Tachyon Mechanics and Tachyon Gravitational Interaction (*).

E. RECAMI (**)

Department of Applied Mathematic, State University at Campinas Campinas, S.P., Brazil

E. GIANNETTO

Dipartimento di Fisica dell'Università - Catania, Italia

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Summary. – By starting from a variational principle, we reformulate some basic questions of tachyon mechanics, improving (and correcting some relevant signs in) various previous equations, and statements, put forth in the past even by ourselves. Partly as an application, we then revisit the theory of the behaviour expected for tachyons when interacting gravitationally. In a gravitational field, tachyons would seem to be repelled *from the kinematical point of view*; but they would actually be attracted *from the dynamical and energetical point of view*. Such considerations might even have a role in astrophysics, as shown elsewhere.

Introduction. – When facing the problem of extending special relativity (SR) to space-like objects (1,2), some authors confined themselves always to refer *all* objects, both subluminal and Superluminal, to *subluminal* observers only (« weak approach »). Other authors attempted on the contrary to generalize SR by introducing both sub- and Superluminal observers, and then by extending the principle of relativity (« strong approach »).

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(**) On leave of absence from the Dipartimento di Fisica, Università Statale di Catania, Catania, Italy.
(1) For a recent review on tachyons, see E. RECAMI: *Classical tachyons*, Report INFN/AE-84/8 (Frascati, 1984), to be published in *Riv. Nuovo Cimento*; for an older review, see E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209, 398 (1974). For partial, recent reviews see ref. (2).
(2) E. RECAMI: *Classical tachyons*, to appear in *Riv. Nuovo Cimento*; E. RECAMI and W. A. RODRIGUES: *A model-theory for tachyons in two dimensions*, Preprint No. 310 (Dept. Appl. Mathem., State Univ. at Campinas, Campinas, S.P., Brazil, 1985), to appear in the *Proceedings of the Sir Arthur Eddington Centenary Symposium*, Vol. **3**: *Gravitational Radiation and Relativity*, edited by T. M. KARADE (World Sci. Pub. Co., Singapore).

The latter approach is theoretically more worth of consideration (tachyons, *e.g.*, get *real* rest masses), but it meets of course the greatest obstacles (3).

Luckily enough, an extensive study of the expected tachyon properties can be performed within the weak approach, and even without assuming the existence of « Superluminal Lorentz transformations » in four dimensions. This is true for instance for the discussion (and solution (2), at least « in microphysics ») of the so-called « causal paradoxes », and for the exploitment of tachyon mechanics.

We already expounded some tachyon kinematics in ref. (4), especially in connection with the two-body interactions via tachyon-exchange, a issue quite relevant both for the correct solution of the causality paradoxes (even if this fact has not yet been widely recognized (*)(5)), and for the possible applications in the realm of elementary-particle physics.

On the variational principle. – Let us now turn a bit our attention to the question of introducing an action integral, S, for the free tachyons. In the case of an ordinary, free particle we have that $S = \alpha\int_a^b \mathrm{d}s$; for a free tachyon let us, rather, write

$$S = \alpha\int_a^b |\mathrm{d}s| \, . \tag{1}$$

By analogy with the bradyonic ($|\boldsymbol{v}| < c^2$) case, we might assume for a free tachyon the Lagrangian ($c = 1$)

$$L = + m_0\sqrt{V^2 - 1} \qquad (V^2 > 1) \tag{2}$$

and therefore evaluate, in the usual way,

$$\boldsymbol{p} \equiv \frac{\partial L}{\partial \boldsymbol{V}} = + \frac{m_0 \boldsymbol{V}}{\sqrt{V^2 - 1}} \equiv m\boldsymbol{V}, \tag{3}$$

which suggests that for tachyons

$$m = \frac{m_0}{\sqrt{V^2 - 1}} \tag{4}$$

in agreement with refs. (1).

If the tachyon is no longer free, we can write as usual

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m_0 \boldsymbol{V}}{\sqrt{V^2 - 1}}\right). \tag{5}$$

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(3) Cf. *e.g.* G. D. MACCARONE and E. RECAMI: *Found. Phys.*, **14**, 367 (1984) and references therein; G. D. MACCARONE, M. PAVŠIČ and E. RECAMI: *Nuovo Cimento B*, **73**, 91 (1983); E. RECAMI: *Int. J. Theor. Phys.* (to appear); A. O. BARUT, G. D. MACCARONE and E. RECAMI: *Nuovo Cimento A*, **71**, 509 (1982).

(4) G. D. MACCARRONE and E. RECAMI: *Nuovo Cimento A*, **57**, 85 (1980).

(*) For a late, remarkable example of wrong perception of the tachyon-dynamics issues involved, see ref. (5).

(5) R. GIRARD and L. MARCHILDON: *Found. Phys.*, **14**, 535 (1984).

By choosing the reference frame, at the considered time-instant t, in such a way that $\boldsymbol{V}$ is parallel to the x-axis, *i.e.* $|\boldsymbol{V}| = V_x$, we then get

$$F_x = + m_0 \left[\frac{1}{\sqrt{V^2 - 1}} - \frac{V^2}{\sqrt{V^2 - 1)^3}}\right] a_x = - \frac{m_0}{(V^2 - 1)^{\frac{3}{2}}} a_x \tag{6a}$$

and

$$F_y = + \frac{m_0}{\sqrt{V^2 - 1}} a_y, \qquad F_z = + \frac{m_0}{\sqrt{V^2 - 1}} a_z . \tag{6b}$$

The sign in eq. (6*a*) is consistent with the ordinary definition of work $\mathscr{L}$,

$$\mathrm{d}\mathscr{L} \equiv + \boldsymbol{F}\cdot \mathrm{d}\boldsymbol{\ell}, \tag{7}$$

and the fact that the total energy of a tachyon *increases* when its speed *decreases* [1].

Notice, however, that the proportionality constant between force and acceleration does *change sign* when passing from the longitudinal to the transverse components.

The tachyon total energy E, moreover, can still be defined as

$$E \equiv \boldsymbol{p}\cdot \boldsymbol{V} - L = \frac{m_0 c^2}{\sqrt{V^2 - 1}} \equiv mc^2 , \tag{8}$$

which, together with eq. (4), extends to tachyons the relation $E = mc^2$.

However, the following comments are in order at this point. An ordinary timelike (straight) line can be bent only in a spacelike direction; and it gets shorter. On the contrary, if you take a spacelike line and, keeping two points on it fixed, bend it slightly in between in a spacelike (time) direction, the bent line is longer (shorter) than the original straight line (see *e.g.* DORLING [6]). For simplicity, let us here skip the generic case when the bending is partly in the time and partly in a spacelike direction (even if such a case looks to be the most interesting). Then, the action integral of eq. (1) along the straight (spacelike) line is *minimal* with respect to the « space-like » bendings and *maximal* with respect to the « timelike » bendings. *A priori*, one might then choose for a free tachyon, instead of eq. (2), the Lagrangian

$$L = - m_0 \sqrt{V^2 - 1} , \tag{2'}$$

which yields

$$\boldsymbol{p} = \frac{\partial L}{\partial \boldsymbol{V}} = - \frac{m_0 \boldsymbol{V}}{\sqrt{V^2 - 1}} \equiv - m\boldsymbol{V} . \tag{3'}$$

Equation (3′) would be rather interesting when tachyons are substituted [1] for the « virtual particles » as the carriers of the elementary-particle interactions (cf. *e.g.* subsect **6**.13 in RECAMI [1]). In fact, the (classical) exchange of a tachyon endowed with a momentum antiparallel to its velocity would generate an *attractive* interaction.

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(6) J. DORLING: *Am. J. Phys.*, **38**, 539 (1970).

For nonfree tachyons, from eq. (3′) one gets

$$F = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m_0 \boldsymbol{V}}{\sqrt{V^2 - 1}}\right) \tag{5′}$$

and therefore, when $|\boldsymbol{V}| = V_x$,

$$F_x = +\frac{m_0}{(V^2-1)^{\frac{3}{2}}}\, a_x , \tag{6′a}$$

$$F_y = -\frac{m_0}{\sqrt{V^2-1}}\, a_y , \qquad F_z = -\frac{m_0}{\sqrt{V^2-1}}\, a_z . \tag{6′b}$$

Due to the sign in eq. (6′a), it is now necessary to define the work $\mathscr{L}$ as

$$\mathrm{d}\mathscr{L} = -\boldsymbol{F}\cdot \mathrm{d}\boldsymbol{l} , \tag{7′}$$

and analogously the total energy E as

$$E = -(\boldsymbol{p}\cdot\boldsymbol{V} - L) = \frac{m_0 c^2}{\sqrt{V^2-1}} = mc^2 . \tag{8′}$$

The brief analysis above allows us some important improvements and corrections of the tachyon mechanics developed by RECAMI and MIGNANI (1) in 1974.

On tachyon mechanics. – Let us consider a force $\boldsymbol{F}$ acting on a tachyon T. If at the considered time-instant t we choose as usual the x-axis, so that $V = |\boldsymbol{V}| = V_x$, then only the force-component F_x will make work. We already mentioned that the total energy of a tachyon decreases when its speed increases, and *vice-versa*; it follows that F_x when applied to a tachyon will actually make a positive, elementary work $\mathrm{d}\mathscr{L}$, only if a_x is antiparallel to the elementary dispacement $\mathrm{d}x$, *i.e.* if sign $(a_x) = -$ sign $(\mathrm{d}x)$. In other words, $\mathrm{d}\mathscr{L}$ in the case of a force $\boldsymbol{F}$ applied to a tachyon *must be defined* so that

$$\mathrm{d}\mathscr{L} = -\frac{m_0}{(V^2-1)^{\frac{3}{2}}}\, a_x \,\mathrm{d}x , \tag{9}$$

where a_x and $\mathrm{d}x$ possess of course their own signs. Equation (9) does agree both with the couple of equations (6a), (7) and with the couple of equations (6′a), (7′).

It is evident that—*with the choice* represented by eqs. (7) and (2)—we shall have ($v_n = v_x$; $V = V_x$):

$$F_x = +\frac{m_0}{(1-v^2)^{\frac{3}{2}}}\, a_x \qquad \text{for bradyons;} \tag{10a}$$

$$F_x = -\frac{m_0}{(V^2-1)^{\frac{3}{2}}}\, a_x \qquad \text{for tachyons.} \tag{10b}$$

On the contrary, still with the choice (7)-(2), we shall have

$$F_{y,z} = +\frac{m_0}{(|1-\beta^2|)^{\frac{1}{2}}}\, a_{y,z} \qquad (\beta^2 \lessgtr 1) \tag{10c}$$

for *both* bradyons and tachyons. Actually, under our hypotheses ($v = v_x$; $V = V_x$), the transverse force components $F_{y,z}$ do not make any work; therefore, one had no reasons *a priori* for expecting any change in eq. (10*c*) when passing from bradyons to tachyons.

Let us repeat that what above improves or corrects what claimed in the past, even by ourselves. As an important consequence of the modifications introduced above (essentially some sign changes), let us consider the interaction of tachyons with a gravitational field.

Before going on, however, let us premise the following consideration. The fundamental law of ordinary (bradyon) dynamics reads

$$F^\mu = c\,\frac{\mathrm{d}}{\mathrm{d}s}\left(m_0 c\,\frac{\mathrm{d}x^\mu}{\mathrm{d}s}\right) \equiv \frac{\mathrm{d}}{\mathrm{d}\tau_0}\left(m_0\,\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau_0}\right) \qquad (\beta^2 < 1)\,. \tag{11}$$

Notice that (since along a spacelike path $\mathrm{d}s^2$ is negative, and then $\mathrm{d}s$ is imaginary) the first form of eq. (11) is invariant only in the timelike case; whilst its second form can be assumed [1] as valid both for bradyons and for tachyons. (Quantity τ_0 is the proper time.)

Even for tachyons, then, we shall have

$$F^\mu = +\,\frac{\mathrm{d}}{\mathrm{d}\tau_0}\,(m_0 u^\mu) \equiv +\,\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau_0} \qquad (\beta^2 > 1)\,, \tag{12}$$

where m_0 is the tachyon (real) proper mass, and we defined [3] $p^\mu \equiv m_0 u^\mu$ also for tachyons. Equation (12) is the relativistic Newton law expected to hold for $\beta^2 \gtrless 1$. It is essential to recall [1], however, that the four-velocity u^μ is to be defined $u^\mu \equiv \mathrm{d}x^\mu/\mathrm{d}\tau_0$. Quantity $\mathrm{d}\tau_0$, in fact, can be assumed to be real and invariant for both bradyons and tachyons; on the contrary, $\mathrm{d}s = \pm\, c\,\mathrm{d}\tau_0$ for bradyons, but $\mathrm{d}s = \pm\, ic\,\mathrm{d}\tau_0$ for tachyons [1]. These last considerations connected with eqs. (11), (12), however, become rigorous only under the assumption that *active* « Superluminal Lorentz transformations » exist [1-3] in four dimensions, which transform timelike into spacelike tangent vectors. Equation (12), incidentally, agrees with eqs. (5) and (5′) and suggests [1] that for tachyons $\mathrm{d}t = \pm\,\mathrm{d}\tau_0/\sqrt{\beta^2 - 1}$, so that for both bradyons and tachyons one may write $\mathrm{d}t = \pm\,\mathrm{d}\tau_0(|1 - \beta^2|)^{-\frac{1}{2}}$.

On the gravitational interaction of tachyons. – In any gravitational field a bradyon feels the (attractive) gravitational four-force

$$F^\mu = -\,m_0 \Gamma^\mu_{\varrho\sigma}\,\frac{\mathrm{d}x^\varrho}{\mathrm{d}s}\,\frac{\mathrm{d}x^\sigma}{\mathrm{d}s} \qquad (\beta^2 < 1)\,. \tag{13}$$

For both bradyons and tachyons, in analogy with the preceding last considerations, eq. (13) may be written as

$$F^\mu = -\,\frac{m_0}{c^2}\,\Gamma^\mu_{\varrho\sigma}\,\frac{\mathrm{d}x^\varrho}{\mathrm{d}\tau_0}\,\frac{\mathrm{d}x^\sigma}{\mathrm{d}\tau_0} \qquad (\beta^2 \gtrless 1)\,, \tag{14}$$

since the Christoffel symbols behave like third-rank tensors under *any* linear transformations of the co-ordinates. Equations (14) are in particular expected to hold for a tachyon in a gravitational field (both when originated by tachyonic and by bradyonic sources). Similarly, the equation of motion for both bradyons and tachyons in a gravitational field may still read

$$a^\mu + \Gamma^\mu_{\varrho\sigma} u^\varrho u^\sigma = 0 \qquad (\beta^2 \gtrless 1) \tag{15}$$

with $a^\mu \equiv \mathrm{d}^2 x^\mu / \mathrm{d}\tau_0^2$.

Passing to general relativity, this does agree with the equivalence principle: bradyons photons and tachyons follow trajectories in a gravitational field, which depend only on the initial (different) four-position and four-velocity and are independent of mass.

In connection with eq. (14), we can say that also tachyons are attracted by a gravitational field. However, such an « *attraction* » has to be understood from the energetical and dynamical point of view *only*. Actually, the fact that the energy of a tachyon decreases (increases) when its speed increases (decreases) engendered a lot of confusion in the literature. We hope to clarify this point here [1].

Let us consider for simplicity's sake a tachyon moving radially with respect to a gravitational source. Due to eq. (10*b*) (*i.e.*, due to the couples of equations either (6*a*)-(7), or (6′*a*)-(7′)) it will *accelerate when receding* from the source, and *decelerate while approaching* to the source. From the kinematical point of view, therefore, we might say that taychons *seem* to be gravitationally *repelled* (analogous results were put forth in ref. [7]). From the more important dynamical and energetical point of view, however, we ought to speak in terms of *attraction.*

In the case of a bradyonic *source*, what precedes agrees with the results obtained within general relativity by various authors [8].

Strictly speaking, eqs. (14)-(15) depend—just as eq. (12)— on the existence of active « Superluminal Lorentz transformations » in four dimensions [1-3], whilst aim of this paper was to emphasize results that do not depend on that assumption. But Superluminal Lorentz transformations surely do exist in two dimensions [1,2], without any problem. Let us corroborate the preceding statements on the tachyon gravitational interaction by a safe (simple, but enlightening) remark in two dimensions. Let us consider two (bradyonic) bodies A, B that for instance—due to mutual attraction—*accelerate while approaching* to each other. The situation is sketched in fig. 1, where A is chosen as the reference-frame $s \equiv (t, x)$ and, for simplicity, only one discrete change in velocity is depicted. From a Superluminal frame they will be described (see the figure) either as two tachyons that *decelerate while approaching* to each other (this is true in the frame $S'' \equiv (t'', x'')$), or as two (anti)tachyons [1,2] *that accelerate while receding* from each other (in the frame $S' \equiv (t', x')$). Therefore, we expect that two tachyons from the *kinematical* point of view will *seem* to suffer a repulsion, if they attract each other in their own rest frames (and in the other frames, of course, in which they are subluminal); this means that such a behaviour of the tachyons is to be considered as due to an attraction from the *energetical and dynamical* point of view. The rigorous analysis of this two-dimensional situation, therefore, perfectly agrees with the results claimed above for tachyon gravitation.

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(7) See *e.g.* P. C. VAIDYA: *Curr. Sci. (India)*, **40**, 651 (1971); A. K. RAYCHAUDHURY: *J. Math. Phys. (N.Y.)*, **15**, 256 (1974); E. HONIG, K. LAKE and R. C. ROEDER: *Phys. Rev. D*, **10**, 3155 (1974).

(8) See *e.g.* F. SALTZMAN and G. SALTZMAN: *Lett. Nuovo Cimento*, **1**, 859 (1969); C. GREGORY: *Nature Phys. Sci.*, **239**, 56 (1972); R. O. HETTEL and T. M. HELLIWELL: *Nuovo Cimento B*, **13**, 82 (1973); C. P. SUM: *Lett. Nuovo Cimento*, **11**, 459 (1974); J. V. NARLIKAR and E. C. G. SUDARSHAN: *Mon. Not. R. Astron. Soc.*, **175**, 105 (1976); J. V. NARLIKAR and S. V. DHURANDHAR: *Pramàna*, **6**, 388 (1976); R. P. COMER and J. D. LATHROP: *Am. J. Phys.*, **46**, 801 (1978); V. K. MALTSEV: *Teor. Mat. Fiz.*, **47**, 177 (1981); J. CIBOROWSKI: Preprint (Inst. Exp. Phys., Warsaw, 1982); A. M. FINKELSTEIN, V. JA. KREINOVICH and S. N. PANDEY: Report (Sp. Astrophys. Obs., Pulkovo, 1983); C. SHENGLIN, X. XINGHUA, L. YONGZHEN and D. ZUGAN: Preprint (Beijing Normal Univ., Peking, 1984).

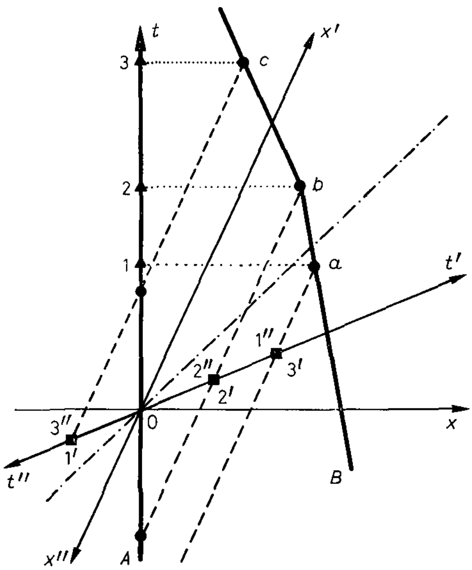

Fig. 1. – If two ordinary bodies A, B attract each other and, for instance, accelerate while approaching to each other, then from Superluminal frames they will be described either as two tachyons that decelerate while approaching to each other (in the frame $S'' \equiv (t'', x'')$), or as two (anti)tachyons [1,2] that accelerate, while receding from each other (in the frame $S' \equiv (t', x')$). In two dimensions this can be made perfectly rigorous [1,2]. For the consequences, see the text. For simplicity's sake, in this figure only a *discrete* velocity change is depicted.

Those results might have a role even in astrophysics, in connection with the observed, *apparent* « Superluminal expansions » in the core of various quasars and galaxies [10], especially if it will be confirmed the observation that in quasar 3C345 a « Superluminal component » does even appear to accelerate away with time [11].

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(9) E. RECAMI, A. CASTELLINO, G. D. MACCARRONE and M. RODONÒ: *Considerations about the apparent superluminal expansions in astrophysics*, Preprint PP/761 (Dept. Phys., Univ. of Catania, Catania, Italy, 1984), submitted for publication.

(10) See *e.g.* M. H. COHEN and S. C. UNWIN: *I.A.U. Symposium*, No. **97** (1982), p. 345; M. J. OOR and I. W. A. BROWNE: *Mon. Not. R. Astron. Soc.*, **200**, 1067 (1982); R. PORCAS: *Nature*, **302**, 753 (1983).

(11) R. L. MOORE, A. C. S. READHEAD and L. BAATH: *Nature*, **306**, 44 (1982).