Diffraction-Attenuation resistant beams in absorbing media

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Abstract: In this work, in terms of suitable superpositions of equal-frequency Bessel beams, we develop a theoretical method to obtain localized stationary wave fields, in *absorbing media*, capable to assume, approximately, any desired longitudinal intensity pattern within a chosen interval $0 \leq z \leq L$ of the propagation axis z. As a particular case, we obtain new nondiffractive beams that can resist the loss effects for long distances. These new solutions can have different and interesting applications, such as optical tweezers, optical or acoustic bistouries, various important medical apparatuses, etc..

OCIS codes: (140.3300) Laser beam shaping; (260.1960) Diffraction theory; (350.7420) Waves; (999.9999) Nondiffracting waves

References and links

1. M. Zamboni-Rached, "Stationary optical wave fields with arbitrary longitudinal shape by superposing equal frequency Bessel beams: Frozen Waves," Opt. Express **12**, 4001-4006 (2004); and references therein.
2. M. Zamboni-Rached, E. Recami, H. Figueroa, "Theory of Frozen Waves: Modelling the Shape of Stationary Wave Fields," J. Opt. Soc. Am. A , **22**, 2465-2475 (2005); and references therein.
3. This paper did first appear as e-print arXiv:physics/0506067, 8 Jun 2005.

1. Introduction

When propagating in a non-absorbing medium, the so called nondiffracting waves [1, 2] maintain their spatial shape for long distances. However, the situation is not the same when dealing with absorbing media. In these cases, both the ordinary and the nondiffracting beams (and pulses) will suffer the same effect: an exponential attenuation along the propagation axis.

Here, we are going to show that, through suitable superpositions of equal-frequency Bessel beams, it is possible to obtain nondiffracting beams, *in absorbing media*, whose longitudinal intensity pattern can assume any desired shape within a chosen interval $0 \leq z \leq L$ of the propagation axis z. As a particular example, we obtain new nondiffracting beams capable to resist the loss effects, maintaining amplitude and spot size of their central core for long distances.

The method that will be developed here [3] is a generalization of a previous one (also developed by us) [1, 2], which was conceived for lossless media. Although we are dealing here with exact solutions of the scalar wave equation, vectorial solutions of the same kind for the electromagnetic field can be obtained, since solutions to Maxwell's equations follow naturally from the scalar wave equation solutions.

It is important to stress that in this new method, there is no active participation of the material medium. Actually, the energy absorption by the medium continues to occur normally, the

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difference being in that these new beams have an initial transverse field distribution, such to be able to reconstruct (even in the presence of absorption) their central cores for distances considerably longer than the penetration depths of ordinary (nondiffracting or diffracting) beams. In this sense, the present method can be regarded as extending, for absorbing media, the auto-reconstruction properties that usual Localized Waves are known to possess in loss-less media.

2. The mathematical methodology

In the same way as for lossless media, we construct a Bessel beam with angular frequency ω and axicon angle θ in the absorbing materials by superposing plane waves, with the same angular frequency ω, and whose wave vectors lie on the surface of a cone with vertex angle θ. The refractive index of the medium can be written as $n(\omega) = n_R(\omega) + in_I(\omega)$, quantity n_R being the real part of the complex refraction index and n_I the imaginary one, responsible for the absorbing effects. With a plane wave, the penetration depth δ for the frequency ω is given by $\delta = 1/\alpha = c/2\omega n_I$, where α is the absorption coefficient.

In this way, a zero-order Bessel beam in dissipative media can be written as $\psi = J_0(k_\rho \rho)\exp(i\beta z)\exp(-i\omega t)$ with $\beta = n(\omega)\omega\cos\theta/c = n_R\omega\cos\theta/c + in_I\omega\cos\theta/c \equiv \beta_R + i\beta_I$; $k_\rho = n_R\omega\sin\theta/c + in_I\omega\sin\theta/c \equiv k_{\rho R} + ik_{\rho I}$, and so $k_\rho^2 = n^2\omega^2/c^2 - \beta^2$. In this way $\psi = J_0((k_{\rho R} + ik_{\rho I})\rho)\exp(i\beta_R z)\exp(-i\omega t)\exp(-\beta_I z)$, where β_R, $k_{\rho R}$ are the real parts of the longitudinal and transverse wave numbers, and β_I, $k_{\rho I}$ are the imaginary ones, while the absorption coefficient of a Bessel beam with an axicon angle θ is given by $\alpha_\theta = 2\beta_I = 2n_I\omega\cos\theta/c$, its penetration depth being $\delta_\theta = 1/\alpha_\theta = c/2\omega n_I\cos\theta$.

Due to the fact that k_ρ is complex, the amplitude of the Bessel function $J_0(k_\rho\rho)$ starts decreasing from $\rho = 0$ till the transverse distance $\rho = 1/2k_{\rho I}$, and afterwards it starts growing exponentially. This behavior is not physically acceptable, but one must remember that it occurs only because of the fact that an ideal Bessel beam needs an infinite aperture to be generated. However, in any real situation, when a Bessel beam is generated by finite apertures, that exponential growth in the transverse direction, starting after $\rho = 1/2k_{\rho I}$, will *not* occur indefinitely, stopping at a given value of ρ. Let us moreover emphasize that, when generated by a finite aperture of radius R, the truncated Bessel beam [2] possesses a depth of field $Z = R/\tan\theta$, and can be approximately described by solution of the previous paragraph, for $\rho < R$ and $z < Z$.

Experimentally, to guarantee that the mentioned exponential growth in the transverse direction *does not* even start, so as to meet only a decreasing transverse intensity, the radius R of the aperture used for generating the Bessel beam should be $R \leq 1/2k_{\rho I}$. However, as noted by Durnin *et al.*, the same aperture has to satisfy also the relation $R \geq 2\pi/k_{\rho R}$. From these two conditions, we can infer that, in an absorbing medium, a Bessel beam with just a decreasing transverse intensity can be generated only when the absorption coefficient is $\alpha < 2/\lambda$, i.e., if the penetration depth is $\delta > \lambda/2$. The method developed in this paper does refer to these cases, i.e., we can always choose a suitable finite aperture size in such a way that the truncated versions of all solutions presented in this work, including the general one given by Eq. (6), will not present any unphysical behavior. Let us now present our method.

Consider an absorbing medium with the complex refraction index $n(\omega) = n_R(\omega) + in_I(\omega)$, and the following superposition of $2N+1$ Bessel beams with the same frequency ω

$$\Psi(\rho,z,t) = \sum_{m=-N}^{N} A_m J_0\left((k_{\rho R_m} + ik_{\rho I_m})\rho\right) e^{i\beta_{R_m} z} e^{-i\omega t} e^{-\beta_{I_m} z}\,, \tag{1}$$

where m are integer numbers, A_m are constant (yet unknown) coefficients, β_{R_m} and $k_{\rho R_m}$ (β_{I_m} and $k_{\rho I_m}$) are the real parts (the imaginary parts) of the complex longitudinal and transverse wave numbers of the m-th Bessel beam in superposition (1), the following relations being satisfied

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$$k_{\rho_m}^2 = n^2\frac{\omega^2}{c^2} - \beta_m^2 \tag{2}$$

$$\frac{\beta_{R_m}}{\beta_{I_m}} = \frac{n_R}{n_I} \tag{3}$$

where $\beta_m = \beta_{R_m} + i\beta_{I_m}$, $k_{\rho_m} = k_{\rho R_m} + ik_{\rho I_m}$, with $k_{\rho R_m}/k_{\rho I_m} = n_R/n_I$.

Our goal is now to find out the values of the longitudinal wave numbers β_m and the coefficients A_m in order to reproduce approximately, inside the interval $0 \le z \le L$ (on the axis $\rho = 0$), a *freely chosen* longitudinal intensity pattern that we call $|F(z)|^2$.

The problem was already solved by us and the method developed for the particular case of lossless media [1, 2], i.e., when $n_I = 0 \to \beta_{I_m} = 0$. For those cases, it was shown that the choice $\beta = Q + 2\pi m/L$, with $A_m = \int_0^L F(z)\exp(-i2\pi mz/L)/L\,dz$ can be used to provide approximately the desired longitudinal intensity pattern $|F(z)|^2$ on the propagation axis, within the interval $0 \le z \le L$, and, at same time, to regulate the spot size of the resulting beam by means of the parameter Q, which can be also used to obtain large field depths and also to validate the linear polarization approximation to the electric field in a TE electromagnetic wave (see details in Refs. [1, 2]).

However, when dealing with absorbing media, the procedure described in the last paragraph does not work, due to the presence of the functions $\exp(-\beta_{I_m}z)$ in the superposition (1), since in this case that series does not became a Fourier series when $\rho = 0$.

On attempting to overcome this limitation, let us write the real part of the longitudinal wave number, in superposition (1), as

$$\beta_{R_m} = Q + \frac{2\pi m}{L} \tag{4}$$

with

$$0 \le Q + \frac{2\pi m}{L} \le n_R\frac{\omega}{c} \tag{5}$$

where this inequality guarantees forward propagation only, with no evanescent waves.

In this way the superposition (1) can be written

$$\Psi(\rho,z,t) = e^{-i\omega t}\,e^{iQz}\sum_{m=-N}^{N} A_m J_0\left((k_{\rho R_m} + ik_{\rho I_m})\rho\right)\,e^{i\frac{2\pi m}{L}z}\,e^{-\beta_{I_m}z}\,, \tag{6}$$

where, by using Eqs. (3), we have $\beta_{I_m} = (Q + 2\pi m/L)n_I/n_R$, and $k_{\rho_m} = k_{\rho R_m} + ik_{\rho I_m}$ is given by Eq. (2). Obviously, the discrete superposition (6) could be written as a continuous one (i.e., as an integral over β_{R_m}) by taking $L \to \infty$, but we have preferred the discrete sum due to the difficult of obtaining closed-form solutions to the integral form.

Now, let us examine the imaginary part of the longitudinal wave numbers. The minimum and maximum values among the β_{I_m} are $(\beta_I)_{\min} = (Q - 2\pi N/L)n_I/n_R$ and $(\beta_I)_{\max} = (Q + 2\pi N/L)n_I/n_R$, the central one being given by $\overline{\beta}_I \equiv (\beta_I)_{m=0} = Qn_I/n_R$. With this in mind, let us evaluate the ratio $\Delta = [(\beta_I)_{\max} - (\beta_I)_{\min}]/\overline{\beta}_I = 4\pi N/LQ$.

Thus, when $\Delta << 1$, there are no considerable differences among the various β_{I_m}, since it holds $\beta_{I_m} \approx \overline{\beta}_I$ for all m. And, in the same way, there are no considerable differences among the exponential attenuation factors, since $\exp(-\beta_{I_m}z) \approx \exp(-\overline{\beta}_I z)$. So, when $\rho = 0$ the series in the r.h.s. of Eq. (6) can be approximately considered a truncated Fourier series *multiplied by* the function $\exp(-\overline{\beta}_I z)$: and, therefore, superposition (6) can be used to reproduce approximately the desired longitudinal intensity pattern $|F(z)|^2$ (on $\rho = 0$), within $0 \le z \le L$, when the coefficients A_m are given by

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$$A_m = \frac{1}{L}\int_0^L F(z)\, e^{\overline{\beta}_I z} e^{-i\frac{2\pi m}{L}z}\, dz \tag{7}$$

the presence of the factor $\exp(\overline{\beta}_I z)$ in the integrand being necessary to compensate for the factors $\exp(-\beta_{I_m} z)$ in superposition (6).

Since we are adding together zero-order Bessel functions, we can expect a good field concentration around $\rho = 0$.

In short, we have shown in this Section how one can get, in an *absorbing medium*, a *stationary* wave-field with a good transverse concentration, and whose longitudinal intensity pattern (on $\rho = 0$) can approximately assume *any desired shape* $|F(z)|^2$ within the predetermined interval $0 \le z \le L$. The method is a generalization of a previous one [1, 2] and consists in the superposition of Bessel beams in (6), the real and imaginary parts of their longitudinal wave numbers being given by Eqs. (4)and (3), while their complex transverse wave numbers are given by Eq. (2), and, finally, the coefficients of the superposition are given by Eq. (7). The method is justified, since $4\pi N/LQ << 1$; happily enough, this condition is satisfied in a great number of situations.

Regarding the generation of these new beams, given a apparatus capable of generating a single Bessel beam, we can use an array of such apparatus to generate a sum of them, with the appropriate longitudinal wave numbers and amplitudes/phases [as required by the method], thus producing the desired beam. For instance, we can use [2] a laser illuminating a array of concentric annular apertures (located at the focus of a convergent lens) with the appropriate radii and transfer functions, able to yield both the correct longitudinal wave numbers (once a value for Q has been chosen) and the coefficients A_n of the fundamental superposition (6).

3. Some examples

For generality sake, let us consider a hypothetical medium in which a typical XeCl excimer laser ($\lambda = 308\text{nm} \rightarrow \omega = 6.12 \times 10^{15}\text{Hz}$) has a penetration depth of 5 cm; i.e., an absorption coefficient $\alpha = 20\text{m}^{-1}$, and therefore $n_I = 0.49 \times 10^{-6}$. Besides this, let us suppose that the real part of the refraction index for this wavelength is $n_R = 1.5$ and therefore $n = n_R + in_I = 1.5 + i0.49 \times 10^{-6}$. Note that the value of the real part of the refractive index is not so important for us, since we are dealing with monochromatic wave fields.

A Bessel beam with $\omega = 6.12 \times 10^{15}\text{Hz}$ and with an axicon angle $\theta = 0.0141\text{rad}$ (so, with a transverse spot of radius $8.4\,\mu\text{m}$), when generated by an aperture, say, of radius $R = 3.5$ mm, can propagate in vacuum a distance (its field depth) equal to $Z = R/\tan\theta = 25$ cm while resisting the diffraction effects. However, in the material medium considered here, the penetration depth of this Bessel beam would be only $z_p = 5$ cm. Now, let us give two interesting applications of the method.

3.1. Almost undistorted beams in absorbing media

Now, we can use the method of the previous Section to obtain, in the same medium and for the same wavelength, an almost undistorted beam capable of preserving its spot size and the intensity of its *central core* for a distance many times larger than the typical penetration depth of an ordinary beam (nondiffracting or not).

With this purpose, let us suppose that, for this material medium, we want a beam (with $\omega = 6.12 \times 10^{15}\text{Hz}$) that maintains amplitude and spot size of its central core for a distance of 25 cm, i.e., a distance 5 times greater than the penetration depth of an ordinary beam with the same frequency. We can model this beam by choosing the desired longitudinal intensity pattern $|F(z)|^2$ (on $\rho = 0$), within $0 \le z \le L$,

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$$F(z) = \begin{cases} 1 & \text{for} \quad 0 \le z \le Z \\ 0 & \text{elsewhere,} \end{cases} \tag{8}$$

and by putting $Z = 25$ cm, with, for example, $L = 33$ cm.

Now, the Bessel beam superposition (6) can be used to reproduce approximately this intensity pattern, and to this purpose let us choose $Q = 0.9999\omega/c$ for the β_{R_m} in Eq. (4), and $N = 20$ (note that, according to inequality (5), N could assume a maximum value of 158.)

Once we have chosen the values of Q, L and N, the values of the complex longitudinal and transverse Bessel beams wave numbers happen to be defined by relations (4), (3) and (2). Eventually, we can use Eq. (7) and find out the coefficients A_m of the fundamental superposition (6), that defines the resulting stationary wave-field.

Let us just note that the condition $4\pi N/LQ << 1$ is perfectly satisfied in this case.

In Fig. 1(a) we can see the 3D field-intensity of the resulting beam. One can see that the field possesses a good transverse localization (with a spot size smaller than 10μm), it being capable of maintaining spot size and *intensity* of its central core till the desired distance (a better result could be reached by using a higher value of N). It is interesting to note that at this distance (25 cm), an ordinary beam would have got its initial field-intensity attenuated 148 times.

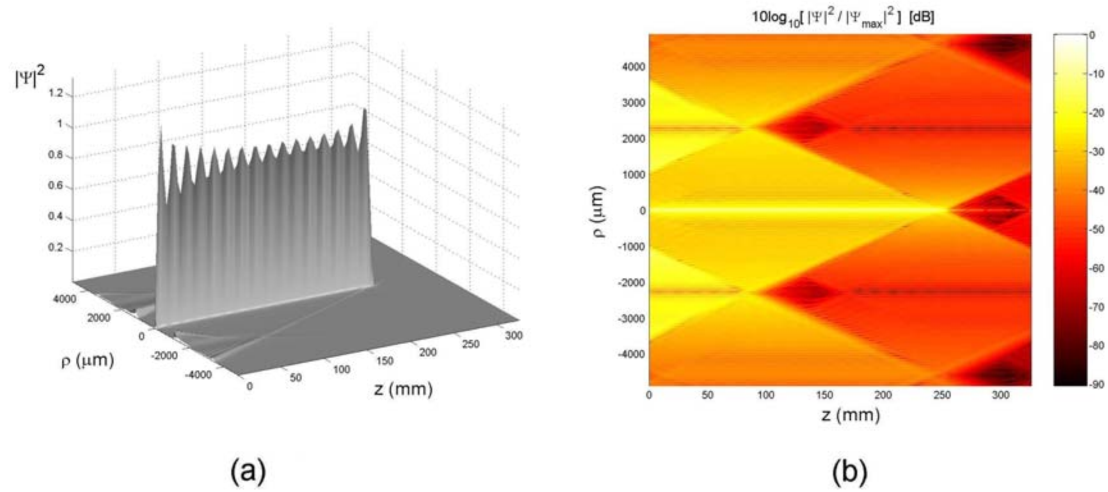

Fig. 1. (a) Three-dimensional field-intensity of the resulting beam. (b) The resulting beam, in an orthogonal projection and in **logaritmic** scale.

As we have said in the Introduction, the energy absorption by the medium continues to occur normally; the difference is that these new beams have an initial transverse field distribution sophisticated enough to be able to reconstruct (even in the presence of absorption) their central cores till a certain distance. For a better visualization of this field-intensity distribution and of the energy flux, Fig. 1(b) shows the resulting beam, in an orthogonal projection and in **logarithmic** scale. It is clear that the energy comes from the lateral regions, in order to reconstruct the central core of the beam. On the plane $z = 0$, within the region $\rho \le R = 3.5$ mm, there is a uncommon field intensity distribution, being very disperse instead of concentrated. This uncommon initial field intensity distribution is responsible for constructing the central core of the resultant beam and for its reconstruction till the distance $z = 25$ cm. Due to the absorption, the beam (total) energy flowing through different z planes, is not constant, but the energy flowing through the beam spot area and the beam spot size itself are conserved till (in this case) the

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distance $z = 25$ cm. Being P_T the incident power on the aperture ($z = 0$, $\rho \leq R = 3.5$ mm) and P_C the incident power on the beam spot area($\approx \pi \times 10^{-10}\,\mathrm{m}^2$), we have, in this example, $P_C/P_T \approx 1.1 \times 10^{-4}$.

3.2. *Beams in absorbing media with a growing longitudinal field intensity*

On considering the previous hypothetical medium, in which an ordinary Bessel beam with $\theta = 0.0141$rad and $\omega = 6.12 \times 10^{15}$Hz has a penetration depth of 5 cm, we aim at constructing now a beam that, instead of possessing a *constant* core-intensity till the position $z = 25$ cm, presents on the contrary a (moderate) exponential *growth* of its intensity, till that distance ($z = 25$ cm).

Let us assume we wish to get the longitudinal intensity pattern $|F(z)|^2$, in the interval $0 < z < L$

$$F(z) = \begin{cases} \exp(z/Z) & \text{for} \quad 0 \leq z \leq Z \\ 0 \quad \text{elsewhere,} \end{cases} \tag{9}$$

with $Z = 25$ cm and $L = 33$ cm.

Using again $Q = 0.9999\omega/c$, $N = 20$, we proceed as in the first example, calculating the complex longitudinal and transverse Bessel beams wave numbers and finally the coefficients A_m of the fundamental superposition (6).

In Fig. 2 we can see the 3D field-intensity of the resulting beam. One can see that the field presents the desired longitudinal intensity pattern with a good transverse localization (a spot size smaller than 10μm).

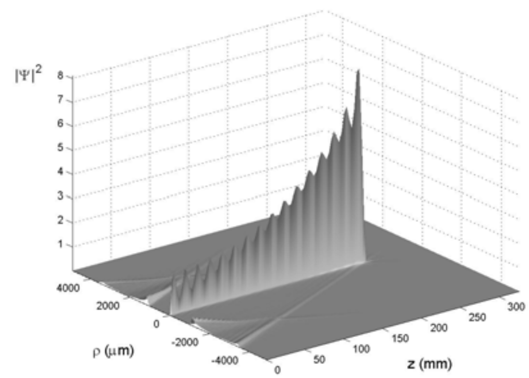

Fig. 2. Three-dimensional field-intensity of the resulting beam, in the absorbing medium, with a growing longitudinal field intensity.

Obviously, the amount of energy necessary to construct these new beams is greater than that necessary to generate an ordinary beam in a non-absorbing medium. And it is also clear that there is a *limitation* on the depth of field of these new beams. In the first example, for distances longer than 10 times the penetration depth of an ordinary beam, besides a greater energy demand, we meet the fact that the field-intensity in the lateral regions would be even higher than that of the core, and the field would loose the usual characteristics of a beam (transverse field concentration).

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