MAGNETIC MONOPOLES AND TACHYONS IN SPECIAL RELATIVITY☆

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The *mere* special relativity does not explicitly predict existence of (sub-luminal) monopoles, but on the contrary *explicitly* predicts existence of super-luminal (tachyon) monopoles, with magnetic charge about 100 times less than usually assumed. This fact is relevant also at the light of current experiments looking for magnetic poles.

Since experiments looking for magnetic monopoles failed till now, but a recent claim [1] appeared about a *possible* monopole-detection, it should be interesting to clearly know – and to take possibly into account – the *separate* predictions on the subject of the mere special relativity, on one hand, and of the mere quantum mechanics, on the other hand.

First, we are going to put forth that *special relativity:*

(i) does *not* explicitly predict existence of (slower-than-light) magnetic monopoles;

(ii) *does* explicitly predict, on the contrary, existence of tachyonic (= faster-than-light) "monopoles";

(iii) in its simplest version, predicts the unit magnetic charge for tachyon monopoles to be about one hundred times less than usually [2] assumed ($g = \pm e$, in Gaussian units);

(iv) many good features of the old hypothesis about magnetic monopoles [2] are reproduced, within relativity, by simply taking account of superluminal ($v^2 > c^2$) speeds. In particular, existence of both subluminal ($v^2 < c^2$) and superluminal "electric" charges brings to *fully* symmetrical Maxwell equations [3] (c.f. eqs. (1) in the following).

On the contrary, as regards quantum mechanics, we can say the following. If one assumes *subluminal* magnetic monopole existence, then it seems that the contemporaneous quantization of both electric and magnetic charges follows [2]. This *might* suggest that even subluminal magnetic poles can exist, with unit charge $g = e/2\alpha$, quantity α being the fine-structure constant. Notice, however, that in the present theory the previous one is the *only* argument in favour of subluminal (and large) magnetic charges, since e.g. Maxwell eqs. don't need them to be written in fully symmetrical form.

In fact, let us rebuild [4] a new theory of special relativity *without* assuming *a priori* $|v| < c$. In other words, let us start from the postulates:

1) *Principle of relativity:* laws of mechanics and electro-magnetism are covariant under transitions between two inertial frames, whose relative speed is *a priori* $-\infty < u < +\infty$.

2) *Space is isotropic and space-time homogeneous.*

3) *Principle of retarded causality:* physical signals are transported only by positive-energy objects[‡1].

There follows an "extended relativity" [4], in which light-speed is invariant with respect to *all* inertial frames, both subluminal (s) and superluminal (S), and in which e.g. tachyons do *not* imply any causality violation (as shown in refs. [4, 5] and refs. therein). What is more, the "extended relativity" already proved to be useful even for *usual* particle physics, since *for*

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‡1 The usefulness of the third postulate – even in *standard* relativity! – has been shown e.g. in ref. [4].

Magnetic Monopoles and Tachyons.

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1. – Recently in ref. (1-3) the existence has been guessed of a link between magnetic monopoles (4) and tachyons (5,6). An experimental group (8) took seriously that suggestion, and searched for «tachyon monopoles», even if they based their experiment on a theoretical assumption that we deem incorrect (6,9).

In the present letter we want preliminarily show the nature of such a connection. We are basing ourselves on special-relativity theory, generalized (6,7) for Superluminal inertial frames and faster-than-light objects. For simplicity we shall refer only to x-axis boosts. The main characteristics of «extended relativity» of interest here are:

i) the generalized Lorentz transformations (GLT), which (in the case of x-axis boosts) read (6,7)

$$(1)\qquad \begin{cases} x = C(x'\cos\varphi + ct'\sin\varphi)\,, & y = (-\eta\delta)y'\,, \\ t = C\left(t'\cos\varphi + \dfrac{x'}{c}\sin\varphi\right), & z = (-\eta\delta)z'\,, \end{cases} \qquad \left[\beta^2 \equiv \left(\frac{u}{c}\right)^2 \gtrless 1\right],$$

(1) L. PARKER: *Phys. Rev.*, **188**, 2287 (1969).
(2) D. WEINGARTEN: Report NBI-HE-71-3, Niels Bohr Institute, Copenhagen (1971), *Ann. of Phys.*, **76**, 510 (1973).
(3) E. RECAMI: in *Enciclopedia EST Mondadori, Annuario 1973* (Milano, 1973), p. 85.
(4) P. A. M. DIRAC: *Proc. Roy. Soc.*, A **133**, 60 (1931); *Phys. Rev.*, **74**, 817 (1948); J. SCHWINGER: *Phys. Rev.*, **144**, 1087 (1966); N. CABIBBO and E. FERRARI: *Nuovo Cimento*, **23**, 1147 (1962). For an extensive bibliography see, *e.g.*, E. AMALDI: in *Old and New Problems in Elementary Particles*, edited by G. PUPPI (New York, 1968); E. AMALDI and N. CABIBBO: in *Aspects of Quantum Theory*, edited by A. SALAM and E. P. WIGNER (Cambridge, 1972).
(5) See, *e.g.*, O. M. P. BILANIUK, V. K. DESHPANDE and E. C. G. SUDARSHAN: *Am. Journ. Phys.*, **30**, 718 (1962).
(6) R. MIGNANI, E. RECAMI: *Riv. Nuovo Cimento* (to appear). See also ref. (3). For an extensive bibliography, see, *e.g.*, ref. (7).
(7) R. MIGNANI and E. RECAMI: *Nuovo Cimento*, **14** A, 169 (1973); **16** A, 208 (1973); E. RECAMI and R. MIGNANI: *Lett. Nuovo Cimento*, **4**, 144 (1972); **8**, 110 (1973); and references therein. See also ref. (1).
(8) D. F. BARTLETT and M. D. LAHANA: *Phys. Rev. D*, **6**, 1817 (1972).
(9) R. MIGNANI and E. RECAMI: *Lett. Nuovo Cimento*, **7**, 388 (1973). See also ref. (6).

where (*)

$$\beta \equiv \operatorname{tg}\varphi \quad [0 \leqslant \varphi \leqslant 2\pi], \qquad \gamma \equiv +|1-\operatorname{tg}^2\varphi|^{-\frac{1}{2}}, \qquad \delta \equiv + \sqrt{\frac{1-\operatorname{tg}^2\varphi}{|1-tg^2\varphi|}},$$

$$\eta \equiv \frac{\cos\varphi}{|\cos\varphi|}\cdot\delta^2, \qquad C \equiv + \sqrt{\frac{1+\operatorname{tg}^2\varphi}{|1-\operatorname{tg}^2\varphi|}};$$

they constitute a new (7) group G;

ii) a « principle of duality » (DP) (6,7), derived from the properties of the group G, which esserts that: « The terms bradyon (7) (B) (usual particles, $v^2 < c^2$), tachyon (T), subluminal inertial frame (s), Superluminal inertial frame (S) do not have an absolute meaning, but only a *relative* one »;

iii) a « rule of tachyonization » (TR) (6,7) (or of the extended relativity), as well derived from the G properties, which states that: « The relativistic laws of mechanics and electromagnetism for tachyons follow by applying a Superluminal Lorentz transformation (SLT)—*e.g.* the *transcendent one* (7) $K_+ \equiv \lim_{\beta\to+\infty} \mathrm{SLT}(\beta>0)$—to the corresponding laws for bradyons ». In fact, particles that are *tachyons* with respect to us (frame s_0) are bradyons with respect to frames S (in particular to *their* own rest frame). And SLT's effect transition from frames S to frames s, and *vice versa.*

2. – The problem of generalizing Maxwell equations for tachyonic sources is not straightforward, since the usual electromagnetic tensor $F_{\mu\nu}$ *a priori* is not expected to be a G-tensor (*i.e.* to behave as a tensor under the extended group G, in particular under SLT's). The problem reduces to generalizing usual Lorentz transformations (10) of the electric field $\boldsymbol{E}$ and of the magnetic field $\boldsymbol{H}$:

$$\text{(2)} \qquad \begin{cases} E_x = E'_x\,, & E_y = \gamma(E'_y + \beta H'_z)\,, \quad E_z = \gamma(E'_z - \beta H'_y)\,, \\ H_x = H'_x\,, & H_y = \gamma(H'_y - \beta E'_z)\,, \quad H_z = \gamma(H'_z + \beta E'_y)\,, \end{cases} \qquad [\beta^2 < 1],$$

for Superluminal velocities. We shall base ourselves (9) on the analogy of « Lorentz transformation » (LT) extension (6,7).

However, let us *first* premise all what follows.

Let us remember that *standard* Maxwell equations read (10)

$$\text{(3)} \qquad \partial_\nu F_{\mu\nu} = j_\mu\,, \qquad \partial_\nu \tilde{F}_{\mu\nu} = 0 \qquad [v^2 < c^2],$$

where $\tilde{F}_{\mu\nu}$ is the *dual* tensor of $F_{\mu\nu}$, and $j_\mu \equiv (\varrho_e, \boldsymbol{j})$ is the *electric* current density four-vector. Notice that we defined (2,11)

$$\text{(4)} \qquad \tilde{F}_{\mu\nu} \equiv \tfrac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta} \qquad [\mu, \nu, \alpha, \beta = 0, 1, 2, 3],$$

(*) By the way, notice that in extended relativity both subluminal observers (s) and Superluminal observers (S), being all equivalent (6,7), will see *the same* Minkowsky space, and will use *the same* metric. However, if an observers s (or S) wants to look at space-time not directly, but through the observations of an observer S (or s), *then* he will have to multiply them by i (since what is spacelike for s is timelike for S, and so on). This is the reason why imaginary units enter (only) *the relations between observations from s and from S* (but *not* the « physics » build up by one and the same observer, through his own measures and observations). See R. MIGNANI and E. RECAMI: *Lett. Nuovo Cimento* (to appear).

(10) See, *e.g.*, L. D. LANDAU and E. M. LIFSHITZ: *Theorie du Champ* (Moscow, 1966).

(11) See, *e.g.*, J. M. LEINAAS: *Nuovo Cimento*, **15 A**, 740 (1973).

where $\varepsilon_{\mu\nu\alpha\beta}$ is the *real* completely antisymmetric Ricci tensor. It is

$$\tilde{\tilde{F}}_{\mu\nu} = F_{\mu\nu}\,. \tag{4 bis}$$

Moreover, present «duality» effects the exchanges $\boldsymbol{E}\to i\boldsymbol{H}$, $\boldsymbol{H}\to -i\boldsymbol{E}$.

Let us now introduce the *autodual* (electromagnetic) tensor

$$T_{\mu\nu} \equiv F_{\mu\nu} + \tilde{F}_{\mu\nu}\,, \qquad \tilde{T}_{\mu\nu} = T_{\mu\nu}\,, \tag{5}$$

where $[\mu, \nu = 0, 1, 2, 3]$

$$(T_{\mu\nu}) = \begin{pmatrix} 0 & iE_x - H_x & iE_y - H_y & iE_z - H_z \\ H_x - iE_x & 0 & iE_z - H_z & H_y - iE_y \\ H_y - iE_y & H_z - iE_z & 0 & iE_x - H_x \\ H_z - iE_z & iE_y - H_y & H_x - iE_x & 0 \end{pmatrix}. \tag{6}$$

We are using the metric $(+ - - -)$; however, we write the generic vector as $x \equiv (ct, ix, iy, iz)$, so that $g_{\mu\nu} \equiv \delta_{\mu\nu}$ and we have no distinction between covariant and contravariant components. Summation is understood over the repeated indices (see, *e.g.*, eqs. (3), (4)). Natural units ($c = 1$) will be used when convenient (*).

In terms of $T_{\mu\nu}$, the standard equations (3) can be written together in an essentially single equation:

$$\partial_\nu T_{\mu\nu} = j_\mu\,, \qquad \tilde{T}_{\mu\nu} = T_{\mu\nu} \qquad [v^2 < c^2]\,. \tag{7}$$

As is well known, if magnetic monopoles are supposed to exist, the previous eqs. (3) and (7) may be written ([4]) in a completely symmetric form, by introduction of the *magnetic*-current density four-vector $g_\mu = (\varrho_m, \boldsymbol{g})$. Equations (3) become

$$\partial_\nu F_{\mu\nu} = j_\mu\,, \qquad \partial_\nu \tilde{F}_{\mu\nu} = ig_\mu \qquad [v^2 < c^2]\,, \tag{3'}$$

and eq. (7) then reads

$$\partial_\nu T_{\mu\nu} = j_\mu + ig_\mu\,, \qquad \tilde{T}_{\mu\nu} = T_{\mu\nu} \qquad [v^2 < c^2]\,. \tag{7'}$$

3. – Let us go back to the beginning of Sect. **2**.

It is easy to recognize that the couple E_y, H_z in eqs. (2) behaves as the couple x, t for subluminal LT's. That is to say, for $0 < \beta < 1$, in the plane (E_y, H_z) the axes E'_y, H'_z rotate from 0° to 45°, in anticlockwise and clockwise sense respectively (E'_y starting from E_y, and H'_z from H_z). It is then straightforward to generalize transformations (2) for $1 < \beta < +\infty$, by allowing axes E'_y, H'_z to rotate with continuity *beyond* 45° until 90° (*i.e.*, until when, for $\beta \to +\infty$, E'_y and H'_z coincide with H_z and E_y, respectively). See Fig. 1*a*)-*b*).

(*) As regards the electromagnetic quantities, Heaviside-Lorentz units will be adopted, for practical purposes.

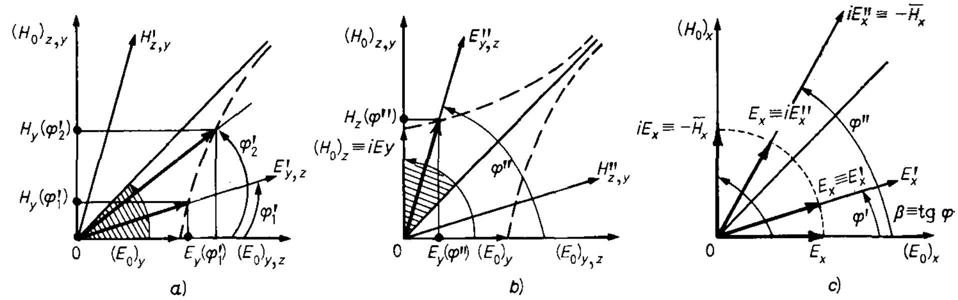

Fig. 1. – Consider an unprimed frame s_0, supposed at rest, and another inertial frame f, to which we shall attribute, every time, all possible collinear velocities u, both subluminal (frames f') and Superluminal (frames f''), along the positive x-axis (*i.e.*, $u \equiv u_x > 0$). Suppose that in f we have a uniform electrostatic field $\boldsymbol{E}$ parallel to the axis y or z (*i.e.*, either $E \equiv E_y$ or $E \equiv E_z$). The cases a), b) represent how s_0 will see the moving field, according to our eqs. (8a), (8b). For $0 < u < c$, we shall have the *usual* case a), with $0 < \varphi < 45°$. For $0 < u < \infty$, we shall have the case b), with $0 < \varphi < 90°$; in particular. for $c < u < \infty$, the angle φ will run from 45° to 90°. We would have met an *analogous* situation when starting from a uniform magnetostatic field, parallel either to the axis y or to the axis z. (See the text.) Notice that here the spaces (E_y, H_z) and (E_z, H_y) are *pseudo-Euclidean*, analogously to space (x, t). On the contrary, c) is only a *symbolic graph*, meaning that for $0 < u < c$ we have $E_x(\varphi') = E'_x$, and that for $c < u < \infty$ we have $E_x(\varphi'') = -\overline{H}_x \equiv iE''_x \equiv H''$. Notice that we are understanding that $\overline{H}_x = E_x$, in Gaussian units. Moreover $(H_0)_x \equiv \overline{H}_x$. Case c) refers for simplicity only to $0 < \varphi < \pi/2$. (See the text.)

Therefore, similarly to eqs. (1), we shall write, for both subluminal *and Superluminal* velocities,

$$(8a)\qquad \begin{cases} E_y = C(E'_y \cos\varphi + H'_z \sin\varphi)\,, \\ H_z = C(H'_z \cos\varphi + E'_y \sin\varphi)\,, \end{cases} \qquad [\beta^2 \gtrless 1;\; 0 \leqslant \varphi \leqslant 2\pi].$$

By analogous, « geometrical » extrapolation, we may write, for both subluminal *and Superluminal* velocities,

$$(8b)\qquad \begin{cases} E_z = C(E'_z \cos\varphi - H'_y \sin\varphi)\,, \\ H_y = C(H'_y \cos\varphi - E'_z \sin\varphi)\,, \end{cases} \qquad [\beta^2 \gtrless 1].$$

Regarding E_x and H_x, as suggested by eqs. (1) we shall write for Superluminal velocities (*)

$$E_x = iE'_x\,, \qquad H_x = iH'_x \qquad [\beta^2 > 1];$$

in fact, as y, z components behaved as co-ordinates x, t, we assumed now x components to behave as co-ordinate y (or z) in eqs. (1). For both subluminal *and Superluminal* velocities we have (*) (see Fig. 1c))

$$(8c)\qquad \begin{cases} E_x = E'_x\delta\,, \\ H_x = H'_x\delta\,, \end{cases} \qquad [\beta^2 \gtrless 1].$$

(*) In the following, we shall for simplicity confine ourselves to $-\pi/2 < \varphi < \pi/2$. Notice since now that, due to invariance of our autodual tensor $T_{\mu\nu}$ under « duality », we may consider $iE'_x \equiv -\overline{H}_x$ (where $\overline{H}_x = E_x$, in Gaussian units) and $iH' \equiv \overline{E}_x$ (where $\overline{E}_x = H_x$, in Gaussian units).

Notice explicitly, from eq. (7′), that *multiplication by the imaginary unit i turns quantities due to the electric current into quantities due to the magnetic current* (*). Our transformations (8) may be as well derived by considering the complex vector ([10])

$$\mathscr{F} \equiv \boldsymbol{E} + i\boldsymbol{H}, \qquad \boldsymbol{E} = \operatorname{Re} \mathscr{F}, \qquad \boldsymbol{H} = \operatorname{Im} \mathscr{F},$$

and suitable rotations (*).

With transformations (8), we get precisely the interesting result that *under* SLT's *electric and magnetic currents are changed one into the other.* In other words, Superluminal « electric charges » contribute to the electromagnetic field as subluminal magnetic monopoles would do, and *viceversa.* If (bradyonic) monopoles do not exist, then *« electrically charged » tachyons will bring into the (generalized) field equation contributions of magnetic-monopole type.*

Namely, let us consider a SLT, *e.g.* the transcendent one (from the unprimed frame to the primed frame) ([6,7]):

$$K_+ = \begin{pmatrix} \sigma_2 & 0 \\ 0 & -i\sigma_0 \end{pmatrix} = -(-\sigma_2 \oplus i\sigma_0), \tag{1 bis}$$

where

$$\sigma_0 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \sigma_2 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

are Pauli's matrices. If, according to DP and TR (Sect. 1), we apply K_+ to eqs. (7), we get (**)

$$\operatorname{div} \boldsymbol{E}' = 0, \qquad \operatorname{div} \boldsymbol{H}' = -\varrho_e' \qquad [\beta^2 > 1], \tag{9}$$

and, by applying K_+ to eqs. (7′),

$$\operatorname{div} \boldsymbol{E}' = \varrho_m', \qquad \operatorname{div} \boldsymbol{H}' = -\varrho_e' \qquad [\beta^2 > 1]. \tag{9′}$$

It is easy to verify that, with our eqs. (8), the (Maxwell) field equations (7′) can be written *in G-covariant* form (*i.e.* in a form covariant under the *whole* group G of GLT's)

(*) Therefore, when passing from a Superluminal frame (with speed corresponding to φ_S) to a collinear subluminal frame (with speed corresponding to φ_s), we can better write

$$\begin{cases} E_x(\varphi_s) = iE_x' = iE_x(\varphi_S) = -\overline{H}_x(\varphi_s), \\ H_x(\varphi_s) = iH_x' = iH_x(\varphi_S) = \overline{E}_x(\varphi_s), \end{cases} \qquad [\beta^2 > 1],$$

where, in Gaussian units, $\overline{H}_x = \overline{E}_x$ and $\overline{E}_x = H_x$. *By adopting a different formalism,* one can, for example, write (for $\beta^2 > 1$)

$$\begin{cases} E_x' \xrightarrow{\text{(SLT)}} -H_x, \\ H_x' \xrightarrow{\text{(SLT)}} E_x, \end{cases}$$

where now, in Gaussian units, $H_x = E_x'$ and $E_x = H_x'$. These considerations do agree with the well-known invariance of the electromagnetic tensor $T_{\mu\nu}$ under the « duality exchanges ».

(**) By the way, the « transcendent Lorentz transformation » K_+ works so that

$$\begin{cases} E_y' = H_z, & H_y' = -E_z, \\ E_z' = -H_y, & H_z' = E_y, \end{cases}$$

as *heuristically* forecast in ref. ([1]).

as follows:

$$(10)\qquad \begin{cases} \partial_\nu T_{\mu\nu} = [j_\mu(s) + g_\mu(S)] + i[g_\mu(s) - j_\mu(S)] , \\ \tilde{T}_{\mu\nu} = T_{\mu\nu} , \end{cases} \qquad [v^2 \gtrless c^2] ,$$

where $j_\mu(s)$ and $g_\mu(S)$ are the *subluminal* electric current and the *Superluminal* magnetic current respectively, and so on.

If is assumed magnetic monopoles do not exist, nevertheless (due to tachyon presence) eq. (10) will read

$$(11)\qquad \begin{cases} \partial_\nu T_{\mu\nu} = j_\mu(s) - ij_\mu(S) , \\ \tilde{T}_{\mu\nu} = T_{\mu\nu} , \end{cases} \qquad [v^2 \gtrless c^2] .$$

Notice that the quantity $T_{\mu\nu}$, which is a tensor under the standard Lorentz group, is no more a usual tensor under the group G, since $T_{\mu\nu}$ under a SLT behaves like a standard tensor, *except*—however—*for* a factor i. That is to say, $T_{\mu\nu}$ is no more supposed to be a standard tensor in eqs. (10)-(11). However, we are assuming $j_\mu \equiv \varrho_0 u_\mu$, where ϱ_0 is the proper charge-density and $u_\mu \equiv dx_\mu/d\tau_0$ is the G-fourvelocity ([4]), so that j_μ is a G-fourvector. Let us now for simplicity reduce to subluminat LT's; in this case eqs. (11) are equivalent to the following ones:

$$(12)\qquad \begin{cases} \nabla\cdot\boldsymbol{D} = \varrho(s) ; \\ \nabla\cdot\boldsymbol{B} = -\varrho(S) ; \\ \nabla\wedge\boldsymbol{E} = \boldsymbol{j}(S) - \dfrac{\partial\boldsymbol{B}}{\partial t} ; \\ \nabla\wedge\boldsymbol{H} = \boldsymbol{j}(s) + \dfrac{\partial\boldsymbol{D}}{\partial t} , \end{cases} \qquad [v^2 \gtrless c^2;\ s = \text{subluminal};\ S = \text{Superluminal}]$$

whose physical meaning is transparent.

All such problems will be dealt with, with more details, in a forthcoming paper, with attention to the experimental consequences too.

Summary. – Connection between magnetic monopoles and faster-than-light particles is shown, in the framework of extended special-relativity theory, by generalizing the Maxwell equations to Superluminal sources.

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