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Two-Body Interactions through Tachyon Exchange (*).

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Summary. — Due to its relevance for the possible applications to particle physics and for causality problems, we thoroughly analyse in this paper the kinematics of (classical) tachyon exchange between two bodies A, B, for all possible relative velocities. In particular, the two cases $\boldsymbol{u}\cdot\boldsymbol{V} \lessgtr c^2$ are carefully investigated, where $\boldsymbol{u}$, $\boldsymbol{V}$ are the body B and tachyon speeds relative to A, respectively.

PART I: Introduction.

1. – Introduction.

It is known since long that, when investigating tachyon dynamics, it is always necessary to take into proper account the tachyon *together with* its emitter A and absorber B (1).

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(1) E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209-290, 398 (1974), and references therein; R. MIGNANI and E. RECAMI: *Phys. Lett. B*, **65**, 148 (1976);

Let us recall that, if two particles or bodies A, B exchange a tachyon T, then suitable subluminal observers always exist, which see the intermediate tachyon T with divergent speed, *i.e.* which judge the tachyon exchange as an instantaneous *symmetrical* interaction [1]. Moreover, other observers always exist, which see an antitachyon $\overline{\mathrm{T}}$ flying from B to A, with the exchange of the emission and absorption rôles. The very « reinterpretation procedure » [1,2] loses its meaning, therefore, if we cannot refer our T ($\overline{\mathrm{T}}$) to some interaction regions.

In other words, tachyons (even when macro-objects) are typical carriers of—*mutual and symmetric—interaction* between A and B [*]. It seems thus probable that in our cosmos tachyons have a rôle as « interaction carriers » rather than as stable « asymptotical » objects [1]. Frequently, attention has been called [4] to the possibile connections, for instance, between tachyons and *internal lines* of the quantum-relativistic processes. Moreover, it is well known [5] that black-holes, in *classical* physics, can emit only tachyons, therefore constituting *a typical kind* of tachyonic sources; from « extended relativity » [1] black-holes then follow to be also absorbers of tachyons: this means that tachyonic matter can *a priori* be exchanged—*for instance*—between black-holes, where we mean both « gravitational black-holes » (the ordinary ones) and possibly « *strong* black-holes » (hadrons).

From what precedes, it follows to be in any case quite important studying in detail the kinematics of tachyon exchange between two (micro- or macro-) bodies.

The kinematics of tachyon exchanges between two bodies A, B has been already investigated in part, but the previous results appeared *scattered* in a

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E. RECAMI, Editor: *Tachyons, Monopoles, and Related Topics* (Amsterdam, 1978), and references therein; Chap. 18 in *Centenario di Einstein: astrofisica e cosmologia, gravitazione, quanti e relatività nello sviluppo del pensiero scientifico di A. Einstein*, edited by M. PANTALEO (Firenze, 1979), p. 1021-1097; also appeared as Chapt. 16 in *Relativity, Quanta and Cosmology in the Development of the Scientific Thought of A. Einstein*, edited by F. DE FINIS, Vol. **2** (New York, N. Y., 1979), p. 537-597.

[2] E. RECAMI: *Found. Phys.*, **8**, 329 (1978), and references therein.

[*] It is not without meaning that WHEELER and FEYNMAN [3] were able to construct, even for the *limiting case* of photons, a theory (equivalent to the usual electromagnetism) in which sources emit photons only if their detectors are, in a sense, (already) ready to absorb them.

[3] J. A. WHEELER and R. P. FEYNMAN: *Rev. Mod. Phys.*, **17**, 157 (1945); **21**, 425 (1949).

[4] See, *e.g.*, P. CASTORINA and E. RECAMI: *Lett. Nuovo Cimento*, **22**, 195 (1978), and references (especially ref. [1,4,5]) therein.

[5] See, *e.g.*, E. RECAMI: in *Tachyons, Monopoles, and Related Topics*, edited by E. RECAMI (Amsterdam, 1968), p. 16; V. DE SABBATA, M. PAVŠIČ and E. RECAMI: *Lett. Nuovo Cimento*, **19**, 441 (1977); E. RECAMI and K. T. SHAH: *Lett. Nuovo Cimento*, **24**, 115 (1979).

series of papers [6]. Due to that fragmentary « spreading » of the past results, increased by the presence of a couple of « errata » [7], we deem useful to expound again—in improved, completed and more organic form—the whole question.

However, before dealing with the tachyon exchange, let us premise an analysis of the *emission* and *absorption* of a tachyon T from a body (or particle) A.

2. – Tachyon emission (description of « intrinsic emission », as seen in the rest frame and in generic frames).

Let us consider first—*in its rest frame*—a body C, with rest mass M, emitting towards a second body D a tachyon (or antitachyon) T, endowed with (real) rest mass m and four-momentum $p \equiv (E_{\rm T}, \boldsymbol{p})$, which travels with speed V for instance in the x-direction.

The four-momentum conservation requires, in natural units, that [1,6]

$$M = \sqrt{\boldsymbol{p}^2 - m^2} + \sqrt{\boldsymbol{p}^2 + M'^2} \qquad \text{(rest frame)}, \tag{1}$$

i.e.

$$2M|\boldsymbol{p}| = \sqrt{[m^2 + (M'^2 - M^2)]^2 + 4m^2M^2}\,, \tag{1'}$$

wherefrom it follows that a body (or particle) C cannot emit in its rest frame any tachyon T (whatever its rest mass m is), unless the rest mass M of C jumps (*classically*) to a *lower* value M', such that [1] $[E_{\rm T} \equiv \sqrt{\boldsymbol{p}^2 - m^2}]$

$$\Delta \equiv M'^2 - M^2 = -m^2 - 2ME_{\rm T} \qquad \text{(emission)}, \tag{2}$$

so that

$$-M^2 < \Delta \leqslant -\boldsymbol{p}^2 \leqslant -m^2 \qquad \text{(emission)}. \tag{3}$$

Equation (1′) can read [1]

$$V = \sqrt{1 + 4m^2M^2/(m^2 + \Delta)^2}\,. \tag{1''}$$

In particular, since infinite-speed T's carry zero energy but nonzero impulse

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([6]) E. RECAMI: ref. ([1]); E. RECAMI: *Lett. Nuovo Cimento*, **21**, 208 (1978); **22**, 591 (1978); E. RECAMI and M. PAVŠIČ: *Int. J. Theor. Phys.*, **17**, 77 (1978); M. PAVŠIČ and E. RECAMI: *Lett. Nuovo Cimento*, **18**, 134 (1977); **19**, 72 (1977); *Nuovo Cimento A*, **36**, 171 (1976); **46**, 298 (1978); M. PAVŠIČ, E. RECAMI and G. ZIINO: *Lett. Nuovo Cimento*, **17**, 257 (1976).

([7]) E. RECAMI: *Lett. Nuovo Cimento*, **22**, 591 (1978). See also M. PAVŠIČ and E. RECAMI: *Nuovo Cimento A*, **46**, 298 (1978).

$|\boldsymbol{p}| = mc$, then C cannot emit any transcendent tachyon without lowering its rest mass; in fact, in the case of infinite-speed T emission, *i.e.* when $E_{\mathrm{T}} = 0$ (in the rest frame of C), eq. (2) yields (1)

$$\Delta = -m^2 \qquad (V = \infty, E_{\mathrm{T}} = 0)\,. \tag{4}$$

Since emission of transcendent tachyons (antitachyons) is equivalent to absorption of transcendent antitachyons (tachyons), we shall again get eq. (4) also as a limiting case of tachyon absorption (cf. eq. (10)).

Notice that Δ is, of course, an invariant quantity. In fact, eq. (2) can be read, in a generic frame f,

$$\Delta = -m^2 - 2p_\mu P^\mu\,, \tag{5}$$

where P^μ is now the four-momentum of body C in the generic frame. Still $-M^2 < \Delta \leqslant -m^2$. The word « *emission* » in eq. (3) aims at indicating an *intrinsic*, *proper* behaviour, in the sense that it refers to « emission (as seen) *in the rest frame* of the emitting body or particle ». In suitably moving frames f, such an « emission » can even appear as an absorption (1,2). (Conversely, other (suitably moving) frames f' can observe a T emission from C (in flight), which does *not* satisfy inequation (3) since it corresponds in the rest frame of C to an (intrinsic) *absorption*.) However, if—in the moving frame f—inequation (3) appears to be satisfied, this implies that in the rest frame of C the process under examination is a tachyon emission, both when f observes an emission and when it observes an absorption. Let us anticipate that, in the case of « intrinsic absorption », relation (8′) will be shown to hold, instead of relation (3). Before going on, let us add here only the following observation: since the (invariant) quantity Δ in relation (8′) *can* assume also *positive* values (contrary to the invariant quantity Δ in eqs. (2)-(3)), if an observer f sees body A to *increase* its rest mass in the process, then the « proper description » of the process can be nothing but an (intrinsic) absorption; see the following.

When Δ in eqs. (2)-(5) can assume only known, discrete, values (so as in elementary-particle physics), then—once M is fixed—eq. (2) imposes *a link* between m and E_{T}, *i.e.* between m and $|\boldsymbol{p}|$.

Let us repeat, at last, that the body C, when in flight, *can appear* to emit (suitable) tachyons without lowering (or even changing) its rest mass. In particular, a particle *in flight* can *a priori* emit a suitable tachyon t transforming *into itself* (but, in such cases, if we go to the rest frame of the initial particle, then the « emitted » tachyon will appear as an *absorbed* antitachyon $\bar{\mathrm{t}}$) (1).

3. – Tachyon absorption.

Secondly, let us consider our body C, with rest mass M, now *absorbing in its rest frame* a tachyon (or antitachyon) T′ endowed with (real) rest mass m, four-momentum $p \equiv (E_{\mathrm{T}}, \boldsymbol{p})$, emitted by a second body D, and travelling with speed V (for instance along the x-direction).

The four-momentum conservation requires, in natural units, that [1,6]

$$M + \sqrt{\boldsymbol{p}^2 - m^2} = \sqrt{\boldsymbol{p}^2 + M'^2} \qquad \text{(rest frame)}, \tag{6}$$

wherefrom it follows that a body (or particle) C at rest can *a priori* absorb (suitable) tachyons both when increasing or lowering its rest mass, and when conserving it. More precisely, eq. (6) yields [1,6]

$$|\boldsymbol{p}| = \frac{1}{2M}\sqrt{(m^2 + \Delta)^2 + 4m^2M^2} \qquad \text{(rest frame)}, \tag{7}$$

which corresponds to

$$\Delta = -m^2 + 2ME_{\mathrm{T}}, \tag{8}$$

so that

$$-m^2 \leqslant \Delta < \infty \qquad \text{(absorption)}. \tag{8'}$$

Equation (7) tells us that body C (in its rest frame) can absorb the tachyon (or antitachyon) T, emitted by the second body D, only when the tachyon speed V is [1,6]

$$V = \sqrt{1 + 4m^2M^2/(m^2 + \Delta)^2}. \tag{9}$$

It should be explicitly noticed that eq. (8) differs from eq. (2). On the contrary, eqs. (7), (9) *formally* coincide with eqs. (1′), (1″), respectively; *but* they refer to different domains of Δ; in fact, *e.g.*, in eq. (1″) we have $\Delta < -m^2$, whilst in eq. (9) we have $\Delta > -m^2$.

In particular, from eq. (9) one observes that C can absorb (in its rest frame) infinite-speed tachyons only when $m^2 + \Delta = 0$, *i.e.*

$$V = \infty \leftrightarrow \Delta = -m^2 \qquad \text{(rest frame)} \tag{10}$$

in agreement with eq. (4).

Quantity Δ is, of course, invariant. Namely, eq. (8) can be written, in a generic frame f [1,6],

$$\Delta = -m^2 + 2p_\mu P^\mu, \tag{11}$$

where P^μ is now the four-momentum of body C in the generic frame f. Still $\Delta \geqslant -m^2$. Notice that the word absorption in eq. (8′) means « intrinsic absorption », since it refers to « absorption (as seen) *in the rest frame* of the absorbing body or particle ». This means that, if a moving observer f sees relation (8′)

to be satisfied, the « intrinsic » description of the process (in the rest frame of C) is a tachyon absorption, *both* when f observes an absorption *and* when it observes an emission. In the particular case $\Delta = 0$, we should simply get

$$2\,ME_{\mathrm{T}} = m^2 \qquad (\Delta = 0)\,.$$

When Δ in eqs. (7)-(11) can assume only known, discrete values (so as in elementary-particle physics), then—once M is fixed—eqs. (7)-(11) state a link between m and E_{T} (or $|\boldsymbol{p}|$, or V).

For further considerations, cf. the end of sect. **2**.

4. – Some remarks.

In view of describing the tachyon exchange between two bodies (or particles) A and B, let us thoroughly write down the implications of the four-momentum conservation *at A and at B*. In order to do that, we need choosing a unique frame for describing the processes both at A and at B. Let us choose to these purposes the rest frame of A.

Before going on, let us since now explicitly mention the important fact that, when bodies A and B exchange *one* tachyon T, the tachyon kinematics (1) is such that the « *intrinsic description* » of the process at A and at B (where the process at A is described in the rest frame of A, and the process at B is described in the rest frame of B) can *a priori* be of the following *four types* (1):

(12)
- i) emission-absorption,
- ii) absorption-emission,
- iii) emission-emission,
- iv) absorption-absorption.

It is noticeable that the possible cases are *not* only i) and ii). Case iii) can happen when the tachyon exchange takes place in the *recession phase* (*i.e.* while bodies A, B are receding one from the other); case iv) can happen when the tachyon exchange takes place in the *approaching phase* (*i.e.* when A, B are approaching one another). For instance, let us consider an *elastic* scattering between two (different) particles a, b. In the c.m.s., as is well known, a and b exchange momentum but no energy. An infinite-speed tachyon T can, therefore, be a suitable carrier of that interaction (T will appear as a *finite-speed* tachyon in the rest frames of a, b).

However, if a, b have to retain their rest mass during the process, then a tachyon exchange can describe that elastic process only when we have « intrinsic absorption » both at a and at b (this can happen only when a, b are approaching one another).

Notice that the descriptions i)-iv) above do *not* refer to one and the same observer, since they on the contrary *add together* the « local » descriptions of observers A and B.

PART II: Tachyon exchange when $\boldsymbol{u}\cdot\boldsymbol{V}\leqslant c^2$ (*).

Let $\boldsymbol{V}$, $\boldsymbol{u}$ be the tachyon and body B velocities, respectively, in the rest frame of A.

Let us now consider A, B to exchange a tachyon (or antitachyon) T when $\boldsymbol{u}\cdot\boldsymbol{V}< c^2$. In the rest frame of A, we can have either intrinsic emission or intrinsic absorption from body A.

5. – Case of « intrinsic emission » at A.

In the case when one observes, in the rest frame of A, an (intrinsic) tachyon emission from A, *both A and B* will see the exchanged tachyon to be emitted by A and absorbed by B. In fact, given a tachyon T with speed V in the frame A, a moving observer B endowed with speed $\boldsymbol{u}$ will see an antitachyon $\overline{\mathrm{T}}$ (travelling the opposite way, according to the *reinterpretation principle* ([1,2])) *only when* $\boldsymbol{u}\cdot\boldsymbol{V}> c^2$, whilst in the present case $\boldsymbol{u}\cdot\boldsymbol{V}< c^2$. Cf. ref. ([1,2,6]).

Imposing the four-momentum conservation at A, we get (in the A rest frame) from eqs. (1), (2)

$$\Delta_A \equiv M_A'^2 - M_A^2 = -m^2 - 2M_A E_{\mathrm{T}} \qquad \text{(rest frame)}, \tag{13}$$

$$2M_A|\boldsymbol{p}| = [(m^2+\Delta_A)^2 + 4m^2 M_A^2]^{\frac{1}{2}}, \tag{13'}$$

$$V = [1 + 4m^2 M_A^2/(m^2+\Delta_A)^2]^{\frac{1}{2}}, \tag{13''}$$

where now we called M_A, M_A' the initial and final rest mass of body A, respectively. According to eq. (5), in a generic frame f, the quantity Δ_A can be written in explicitly covariant form as follows:

$$\Delta_A = -m^2 - 2p_\mu P_A^\mu, \tag{14}$$

wherefrom

$$-M_A^2 < \Delta_A \leqslant -m^2 \qquad \text{(intrinsic emission)}, \tag{14'}$$

(*) For instance, this includes tachyon exchanges in the « approaching phase » (for intrinsic T emission at A) and in the « receding phase » (for intrinsic T absorption at A).

where now p_μ and P^μ_A are tachyon T and body A four-momenta, respectively, in the frame f. Remember that, whatever the process description be in f, eq. (14′) holds *if and only if* the process « intrinsic description » in A is a (tachyon) emission. Remember also inequation (3).

Let us remain in the rest frame of A, and now study the kinematical conditions under which the tachyon T emitted by A *can* be absorbed by a second body B moving—in general—with speed u along a generic direction (with respect to A).

Let M_B and $P_B \equiv (E_B, \boldsymbol{P}_B)$ be rest mass and four-momentum of body B, respectively. If T must be absorbed by B, then [6]

$$\sqrt{\boldsymbol{P}_B^2 + M_B^2} + \sqrt{\boldsymbol{p}^2 - m^2} = \sqrt{(\boldsymbol{P}_B + \boldsymbol{p})^2 + M_B'^2}\,, \tag{15}$$

where M_B' is the final rest mass of B.

Let us define

$$\Delta_B \equiv M_B'^2 - M_B^2\,,$$

which reads [1,6]

$$\Delta_B = -m^2 + 2\tilde{m}\tilde{M}_B(1 - uV\cos\alpha)\,, \tag{16}$$

where $\tilde{m} \equiv E_\text{T}$ and $\tilde{M}_B \equiv E_B \equiv \sqrt{\boldsymbol{P}_B^2 + M_B^2}$ are the relativistic masses of T and B, respectively, and $\alpha \equiv \widehat{\boldsymbol{u}\cdot\boldsymbol{V}}$ is the angle between $\boldsymbol{u}$ and $\boldsymbol{V}$. The invariant quantity Δ_B, in a generic frame f, can be written [1,6]

$$\Delta_B = -m^2 + 2p_\mu P^\mu_B\,, \tag{17}$$

where now p_μ, P^μ_B are T and B four-momenta *in the generic frame f*. Differently from the (intrinsic) emission case, Δ_B can *a priori* assume both negative and positive or null values:

$$-m^2 \leqslant \Delta_B < \infty \qquad \text{(intrinsic absorption)}\,. \tag{18}$$

Notice that, if in the generic frame f relation (18) is verified, then (whatever be the description of the process at B given by f) the process will appear in the rest frame or B as an (intrinsic) absorption. Of course, the kinematics connected with eq. (15) is such that Δ_B can even be smaller than $-m^2$ (cf. eq. (16)); but such a case $[uV\cos\alpha > 1]$ would correspond to intrinsic emission (and no longer to intrinsic absorption).

In conclusion the tachyon exchange, in the case of « intrinsic emission » at A with $\boldsymbol{u}\cdot\boldsymbol{V} < c^2$ in the A rest frame, is kinematically allowed when the following equations are simultaneously satisfied:

$$\begin{cases} \Delta_A = -m^2 - 2M_A E_\text{T} & (-M_A^2 < \Delta_A \leqslant -m^2)\,, \\ \Delta_B = -m^2 + 2E_\text{T}E_B(1 - \boldsymbol{u}\cdot\boldsymbol{V}) & (\Delta_B \geqslant -m^2)\,. \end{cases} \tag{19}$$

In the particular case in which B moves along the direction line of tachyon T (in the negative or positive x-direction, let us say), so that $\boldsymbol{P}_B \| (\pm \boldsymbol{p})$, then the second of eqs. (19) can also be written [1,6]

$$2M_B^2|\boldsymbol{p}| = E_B\sqrt{(m^2+\Delta_B)^2+4m^2M_B^2} \pm (m^2+\Delta_B)|\boldsymbol{P}_B| \qquad (\boldsymbol{P}_B\|(\pm\boldsymbol{p}))\,. \tag{20}$$

When B is at rest with respect to A (*i.e.* when $\boldsymbol{P}_B = 0$) we are back to sect. **3** and recover eqs. (7), (8), (9).

Finally, let us add the consideration that in this case (« intrinsic absorption » at B) the quantity Δ_B *can a priori* vanish—differently from the quantity Δ_A which has always to be negative (cf. eq. (3))—. In the case in which $\Delta_B = 0$, the second of eqs. (19) simplifies into

$$2E_{\text{T}}E_B(1-\boldsymbol{u}\cdot\boldsymbol{V}) = m^2 \qquad (\Delta_B = 0),$$

and eqs. (20) become

$$|\boldsymbol{p}| = \frac{m}{2M_B^2}\left[E_B\sqrt{m^2+4M_B^2} \pm m|\boldsymbol{P}_B|\right] \qquad (\boldsymbol{P}_B\|(\pm\boldsymbol{p});\ \Delta_B = 0)\,. \tag{21}$$

In the very particular case in which both $\boldsymbol{P}_B = 0$ and $\Delta_B = 0$, eqs. (20), (21) yield [1,6] (cf. eq. (9))

$$V = \sqrt{1+4M_B^2/m^2} \qquad (\boldsymbol{P}_B = 0;\ \Delta_B = 0)\,. \tag{22}$$

6. – Case of « intrinsic absorption » at A.

Let us consider tachyon exchanges such that the process at A appears, in the A rest frame, as an (intrinsic) absorption. Observer A will see the (exchanged) tachyon T to be emitted by B. The condition $\boldsymbol{u}\cdot\boldsymbol{V} < c^2$ implies [1,2,6] in this case that body B appears to emit tachyon T *also* in its own rest frame.

The present case, therefore, is just the symmetrical of the previous one in sect. **5**. The only difference is that now we are in the *rest frame* of A, *i.e.* of the *absorbing* body.

For the process at A we have

$$M_A + \sqrt{\boldsymbol{p}^2 - m^2} = \sqrt{\boldsymbol{p}^2 + M_A'^2} \qquad (A \text{ rest frame}), \tag{23}$$

where now

$$\Delta_A \equiv M_A'^2 - M_A^2 = -m^2 + 2M_AE_{\text{T}} \qquad (\text{intrinsic absorption})\,. \tag{24}$$

In a generic frame f, the invariant quantity Δ_A can read

$$\Delta_A = -m^2 + 2p_\mu P_A^\mu, \tag{25}$$

wherefrom

$$-m^2 \leqslant \Delta_A < \infty \qquad \text{(intrinsic absorption)}, \tag{26}$$

where now p_μ, P_A^μ are the four-momenta of T and A, respectively, *in the generic frame f*. Let us recall that f will see relation (26) to be satisfied *if and only if* it refers to a process (at A) which is « intrinsically » a tachyon absorption, whatever his description from f be.

Let us recall that, in the particular case $\Delta_A = 0$, we get $2M_A E_T = m^2$.

For the process at B, in the rest frame of A, we have

$$\sqrt{P_B^2 + M_B^2} = \sqrt{\boldsymbol{p}^2 - m^2} + \sqrt{(\boldsymbol{P}_B - \boldsymbol{p})^2 + M_B'^2}\,, \tag{27}$$

where (cf., for the symbols, eq. (16))

$$\Delta_B \equiv M_B'^2 - M_B^2 = -m^2 - 2\tilde{m}\tilde{M}_B(1 - \boldsymbol{u}\cdot\boldsymbol{V})\,. \tag{28}$$

In a generic frame f, the invariant quantity Δ_B can be written ($\boldsymbol{p}_\mu$, $\boldsymbol{P}_B$ now being the four-momenta of T and B, respectively, *in the generic frame f*)

$$\Delta_B = -m^2 - 2p_\mu P_B^\mu\,, \tag{29}$$

wherefrom

$$-M_B^2 < \Delta_B \leqslant -m^2 \qquad \text{(intrinsic emission)}\,. \tag{30}$$

Let us observe that, in a frame f, relation (30) holds *if and only if* the process at B (no matter how it may appear to f) is « intrinsically »—*i.e.* in the B rest frame—a tachyon emission. We have already seen that it would be an « intrinsic absorption » only if we had $\Delta_B \geqslant -m^2$; that is to say, in general,

$$[\boldsymbol{u}\cdot\boldsymbol{V} \leqslant c^2] \therefore \Delta_B = \begin{cases} -m^2 + 2p_\mu P_B^\mu \geqslant -m^2 \Rightarrow \text{intrinsic absorption}, \\ -m^2 - 2p_\mu P_B^\mu \leqslant -m^2 \Rightarrow \text{intrinsic emission}\,. \end{cases} \tag{31}$$

For clarity's sake, let us explicitly repeat that: *Necessary condition in order that the tachyon (or antitachyon)* T, *seen by A to be* absorbed *by B, can be seen in the rest frame of B as an antitachyon (or tachyon)* $\overline{\text{T}}$ *actually* emitted *by B, is that during the process B* lowers *its rest mass (invariant statement!) in such a way that* $-M_B^2 < \Delta_B \leqslant -m^2$.

In conclusion the tachyon exchange, in the case of « *intrinsic* » *absorption* at A and $\boldsymbol{u}\cdot\boldsymbol{V} < c^2$ (in the rest frame of A), is kinematically allowed when the following equations are simultaneously satisfied:

$$\begin{cases} \Delta_A = -m^2 + 2M_A E_T & (\Delta_A \geqslant -m^2)\,, \\ \Delta_B = -m^2 - 2E_T E_B(1 - \boldsymbol{u}\cdot\boldsymbol{V}) & (-M_B^2 < \Delta_B \leqslant -m^2)\,. \end{cases} \tag{32}$$

In the particular case in which B moves along the direction line of tachyon T (in the x-direction, let us say), so that $\boldsymbol{P}_B \| (\pm \boldsymbol{p})$, then the second of eqs. (32) can be written

$$2M_B^2|\boldsymbol{p}| = E_B\sqrt{(m^2 + \Delta_B)^2 + 4m^2 M_B^2} \mp (m^2 + \Delta_B)|\boldsymbol{P}_B| \qquad (\boldsymbol{P}_B \| (\pm \boldsymbol{p})), \tag{33}$$

where attention should be paid to the fact that the signs in the r.h.s. of eq. (33) are opposite to the ones entering eq. (20), as it should be also for self-evident symmetry reasons.

When B is at rest with respect to A, so that $\boldsymbol{P}_B = 0$, the second one of eqs. (33) transforms into

$$|\boldsymbol{p}| = \frac{1}{2M_B}\sqrt{(m^2 + \Delta_B)^2 + 4m^2 M_B^2} \qquad (\boldsymbol{P}_B = 0) \tag{34}$$

in obvious agreement with eq. (1′). And so on: cf. eq. (1″), (2). Let us repeat the observation that, in the present case of *intrinsic emission*, eq. (34) corresponds to values of Δ_B in the range $-M_B^2 < \Delta_B \leqslant -m^2$, whilst eq. (7), which holds in the opposite case of intrinsic absorption, corresponds to $\Delta_B \geqslant -m^2$.

PART III: Tachyon exchange when $\boldsymbol{u}\cdot\boldsymbol{V} \geqslant c^2$.

Still in the rest frame of A, let us now consider A, B to exchange a tachyon (or antitachyon) T when $\boldsymbol{u}\cdot\boldsymbol{V} > c^2$. Under the present condition, again we can have either « intrinsic emission » or « intrinsic absorption » by body A.

The present cases *differ* from the ones previously considered in Part II for the fact that now—due to the reinterpretation procedure [1,2]—the T emission (at A) and T absorption (at B) are described in the rest frame of B as a $\overline{\text{T}}$ absorption (at A) and a $\overline{\text{T}}$ emission (at B), respectively [6].

7. – Case of « intrinsic emission » at A.

If, in the rest frame of A, we observe that body A (intrinsically) emits tachyon T, then in the B rest frame we would observe an antitachyon $\overline{\text{T}}$ absorbed by A—due to the present condition $\boldsymbol{u}\cdot\boldsymbol{V} > c^2$, and to the reinterpretation procedure [1,2,6]—.

Necessary condition for this case to exist is that A, B are *receding* one from the other (*i.e.* are in the « recession phase »).

In any case, in the A rest frame, we get for the process at A the same kinematics already expounded in sect. **5**. Here we confine ourselves, therefore, to quote eqs. (1)-(5), or rather eqs. (13), (14).

As to the process at B, in the A rest frame body B is observed to absorb a tachyon T:

$$\sqrt{\boldsymbol{P}_B^2 + M_B^2} + \sqrt{\boldsymbol{p}^2 - m^2} = \sqrt{(\boldsymbol{P}_B + \boldsymbol{p})^2 + M_B'^2}\,. \tag{15}$$

In the B rest frame, however, one would observe an (« intrinsic ») $\overline{\mathrm{T}}$ emission, so that what we stated between eqs. (31) and (32) is here in order. Namely, relation (30) has to hold in this case (even if it is now associated to eq. (15) and *not* to eq. (27), in the A rest frame). Notice that, when passing from the A rest frame to the B rest frame (and applying the reinterpretation procedure ([1,2])), in eq. (15) one has that: i) Quantity E_{T} changes its sign, so that quantity $\sqrt{\boldsymbol{p}^2 - m^2}$ appears added to the r.h.s. (and no longer to the l.h.s.); ii) the tachyon three-momentum $\boldsymbol{p}$ changes its sign (since we go from a tachyon T with impulse $\boldsymbol{p}$ to its antitachyon $\overline{\mathrm{T}}$ with impulse $-\boldsymbol{p}$).

In any case, *from* eq. (15) with the condition $\boldsymbol{u}\cdot\boldsymbol{V} \geqslant c^2$ it *directly follows*

$$\Delta_B = -m^2 + 2\tilde{m}\tilde{M}_B(1 - \boldsymbol{u}\cdot\boldsymbol{V}) \leqslant -m^2\,, \tag{35}$$

which can read

$$\Delta_B = -m^2 + 2p_\mu P_B^\mu \leqslant -m^2\,. \tag{36}$$

In other words, eq. (15) with $\boldsymbol{u}\cdot\boldsymbol{V} \geqslant c^2$ yields

$$-M_B^2 < \Delta_B \leqslant -m^2 \qquad \text{(intrinsic emission)}\,. \tag{37}$$

In conclusion the tachyon exchange, in the case of « intrinsic emission » at A and $\boldsymbol{u}\cdot\boldsymbol{V} \geqslant c^2$ (in the A rest frame), is kinematically allowed when the following equations are simultaneously satisfied:

$$\begin{cases} \Delta_A = -m^2 - 2M_A E_{\mathrm{T}} & (\Delta_A \leqslant -m^2)\,, \\ \Delta_B = -m^2 + 2E_{\mathrm{T}} E_B(1 - \boldsymbol{u}\cdot\boldsymbol{V}) & (\Delta_B \leqslant -m^2)\,. \end{cases} \tag{38}$$

In the particular case in which $\boldsymbol{P}_B$ and $\boldsymbol{p}$ are collinear, we can have only $\boldsymbol{P}_B \| \boldsymbol{p}$ (« recession phase ») and the second of eqs. (38) can be written

$$2M_B^2|\boldsymbol{p}| = E_B\sqrt{(m^2 + \Delta_B)^2 + 4m^2M_B^2} + (m^2 + \Delta_B)|\boldsymbol{P}_B| \qquad (\boldsymbol{P}_B \| \boldsymbol{p})\,. \tag{39}$$

Equation (39) is formally identical to part of eq. (20), but refers to values of Δ_B in the range $-M_B^2 < \Delta_B \leqslant -m^2$. It refers, therefore, to the same range of Δ_B values as eq. (33), but its r.h.s. contains a sign which is at variance with eq. (33).

8. – Case of « intrinsic absorption » at A.

Due to the present condition $\boldsymbol{u}\cdot\boldsymbol{V}\geqslant c^2$ and to the reinterpretation procedure [1,2], if we observe in the A rest frame body A (intrinsically) to absorb a tachyon T, then in the B rest frame we should observe an antitachyon $\overline{\text{T}}$ emitted by A.

Necessary condition for this case to exist is that A, B are *approaching* each other (*i.e.* are in the « approaching phase »).

In any case, in the rest frame of A, we get (for the process at A) the same kinematics already expounded in sect. **6**. Here we confine ourselves, therefore, to quote eqs. (6)-(11), or rather eqs. (23)-(26).

As to the process at B, in the A rest frame body B is observed to emit a tachyon T:

$$\sqrt{\boldsymbol{P}_B^2+M_B^2}=\sqrt{\boldsymbol{p}^2-m^2}+\sqrt{(\boldsymbol{P}_B-\boldsymbol{p})^2+M_B'^2}\,. \tag{27}$$

In the B rest frame, however, one would observe an (« intrinsic ») $\overline{\text{T}}$ absorption, so that it must be

$$-m^2\leqslant\Delta_B<\infty \qquad \text{(intrinsic absorption)}\,. \tag{40}$$

In fact, *at variance* with eqs. (31), in the case $\boldsymbol{u}\cdot\boldsymbol{V}\geqslant c^2$ we have

$$[\boldsymbol{u}\cdot\boldsymbol{V}\geqslant c^2]\;.\dot{.}.\;\Delta_B=\begin{cases}-m^2-2p_\mu P_B^\mu\geqslant-m^2\Rightarrow\text{intrinsic absorption,}\\ -m^2+2p_\mu P_B^\mu\leqslant-m^2\Rightarrow\text{intrinsic emission}\,.\end{cases} \tag{41}$$

Namely, from eq. (27) with the condition $\boldsymbol{u}\cdot\boldsymbol{V}\geqslant c^2$, it directly follows

$$\Delta_B=-m^2-2\tilde{m}\tilde{M}_B(1-\boldsymbol{u}\cdot\boldsymbol{V})\geqslant-m^2\,, \tag{42}$$

which can read

$$\Delta_B=-m^2-2p_\mu P_B^\mu\geqslant-m^2\,. \tag{43}$$

In other words, eq. (27) with the condition $\boldsymbol{u}\cdot\boldsymbol{V}\geqslant c^2$ yields

$$-m^2\leqslant\Delta_B<\infty \qquad \text{(intrinsic absorption)}\,. \tag{44}$$

In conclusion the tachyon exchange, in the case of « intrinsic absorption » at A and $\boldsymbol{u}\cdot\boldsymbol{V}\geqslant c^2$ (in the A rest frame), is kinematically allowed when the followings equations are simultaneously verified:

$$\begin{cases}\Delta_A=-m^2+2M_AE_\text{T} & (\Delta_A\geqslant-m^2)\,,\\ \Delta_B=-m^2-2E_\text{T}E_B(1-\boldsymbol{u}\cdot\boldsymbol{V}) & (\Delta_B\geqslant-m^2)\,.\end{cases} \tag{45}$$

In the particular case in which $\boldsymbol{P}_B$ and $\boldsymbol{p}$ are collinear, we can have *only* $\boldsymbol{P}_B \| \boldsymbol{p}$ (« approaching phase »), and the second of eqs. (45) can be written

$$2M_B^2|\boldsymbol{p}| = E_B\sqrt{(m^2+\Delta_B)^2+4m^2M_B^2}-(m^2+\Delta_B)|\boldsymbol{P}_B| \qquad (\boldsymbol{P}_B\|\boldsymbol{p}) \tag{46}$$

with $-m^2 \leqslant \Delta_B < \infty$.

Finally, let us recall that in the present case (« intrinsic absorptions » at B and at A) *both* quantities Δ_A, Δ_B *can* vanish. When $\Delta_A = 0$, we simply get $2M_A E_T = m^2$. In the particular case in which $\Delta_B = 0$, we would get

$$2E_T E_B(\boldsymbol{u}\cdot\boldsymbol{V}-1) = m^2 \qquad (\Delta_B = 0)\,,$$

and eqs. (46) would become (when $\boldsymbol{P}_B\|\boldsymbol{p}$)

$$|\boldsymbol{p}| = \frac{m}{2M_B^2}\left[E_B\sqrt{m^2+4M_B^2}-m|\boldsymbol{P}_B|\right] \qquad (\boldsymbol{P}_B\|\boldsymbol{p};\ \Delta_B=0)\,. \tag{47}$$

At this point let us remember that, when elementary interactions are considered to be mediated by the strong-field quanta, no (« realistic ») ordinary particles can actually be the carriers of the transferred energy-momentum (8). On the contrary, tachyons (instead of the so-called *virtual* particles) *can a priori* work as the actual carriers of the strong interactions (8,9) (*).

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(8) E. RECAMI: Chapt. 18 in *Centenario di Einstein: astrofisica e cosmologia, gravitazione, quanti e relatività nel pensiero scientifico di A. Einstein*, edited by M. PANTALEO (Firenze, 1979), p. 1071 folls.; Chapt. 16 in *Relativity, Quanta and Cosmology in the Development of the Scientific Thought of A. Einstein*, edited by F. DE FINIS, Vol. **2** (New York, N. Y., 1979), p. 575 folls.; in *Tachyons, Monopoles, and Related Topics*, edited by E. RECAMI (Amsterdam, 1978), p. 17; P. CASTORINA and E. RECAMI: *Lett. Nuovo Cimento*, **22**, 195 (1978); M. PAVŠIČ and E. RECAMI: *Nuovo Cimento A*, **36**, 171 (1976), particularly footnotes (17,21,32); E. RECAMI, R. MIGNANI and G. ZIINO: in *Recent Development in Relativistic Quantum Field Theory and its Application*, edited by W. KARWOWSKI, Vol. **2** (Wroclaw, 1976), p. 269; E. RECAMI and R. MIGNANI: *Phys. Lett. B*, **62**, 41 (1976), p. 43; R. MIGNANI and E. RECAMI: *Phys. Lett. B*, **65**, 149 (1976), footnotes on p. 149; *Nuovo Cimento A*, **30**, 533 (1975), Sect. **4**, p. 538-539; M. BALDO and E. RECAMI: *Lett. Nuovo Cimento*, **2**, 643 (1969), p. 646; V. S. OLKHOVSKI and E. RECAMI: *Nuovo Cimento A*, **63**, 814 (1969), p. 122; E. RECAMI: *G. Fis.*, **10**, 195 (1969), p. 203-205; *Possible ... and comments on tachyons, virtual particles, resonances*, Report IFUM-088/S.M., University of Milano (August 1968), p. 4-8.

(9) H. C. CORBEN: in *Tachyons, Monopoles, and Related Topics*, edited by E. RECAMI (Amsterdam, 1978), p. 31; *Lett. Nuovo Cimento*, **22**, 116 (1978); **20**, 645 (1977); three preprints, Scarborough College, West Hill, Ont. (August, September and November 1977); P. CASTORINA and E. RECAMI: *Lett. Nuovo Cimento*, **22**, 195 (1978); J. D. EDMONDS jr.: *Specul. in Sc. and Techn.* (to appear); N. ROSEN and G. SZAMOSI: preprint (Technion, Haifa, 1979); G. SZAMOSI and D. TREVISAN: preprint (Windsor University, Ont., 1978); K. RAFANELLI: *Phys. Rev. D*, **17**, 640 (1978); T. AKIBA: report

For instance, let us recall that any *elastic scattering can* be considered as « realistically » (classically) mediated by a suitable tachyon exchange during the approaching phase of the two bodies. In such a case eqs. (45) read (always in the A rest frame)

$$\text{(48)} \qquad \begin{cases} E_T = m^2/2M_A\,, \\ E_B = M_A/(\boldsymbol{u}\cdot\boldsymbol{V}-1)\,, \end{cases} \qquad (\Delta_A = \Delta_B = 0)\,;$$

we are neglecting the angular-momentum conservation.

In the c.m.s., for instance, we would have $|\boldsymbol{P}_A| = |\boldsymbol{P}_B| \equiv |\boldsymbol{P}|$, and

$$\text{(49)} \qquad \cos\theta_{\text{c.m.}} = 1 - \frac{m^2}{2|\boldsymbol{P}|^2} \qquad \text{(elastic scattering)}\,,$$

so that (once $|\boldsymbol{P}|$ is fixed), for each tachyon mass m, we should get one particular $\theta_{\text{c.m.}}$; if m assumes only discrete values (according to the *duality principle* [1,2]), then $\theta_{\text{c.m.}}$ results to be (classically) « quantized » [8,9], except for a cylindrical symmetry. More in general, for each discrete value of the tachyon mass m, the quantity $\theta_{\text{c.m.}}$ assumes too a discrete value, which is merely a

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TU/76/138 (Tohoku University, Sendai, 1976); S. HAMAMOTO: *Prog. Theor. Phys.*, **51**, 1977 (1974); E. VAN DER SPUY: *Phys. Rev. D*, **7**, 1106 (1973); in *Tachyons, Monopoles, and Related Topics*, edited by E. RECAMI (Amsterdam, 1978), p. 175; C. JUE: *Phys. Rev. D*, **8**, 1757 (1973); A. M. GLEESON, M. G. GUNDZIG, E. C. G. SUDARSHAN and A. PAGNAMENTA: *Phys. Rev. A*, **6**, 807 (1972); *Part. Nucl.*, **1**, 1 (1970); E. C. G. SUDARSHAN: *Phys. Rev. D*, **1**, 2428 (1970); *Ark. Fys.*, **39** (40), 585 (1969). See also A. I. BUGRIJ, L. L. JENKOVSKY and N. A. KOBYLINSKY: *Lett. Nuovo Cimento*, **5**, 389 (1972); F. T. HADJIOANNOU: *Nuovo Cimento*, **44**, 185 (1966); I. FERRETTI and M. VERDE: *Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Nat.* (1966), p. 318.

(*) For instance, let us consider the vertex $\Delta_{33} \to p + \pi_T$ of a suitable one-particle exchange diagram, and suppose the exchanged particle π_T (« internal line ») to be a tachyon pion (instead of a virtual object). Then, from eqs. (2), (13) we should get

$$(1232)^2 - (938)^2 = (140)^2 + 2\times 1232 \times \sqrt{c^2|\boldsymbol{p}|^2 - (140)^2}$$

and, therefore,

$$|\boldsymbol{p}|_{\pi_T} = 287\ \text{MeV/c}\,, \qquad E_{\pi_T} = 251\ \text{MeV}\,,$$

so that, in the c.m.s. of the $\Delta(1232)$, the total energy of the tachyonic pion—under the present hypotheses—should be centred around 251 MeV.

Again, let us consider the decay $\pi \to \mu + \nu_T$ under the hypothesis that ν_T be a tachyon-neutrino (vith $m_\nu \geqslant 0$, $v_\nu \geqslant c$). It has been shown, *e.g.* by R. G. CAWLEY (*Lett. Nuovo Cimento*, **3**, 523 (1972)), that this hypothesis is not inconsistent with the experimental data and implies for the muon-neutrino $m_{\nu_T} \leqslant 1.7$ MeV. In the two limiting cases ($m_\nu = 0$ and $m_\nu = 1.7$ MeV), from eqs. (2), (13) one gets in the c.m.s. of the pion

$$m_\nu = 0 \quad \Rightarrow |\boldsymbol{p}|_\nu = 29.79\ \text{MeV/c} \qquad (v = c)\,,$$
$$m_\nu = 1.7 \Rightarrow |\boldsymbol{p}|_\nu = 29.83\ \text{MeV/c} \qquad (v \simeq 1.0016c)\,,$$

where the first result coincides, of course, with the standard one.

function of $|\boldsymbol{P}|$. Such naïve considerations are neglecting the mass width of the tachyonic (« mesonic ») resonances [8,9]. Let us recall that in the c.m.s. any elastic scattering appears classically as mediated by an infinite-speed tachyon having $p_\mu \equiv (0, \boldsymbol{p})$, where $|\boldsymbol{p}| = m$. Moreover, eqs. (48) impose a link between m and the direction of $\boldsymbol{p}$, *i.e.* between m and $\alpha \equiv \widehat{\boldsymbol{pP}}$ (where, *e.g.*, we can choose $\boldsymbol{P} = \boldsymbol{P}_B$; remember that $\boldsymbol{P}_B = -\boldsymbol{P}_A$):

$$\cos\alpha = \frac{m}{2|\boldsymbol{P}|}\,; \tag{50}$$

again we find that (once $|\boldsymbol{P}|$ is fixed), if the tachyon-meson masses are discrete, then also the exchanged three-momentum results to be (classically) « quantized » in both its magnitude and direction.

This means again that, for each discrete value of m, also the exchanged three-momentum assumes one discrete direction (except for a cylindrical symmetry), which is a function only of $|\boldsymbol{P}|$. Notice that such a result *cannot* be obtained *at the classical level* when confining ourselves only to bradyons, since ordinary particles *cannot*, from the kinematical viewpoint, be the interaction carriers.

Of course, also nonelastic scatterings can be considered as mediated by suitable tachyon exchanges [8].

9. – Final considerations.

Roughly speaking, we can summarize what precedes by saying that

a) in the case of « intrinsic emission » at A

$$\boldsymbol{u}\cdot\boldsymbol{V} \lessgtr c^2 \Rightarrow \Delta_B \gtrless -m^2\,, \tag{51}$$

b) in the case of « intrinsic absorption » at A

$$\boldsymbol{u}\cdot\boldsymbol{V} \lessgtr c^2 \Rightarrow \Delta_B \lessgtr -m^2\,. \tag{52}$$

At this point, let us recall [1,2] that *no* causal problems arise in tachyon microphysics, some problems remaining *possibly* open only in tachyon macrophysics.

More precisely, when $\boldsymbol{u}\cdot\boldsymbol{V} \leqslant c^2$, *no* causality problem arises even in tachyon

[10] Cf. ref. [1,2], and P. CALDIROLA and E. RECAMI: *Causality and tachyons in relativity*, in *Italian Studies in the Philosophy of Science*, edited by M. DALLA CHIARA (Boston, Mass., 1980); E. RECAMI: in *Annuario '73, Enciclopedia EST-Mondadori* (Milano, 1973), p. 85.

macrophysics [6,10]. Only when $\boldsymbol{u}\cdot\boldsymbol{V} > c^2$, it *seems* that some interesting causal problems remain to be exploited *in tachyon macrophysics* [10] (but *not* in microphysics), whose discussion apparently requires taking into account different subjects which may range from the peculiar behaviour of tachyon sources and detectors, to the spontaneous tachyon emission properties of matter, to information theory, and so on (and even to the question whether Minkowski space-time is enough for allocating the « free-will » behaviour).

In any case, let us warn once more that the correct procedure for getting physically realizable processes among tachyons is: i) to start from any possible processes among bradyons, ii) to apply to them a Superluminal Lorentz transformation (thus obtaining any « real » interactions among tachyons).

As to the possible applications of the present work, here let us confine ourselves to refer—besides to what already sketched in sect. **8**—to the hints contained in ref. [1,8,9].

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● RIASSUNTO

A causa della sua possibile rilevanza per le applicazioni alla fisica delle particelle e ai problemi causali, in questo articolo si analizza dettagliatamente la cinematica dello scambio di un tachione tra due corpi A, B, per tutte le velocità relative possibili. In particolare si studiano accuratamente i due casi $\boldsymbol{u}\cdot\boldsymbol{V} \lessgtr c^2$, dove $\boldsymbol{u}$, $\boldsymbol{V}$ sono rispettivamente le velocità del corpo B e del tachione rispetto ad A.

Двухчастичные взаимодействия через обмен тахионом.

Резюме (*). — Исходя из возможных применений к физике частиц и проблемам причинности, мы проводим анализ кинематики (классического) обмена тахионами между двумя телами A, B, для всех возможных относительных скоростей. B частности, исследуются два случая $\boldsymbol{u}\cdot\boldsymbol{V} \lessgtr c^2$, где $\boldsymbol{u}$, $\boldsymbol{V}$ скорости тела B и тахиона относительно A.

(*) *Переведено редакцией.*