Portfolio Choice based on Third-degree Stochastic Dominance

Thierry Post and Miloš Kopa

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Motivation
Summary

- Optimization method for building portfolios that dominate a benchmark by the third order stochastic dominance (TSD).
- Relies on the properties of semivariance, ‘super-convex’ dominance and QCP.
- Applied to stock market data using an industry momentum strategy.
- Important performance improvements compared with MV dominance and SSD.
- Average out-of-sample outperformance ≈ 7% p.a. with less downside risk, quarterly rebalancing and no short selling.
1. Stochastic Dominance

2. SD Portfolio analysis

3. TSD optimization

4. Industry momentum strategy

5. Conclusions
Stochastic Dominance

- Model-free decision rules for DMuR
- Based on general regularity conditions about risk preferences
- Related to majorization in mathematical order theory
- Under Gaussianity, similar to M-V analysis
- Applies more generally for any probability distribution
### Genealogy

<table>
<thead>
<tr>
<th>Crit.</th>
<th>Reference</th>
<th>Distr</th>
<th>Utility</th>
<th>Risk</th>
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<tbody>
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<td>MV</td>
<td>Mark52 JF</td>
<td>Normal</td>
<td>$x + bx^2$</td>
<td>$\mathbb{E}[(x - \mu)^2]$</td>
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<td>FSD</td>
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<td>$\mathbb{E}[(z - x)I(x \leq z)]$</td>
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Numerical example

<table>
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<tr>
<th></th>
<th>$\tau = 1$</th>
<th>$\tau = 2$</th>
<th>$\tau = 3$</th>
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<th>$\sigma$</th>
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<td>0.90</td>
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<td>1.30</td>
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<tr>
<td>$\nu_2$</td>
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### Numerical example

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<td>0.97</td>
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<td>1.40</td>
<td>1.12</td>
<td>0.20</td>
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FSD portfolio analysis

- Kuosmanen (2001, 2004) formulates FSD optimization as a large MILP problem
- Subsequent OR studies by Ruszczynski c.s. analyze various algorithms and approximations
- The combinatorial optimization problem is computationally expensive
- Solved for numerical examples and choice experiments but not for realistic data dimensions
- Presumably, FSD is also too weak to detect many risk arbitrage opportunities
- Kopa and Post (2009) distinguish between admissibility and optimality

Subsequent OR studies analyze various generalizations, problem reductions and algorithms; Fabian, Mitra, Roman and Zverovich (2011), Roman, Mitra, Zverovich (2013), ....

The LP problem is nowadays easy to solve for realistic data dimensions.

A recent application is Hodder, Jackwerth and Kolokolova (2015).

SSD optimization seems superior to FSD optimization and MV optimization.
TSD portfolio analysis

- 1970s literature on algorithms for pairwise analysis
- Bawa c.s. (1985) develop an LP problem for comparison with a discrete choice set
- Gotoh and Konno (2000) develop a mean-risk model
- Post and Versijp (2007) develop a GMM test to detect incremental improvement possibilities
- Armbruster and Delage (2015) develop an XXL LP problem to approximate TSD optimization
- A tractable approach for realistic data dimensions and applications do not exist
Problem definition

- Let $S^2_\lambda(z) := \mathbb{E}[(z - x^T \lambda)^2 \mathbb{1}(x^T \lambda \leq z)]$ for portfolio $\lambda \in \Lambda$ and threshold $z \in [a, b]$

- **Definition 2.2:** Portfolio $\lambda \in \Lambda$ dominates the benchmark $\tau \in \Lambda$ by third-degree stochastic dominance (TSD), or $\lambda \succeq_{TSD} \tau$, if

$$S^2_\lambda(z) \leq S^2_\tau(z), \ \forall z \in [a, b]; \quad (1)$$

$$\mathbb{E}[x^T \lambda] \geq \mathbb{E}[x^T \tau].$$

- **Analytical challenges:**
  - Infinitely many threshold levels $z \in [a, b]$ - unlike in SSD
  - Truncation at the threshold requires binary 0-1 variables
Our solution in 5 steps

1. Discrete state-dependent distribution
2. 'Super-convexity' conditions
3. Quadratic problem for $S_\lambda^2(z)$ given $\lambda$ and $z$
4. One large convex QCP problem
5. Problem reduction by fixing the values of most 0-1 variables
Step 1: Discrete distribution

- To obtain a tractable problem of finite dimensions, we assume a discrete state-dependent distribution.
- Scenarios with realizations $X_t := (X_{1,t} \cdots X_{K,t})^T$ and probabilities $p_t := \mathbb{P}[x = X_t], \ t = 1, \cdots, T$.
- Probs can be estimated using historical freqs, GMM/GEL implies probs or Bayes posterior probs.
- Flexibility to include realistic multivariate scenarios of market sell-offs and momentum crashes.
- Continuous distributions can be approximated using a finite number of random draws (MC sim).
- SV becomes a non-decreasing, convex, piece-wise quadratic function:
  $$S^2_\lambda (x) := \sum_{t=1}^T p_t (x - X_t^T \lambda)^2 \mathbb{1}(X_t^T \lambda \leq x).$$
Step 2: Super-convexity

- BBRS (1985): 'super-convex' TSD if the SV restrictions hold with sufficient slack at grid points:
  \[(1 + \varepsilon)S^2_\lambda(z_s) \leq S^2_\tau(z_s), \quad s = 1, \cdots, T,\]
  with \(\varepsilon > 0\) such that \((1 + \varepsilon)S^2_\tau(z_s) \geq S^2_\tau(z_{s+1}), \quad s = 1, \cdots, T - 1,\).

- We refine this condition to
  \[(1 + \varepsilon_s)S^2_\lambda(z_s) \leq S^2_\tau(z_s), \quad s = 1, \cdots, S,\]
  with \(\varepsilon_s = f(S^2_\tau(z_{s-1}), S^2_\tau(z_s), E_\tau(z_{s-1}))\)

- This amounts to using a piece-wise linear convex lower envelope for the SV function

- The approximation achieves machine precision for relatively rough partitions
### Step 2: Super-convexity - simple example

<table>
<thead>
<tr>
<th></th>
<th>( s = 1 )</th>
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<th>( s = 3 )</th>
<th>( \mu )</th>
<th>( \sigma )</th>
<th>( sk )</th>
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</thead>
<tbody>
<tr>
<td>( \mathbb{P} [s] )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
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<tr>
<td>( Y )</td>
<td>0.90</td>
<td>1.10</td>
<td>1.30</td>
<td>1.10</td>
<td>0.16</td>
<td>0.00</td>
</tr>
<tr>
<td>( Z )</td>
<td>0.97</td>
<td>1.00</td>
<td>1.40</td>
<td>1.12</td>
<td>0.20</td>
<td>1.70</td>
</tr>
</tbody>
</table>

![Graph showing comparison between S_Y, Approx_Y, and S_Z](image)
Step 2: Formalization

- **Definition:** Portfolio $\lambda \in \Lambda$ dominates the benchmark $\tau \in \Lambda$ by super-convex third-degree stochastic dominance (SCTSD), or $\lambda \succeq_{SCTSD} \tau$, if

  $$(1 + \varepsilon_s)S^2_\lambda(y_s) \leq S^2_\tau(y_s), \ s = 1, \cdots, T;$$

  $$\sum_{t=1}^{T} p_t X_t^T \lambda \geq \sum_{t=1}^{T} p_t y_t.$$ 

- **Proposition:** If portfolio $\lambda \in \Lambda$ dominates portfolio $\tau \in \Lambda$ by SCTSD, then $\lambda$ also dominates $\tau$ by TSD: $$(\lambda \succeq_{SCTSD} \tau) \Rightarrow (\lambda \succeq_{TSD} \tau).$$
Step 3: Quadratic form SV

- Similarly, we can formulate the restriction
  
  \[(1 + \varepsilon_s)S^2_\lambda(y_s) \leq S^2_\tau(y_s), \] 
  
  for given \( \lambda \in \Lambda \) and \( s = 1, \ldots, T \), by the following convex quadratic system:

  \[
  (1 + \varepsilon_s)\sum_{t=1}^{T} p_t \theta^2_t \leq S^2_\tau(y_s); \tag{2}
  \]

  \[
  \theta_t \geq y_s - X_t^T \lambda, \quad t = 1, \ldots, T; \tag{3}
  \]

  \[
  \theta_t \geq 0, \quad t = 1, \ldots, T. \tag{4}
  \]

- This formulation avoids binary variables and is linear in \( \lambda \) which appears as the RHS of linear constraints.
Step 4: QCP problem

- We apply the system for every $y_s$, $s = 1, \ldots, T$, and endogenize the portfolio weights:

$$
(1 + \varepsilon_s) \sum_{t=1}^{T} p_t \theta_{s,t}^2 \leq S_\tau^2(y_s), \quad s = 1, \ldots, T; \quad (5)
$$

$$
- \theta_{s,t} - \mathbf{X}_t^T \lambda \leq -y_s, \quad s, t = 1, \ldots, T;
$$

$$
- \sum_{t=1}^{T} p_t \mathbf{X}_t^T \lambda \leq -\sum_{t=1}^{T} p_t y_t;
$$

$$
1_T^T \lambda = 1;
$$

$$
\theta_{s,t} \geq 0, \quad s, t = 1, \ldots, T;
$$

$$
\lambda_k \geq 0, \quad k = 1, \ldots, K.
$$

- Maximizing $g(\lambda) := \mathbb{E}[\mathbf{x}^T \lambda] - \sum_{s=1}^{T} w_s S_\lambda^2(y_s), \quad w_s \geq 0, \quad s = 1, \ldots, T$, s.t. this system is a QCP problem.
Step 5: Problem reduction

- In MV analysis, \#var and \#constr are $O(K)$; in our case, $O(T^2)$
- The large size stems from relaxation of all binary vars $\mathbb{I}(X_t^T \lambda \leq y_s), t = 1, \cdots , T; s = 1, \cdots , T$
- A preliminary analysis can determine the value of most binary vars $\mathbb{I}(X_t^T \lambda \leq y_s), t = 1, \cdots , T; s = 1, \cdots , T$
- Our optimal portfolio must be an element the polytope $\Omega := \{ \lambda \in \Lambda : (\sum_{t=1}^T p_t x_t^T \lambda) \geq (\sum_{t=1}^T p_t y_t); x_1^T \lambda \geq y_1 \}$
- For every scenario $s = 1, \cdots , T$, we compute the min and max return for portfolios $\lambda \in \Omega$
- This allows us to fix most binary vars and eliminate the corresponding vars and constrs
Step 5: Problem size

- In our application \((K = 49, T > 250)\), the original problem has \(>62,500\) vars and \(>62,500\) constrs
- The reduced problem typically has \(<15,625\) vars and \(<15,625\) constrs
- We solved it on a desktop PC (Intel i7; 2.93 GHz; 16GB) with the IPOPT 3.12.3 solver in GAMS
- The median run time (using the reduction) was about two minutes
- Further reductions obtained through lessening the partition
Step 5: Problem size

<table>
<thead>
<tr>
<th>S</th>
<th>Relative error</th>
<th>Computer time(s)</th>
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<tbody>
<tr>
<td>S=1000</td>
<td>0%</td>
<td>475</td>
</tr>
<tr>
<td>S=800</td>
<td>0.001%</td>
<td>420</td>
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<tr>
<td>S=600</td>
<td>0.002%</td>
<td>280</td>
</tr>
<tr>
<td>S=400</td>
<td>0.004%</td>
<td>182</td>
</tr>
<tr>
<td>S=200</td>
<td>0.020%</td>
<td>102</td>
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<tr>
<td>S=100</td>
<td>0.080%</td>
<td>63</td>
</tr>
<tr>
<td>S=50</td>
<td>0.323%</td>
<td>28</td>
</tr>
<tr>
<td>S=25</td>
<td>1.165%</td>
<td>14</td>
</tr>
<tr>
<td>SSD</td>
<td>3.140%</td>
<td>85</td>
</tr>
</tbody>
</table>

Table: Numerical results using GAMS software with solver IPOPT
Motivation

■ Stock price momentum was documented first by Jegadeesh and Titman (1993)
■ It appears also for industries; Moskowitz and Grinblatt (1999)
■ Typical momentum strategies rely on heuristics such as buy D10 and sell D01
■ It seems interesting to use decision theory and optimization to improve on such heuristics
■ Hodder, Jackwerth and Kolokolova (2015) use SSD enhancement
■ Concentration in winner industries creates positive skewness
■ To exploit skew, we apply TSD enhancement
Data

- Benchmark = CRSP all-share index
- Base assets = 49 vw industry portfolios from Ken French’ library
- No concentration in individual stocks & no short positions
- Daily excess returns 1927-2014
- Same data as Hodder, Jackwerth and Kolokolova (2015)
- Other data sets are work in progress (IND10, 5MEx5BtM)
We compare 4 alternative enhanced portfolios:

- Top-15 = EWA of 15 recent winner industries
- 3 optimized portfolios maximize the mean s.t. benchmark risk restrictions:
  1. MV (variance)
  2. SSD (expected shortfall)
  3. SCTSD (semi-variance)

- Formation period = a 12-month trailing window of daily returns ($T > 250$)
- Portfolios are held for 3 months and then rebalanced (Jan - Apr - Jul - Oct)
Performance Evaluation

- We illustrate the features of the method using in-sample performance.
- Out-of-sample performance is evaluated on an annual basis ($N = 87$ Jan - Dec returns).
- We focus on the raw outperformance ($X - X_{Bench}$) of annual returns.
- We do not report alphas of factor models:
  - The market betas of the portfolios are smaller than 1.
  - The SMB and HML loadings are limited (dynamic & diversified).
  - Even MOM explains only part of the outperformance (industry-level & no short).
Performance Evaluation

- We decompose the outperformance \((X_{SCTSD} - X_{Bench})\) in components of
  \[\begin{align*}
  1 \quad & (X_{Top15} - X_{Bench}) \\
  2 \quad & (X_{MV} - X_{Top15}) \\
  3 \quad & (X_{SSD} - X_{MV}) \\
  4 \quad & (X_{SCTSD} - X_{SSD})
  \end{align*}\]
- We report t-stats for statistical significance
- We report also certainty equivalents (using logarithmic utility function)
### Performance Summary 1/3

<table>
<thead>
<tr>
<th></th>
<th>In-sample</th>
<th></th>
<th></th>
<th>Remarks</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Daily</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>((\bar{X}))</td>
<td>(s_{X})</td>
<td>(sk_{X})</td>
</tr>
<tr>
<td>(X_{Bench} - X_{Bond})</td>
<td>0.028</td>
<td>0.943</td>
<td>-0.325</td>
<td>Downs correl</td>
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<tr>
<td>(X_{Top15} - X_{Bond})</td>
<td>0.091</td>
<td>0.981</td>
<td>-0.434</td>
<td>Excess risk</td>
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<tr>
<td>(X_{MV} - X_{Bond})</td>
<td>0.128</td>
<td>0.923</td>
<td>-0.253</td>
<td>Max Sharpe</td>
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<tr>
<td>(X_{SSD} - X_{Bond})</td>
<td>0.131</td>
<td>0.965</td>
<td>-0.019</td>
<td>(\sigma \neq \text{risk})</td>
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<tr>
<td>(X_{SCTSD} - X_{Bond})</td>
<td>0.134</td>
<td>0.984</td>
<td>0.032</td>
<td>Upside pot</td>
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<tr>
<td>(X_{Top15} - X_{Bench})</td>
<td>0.063</td>
<td>0.352</td>
<td>-0.059</td>
<td>Form&amp;Hold</td>
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<tr>
<td>(X_{MV} - X_{Top15})</td>
<td>0.038</td>
<td>0.553</td>
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<tr>
<td>(X_{SSD} - X_{MV})</td>
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<td>0.213</td>
<td>0.353</td>
<td>Downs risk</td>
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<tr>
<td>(X_{SCTSD} - X_{SSD})</td>
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<td>0.080</td>
<td>0.183</td>
<td>Skewness</td>
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<tr>
<td>(X_{SCTSD} - X_{Bench})</td>
<td>0.106</td>
<td>0.662</td>
<td>0.400</td>
<td>Hindsight</td>
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## Performance Summary 2/3

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### In-sample Performance

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<th>tX</th>
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<td>29.17</td>
<td>8.41</td>
<td>25.90</td>
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<td>X_{MV} - X_{Bond}</td>
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<td>X_{SCTSD} - X_{Bond}</td>
<td>43.73</td>
<td>15.17</td>
<td>41.45</td>
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### Annual Performance

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<td>35.56</td>
<td>18.63</td>
<td>35.26</td>
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### Remarks

- Form&Hold
- Risk constr
- Downs risk
- Skewness
- Hindsight
## Performance Summary 3/3

### Out-of-sample

<table>
<thead>
<tr>
<th></th>
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<tr>
<td>( X_{MV} - X_{Bond} )</td>
<td>14.55</td>
<td>12.62</td>
</tr>
<tr>
<td>( X_{SSD} - X_{Bond} )</td>
<td>14.79</td>
<td>12.71</td>
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<tr>
<td>( X_{SCTSD} - X_{Bond} )</td>
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<td>12.86</td>
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<td>( X_{Top15} - X_{Bench} )</td>
<td>4.50</td>
<td>Form&amp;Hold</td>
</tr>
<tr>
<td>( X_{MV} - X_{Top15} )</td>
<td>1.88</td>
<td>Risk constr</td>
</tr>
<tr>
<td>( X_{SSD} - X_{MV} )</td>
<td>0.24</td>
<td>Downs risk</td>
</tr>
<tr>
<td>( X_{SCTSD} - X_{SSD} )</td>
<td>0.19</td>
<td>Skewness</td>
</tr>
<tr>
<td>( X_{SCTSD} - X_{Bench} )</td>
<td>6.81</td>
<td>No short, Hold =</td>
</tr>
</tbody>
</table>
We formed 21 benchmark portfolios:

- Market portfolio
- 10 convex combinations of global minimal variance portfolio and market portfolio
- 10 convex combinations of global maximal mean portfolio and market portfolio

- We found MV, SSD and TSD solution portfolios for each benchmark
- We present the results in mean-st.dev. figure
Close-up of 2013 in-sample results
Conclusions

■ Our contributions:

1. Refinement of SCTSD
2. QP for SV
3. CQP for SCTSD enhancement
4. Outperformance of Bench, Top15, MV and SSD in application

■ Follow-up ideas:

1. Better estimates:
   1. Conditioning on business cycle and market conditions
   2. GMM/GEL implied probabilities
   3. Bayesian posterior distribution

2. More data sets (IND30, 5ME\times5BtM)
3. Consider only decreasing absolute risk aversion utility functions
DARA SD

- Arrow-Pratt coefficient of absolute risk aversion:
  \[ r(x) = - \frac{u''(x)}{u'(x)} \]

- \( r(x) \) is typically decreasing (non-increasing)

- A recent analysis of DARA SD in Post, Fang, Kopa (2015)

- TSD implies DARA SD

- DARA SD enhancement - mean maximization over larger set of portfolios (more portfolios dominates the benchmark)

- DARA SD outperforms TSD mainly for the benchmarks with relatively low returns
Close-up of 2013 in-sample results + DARA

Portfolio Choice based on Third-degree Stochastic Dominance

Thierry Post & Miloš Kopa

Stochastic Dominance
SD Portfolio analysis
TSD optimization
Industry momentum strategy
Conclusions
http://dx.doi.org/10.1287/mnsc.2016.2506

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