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Properties of localized pulses through the analysis of temporal modulation effects in Bessel beams and the convolution theorem

C.A. Dartora [a,*,1], K.Z. Nóbrega [b,1], H.E. Hernández-Figueroa [b], Erasmo Recami [c]

[a] *DFMC, Institute of Physics, University of Campinas, Campinas-SP 13083-970, Brazil*
[b] *DMO, FEEC, University of Campinas, Campinas-SP 13083-970, Brazil*
[c] *Faculty of Engineering, Bergamo State University, Italy, INFN-Sezione di Milano, Milan, Italy*

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Abstract

In this paper we analyze the effects of the time modulation of (zeroth-order) Bessel beams, by considering a few different pulse shapes. Namely, three modulating functions are considered: a train of rectangular waves, a single rectangular pulse, and a gaussian pulse. The influence of the carrier frequency, and of shape and spectral bandwidth of the modulating function, is also discussed; while further support to our results is met by using the convolution technique in the time domain. At the beginning, a brief review of the X-shaped solutions to the wave equation, and of some properties of theirs, is presented.

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1. Introduction

Since many years it has been known that localized solutions exist to the wave equation which, in the ideal case, do not suffer diffractive or dispersive effects during their propagation: see, e.g., [1–9]. In practice, such waves remain unchanged in their space-time structure, i.e., maintain their transverse intensity profile and temporal shape for a (long) distance only: namely, along their "depth of field" only [2]. Localized pulses

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[*] Corresponding author. Tel.: +55-1932376533; fax: +55-1937883704.
E-mail address: dartora@ifi.unicamp.br (C.A. Dartora).
[1] Supported by Fapesp.

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have been experimentally produced in acoustics [10], in optics [11] and recently in microwave physics [12]: such pulses can be obtained by an adequate superposition of diffraction-free beams [1,2,13–16]. For cylindrical symmetry, a set of basis functions for the mentioned localized waves is known to be constituted by the Bessel beams.

There are potential applications of them in a number of fields, from wireless communications, to laser surgery and machining, to imaging, etc. [13–16]. Since the beginning, much effort has been devoted in attempts at constructing a laser which emits beams endowed with a Bessel pattern in the transverse direction; an advantage being that Bessel beams propagate a long distance without spreading, when compared to gaussian beams. Indeed, many ways are today available for producing Bessel beams. In this paper, our main aim is a preliminary study of the effects of *modulating* (not ordinary waves, but) Bessel beams.

The content of this paper can be described as follows: in the next Section we develop a theoretical basis for analyzing localized pulses (in terms of Bessel beams), presenting the X-shaped wave. In Section 3, we analyze localized pulses generated by modulation of a zeroth-order Bessel beam; three modulating functions will be considered: a train of rectangular waves, a single rectangular pulse, and a gaussian pulse. In Section 4, the convolution theorem is applied to explain some of the properties of these localized pulses, and finally, in Section 5, a few conclusions and remarks are added.

2. Theoretical preliminaries

Let us provide, first, a sketchy description of localized waves in the vacuum ($\epsilon = \epsilon_0$), by referring ourselves to the homogeneous scalar wave equation (utilized, as we know, in optics, microwaves, acoustics, and so on) [1,2,9]

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi(\rho,\varphi,z;t) = 0, \tag{1}$$

where $\nabla^2 \equiv \nabla_\perp^2 + \partial^2/\partial z^2$ is the laplacian operator, c the light speed in vacuum, and (ρ,φ,z) are the cylindrical coordinates. Quantity Ψ can represent, e.g., a component of the electric or magnetic field. Then, let us perform a change of coordinates: we adopt, more precisely, the "generalized bidirectional decomposition" introduced in [1] in terms of the so-called V-cone variables

$$\eta = z + Vt, \;\; \zeta = z - Vt,$$

where V is an arbitrary propagation speed along the z-axis. In this new set of coordinates the wave equation becomes

$$\left[\nabla_\perp^2 - \frac{1}{\gamma^2}\left(\frac{\partial^2}{\partial\eta^2} + \frac{\partial^2}{\partial\zeta^2}\right) + 2\left(1 + \frac{V^2}{c^2}\right)\frac{\partial^2}{\partial\eta\partial\zeta}\right]\Psi(\rho,\varphi,\eta,\zeta) = 0, \tag{2}$$

it being [1]

$$\gamma = \frac{1}{\sqrt{\frac{V^2}{c^2} - 1}}, \quad V > c.$$

If we are interested in solutions propagating forward in the z-direction only, which means that $\partial\Psi/\partial\eta = 0$, we reduce the four-dimensional $(\rho,\varphi,z;\,t)$ problem to only three dimensions $(\rho,\varphi,\zeta = z - Vt)$, while the wave equation is now given by

$$\left(\nabla_\perp^2 - \frac{1}{\gamma^2}\frac{\partial^2}{\partial\zeta^2}\right)\Psi(\rho,\varphi,\zeta) = 0. \tag{3}$$

The solution to this problem can be straightforwardly obtained using Fourier transforms and following the approach in [1]. On defining

$$\Phi(\rho,\varphi,\omega)=\frac{1}{2\pi V}\int_{-\infty}^{\infty}\mathrm{d}\zeta\Psi\left(\rho,\varphi,\frac{\zeta}{V}\right)\exp\left[-\mathrm{i}\omega\frac{\zeta}{V}\right], \tag{4}$$

$$\Psi\left(\rho,\varphi,\frac{\zeta}{V}\right)=\int_{-\infty}^{\infty}\mathrm{d}\omega\Phi(\rho,\varphi,\omega)\exp\left[\mathrm{i}\omega\frac{\zeta}{V}\right], \tag{5}$$

the insertion of Eq. (5) into Eq. (3) yields the equation for $\Phi(\rho,\varphi,\omega)$:

$$\left(\nabla_{\perp}^{2}+\frac{\omega^{2}}{\gamma^{2}V^{2}}\right)\Phi(\rho,\varphi,\omega)=0, \tag{6}$$

whose solutions (required to be regular at the origin $\rho=0$) are known to be the Bessel functions of first kind

$$\Phi_n(\rho,\varphi,\omega)=A_n(\omega)\mathrm{e}^{\mathrm{i}n\varphi}J_n\left(\frac{\omega}{\gamma V}\rho\right),\quad n=0,\pm1,\pm2\ldots$$

When restricting to zeroth-order ($n=0$) Bessel functions only, which are radially symmetric, the solution is represented by the pulses

$$\Psi\left(\rho,\frac{\zeta}{V}\right)=\int_{-\infty}^{\infty}\mathrm{d}\omega\,A(\omega)J_0\left(\frac{\omega}{\gamma V}\rho\right)\exp\left[\mathrm{i}\omega\frac{\zeta}{V}\right], \tag{7}$$

where $A(\omega)$ is an arbitrary spectral function; and $J_0(\omega\rho/\gamma V)$ can be understood as the (monochromatic) response of the physical apparatus.

When all the wave-vectors lie on the cone corresponding to the angle θ, it holds $k_z=(\omega/c)\cos\theta=\omega/V$ and $k_\rho=(\omega/c)\sin\theta$. The velocity in this case is $V=c/\cos\theta$. One can write the solution of Eq. (6) into the more general form [1]:

$$\Psi\left(\rho,\varphi,\frac{\zeta}{V}\right)=\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{d}\omega\,A_n(\omega)\mathrm{e}^{\mathrm{i}n\varphi}J_n\left(\frac{\omega}{\gamma V}\rho\right)\exp\left[\mathrm{i}\omega\frac{\zeta}{V}\right]. \tag{8}$$

But Eq. (8) is an ideal solution, obtainable with an infinite aperture and thus possessing infinite energy. One should therefore investigate the physical solutions (produced by finite apertures) by means of the diffraction theory [17,18]. Indeed, considering the solution for a flat circular aperture, and using the Green propagator, one can write:

$$\Psi(\rho,\varphi,z;t)=\int_{S'}\mathrm{d}S'\frac{\Psi\left(\rho',\varphi',z'=0;t'=t-\frac{|\mathbf{r}-\mathbf{r}'|}{c}\right)}{|\mathbf{r}-\mathbf{r}'|}, \tag{9}$$

where $\Psi(\rho',\varphi',z'=0;t'=t-|\mathbf{r}-\mathbf{r}'|/c)$ is the ideal solution given by the above Eq. (8), and vectors $\mathbf{r}$, $\mathbf{r}'$ refer to the observation point and the aperture, respectively. Various papers already appeared setting forth finite-energy solutions; but in this work we shall consider only the ‘‘ideal’’ solutions, generated by means of infinite apertures.

The so-called X-*shaped* solutions to the wave equation are the most studied in the current literature [1,2,6,7,10–12,19–23], since they can be *easily* obtained through suitable superpositions of Bessel beams [1,2,10]. First of all, consider the following frequency spectral function:

$$A_n(\omega)=\delta_{mn}B_m(\omega)\mathrm{e}^{-a\omega};\quad \omega\geqslant0;\;\; m=0,1,2,\ldots \tag{10}$$

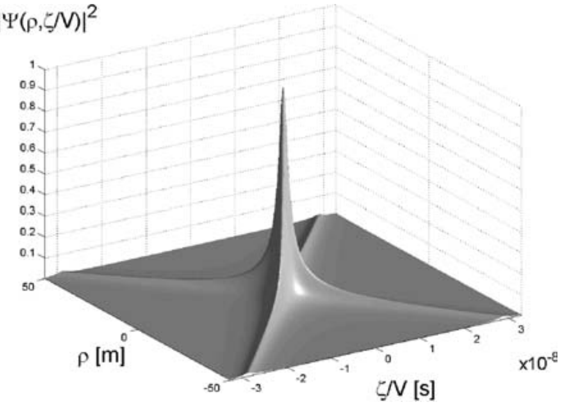

Fig. 1. The intensity distribution (in normalized units) of a classical X-wave in the plane $\rho - \zeta/V$. $a = 10^{-9}$ and $\theta = 10°$.

With this spectral function, substituted into Eq. (8), one can get [1] X-shaped waves of any orders:

$$\Psi_{X_m}\left(\rho, \varphi, \frac{\zeta}{V}\right) = \mathrm{e}^{\mathrm{i}m\varphi} \int_0^{\infty} \mathrm{d}\omega B_m(\omega) J_m\left(\frac{\omega}{\gamma V}\rho\right) \exp\left(-\omega\left[a - \mathrm{i}\frac{\zeta}{V}\right]\right). \tag{11}$$

As a special case, when we consider [1] $m = 0$ and $B_0(\omega) = 1$, with the help of tables [24], one obtains the *ordinary* X-wave [2,20] in its celebrated analytic form:

$$\Psi_{X_0}\left(\rho, \frac{\zeta}{V}\right) = \frac{V}{\sqrt{(aV - \mathrm{i}\zeta)^2 + (\frac{V^2}{c^2} - 1)\rho^2}}. \tag{12}$$

By Fig. 1 we recall the interesting shape of the classical solution (12). Its space-time localization appears evident, but it is better suited to low frequencies [1]. On the other side, most of the optical systems deal with pulses having a well defined carrier frequency ω_0. It is therefore difficult to generate the X-waves in optics.

3. Modulation of zero-order Bessel beams

We shall restrict our attention, in the following, to the *zeroth-order* Bessel beams.

As we were saying in the previous section, one can regard the X-wave as the result of a specific *modulation* of Bessel beams in the low frequency range, which generates a wave packet that does not suffer diffraction. The main aim of this paper will be addressing the problem of the modulation of (zeroth-order) Bessel beams by means of other, different spectral functions, in order to be able to define an optical carrier frequency, if desired.

3.1. Wave train of rectangular pulses

First, let us modulate a laser, with carrier frequency ω_0, by a train of waves with duty cycle $U = \tau/T$, where τ is the temporal width of each pulse, and the bit rate is given by $1/T$. Its representation by a Fourier series is:

$$a(t) = \{\exp[\mathrm{i}\omega_0 t]\}\left\{1 + 2\sum_{n=1}^{\infty} \frac{\sin(\pi n U)}{\pi n U} \cos(2\pi n U t)\right\}, \tag{13}$$

where the first (second) term between {.} is related to the laser (wave train).

Using the Fourier property of the frequency shifting, the spectrum of (13) will become

$$A(\omega) = \delta(\omega - \omega_0) + \sum_{n=1}^{\infty} \frac{\sin(\pi n U)}{\pi n U} \left(\delta\left(\omega - \omega_n^+\right) + \delta\left(\omega - \omega_n^-\right) \right) \tag{14}$$

being $\delta(.)$ the Dirac delta function and $\omega_n^+ = \omega_0 + 2\pi n/T$ and $\omega_n^- = \omega_0 - 2\pi n/T$. The substitution of this spectral function into Eq. (7) yields the final solution in its closed form

$$\begin{aligned}\Psi\left(\rho, \frac{\zeta}{V}\right) = {} & J_0\left(\frac{\omega_0}{\gamma V}\rho\right) \exp\left[\mathrm{i}\omega_0 \frac{\zeta}{V}\right] + \sum_{n=1}^{N=\infty} \frac{\sin(\pi n U)}{\pi n U} \left(J_0\left(\frac{\omega_n^+}{\gamma V}\rho\right) \exp\left[\mathrm{i}\omega_n^+ \frac{\zeta}{V}\right] \right. \\ & \left. + J_0\left(\frac{\omega_n^-}{\gamma V}\rho\right) \exp\left[\mathrm{i}\omega_n^- \frac{\zeta}{V}\right] \right). \end{aligned} \tag{15}$$

Let us analyze three cases: $U = 1$, $U \to 0$ and $U = 0.5$ by numerical simulations. We assume angle θ to be $\theta = 10^{\circ}$, the carrier wavelength $\lambda_0 = 632.8$ nm, quantity $1/T = 10^{13}$ Hz, and a number of terms $N = 100$ in the series. Figs. 2(a) and (b) show results consistent with the mathematical and physical expectations: see below.

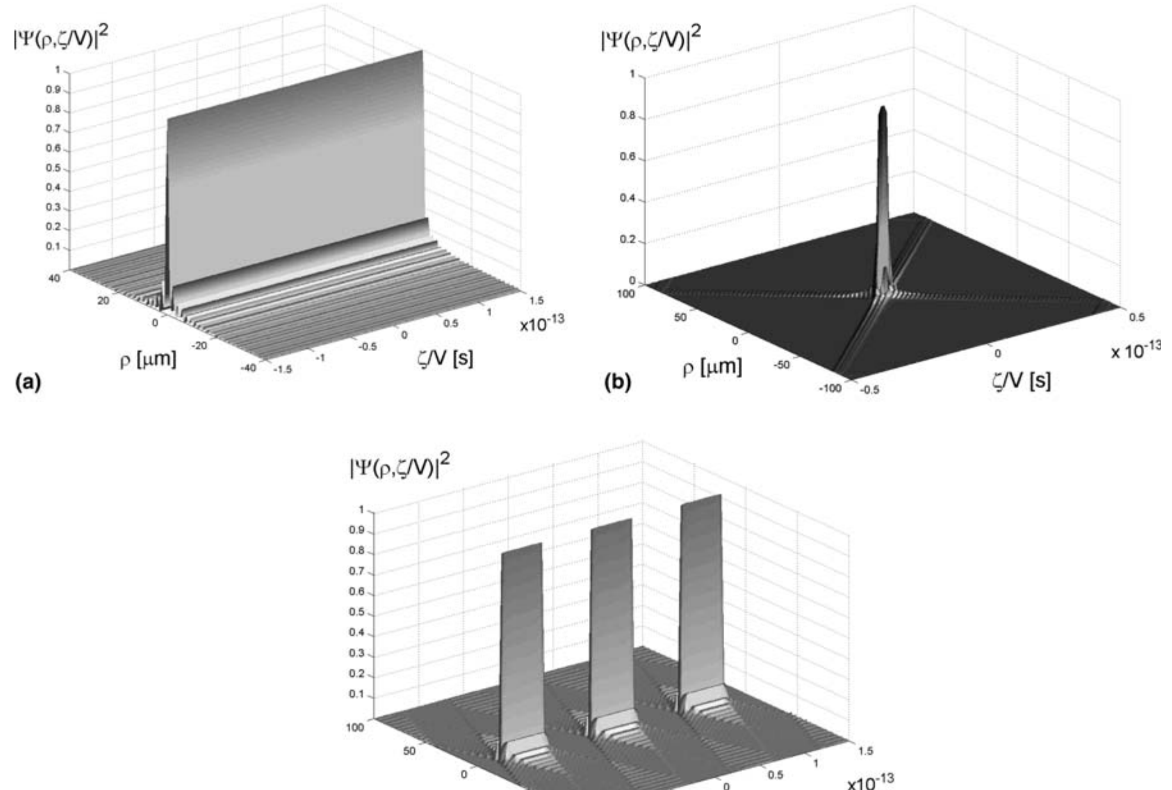

Fig. 2. (a) 3D Intensity pattern (in normalized units) of a Bessel beam modulated by a wave train in the plane $\rho - \zeta/V$ using the following parameters: $U = 1$, $\lambda_0 = 632.8$ nm , $\theta = 10^{\circ}$, $1/T = 10^{13}$ Hz and $N = 100$. (b) View of a single pulse of the intensity pattern (in normalized units) of a Bessel beam modulated by a wave train in the plane $\rho - \zeta/V$ using the following parameters: $U = 0.05$, $\lambda_0 = 632.8$ nm , $\theta = 10^{\circ}$, $1/T = 10^{13}$ Hz and $N = 100$. (c) 3D intensity pattern (in normalized units) of a Bessel beam modulated by a wave train in the plane $\rho - \zeta/V$ using the following parameters: $U = 0.5$, $\lambda_0 = 632.8$ nm , $\theta = 10^{\circ}$, $1/T = 10^{13}$ Hz and $N = 100$.

As a matter of fact, for Fig. 2(a), when $U = 1$, one can infer from Eq. (15) that $\sin(n\pi) = 0$, which cancels out the summation, so much so $\Psi(\rho, \zeta/V)$ results to be just a monochromatic Bessel beam. From the physical point of view, one already knows that, when $U = 1$, we end up with a monochromatic laser ($A(\omega) = \delta(\omega - \omega_0)$), which produces a Bessel beam, as seen above.

As to Fig. 2(b), when $U \to 0$, it is $\lim_{U\to 0} \sin(n\pi U)/(n\pi U) = 1$, and $\Psi(\rho, \frac{\zeta}{V})$ appears nearly as a train of impulses, axially symmetric w.r.t. $\rho = 0$. Physically speaking, the case $U \to 0$ represents the apparatus response just to a train of impulses. In this figure we depict a single pulse of the wavetrain.

The last case is $U = 0.5$: see Fig. 2(c). One can clearly see, therein, the influence of the modulation of the Bessel beam: its temporal behavior is the same as that of the modulating function, while the X-shape is clearly present (due to the superposition of Bessel beams). On the other side, due to the existence of a well-defined carrier frequency, its transverse pattern does still show the same oscillations of a Bessel beam: something that is completely absent, of course, for a classical X-wave (when it is not possible to define a carrier frequency).

3.2. Single rectangular pulse

In order to eliminate the interference phenomena present in any pulsed system (wave train), now we go on to the laser modulation by a single pulse.

Let us define a single rectangular pulse in the time domain, given by $a(t) = 1(0)$; $|t| \leqslant b$ (otherwise), being $2b$ the time width of the pulse. Apart from constants, the Fourier transform of this function, shifted by the carrier frequency is known to be

$$A(\omega) = \frac{\sin[b(\omega - \omega_0)]}{(\omega - \omega_0)}. \tag{16}$$

Substitution of Eq. (16) into Eq. (7) results in the new equation

$$\Psi\left(\rho, \frac{\zeta}{V}\right) = \int_{-\infty}^{\infty} \mathrm{d}\omega \frac{\sin[b(\omega - \omega_0)]}{(\omega - \omega_0)} J_0\left(\frac{\omega}{\gamma V}\rho\right) \exp\left[\mathrm{i}\omega\frac{\zeta}{V}\right]. \tag{17}$$

In Fig. 3 the intensity profile of Eq. (17) is shown. We have adopted the following values for the parameters: $\lambda_0 = 632.8$ nm, $\theta = 10°$ and $1/b = 10^{13}$ Hz. In accordance with our previous results, in the top

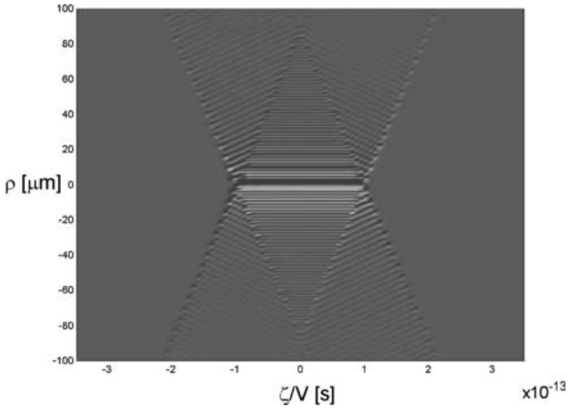

Fig. 3. Top view of the intensity pattern of a Bessel beam modulated by a single rectangular modulating function in the plane $\rho - \zeta/V$ using the following parameters: $\lambda_0 = 632.8$ nm , $\theta = 10°$ and $1/b = 10^{13}$ Hz.

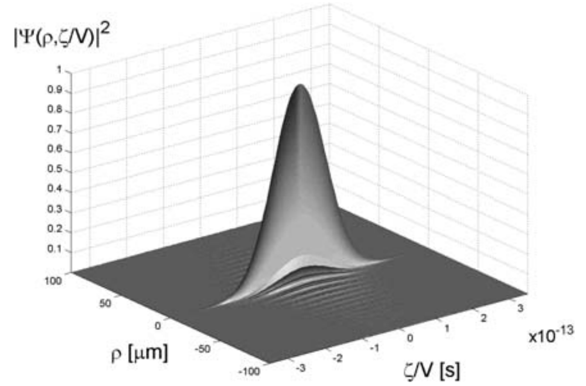

Fig. 4. 3D intensity pattern (in normalized units) of a Bessel beam modulated by a gaussian modulating function in the plane $\rho - \zeta/V$ using the following parameters: $U = 1$, $\lambda_0 = 632.8$ nm , $\theta = 10^{\circ}$ and $1/b = 10^{13}$ Hz.

view of the intensity profile (17) one can easily observe its X-shape and the oscillations in its transverse pattern, while along the time axis it keeps the modulating function profile.

3.3. Gaussian pulse

Following the same steps of the previous case, we may analyze a gaussian pulse defined as $a(t) = \mathrm{e}^{\frac{-t^2}{2b^2}}$ whose Fourier transform (shifted in frequency) is:

$$A(\omega) = \mathrm{e}^{-\frac{b^2(\omega-\omega_0)^2}{2}}, \tag{18}$$

and, by substituting it into Eq. (7), one gets this time

$$\Psi\left(\rho, \frac{\zeta}{V}\right) = \int_{-\infty}^{\infty} \mathrm{d}\omega \exp\left[-\frac{b^2(\omega-\omega_0)^2}{2}\right] J_0\left(\frac{\omega}{\gamma V}\rho\right) \exp\left[\mathrm{i}\omega\frac{\zeta}{V}\right]. \tag{19}$$

The numerical simulation of Eq. (19) and its intensity profile are illustrated in Fig. 4. We have kept the same parameters as in the case of a single rectangular pulse. In Fig. 4 we can see that its Bessel oscillations are similar to those in Fig. 3. Although both spectra are well localized around a carrier frequency, the gaussian spectrum does *not* imply significative values in a *wide* range of frequencies, when compared to the spectrum given by (17), and consequently in Fig. 4 the X-shape is not evident (with the chosen parameters).

4. Convolution theorem and some properties of localized pulses

On using the convolution theorem of the Fourier transform [2,10],

$$\int_{-\infty}^{\infty} \mathrm{d}\omega F(\omega) G(\omega) \exp\left[\mathrm{i}\omega\frac{\zeta}{V}\right] = \int_{-\infty}^{\infty} \mathrm{d}\tau f\left(\frac{\zeta}{V} - \frac{\tau}{V}\right) g\left(\frac{\tau}{V}\right) = f\left(\frac{\zeta}{V}\right) * g\left(\frac{\zeta}{V}\right),$$

(where * denotes convolution w.r.t. ζ/V), and the following definitions:

$$F(\omega) = A(\omega),$$

$$G(\rho, \omega) = J_0\left(\frac{\omega}{\gamma V}\rho\right),$$

the expression (7) assumes the form

$$\Psi\left(\rho,\frac{\zeta}{V}\right)=f\left(\frac{\zeta}{V}\right)*g\left(\rho,\frac{\zeta}{V}\right). \tag{20}$$

The functions f and g are then easily calculated, resulting to be

$$f\left(\frac{\zeta}{V}\right)=\int_{-\infty}^{\infty}\mathrm{d}\omega A(\omega)\exp\left(\mathrm{i}\omega\frac{\zeta}{V}\right) \tag{21}$$

and

$$g\left(\rho,\frac{\zeta}{V}\right)=\int_{-\infty}^{\infty}\mathrm{d}\omega J_0\left(\frac{\omega}{\gamma V}\rho\right)\exp\left(\mathrm{i}\omega\frac{\zeta}{V}\right). \tag{22}$$

From Eq. (20) one can think the space-time shape of a pulse as the convolution of a modulating function (21) with the transmittance function (22) of an experimental apparatus. The modulating function strongly contributes to the temporal behavior of the resulting pulse, while the apparatus is the main responsible for the transverse pattern. These observations give further support to our results in Section 3.

Notice that one is free to choose the functions f and g in any convenient ways. One could have chosen, for e.g.,

$$f\left(\frac{\zeta}{V}\right)=\int_{-\infty}^{\infty}\mathrm{d}\omega A(\omega)\mathrm{e}^{a\omega}\exp\left(\mathrm{i}\omega\frac{\zeta}{V}\right) \tag{23}$$

and

$$g\left(\rho,\frac{\zeta}{V}\right)=\int_{-\infty}^{\infty}\mathrm{d}\omega\mathrm{e}^{-a\omega}J_0\left(\frac{\rho}{\gamma V}\omega\right)\exp\left(\mathrm{i}\omega\frac{\zeta}{V}\right)=\frac{V}{\sqrt{(aV-\mathrm{i}\zeta)^2+\left(\frac{V^2}{c^2}-1\right)\rho^2}}. \tag{24}$$

Eq. (24) being nothing but the classical X-wave, already met in Section 2. With these definitions, one meets another way to look at the problem: namely, one may now think of a system whose impulsive (modulating function) response is an X-wave, which can be shifted in its spectrum towards higher frequencies, by means of a carrier frequency ω_0 and of the modulating function $a(t)$:

$$\Psi\left(\rho,\frac{\zeta}{V}\right)=\int_{-\infty}^{\infty}\mathrm{d}\tau\ a(\tau)\mathrm{e}^{\mathrm{i}\omega_0\frac{\tau}{V}}\frac{V}{\sqrt{(aV-\mathrm{i}(\zeta-\tau))^2+\left(\frac{V^2}{c^2}-1\right)\rho^2}}. \tag{25}$$

But this procedure has no practical interest, it being more realistic the modulation of a Bessel beam generator, as attempted in this work.

5. Conclusion

In this paper, after a brief presentation of the X-shaped solutions to the wave equation, and of some properties of theirs, we have analyzed effects of the *modulation* of a (zeroth-order) Bessel beam by having recourse to some of the pulse shapes most used in communication systems. Three modulating functions have been considered: a rectangular wavetrain, a single rectangular pulse, and a gaussian pulse. We have seen the temporal behavior to be strongly influenced by the modulating function shape; while the presence of a well-defined carrier frequency is responsible for oscillations in the transverse direction of the pulse (which resembles a Bessel beam of such a frequency). Moreover, the spectral bandwidth is directly related to the appearance of the X-shape. Finally, a brief analysis in terms of the convolution theorem has been added, which supports the previous results.

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