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On superluminal motions in photon and particle tunnellings

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Abstract

It is shown that the Hartman–Fletcher effect is valid for all known expressions of the mean tunnelling time, in the various nonrelativistic approaches, for the case of finite width barriers without absorption. Then, we show that the same effect is not valid for the mean-square tunnelling time fluctuations. On the basis of the Hartman–Fletcher effect and the known analogy between photon and nonrelativistic-particle tunnelling, one can explain the superluminal group-velocities observed in various photon tunnelling experiments (without violation of the so-called "Einstein causality"). © 1998 Elsevier Science B.V.

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1. The Hartman–Fletcher effect (HFE) was first studied in Refs. [1,2] within the stationary-phase method for quasi-monochromatic nonrelativistic particles tunnelling through potential barriers. It is the phenomenon that the tunnelling phase-time

$$\tau_{\rm tun}^{\rm Ph} = \hbar(\partial(\arg A_{\rm T} + ka))/(\partial E) \qquad (1)$$

is independent of the barrier width a for sufficiently large a; $\tau_{\rm tun}^{\rm Ph}$ is the mean tunnelling time $\langle\tau_{\rm tun}\rangle$ evaluated by the stationary-phase method, when it is possible to neglect the interference between incident and reflected waves before the barrier; $A_{\rm T}$ and $E = \hbar^2k^2/2\mu$ are transmission amplitude and particle kinetic energy, respectively. In particular, for a rectangular potential barrier $\tau_{\rm tun}^{\rm Ph} \to 2/v\kappa$ when $\kappa a \gg 1$, $v = \hbar\kappa/\mu$ being the particle velocity. It is known that, within different approaches and under various conditions, many different theoretical expressions for the mean tunnelling time were deduced in the literature (see, for instance, Ref. [3]). The first goal of this paper is to analyze the role and validity of the HFE for all its known theoretical expressions. Our second goal is to explain the recent photon tunnelling experiments, in which superluminal phenomena were observed, on the basis of the mentioned HFE and the analogy between photon and nonrelativistic-particle tunnelling.

2. If one chooses, out of all the known definitions of tunnelling times, the mean dwell time $\langle\tau_{\rm tun}^{\rm dw}\rangle$ [4], the first mean Larmor time $\langle\tau_{\rm tun}^{\rm L}\rangle$ [4,5], and the real part of the complex tunnelling time obtained by averaging over the Feynman paths ${\rm Re}\,\tau_{\rm tun}^{\rm F}$ [6], which equal $\hbar k/\kappa V_0$ for quasi-monochromatic particles and opaque rectangular barriers, one immediately and easily sees

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that also in such cases there is no dependence on the barrier width, and consequently the HFE is present.

The validity of the HFE for the mean tunnelling time, within the general nonrelativistic approach developed in Refs. [3,7-11], can be directly inferred from the expression

$$\langle \tau_{\rm tun} \rangle = \langle t_+(a) \rangle - \langle t_+(0) \rangle = \langle \tau_{\rm tun}^{\rm Ph} \rangle_E - \langle t_+(0) \rangle \quad (2)$$

that was found in Refs. [8-11], and was additionally confirmed in Refs. [7-11] by numerous calculations for various cases of Gaussian electron wavepackets. In Eq. (2), $\langle\ \rangle_E$ denotes the average over the initial wavepacket energy spread, and $\langle t_+(x) \rangle = \int_{-\infty}^{\infty} t J_\pm(x,t)\,{\rm d}t / \int_{-\infty}^{\infty} J(x,t)\,{\rm d}t$, with $J_\pm(x,t) = \theta(\pm J) J(x,t)$, where $J(x,t)$ is the probability flux density for a wavepacket moving through a barrier located in the interval $(0,a)$ along the x-axis. Since our conclusion about the validity of the HFE is true for rectangular barriers, one can easily see that it holds also for more general barriers, localized in the interval $(0,a)$.

Let us stress that up to now there is no a unique general formulation of the causality condition, necessary and sufficient for all possible cases of collisions. Requiring non-negative values for the mean durations represents the most simple causality condition (which is sufficient, but not necessary). Although in the approach presented in Refs. [3,7-11] there is no direct causal connection between the peaks of the incoming and of the transmitted or reflected wavepackets (because of the nontrival motions inside the barrier), the previous causality condition seems to be valid for the whole tunnelling [10,11]. An analysis of various formulations of the causality condition is presented in Refs. [10,11] for both nonrelativistic and relativistic tunnelling.

In any case, concluding this section, we may assert that no exception is known of the validity of the HFE, for whatever expression of the mean tunnelling time.

3. Let us recall that the second Larmor time [4]

$$\tau_{z,{\rm tun}}^{\rm L} = \hbar [\langle (\partial \mid A_{\rm T} \mid / \partial E)^2 \rangle / \langle \mid A_{\rm T} \mid^2 \rangle]^{1/2} \quad (3)$$

as well as the Büttiker-Landauer time $\tau_{\rm tun}^{\rm B-L}$ [12] and the imaginary part of the complex tunnelling time ${\rm Im}\,\tau_{\rm tun}^{\rm F}$ (obtained within the Feynman approach [6]), both of which are equivalent to expression (3), in the limiting case of opaque rectangular barriers yield all the same value $a\mu/\hbar\kappa$, i.e. become all proportional to the barrier width a. It was shown in Refs. [10,11], however, that those times are not mean times, but mean-square fluctuations of the tunnelling-time distribution. In fact, they are equal to $(D_{\rm dyn}\tau_{\rm tun})^{1/2}$, where $D_{\rm dyn}\tau_{\rm tun}$ is the dynamical tunnelling-time variance caused by the barrier only and defined by the equation $D_{\rm dyn}\tau_{\rm tun} = D\tau_{\rm tun} - Dt_+(0)$, with $D\tau_{\rm tun} = \langle \tau_{\rm tun}^2 \rangle - \langle \tau_{\rm tun} \rangle^2$; and $\langle \tau_{\rm tun}^2 \rangle = \langle [t_+(a) - \langle t_+(0) \rangle]^2 \rangle + Dt_+(0)$. Hence they are not connected with the peak (or group) velocities of tunnelling particles, but rather with the relevant tunnelling velocity distribution along the barrier region.

4. All these results have been obtained for transparent media (without absorption and/or dissipation). As it was theoretically demonstrated in Ref. [13] within nonrelativistic quantum mechanics, the HFE vanishes for barriers with strong enough absorption. This was confirmed experimentally in the case of electromagnetic (microwave) tunnelling in Ref. [14]. Namely, it follows from Ref. [13] that, if one describes the absorption by adding the imaginary term $-iV_1$ ($V_1 > 0$) to V_0, then the HFE does not disappear only for very small absorptions, when $V_1 \ll V_0$ and $V_1 \mu^{1/2} \nu \kappa a / [2(V_0 - E)]^{3/2} \ll 2$.

5. Let us now propose, here, another description of tunnelling which appears convenient for applications to media without absorption and dissipation, and for Josephson junctions. In our new representation the transmission and reflection amplitudes are rewritten (for the same external boundary conditions [15]) as

$$A_{\rm T} = {\rm i}\,{\rm Im}\,(\exp({\rm i}\varphi_1))\exp({\rm i}\varphi_2 - {\rm i}ka)\ ,$$
$$A_{\rm R} = {\rm Re}\,(\exp({\rm i}\varphi_1))\exp({\rm i}\varphi_2 - {\rm i}ka)\ , \quad (4)$$

where for rectangular potential barriers

$$\varphi_1 = \arctan\{2\sigma/[(1+\sigma^2)\sinh(\kappa a)]\}\ ,$$
$$\varphi_2 = \arctan\{\sigma\sinh(\kappa a)/[\sinh^2(\kappa a/2) - \sigma^2\cosh^2(\kappa a/2)]\}\ ,$$

with $\sigma = \kappa/k$, and $\kappa^2 = \kappa_0^2 - k^2$, $\kappa_0 = [2\mu V_0]^{1/2}/\hbar$ being the barrier height. We have introduced two new phases, φ_1 and φ_2, in terms of which the expressions for $\tau^{\rm Ph}_{\rm tun}$ and for $\tau^{\rm L}_{z,\rm tun} = \tau^{\rm B-L}_{\rm tun}$ acquire the following form,

$$\tau^{\rm Ph}_{\rm tun} = \hbar\frac{\partial(\arg A_{\rm T})}{\partial E} = \hbar\frac{\partial(\varphi_2 - ka)}{\partial E},$$
$$\tau^{\rm L}_{z,\rm tun} = \tau^{\rm B-L}_{\rm tun} = \hbar\frac{\partial\varphi_1}{\partial E}\cot(\varphi_1). \qquad (5)$$

So, we see that, in the rectangular opaque potential limit, without absorption, the phase $\varphi_2 - ka$ leads to the HFE for tunnelling times, while the phase φ_1 leads to a dependence of tunnelling times on a. For the times $\langle\tau^{\rm dw}_{\rm tun}\rangle = \langle\tau^{\rm L}_{y,\rm tun}\rangle$ we obtain a complicated formula, which can be expressed in terms of φ_2 and of $\partial\varphi_2/\partial E$ only in the limit $\kappa a \gg 1$. In the presence of absorption, both phases become complex and expressions (5) are much bulkier and, in general, depend on a (with a violation of the HFE, in accordance with Refs. [13,14]).

6. These conclusions can be easily extended for photon tunnelling, as already mentioned, if one takes account of the analogy between the photon and nonrelativistic-particle tunnelling. Such an analogy was first examined in Refs. [16,17] for the case of the time-independent Schrödinger and Helmholtz equations, and in Refs. [10,11] for the case of the time-dependent Schrödinger and Maxwell equations (on the basis, also, of a suitable generalization of our operator for time from quantum mechanics to first-quantization quantum electrodynamics).

While for nonrelativistic particle tunnelling is associated with potential barriers, in the photon experiments tunnelling takes place through different kinds of regions generating evanescent (decreasing) and anti-evanescent (increasing) waves: for instance, waveguides with cutoff (undersized) regions [18,19], multilayer dielectric mirrors as realizations of one-dimensional (1D) photonic band gaps [20,21], and frustrated total internal reflection regions [22]. In all these cases the HFE can cause to such large group (or peak, or effective) velocities of the tunnelling photons that they can acquire superluminal [18-22] and even infinite [23], or negative [24,29,31], speed.

Here we shall briefly analyze the superluminal phenomena observed in the 1D microwave tunnelling experiment and 2D optical tunnelling experiment described in Refs. [14] and [22] and depicted in Fig. 1 and 2, respectively.

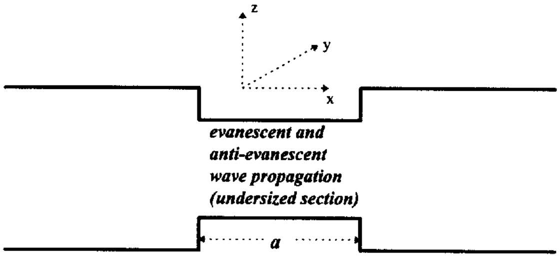

Fig. 1. Waveguide scheme of the microwave tunnelling experiment [14].

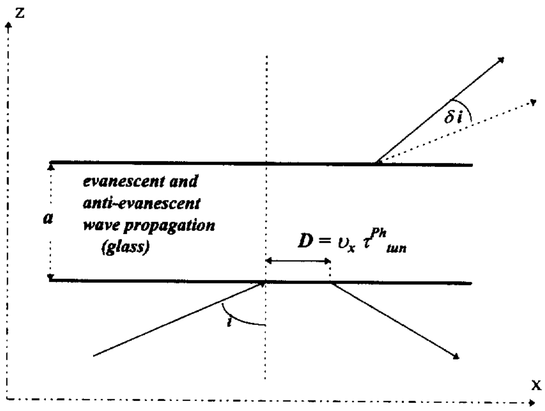

Fig. 2. Frustrated total-reflection scheme of the optical tunnelling experiment [18].

In both cases, inside the "photonic barrier" (similarly to what is done for the quantum-mechanical wave functions) the electromagnetic waves are to be described by the superposition of evanescent and anti-evanescent waves: $\alpha\exp(-\kappa_{\rm e}x) + \beta\exp(\kappa_{\rm e}x)$, with non-zero fluxes for non-zero complex coefficients α and β, which depend on the boundary conditions. The evanescent-wave wave number $\kappa_{\rm e}$ is defined by the difference between the incident wave frequency and the cutoff frequency in the microwave experiment [14,18] (Fig. 1): $\kappa_{\rm e} = 2\pi(\lambda^{-2} - \lambda_{\rm c}^{-2})^{1/2}$, $\lambda_{\rm c}$ being the cutoff wave length, and by the difference between the incidence angle $\hat{\imath}$ and the critical angle $\hat{\imath}_{\rm c} = \sin^{-1}(1/n)$ of total reflection ($\hat{\imath} > \hat{\imath}_{\rm c}$; $n > 1$) in the optical experiment [22] (Fig .2). For quasi-monochromatic wavepackets and appropriate measurement conditions (which were realized in the experiments described in Refs. [18,20-22]), one can use the stationary-phase method, and the phase tunnelling time is defined by the expression [10,11]

$$\tau_{\rm tun}^{\rm Ph} = \frac{2}{c\kappa} \quad (6)$$

for $\kappa a \gg 1$. The quantity $v_{\rm tun}^{\rm eff} = a/\tau_{\rm tun}^{\rm Ph}$, which has the meaning of the effective photon tunnelling velocity, turned out in both experiments [18,22] (and also in Refs. [20,21]) to be superluminal, since $\kappa_e > 2$. In the microwave experiment [18] the time $\tau_{\rm tun}^{\rm Ph}$ was found by pulsed measurements in the time domain, and expression (6) was directly tested [10,11]. In the optical experiment [22], the quantity $\tau_{\rm tun}^{\rm Ph}$ was calculated with the help of the simple formula

$$\tau_{\rm tun}^{\rm Ph} = \frac{D}{v_x}, \quad v_x = \frac{c}{n\sin(\hat{\imath})} \quad (7)$$

by measuring D and $\hat{\imath}$.

It was also shown in Ref. [22] that the angular deviation $\delta\hat{\imath}$ of the emerging beam is related to the "loss time" $\tau_{\rm tun}^{\rm loss}$, which is precisely equal to expression (2), and so has the meaning of a mean-square tunnelling-time fluctuation (i.e., of the square-root of the dynamical tunnelling-time variance), by the formula

$$\delta\hat{\imath} = \Omega\, \tau_{\rm tun}^{\rm loss}, \quad (8)$$

where the frequency Ω is determined by $\hat{\imath}$, n and the beam Rayleigh length. Let us recall again that the quantity $\tau_{\rm tun}^{\rm loss}$ is connected not with the effective photon tunnelling velocity, but with the tunnelling-velocity distribution.

7. Thus, we have seen that the HFE is valid for all the known expressions of the mean tunnelling times (without absorption and dissipation), while it is not valid for the mean-square tunnelling-time fluctuations. Then, the HFE is a good basis for explaining the superluminal group velocities of tunnelling photons. These are our main conclusions, substantiated in Sections 2–6.

Finally, let us mention that many discussions (see e.g. Refs. [25-28]) were generated by the superluminal phenomena, observed e.g. in the experiments [18-22], and revived by the results observed in similar phenomena, namely the electromagnetic pulse propagation in dispersive media [29-31]. In fact, on the other hand, it has been known since long that the wave-front velocity for electromagnetic pulse propagation cannot exceed the vacuum light speed c [32,33]. This is a conclusion confirmed by various methods and in various processes, including tunnelling, [25-28] and that seems to guarantee the so-called (naive) "Einstein causality". We shall come back to these points [23] elsewhere.

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