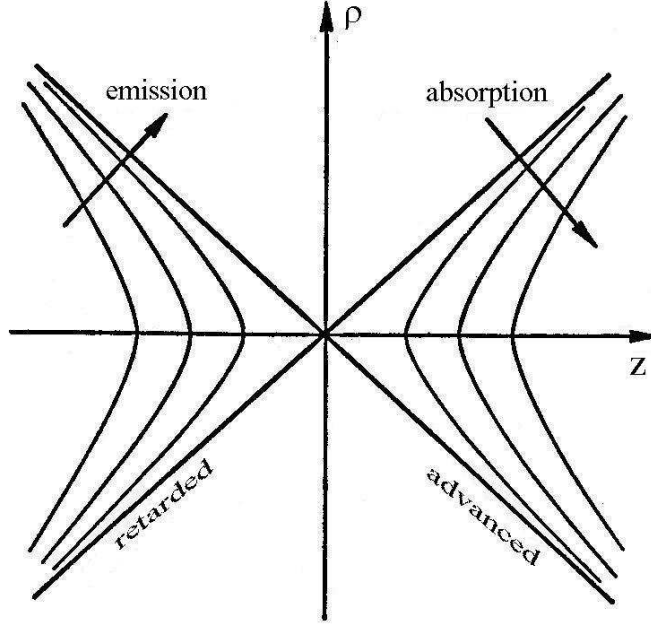


Figure 2: This figure —appeared in Ref.[3], but taken from Ref.[8]— does intuitively show, among the others, that a Superluminal charge[17, 18, 7, 8, 13, 14] traveling at constant speed in the vacuum, would *not* lose energy: see [3] and the text.

$$\zeta \equiv z - Vt; \quad \eta \equiv z - ct .$$

In terms of the new variables* and confining ourselves now to $V > c$, equation (11) can be rewritten as

$$\psi(\rho, \zeta, \eta) = (V - c) \int_0^\infty d\sigma \int_{-\infty}^\sigma d\alpha J_0 \left(\rho \sqrt{\gamma^{-2}\sigma^2 - 2(\beta - 1)\sigma\alpha} \right) \exp[-i\alpha\eta] \exp[i\sigma\zeta] A(\alpha, \sigma)$$

where $\alpha \equiv (\omega - V k_z)/(V - c)$ and $\sigma \equiv (\omega - c k_z)/(V - c)$.

*The same variables were adopted in Ref.[22] in the paraxial approximation context, while we are addressing the general exact case

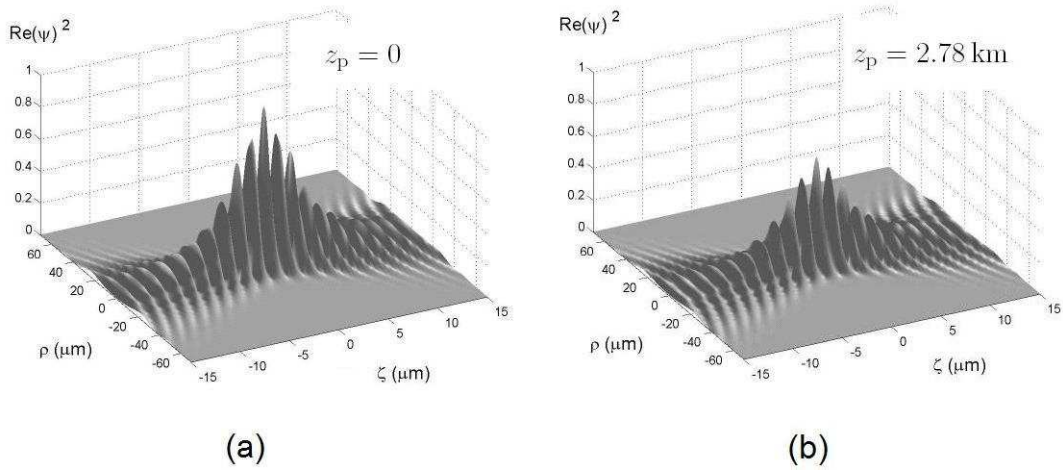


Figure 3: Example of a finite-energy X-type LW, corresponding to an exact, analytic solution of Eq.18, totally free from backward-components. This figure represents the real part of the field, normalized at $\rho = z = 0$ for $t = 0$, with the choices $a = 3.99 \times 10^{-6}$ m, $d = 20$ m, $V = 1.005 c$, and $\alpha_0 = 1.26 \times 10^7 \text{ m}^{-1}$. In this case the frequency spectrum starts at $\omega \equiv \omega_{\min} \approx 3.77 \times 10^{15}$ Hz and afterwards decays exponentially with the bandwidth $\Delta\omega \approx 7.54 \times 10^{13}$ Hz. The value ω_{\min} can be regarded as the pulse central frequency; since $\Delta\omega/\omega_{\min} \ll 1$, it exists a well-defined carrier wave, which does clearly show up in the plots. Any finite-energy LW gets deformed while propagating, and (b) represents the pulse depicted in (a) after it has traveled 2.78 km.

As mentioned above, ideal (infinite energy) superluminal LWs are got by imposing the linear constraint $A(\alpha, \sigma) = B(\sigma) \delta(\alpha + \alpha_0)$. By contrast, finite-energy superluminal LWs are obtained by concentrating the spectrum $A(\alpha, \sigma)$ in the vicinity of the straight line $\alpha = -\alpha_0$. By choosing for example

$$A(\alpha, \sigma) = \frac{\Theta(-\alpha - \alpha_0)}{V - c} e^{d\alpha} e^{-a\sigma}, \quad (17)$$

we get the *finite-energy exact solutions* (free from any backward-components):

$$\psi(\rho, \zeta, \eta) = \frac{X}{VZ} e^{-\alpha_0 Z}, \quad (18)$$

where X is defined in eq.(8), quantities a , d and α_0 are positive constants, and

$$Z \equiv (d - i\eta) - \frac{c}{V + c} (a - i\zeta - VX^{-1}).$$

In Figs.3 we show one of such finite-energy Superluminal LWs, corresponding to an exact, analytic solution totally free from backward-travelling components. As we know, any finite-energy X-wave gets deformed while propagating: and Fig.3(b) shows the pulse in Fig.3(a) after it has travelled 2.78 km.

6) Acknowledgements — The authors are greatly indebted to I.M.Besieris for having called their attention to Ref.[1] and for a very careful, extensive and painstaking revision of this work; and to I.M.Besieris, P.Saari, and C.A.Dartora for many stimulating discussions.

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