

DOES THERMODYNAMICS REQUIRE A NEW EXPANSION AFTER THE "BIG CRUNCH" OF OUR COSMOS? ^{*}

Erasmus RECAMI

*Dipartimento di Fisica, Università Statale di Catania, Catania, Italy*¹
*and Department of Applied Mathematics, State University at Campinas, Campinas, SP, Brazil*²

and

Vilson Tonin ZANCHIN

"Curso de Mestrado", Department of Physics, State University at Campinas, Campinas, SP, Brazil

Received 31 January 1986

Recently, a unified geometrical approach to gravitational and strong interactions was proposed, based on the methods of general relativity. According to it, hadrons can be regarded as "black-hole type" solutions of new field equations describing *two* tensorial metric fields (the ordinary gravitational, and the "strong" one). By extending the Bekenstein-Hawking thermodynamics to those "strong black holes" (SBH), we show that (i) SBH thermodynamics seems to require a new expansion of our cosmos after its "Big Crunch", (This thermodynamical indication being rather unique, up to now, in showing that a recontraction of our cosmos has to be followed by a new "creation"); (ii) a collapsing star with masses approximately in the range 3-5 solar masses, once reaching the neutron-star density, could re-explode *tending* to form a (radiating) object with a diameter of the order of 1 light-day: thus failing to create a gravitational black hole.

Introduction. Inspired by the Weyl-Eddington-Dirac "large numbers coincidences", recently we proposed ^{‡1} a unified theory of strong and gravitational interactions, that (even if geometrical and classical, i.e. based on the methods of general relativity) yields results similar to the "strong gravity" ones by Salam et al. [9].

This geometrical approach is twofold. On one side, it tries to describe the strong field inside a hadron just by means of "scaled down" Einstein equations (with attractive cosmological term) [1-4,7,8]. One thus naturally gets the confinement of the hadron constituents, and is allowed e.g. to calculate the hadron masses (for the meson mass-spectra, see refs. [8,10]).

On the other hand, one is led to describe the strong field surrounding a hadron as a new tensorial field, to be "added" to the gravitational one; so that a *bi-scale* theory arises [1,2,5,6] where the total metric $g_{\mu\nu}$ is regarded as the sum of the ordinary gravitational metric field $g_{\mu\nu}^{(G)} \equiv e_{\mu\nu}$ and of the "strong" metric field $s_{\mu\nu}$, at least *in the linear approximation* ^{‡2}. In the surroundings of a hadron (in suitable coordinates and - as we said - in the "weak field" approximation), therefore:

$$g_{\mu\nu} \approx e_{\mu\nu} + s_{\mu\nu} \approx \eta_{\mu\nu} + s_{\mu\nu}, \quad (1)$$

where $\eta_{\mu\nu} \equiv \text{diag}(+1, -1, -1, -1)$. Anyway, the new field equations are [1,2,5,6]

$$R_{\mu\nu} + \lambda s_{\mu\nu} \approx - (8\pi/c^4) (S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S_{\rho}^{\rho})$$

$$[\lambda > 0], \quad (2)$$

^{*} Work supported in part by CIME/IILA, INFN, CNPq and IBM-do Brasil.

¹ Permanent address.

² Temporary address.

^{‡1} For an extended review of it, see refs. [1,2], see also refs. [3-8].

^{‡2} For a more general approach, going beyond the linear approximation, see the recent paper [11].

(B)

Does Thermodynamics Require Our Cosmos to Undergo a Series of Contraction/Expansion Cycles?*

ERASMO RECAMI,[‡] J. M. MARTINEZ[§]
and V. TONIN ZANCHIN[‡]

[‡]"Curso de Mestrado" Department of Physics, State
University at Campinas, Campinas, S.P., Brazil

[§]IMECC, UNICAMP, Campinas, S.P., Brazil

<<Matter possesses a cohesion that keeps it together and against which the vacuum is impotent. Actually, the material world is supported by an immense force, and alternately it contracts and expands following its physical transmutations, now consumed by fire, now giving rise to a new cosmos creation>>.

Posidonius (~135 ÷ 51 B.C.).

ABSTRACT

A unified geometrical approach to strong and gravitational interactions has been recently proposed, based on the classical methods of General Relativity. According to it, hadrons can be regarded as "black-hole type" solutions of new field equations describing *two* tensorial metric-fields (the ordinary gravitational field, and the "strong" one). In this paper, we first seize the opportunity for an improved exposition of some elements of the theory relevant to our present scope. Secondly, by extending the Bekenstein-Hawking thermodynamics to the abovementioned "strong black-holes" (SBH), we show: (i) that SBH thermodynamics seems to require a new expansion of our cosmos after its *Big Crunch* (i.e., that a recontraction of our cosmos has to be followed by a new "creation"); (ii) that a collapsing star with mass M approximately in the range 3 to 5 solar masses, once reached the neutron-star density, could re-explode *tending* to form a (radiating) object with a diameter of the order of 1 light-day: thus failing to create a gravitational black-hole.

KEYWORDS

Unified theories; classical "strong gravity"; strong interactions; gravitational interactions; General Relativity; black-hole thermodynamics; "strong black-holes"; Big Bang; collapsing stars; neutron stars; cosmology; "Big Crunch".

(*) Talk delivered by E.R. Work supported in part by I.N.F.N.-Sezione di Catania, CIME/IILA, CNPq, and IBM-do-Brasil.

1. INTRODUCTION

Inspired by the Weyl-Klein-Eddington-Dirac "large numbers coincidences", we proposed a few years ago a unified approach to strong and gravitational interactions (for an extended summary of it see Recami, 1982a,b), that happens to be *formally* analogous to N.Rosen's (1979, 1980a,b) *bi-metric General Relativity theory*, and that—even if geometrical, i.e. based on the classical methods of General Relativity (GR)—yields results similar to the "*strong gravity*" ones by Salam and coworkers (Salam, 1978; Salam & Strathdee, 1978).

Our geometrical approach is twofold:

- (i) on one side, it tries to describe the strong field inside a hadron just by means of "scaled down" Einstein equations, with attractive cosmological term (Recami & Castorina, 1976; Caldirola, Pavsic & Recami, 1978a,b; Recami, 1983, 1984). One thus naturally gets the *confinement* of the hadron constituents, and e.g. is allowed to calculate the meson mass-spectra (Recami, 1983; Italiano & Recami, 1984; Italiano, Lattuada, Maccarrone, Recami, Riggi & Vinciguerra, 1984 and work in progress in coll. also with L.A.B. Annes, P. Ammiraju, J.S. Roversi and W.A. Rodrigues);
- (ii) on the other hand, one is led to describe the strong field surrounding a hadron as a new tensorial field, to be "added" to the gravitational one: so that a *bi-scale theory* arises (Caldirola & Recami, 1979; Ammiraju, Recami & Rodrigues, 1983), in which the total metric $g_{\mu\nu}$ is the superposition of the ordinary gravitational metric-field $g_{\mu\nu}$ (Grav.) $\equiv e_{\mu\nu}$ and of the "strong" metric-field $s_{\mu\nu}$.

Here we shall stick to question (ii) only.

2. THEORETICAL FRAMEWORK

1) The strong field surrounding a hadron is a (second) tensorial field $s_{\mu\nu}$ such that $s_{\mu\nu} \rightarrow 0$ for $r \gg 1$ fm. All tensorial indices, incidentally, are raised (lowered) by the *total metric-tensor* $g^{\mu\nu}$ ($g_{\mu\nu}$).

2) Hadrons possess, besides the ordinary (gravitational) mass m , a "strong mass" g (or *strong charge*) which *directly* affects (deforms) space-time via the constant $N \equiv \rho^{-1}G$, where ρ^{-1} is the ratio of the strong interaction strength S to the gravitational interaction strength s :

3) $\rho^{-1} \equiv S/s \equiv (Ng^2/\hbar c) \cdot (Gm^2/\hbar c)^{-1} \approx 10^{41}$,

and where, by "coincidence", ρ is approximately equal to the ratio of the typical hadron radius $r(h)$ to the Hubble radius $R(U)$ of our cosmos. If, e.g., we put conventionally $g=m$, then $N \approx 10^{41}G \approx hc/m^2$.

4) Far from a source-hadron, $g_{\mu\nu} = e_{\mu\nu}$, and both hadrons and non-hadrons will feel only $e_{\mu\nu}$: So that our approach does correctly yield the theory of GR.

5) Near a source-hadron, the test-hadrons will feel *both* $e_{\mu\nu}$ and $s_{\mu\nu}$. For a test-hadron endowed with mass m' and strong-mass g' in the surroundings of a source-hadron, the simplest field equations would be

$$R_{\mu\nu} + \Lambda g_{\mu\nu} + \lambda s_{\mu\nu} = -8\pi c^{-4} (GT_{\mu\nu} + S_{\mu\nu} - \frac{1}{2}Gg_{\mu\nu} T^{\rho}_{\rho} - \frac{1}{2}g_{\mu\nu} S^{\rho}_{\rho}).$$

Disregarding the terms negligible in the neighbourhood of the source-hadron, we end up with the *field equations* $[\mu, \nu = 0, 1, 2, 3]$:

$$R_{\mu\nu} + \lambda s_{\mu\nu} \approx -8\pi c^{-4} (S_{\mu\nu} - \frac{1}{2}g_{\mu\nu} S^{\rho}_{\rho}) \quad (1)$$

with

$$\lambda \equiv \rho^{-2}\Lambda; \quad S_{\mu\nu} \equiv NT_{\mu\nu}; \quad N \equiv \rho^{-1}G, \quad (1')$$

where: $S_{\mu\nu}$ is the "strong-matter tensor", specifying the strong-matter distribution associated with the source-hadron (just as $T_{\mu\nu}$ is the "ordinary-matter tensor"); G is the ordinary gravitation universal constant; Λ is the ordinary cosmological constant ($|\Lambda| \approx 10^{-56} \text{cm}^{-2}$); and—as already mentioned—by "coincidence" $\rho \equiv s/S \approx r(h)/R(U) \approx 10^{-15}/10^{26} = 10^{-41}$. Quantity λ can be called the "strong (or hadronic) cosmological constant". By assuming spherical symmetry

(and a static source, i.e. $\partial g_{ik}/\partial x_a = 0$; $g_{ok} = 0$; $[i,k=1,2,3]$), the *total* metric acting on the test-hadron—in the surroundings of the source-hadron— writes in polar coordinates, as usual,

$$ds^2 = g_{00} dt^2 + g_{11} dr^2 - r^2(d\theta^2 + \sin^2\theta \cdot d\phi^2) .$$

5) *Very* near the source-hadron, we may assume that $s_{\mu\nu} \rightarrow g_{\mu\nu}$. This condition can be useful when looking for explicit solutions of our new field-equations.

6) On the contrary, sufficiently far from the source hadron (when our field equations can be linearized), we may write

$$g_{\mu\nu} \approx e_{\mu\nu} + s_{\mu\nu} , \tag{2}$$

that is to say, in suitable coordinates,

$$g_{\mu\nu} \approx \eta_{\mu\nu} + s_{\mu\nu} ; \quad [\eta_{\mu\nu} \equiv \text{diag}(+1,-1,-1,-1)] . \tag{2'}$$

7) At last, let us recall that—as we saw under point 4)—very far from the source-hadron our field equations (1) must imply that $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$ when $r \rightarrow \infty$.

3. WEAK FIELD APPROXIMATION (Linear Approximation)

In the "weak-field approximation", we can assume eqs.(2') to hold: $g_{\mu\nu} \approx \eta_{\mu\nu} + s_{\mu\nu}$. Then, if $\partial_\alpha \partial^\alpha \equiv \square \equiv \eta^{\rho\sigma} \partial_{,\rho\sigma}$ is the D'Alembertian operator, where the *comma* represents the ordinary (partial) derivative, it follows that

$$R_{\mu\nu} \approx \frac{1}{2} \partial_\alpha \partial^\alpha s_{\mu\nu}$$

$$\partial_\alpha \partial^\alpha s_{\mu\nu} \equiv s_{\mu\nu, \rho\sigma} \eta^{\rho\sigma} = [(\partial^2/\partial t^2) - \nabla^2] s_{\mu\nu}$$

with the conditions $(s_{\mu}^{\nu} - \frac{1}{2} s_{\rho}^{\rho} \delta_{\mu}^{\nu})_{, \nu} = 0$; so that our field equations (1) yield

$$\partial_\alpha \partial^\alpha s_{\mu\nu} + 2\lambda s_{\mu\nu} = -16\pi c^{-4} (S_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} S_{\rho}^{\rho}) . \tag{3}$$

For a source-hadron at rest, at the origin of the reference frame, in the *static* case (with zero pressure-terms),—i.e., when $\partial_\alpha \partial^\alpha s_{\mu\nu} = -\nabla^2 s_{\mu\nu}$ —, in polar coordinates it is $S_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} S_{\rho}^{\rho} = \text{diag } \frac{1}{2} c^2 \delta_S (c^2, 1, r^2, r^2 \sin^2\theta)$; and in Cartesian coordinates:

$$S_{\mu\nu} = \text{diag } (c^4 \delta_S, 0, 0, 0) ;$$

$$S_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} S_{\rho}^{\rho} = \text{diag } \frac{1}{2} c^2 \delta_S (c^2, 1, 1, 1) ,$$

with $\delta_S = N\gamma$ being the strong-matter density. So that eq.(3) reduces to

$$\nabla^2 s_{00} - 2\lambda s_{00} = 8\pi N\gamma . \tag{4}$$

For a point-like *nucleon* (for instance) at the origin, s_{00} must be a function only of $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$, and moreover $\gamma = m\delta(r)$; $\delta_S = Nm\delta(r) \equiv g\delta(r)$; notice that $\delta(r)$ is nothing but the Dirac delta; we get, therefore,

$$s_{00} \approx 2Nm (\exp\{-\sqrt{2\lambda} r\})/r = 2g \frac{\exp\{-\sqrt{2\lambda} r\}}{r} . \quad [g \equiv Nm]$$

By identifying $\sqrt{2\lambda} \equiv m_S c/\hbar$, we can calculate

$$m_S = \hbar \sqrt{2\lambda}/c \approx m_{\pi} . \tag{5}$$

In conclusion, by linearizing our field eqs.(1), at the static limit (when the *tensorial* strong potential goes into a *scalar* strong potential) one obtains the CORRECT YUKAWIAN BEHAVIOUR:

$$V \equiv \frac{1}{2}(g_{00} - 1) = -g \frac{e^{-m_{\pi} r c / \hbar}}{r}, \quad (6)$$

where we had to remember that (for weak fields and not too high speeds) $g_{00} \equiv 1 + 2V$.

Still in the weak field approximation, when eq.(2') holds: $g_{\mu\nu} \approx \eta_{\mu\nu} + s_{\mu\nu}$, but relaxing the condition $v \ll c$, it is easy to verify that (in Cartesian coordinates) all the components $s_{\mu\nu} \rightarrow 0$ for $r \gg 1$ fm. And in polar coordinates we simply get that $s_{11} = s_{00}/c^2$, so that both s_{00} and s_{11} are exponentially damped.

4. THE SCHWARZSCHILD-TYPE (spherically symmetric) PROBLEM FOR THE NEW FIELD-EQUATIONS

Let us eventually abandon the weak-field approximation; but still consider the source-hadron to be a spherically symmetric distribution of strong-matter. In polar coordinates, then, let us explicitly put $ds^2 = \exp\{\nu(r)\}dt^2 - \exp\{-\lambda(r)\}dr^2 - d\Omega$, that is to say

$$g_{\mu\nu} = (e^{\nu(r)}, -e^{-\lambda(r)}, -r^2, -r^2 \sin^2\theta), \quad (7)$$

where $\nu=\nu(r)$ and $\lambda=\lambda(r)$ are functions still to be determined.

We cannot make recourse any longer to approximations (2), and finding out exact solutions of our field eqs.(1) is a difficult task.

We shall therefore use the trick of following the procedures usually adopted (in connection with the ordinary Einstein equations) when in presence of an electric charge and field, besides a mass. We purport to follow an iterative procedure; and, as the starting point, we take from what precedes the model of a static strong-field source, whose field at the zero-order approximation is Yukawian:

$$V = -g \frac{e^{-m_0 r}}{r} \quad [m_0 \equiv m_{\pi} c / \hbar]$$

Let us define an energy-density $u=u(r)$ for the Yukawa potential (by following, however, our philosophy):

$$W = \frac{1}{2} \iint \delta(x) \frac{\delta(x')}{|x-x'|} e^{-m_0 |x-x'|} d^3x d^3x' = \frac{1}{2} \int \delta(x) V(x) d^3x, \quad (8)$$

where $V(x) = \int \delta(x') \cdot \exp[-m_0 |x-x'|] \cdot |x-x'|^{-1} d^3x'$ is the Yukawa potential. Therefore:

$$\nabla^2 V(x) = m_0^2 V(x) - 4\pi\delta(x),$$

and eq.(8) becomes

$$W = \frac{1}{2} \int [-\nabla^2 V(x) + m_0^2 V(x)] \cdot \frac{V(x)}{4\pi} d^3x = \frac{1}{8\pi} \int [|\nabla V|^2 + m_0^2 |V|^2] d^3x,$$

so that

$$u(r) = \frac{1}{8\pi} [|\nabla V|^2 + m_0^2 |V|^2] = \frac{1}{8\pi} g^2 \frac{e^{-2m_0 r}}{r} (r^{-2} + 2m_0 r^{-1} + 2m_0^2). \quad (9)$$

Formally, we are dealing with a strong-energy tensor $t_{\mu\nu} \equiv \text{diag}(u(r), 0, 0, 0)$, and with the "Einstein" equations:

$$R_{\mu}^{\nu} - \frac{1}{2} \delta_{\mu}^{\nu} R^{\rho}_{\rho} = -\frac{8\pi}{c^4} t_{\mu}^{\nu}, \quad (\text{in the vacuum}),$$

which become

$$\begin{cases} -8\pi c^{-4} u(r) = -\frac{e^{-\lambda}}{r} \lambda' + \frac{e^{-\lambda}}{r^2} - \frac{1}{r^2}; \\ 0 = \frac{e^{-\lambda}}{r} \nu' + \frac{e^{-\lambda}}{r^2} - \frac{1}{r^2}. \end{cases} \quad \left[' \equiv \frac{d}{dr} \right] \quad (10)$$

The *first* one of eqs.(10) is exactly solvable: if we set $\exp\{-\lambda\} \equiv y$, it writes

$$y'r + y = 1 - \alpha^2 e^{-2m_0 r} (r^{-2} + 2m_0 r^{-1} + 2m_0^2), \quad [\alpha^2 \equiv g^2/c^4]$$

whose solution is

$$e^{-\lambda(r)} \equiv y = 1 - \frac{2\ell}{r} + \frac{m_0}{r} \alpha^2 e^{-2m_0 r} + \frac{\alpha^2}{r^2} e^{-2m_0 r}, \quad (11)$$

ℓ being an integration constant related to the value of the strong-charge g of the considered source-hadron.

The *second* one of eqs.(10) can be written:

$$v' = \frac{1}{r} \left(\frac{1}{y} - 1 \right). \quad (12)$$

By recalling eq.(7), in the present "strong Schwarzschild problem" the *strong Schwarzschild radii* will be given by the conditions

$$\exp\{-\lambda(r)\} = 0; \quad \exp\{v(r)\} = 0. \quad (13)$$

The first one of eqs.(13), *i.e.* $\exp\{\lambda(r)\} = \infty$, when associated with eq.(11), does yield *two* "strong Schwarzschild radii", r_1 and r_2 . In the case when the source-hadron is a nucleon, we get

$$r_1 \approx 1 \text{ fm}; \quad r_2 \approx 10^{-2} \text{ fm}. \quad (14)$$

In general:

$$r_1 \approx 2\ell; \quad r_2 \ll r_1.$$

From eq.(12), not exactly solvable, we can for instance *verify* that for $r = r_1$

$$\exp\{v(r_1)\} \approx \exp\{-\lambda(r_1)\}.$$

5. APPLICATIONS

For a possible application to the $E > 100$ TeV cosmic-ray phenomenology, we refer to Ammiraju, Recami & Rodrigues (1983). That phenomenology seems to require the existence of a few "fireballs", whose decay-modes — called *Centeuros*, *Geminions*, *Chirons*, ... — have been tentatively associated by the abovementioned authors with the "evaporation" of some exotic "strong black-holes".

Here let us only recall the following. Just as black bodies (defined as objects absorbing all electromagnetic radiation impinging on them) have been discovered to emit a characteristic, thermal spectrum of photons, so the gravitational *black-holes* (BH) — which are objects absorbing all kinds of particles impinging on them — are expected to emit characteristic, Planck-type spectra of every kind of particles (Bekenstein, 1973,1974; Hawking, 1976). Analogously, our *strong black-holes* (SBH) — which will absorb and confine only particles sensitive to the strong field — are expected to emit characteristic spectra of hadrons only (Recami 1979,1982b).

6. OUR SPACE-TIME MANIFOLD AND STRONG BLACK-HOLE (SBH) THERMODYNAMICS

The results met in Sect.3 and Sect.4 allow us to associate hadrons with the "SBH" solutions of our (new) field equations (1); solutions related to the presence of the strong-gravity field $s_{\mu\nu}$ (besides $e_{\mu\nu}$). Such SBHs will *confine* — let us repeat it — only objects carrying a strong charge: *i.e.*, only hadronic particles.

Let us recall, for instance, that on the basis of our eqs.(1) we derived: (a) that $s_{\mu\nu}$ vanish for r larger than a few fm; (b) the correct Yukawa behaviour (under the suitable hypotheses) of the *scalar* strong potential; (c) the fact that the external "strong Schwarzschild radius" for ordinary particles is of the order of 1 fm. Moreover, we can geometrize also the strong field since the *inertia* m_I of the test-hadron in the neighbourhood of the source-hadron, when it is subjected to the strong field, coincides (cf. e.g. Caldirola et al., 1978a) with its *strong*

-mass g' , and *not* with its gravitational mass m' , so that $m' \equiv g'$; and, in the special relativistic limit, it is for example $g' = g'_0 / \sqrt{1-v^2}$, quantity g'_0 being the *rest* strong-mass. At last, let us remind the reader that in the present bi-scale approach two kinds of particles are taken into consideration: the ones —carrying ordinary gravitational mass, i.e. with intrinsic scale-factor $\kappa=1$ — that are subjected (only) to the gravitational metric $e_{\mu\nu}$; and the ones —carrying a strong-charge or "strong mass", i.e. with intrinsic scale-factor $\kappa=\rho \approx 10^{-41}$ — that are subjected also to the strong metric $s_{\mu\nu}$ (the latter being of course the hadrons).

We already mentioned elsewhere (Caldirola et al., 1978a; Recami, 1979, 1982b) that a fascinating consequence would follow by assuming the Bekenstein-Hawking ordinary BH thermodynamics to hold good also for our SBHs. Let us approach the question more closely.

Since in the surroundings of a hadron the cosmological constant Λ plays no role, and g_{00}, g_{11} tend to 1, -1, respectively, for $r \gg 1$ fm, we can assume the space-time manifold V —associated with our field equations (1)— to be: (i) *asymptotically flat*, such a property implying also the *positivity* of energy (see e.g. Choquet-Bruhat, 1984); (ii) and moreover (see e.g. Hawking & Ellis, 1973) *time-orientable* and *Lorentzian* (= pseudo-Riemannian), with a metric $g_{\mu\nu}$ of Lorentz signature -2 and satisfying the two ordinary postulates of: (iii) *local causality*, and (iv) *local conservation of energy-momentum*.

As to our field eqs.(1), we first notice that the present approach implies gravity and strong-gravity to be attractive for positive matter and strong-matter densities, respectively. Secondly, we explicitly assume (see e.g. Misner, Thorn & Wheeler, 1973) that: (v) quantity $R_{\mu\nu} K^\mu K^\nu$ is *non-positive* for any non-spacelike Killing vector K^μ . Notice, incidentally, that all the above "assumptions" (i)÷(v) do agree with our physical model (even if the question, whether the form actually chosen for our field equations is *fully* compatible with them, is still *partially* open).

All the ordinary theorems on singularities (Hawking & Ellis, 1973), then, hold good also for our manifold V . Let us deal, in particular, with the Penrose theorem (1968) and the Second Law of BH Thermodynamics (Hawking, 1971, 1972, 1973; Bekenstein, 1973, 1974), in order to be able to draw briefly the most relevant consequences.

Let us consider, e.g., a SBH solution of our field equations in the static case, with its strong horizon (Ammiraju et al., 1983, and references therein). Always with reference to objects endowed with strong-mass, such solutions will present at least one asymptotically flat region, or "external universe", possessing a timelike (strong) future infinity I_S^+ and past infinity I_S^- , a spacelike (strong) infinity I_S^0 , and a null (strong) future infinity J_S^+ and past infinity J_S^- . The strong-horizon divides of course the external universe (which can send hadronic particles —"strong signals"— out to J_S^+) from the SBH interior (which cannot). Let us define the union of the future SBH horizons as the *boundary* $B \equiv J_S^-(J_S^+)$ of the causal past —via strong-signals— of the future null strong-infinity (i.e., of the domain $J_S^-(J_S^+)$ that can send future-directed causal strong-curves to future null strong-infinity).

From assumptions (i)÷(iv) above, it is then straightforward to generalize the Penrose (1968) theorem on the structure of the future strong-horizons; i.e., to conclude that B is generated by null strong-geodesics which do not have future end-points.⁽¹⁾ Let us clarify that the strong-geodesics are the ones followed by the hadronic test-particles, i.e. by particles affected

(1) Let us recall more specifically that Penrose's theorem states that: (a) the "generators" of B are defined as null strong-geodesics that lie in B , at least for some finite interval of the affine parameter; (b) a generator, followed into the past, may leave B entering $J_S^-(J_S^+)$; (c) once a generator, followed into the future, enters the boundary B of $J_S^-(J_S^+)$ at point S_C (a "caustic" of B), it'll never be able thereafter to leave B , nor to intersect another generator (the generators can intersect each other only at the caustics); (d) only *one* generator (apart from a normalization of the geodesic affine-parameter) can pass through a non-caustic event.

(2) Remember that the global space-time dilations, considered by us (cf. e.g. Caldirola et al., 1978a), do *not* affect the speeds, in particular do not affect the value of the light-speed c .

by the strong metric $s_{\mu\nu}$ besides the gravitational metric $e_{\mu\nu}$ (negligible in the vicinity of the source hadron). The *null* strong-geodesics, therefore, are geometrically well defined, even if test-hadrons would not exist travelling exactly at the speed of light.⁽²⁾ But in the naïve version of our approach there should exist, for instance, "strong gravitons".

At this point, let us consider our asymptotically-flat space-time, endowed with gravitational and strong charges, i.e. not only "gravitationally" but also "strongly" (locally) curved; and the union B of all its future strong-horizons. Let us divide the null strong-geodesics, which generate B , into a large number of infinitesimal bundles, each one identified by a number α . Consider along a generic bundle α of strong-generators (all lying in a surface of constant phase), a particular event P and various observers (endowed with different velocities) all measuring the cross-section $a(\alpha, P)$ at P of α . Then, from assumptions (i)-(v) above and following e.g. Misner et al. (1973) —taking into account in particular that the geometric optics in curved space-times exploited therein holds good also in our approach—, one can show that the area $a(\alpha, P)$ does *not* depend on the observer velocity; and that, in general, $a(\alpha, P)$ depends only on the localization of P along the bundle α , according to the "focusing theorem": If condition (v) holds for any one of the observers measuring $a(\alpha, P)$, then for any point P along α :

$$d^2 \sqrt{a(\alpha, P)} / dk_\alpha^2 \leq 0, \quad \forall P \in \alpha, \tag{15}$$

where $k_\alpha \equiv k(\alpha, P)$ is the affine parameter along the bundle α . Let us discuss, in connection with eq. (15), the sign of the *first derivative* $d\sqrt{a_\alpha} / dk_\alpha$, where $a_\alpha \equiv a(\alpha, P)$.

If $d\sqrt{a_\alpha} / dk_\alpha$ were negative at a certain event P along α , then it would thereafter go on being negative, and fatally $\sqrt{a_\alpha}$ would reach the value zero, after a finite interval of the affine parameter. At the point where $\sqrt{a_\alpha}$ vanishes, all the (adjacent) null strong-geodesics belonging to α cross each other, thus violating the Penrose theorem; unless those null geodesics hit a real strong-singularity, and disappear, before intersecting each other. But the last hypothesis implies the existence of a naked singularity. We wish, on the contrary, to associate hadrons with SBHs, excluding the existence —just as it is generally done in GR— of naked point-like singularities [a hadron associated with a naked singularity would appear to be pointlike, whilst experimentally —as well as in our theoretical approach— they are *extended-type* objects]. We may therefore conclude that

$$d\sqrt{a_\alpha} / dk_\alpha \geq 0. \tag{16}$$

Since new bundles can be born, but none destroyed, when moving towards the future the total cross-section of B cannot decrease.

Therefore, we can believe the Second Law of the ordinary BH Thermodynamics, by Bekenstein (1973, 1974) and Hawking (1971, 1972, 1973), to hold good also for our SBHs. This conclusion is confirmed by the fact that the purely thermodynamical considerations elegantly developed by Hawking in 1976 (by which he generalized the results above) seem to be easily translatable into the SBH language.

In particular, if two SBHs, forming an isolated system, *merge together* so to originate a single new SBH, the final strong Schwarzschild area A_S must be larger than the sum of the initial strong Schwarzschild areas:

$$A_S \geq A'_S + A''_S. \tag{17}$$

7. CONSEQUENCES OF SBH THERMODYNAMICS FOR THE "BIG CRUNCH" OF OUR UNIVERSE

Let us first consider an ordinary star S , collapsing towards its gravitational Schwarzschild radius $r_{BH} = 2GM/c^2$. We are interested in the case in which S —or rather its core— reaches the nuclear-matter density *before* collapsing beyond its gravitational horizon; i.e., in which its BH-density $\rho_{BH} = 3c^6 / (32\pi G^3 M^2)$ is *higher* than the density $\rho_n \approx 10^{15}$ g/cm³ of nuclear matter. Here, and in what follows, M should represent the S core mass. However, under the simplifying hypothesis of a homogeneous collapse (when M is just the mass of S), we get: $M \lesssim 5M_\odot$.

its contraction, has to <<give rise to a new cosmos creation>>, —to use Posidonius' words (about 100 B.C.). But one should not be surprised that the laws of physics seem to allow us to make predictions beyond the "Big Crunch" : in fact, in our approach, no final (nor initial) singularity exists in the cosmos history (cf. also Rosen, 1979, 1980a,b), so that the physical laws do not break down in the temporal vicinity of such a *non-singular* Big Crunch.

ACKNOWLEDGEMENTS

One of the authors (ER) is grateful to A.Faessler and D.Wilkinson for their very kind interest. He also acknowledges many stimulating discussions over the years with P.Caldirola, G.D.Maccarrone, R.Mignani, M.Pavšič, and particularly with E.Giannetto and W.A.Rodrigues, as well as with M.Anile, A.Insolia, L.R.Baldini, M.Tenório, and some of the participants in this Erice meeting.

REFERENCES

- Amiraju, P., E. Recami and W.A. Rodrigues (1983). Nuovo Cimento, A78, 172-204.
- Bekenstein, J.D. (1973). Phys.Rev., D7, 2333.
- Bekenstein, J.D. (1974). Phys.Rev., D9, 3292.
- Caldirola, P., M. Pavšič and E. Recami (1978a). Nuovo Cimento, B48, 205-271.
- Caldirola, P., M. Pavšič and E. Recami (1978b). Phys.Letters, A66, 9.
- Caldirola, P. and E. Recami (1979). Lett.Nuovo Cimento, 24, 565.
- Choquet-Bruhat, Y. (1984). In B. de Witt and R.Stora (Eds.), "Relativity, Groups and Topology - II: Proceedings of the 40th Les Houches Summer School in Theoretical Physics". North-Holland, Amsterdam. pp.739-785 and references therein.
- de Witt, B. (1975). Phys.Reports, C19, 295.
- Italiano, A., M. Lattuada, G.D. Maccarrone, E. Recami, F. Riggi and D. Vinciguerra (1984). Hadronic J., 7, 1321.
- Italiano, A. and E. Recami (1984). Lett.Nuovo Cimento, 40, 140.
- Hawking, S.W. (1971). Phys.Rev.Lett., 26, 344.
- Hawking, S.W. (1972). Comm.Math.Phys., 25, 152.
- Hawking, S.W. (1973). In B. de Witt and C. de Witt (Eds.), "Black-Holes". Gordon & Breach, New York.
- Hawking, S.W. (1974). Nature, 248, 30.
- Hawking, S.W. (1975). Comm.Math.Phys., 43, 199.
- Hawking, S.W. (1976). Phys.Rev., D13, 191.
- Hawking, S.W. and G.F.R. Ellis (1973). "The Large-Scale Structure of Space-Time". Cambridge University Press, Cambridge.
- Misner, C.W., K.S. Thorne and J.A. Wheeler (1973). "Gravitation". Freeman, San Francisco.
- Penrose, R. (1968). In C. de Witt and J.A.Wheeler (Eds.), "Battelle Rencontres: 1967 Lectures in Mathematics and Physics". Benjamin, New York.
- Recami, E. (1979). In F. de Finis and M.Pantaleo (Eds.), "Relativity, Quanta and Cosmology in the development of the scientific thought of A.Einstein", Vol.2. Johnson Rep. Co., New York. pp.537-597. Russian translation: in F.Fedorova (Ed.), "Astrofizika, Kvanti i Teoriya Otnositelnosti". MIR, Moscow. pp.53-128.
- Recami, E. (1982a). In D.Wilkinson (Ed.), "Quarks and the Nucleus: vol.8 of Progress in Particle and Nuclear Physics". Pergamon Press, Oxford. pp.401-411.
- Recami, E. (1982b). In A. van der Merwe (Ed.), "Old and New Questions in Physics, Cosmology,...: Essays in Honor of W.Yourgrau". Plenum, New York. Chapt.24, pp.377-396.
- Recami, E. (1983). Found.Phys., 13, 341.
- Recami, E. (1984). In Yu.I. Zaporovannyi and Yu.P.Rybakov (Eds.), "Problemy Statisticheskoi i Kvantovoi Fiziki". Peoples' Friendship Univ.Press, Moscow. pp.3-22.
- Recami, E. and P. Castorina (1976). Lett.Nuovo Cimento, 15, 357.
- Rosen, N. (1979). Lett.Nuovo Cimento, 25, 266.
- Rosen, N. (1980a). In P.G. Bergmann and V. De Sabbata (Eds.), "Proceedings of the International School of Cosmology and Gravitation". Plenum Press, New York.

Rosen, N. (1980b). Astrophys.J., 239, 1024, 1032.

Salam, A. (1978). In "Proceedings of the 19th International Conference on High-Energy Physics (Tokyo, 1978)". p.937.

Salam, A. and J. Strathdee: Phys.Rev., D18, 4598.

also:

Phys Lett. B 177/1986/304