On the construction of the HPFC for electricity prices

Conference on Computational Management Science 2017/Bergamo

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HPFC

Definition

Is the current price of electricity with delivery at a certain point (on an hourly scale) in the future.

- Is an OTC product used by producers of electricity to sell specialized electricity contracts
- Products traded on the exchanges are typically insufficient for most producers/consumers, making the HPFC crucial!
- Should reflect the believes about how electricity prices evolve, and be arbitrage free to traded Futures
The goal of this talk is to explain how the HPFC is constructed

We will present three different methods, two from the Literature and one novel approach

We will make a comparative analyzes of these models explaining the strengths and weaknesses, and how these methods can be improved.
▶ Consists of two parts:
▶ Seasonality curve:
  ▶ Represents historical spot prices, weather etc.
▶ Adjustment function:
  ▶ Makes sure the curve is arbitrage free to traded Futures products
Construction Methods
Seasonality Curve (Blöchlinger, 2008, PHD-Thesis) [1]:

- Dummy Variables:

\[ f_{2y_d} = a_0 + \sum_{i=1}^{6} b_i D_{di} + \sum_{i=1}^{12} c_i M_{di} + \sum_{i=1}^{3} d_i CDD_{di} + \sum_{i=1}^{3} e_i HDD_{di} \]

- Assigns different days into blocks and assigns a mean price to those blocks of days
- Problem: A non-smooth change between months
Adjustment Curve

- Has as a primary focus to make sure that the PFC fits to the observed Futures products
- Our methods:
  - Fleten et al. (2003) [2]
  - Benth et al. (2007) [3]
  - Novel approach based on a combined fitting of the seasonal directly to the Futures prices
- Our three main questions:
  - Do they take care of the smoothing?
  - What happens when the price of a Futures product change?
  - What happens when a Futures product is cascaded into several smaller products?
Fletens approach

The approach suggested by Fleten et al. (2003)[2] reads as follows:

- For a seasonality curve $s_t$
  
  $\min_{f_t} \left[ \sum_{t=1}^{T} (f_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} (f_{t-1} - 2f_t + f_{t+1})^2 \right]$  
  
  (1)

- given: $\sum_A f_t = FuturesPrice(A)$

- Problem:
  - Suppresses the weekly/daily seasonality (Blöchlinger, 2008, PHD-Thesis) [1]
  - $\lambda$ seems arbitrary
Daily Profile
Benth Approach

Benth et al. (2007) [3] suggests a method as follow:

$\text{FC}(t) = s_t + \varepsilon_t$

$\varepsilon_t = \begin{cases} 
  a_1 t^4 + b_1 t^3 + c_1 t^2 + d_1 t + e_1 & t \in [t_0, t_1) \\
  a_2 t^4 + b_2 t^3 + c_2 t^2 + d_2 t + e_2 & t \in [t_1, t_2) \\
  \vdots \\
  a_n t^4 + b_n t^3 + c_n t^2 + d_n t + e_n & t \in [t_{n-1}, t_n] 
\end{cases}$

$x = \{ a_1, \cdots, b_1, \cdots, c_1, \cdots \}$

$\min_x \int_{t_0}^{t_n} [\varepsilon''(t; x)]^2 dt$
Introducing a new Futures Product

![Graph showing original curve, implied curve, and implied futures over 300 days. The graph illustrates the fluctuation of Euro/Mwh prices over time.]

Audun Sætherø | Bergamo, Italy | June 01, 2017
Spline Curve

\[ FC(t) = C + \]
\[ \sum_{i=1}^{6} a_i \sin\left(\frac{2\pi i(t + S(m))}{12 \cdot M(m)}\right) + b_i \cos\left(\frac{2\pi i(t + S(m))}{12 \cdot M(m)}\right) + \]
\[ a_{4}^{Q(m)} \sin\left(\frac{8\pi(t + S(m))}{12 \cdot M(m)}\right) + b_{4}^{Q(m)} \cos\left(\frac{8\pi(t + S(m))}{12 \cdot M(m)}\right) + \]
\[ a_{12}^{Q(m)} \sin\left(\frac{24\pi(t + S(m))}{12 \cdot M(m)}\right) + b_{12}^{Q(m)} \cos\left(\frac{24\pi(t + S(m))}{12 \cdot M(m)}\right) \]
\[ T(m) + S(m) \quad M(m) = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 \]
\[
\begin{align*}
\text{minimize} & \quad \|Ax - y\|_2 \\
\text{subject to} & \quad Cx = d \\
\end{align*}
\]

where \( x \) are the 26 coefficients, \( C \) and \( d \) assures the resulting curve is arbitrage free with respect to the Futures products

\[
x = (a_1, \ldots, b_1, \ldots, a_4^1, \ldots a_4^4, b_4^1, \ldots, b_4^4, a_{12}^1, \ldots a_{12}^4, b_{12}^1, \ldots, b_{12}^4)
\]

- Requires a higher than normal number of parameters
  - May lead to overfitting!
- Needs to pre-specify the number of Futures products
Derivative of Adjustment Function

**Figure:** The derivative of the adjustment functions with respect to the March Future, with 3-12 monthly products as input.
<table>
<thead>
<tr>
<th>Test</th>
<th>Data</th>
<th>Benth</th>
<th>Novel</th>
<th>Fleten 1</th>
<th>Fleten 2</th>
</tr>
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<tr>
<td>MAPE</td>
<td>IS</td>
<td>32%</td>
<td>29%</td>
<td>45%</td>
<td>32%</td>
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<tr>
<td>MAPE</td>
<td>OoS</td>
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<td>42%</td>
<td>57%</td>
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<td>AD</td>
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<tr>
<td>AD</td>
<td>OoS</td>
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<td>6.79</td>
<td>7.15</td>
<td>6.32</td>
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<tr>
<td>AD</td>
<td>IS</td>
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<td>5.83</td>
<td>7.15</td>
<td>5.98</td>
</tr>
<tr>
<td>AD</td>
<td>OoS</td>
<td>9.92</td>
<td>8.09</td>
<td>8.55</td>
<td>7.67</td>
</tr>
<tr>
<td>SD</td>
<td>IS</td>
<td>65.71</td>
<td>61.69</td>
<td>91.46</td>
<td>65.89</td>
</tr>
<tr>
<td>SD</td>
<td>OoS</td>
<td>181.69</td>
<td>116.86</td>
<td>139.76</td>
<td>109.25</td>
</tr>
</tbody>
</table>

**Table:** Comparing the different models to the realized spot prices, Test 1-4 is for a daily scale, while Test 5-8 is on an hourly scale. AD=Absolute Difference, SD=Square Difference, IS=In Sample, OoS=Out of sample
Conclusion

► Important to know what your construction method does, and what you actually want it to do
► One should be careful about what the different parts of the seasonality curve does
► One should be careful about adding new parameters when the Futures are cascading (or when other new information appears)
► If your adjustment function has a smoothing term, you have to be sure what you want to smooth (and what not to smooth)
References


Thank you for your attention...