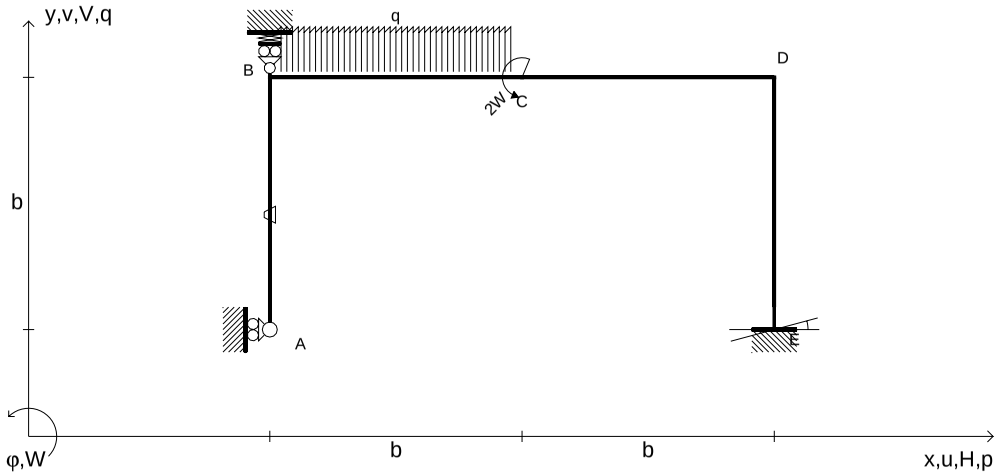

$W_C = 2W = 2Fb$ $\varphi_E = \delta/b = b^2F/EJ$ $EJ_{BC} = EJ$

$q_{BC} = q = F/b$ $k_B = 2EJ/b^3$ $EJ_{CD} = EJ$

$\theta_{AB} = 2\theta = 2\alpha T/b = 2bF/EJ$ $EJ_{AB} = EJ$ $EJ_{ED} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio.
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Rotazione assoluta W imposta al nodo E.

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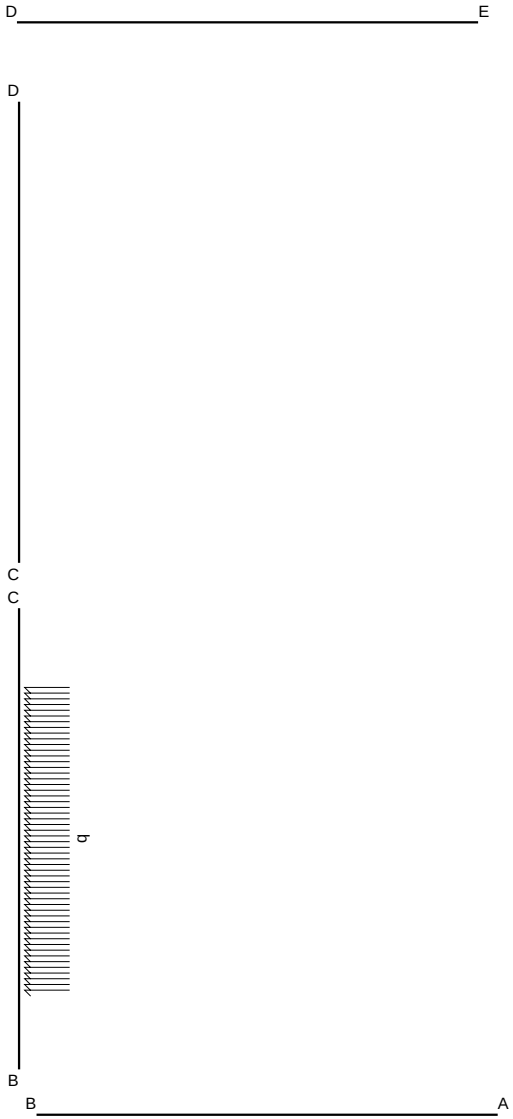

DEFORMATA (coordinate locali)
AB y(x)EJ =
BC y(x)EJ =
CD y(x)EJ =
ED y(x)EJ =

SPOSTAMENTI NODALI

u_A = u_B = u_C = u_D =
v_{AAB} = v_B = v_C = v_D =
φ_{AAB} = φ_B = φ_C = φ_D =

u_E =
v_E =
φ_E =

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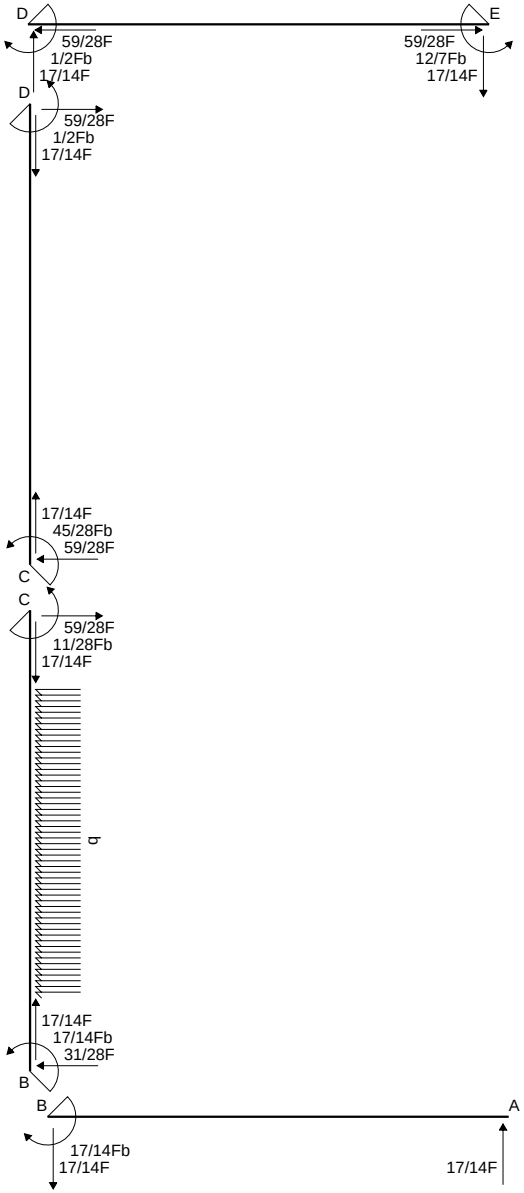

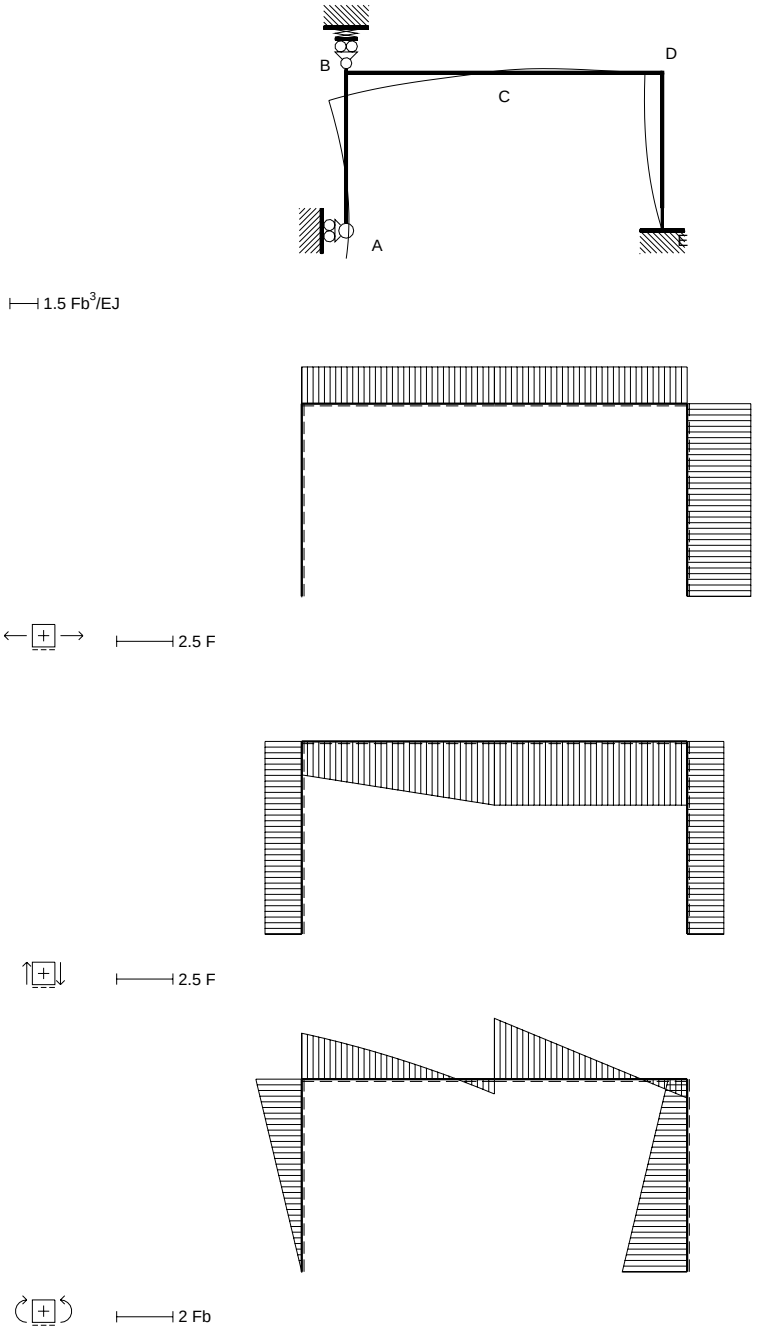

REAZIONI IPERSTATICHE

$X = W_{BC}$ $Y = W_{DE}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{BC} K_{BC} φ_{CD} K_{CD} φ_{ED} K_{ED}

Relazioni di congruenza

$y'_{AB}(b) - y'_{BC}(0) = 0$
$y'_{BC}(b) - y'_{CD}(0) = 0$
$y'_{CD}(b) - y'_{ED}(b) = 0$
$y'_{ED}(0) - \delta/b = 0$
$y_{AB}(0) = 0$
$y_{BC}(0) - 1/2V_B b^3/EJ = 0$
$y_{CD}(0) - y_{BC}(b) = 0$
$y_{CD}(b) = 0$
$y_{ED}(0) = 0$
$y_{ED}(b) - y_{AB}(b) = 0$

$M_{AB} = -Xx/b$
$EJy'' = 2EJ\theta - Xx/b$
$EJy' = 2EJ\theta x - 1/2Xx^2/b + EJ\varphi_{AB}$
$EJy = EJ\theta x^2 - 1/6Xx^3/b + EJ\varphi_{AB}x + EJK_{AB}$

$M_{BC} = 1/4Fx + 1/2qx^2 + 1/2Xx/b - X - 1/2Yx/b$
$EJy'' = 1/4Fx + 1/2qx^2 + 1/2Xx/b - X - 1/2Yx/b$
$EJy' = 1/8Fx^2 + 1/6qx^3 + 1/4Xx^2/b - Xx - 1/4Yx^2/b + EJ\varphi_{BC}$
$EJy = 1/24Fx^3 + 1/24qx^4 + 1/12Xx^3/b - 1/2Xx^2 - 1/12Yx^3/b + EJ\varphi_{BC}x + EJK_{BC}$

$M_{CD} = 5/4Fx - 5/4Fb + 1/2Xx/b - 1/2X - 1/2Yx/b - 1/2Y$
$EJy'' = 5/4Fx - 5/4Fb + 1/2Xx/b - 1/2X - 1/2Yx/b - 1/2Y$
$EJy' = 5/8Fx^2 - 5/4Fbx + 1/4Xx^2/b - 1/2Xx - 1/4Yx^2/b - 1/2Yx + EJ\varphi_{CD}$
$EJy = 5/24Fx^3 - 5/8Fbx^2 + 1/12Xx^3/b - 1/4Xx^2 - 1/12Yx^3/b - 1/4Yx^2 + EJ\varphi_{CD}x + EJK_{CD}$

$M_{ED} = Xx/b - X + Y$
$EJy'' = Xx/b - X + Y$
$EJy' = 1/2Xx^2/b - Xx + Yx + EJ\varphi_{ED}$
$EJy = 1/6Xx^3/b - 1/2Xx^2 + 1/2Yx^2 + EJ\varphi_{ED}x + EJK_{ED}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{BC}b$	K_{BC}	$\varphi_{CD}b$	K_{CD}	$\varphi_{ED}b$	K_{ED}	Xb^2/EJ	Yb^2/EJ		Wb^2/EJ
y'_{BA}	1	0	-1	0	0	0	0	0	-1/2	0		0
y'_{CB}	0	0	1	0	-1	0	0	0	-3/4	-1/4		-7/24
y'_{DC}	0	0	0	0	1	0	-1	0	1/4	-7/4		5/8
y'_{ED}	0	0	0	0	0	0	1	0	0	0		0
y_{AB}	0	1	0	0	0	0	0	0	0	0		0
y_{BC}	0	0	0	1	0	0	0	0	1/4	-1/4	=	-1/8
y_{CD}	0	0	-1	-1	0	1	0	0	5/12	1/12		1/12
y_{DC}	0	0	0	0	1	1	0	0	-1/6	-1/3		5/12
y_{ED}	0	0	0	0	0	0	0	1	0	0		0
y_{DE}	-1	-1	0	0	0	0	1	1	-1/6	1/2		0

Condizioni al contorno

αTb	δ
-2	0
0	0
0	0
0	1
0	0
0	0
0	0
0	0
0	0
1	0

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		-19/42
$\varphi_{BC}b$		79/84
$\varphi_{CD}b$		25/56
$\varphi_{ED}b$		1
K_{AB}		0
K_{BC}	=	-31/56
K_{CD}		1/168
Xb^2/EJ		17/14
K_{ED}		0
Yb^2/EJ		-1/2

DEFORMATA (coordinate locali)

AB $y(x)EJ = -19/42xFb^2 + x^2Fb - 17/84x^3F$
BA $y(x)EJ = 29/84Fb^3 - 79/84xFb^2 + 11/28x^2Fb + 17/84x^3F$
BC $y(x)EJ = -31/56Fb^3 + 79/84xFb^2 - 17/28x^2Fb + 31/168x^3F + 1/24x^4q$
CB $y(x)EJ = 1/168Fb^3 - 25/56xFb^2 + 11/56x^2Fb - 59/168x^3F + 1/24x^4q$
CD $y(x)EJ = 1/168Fb^3 + 25/56xFb^2 - 45/56x^2Fb + 59/168x^3F$
DC $y(x)EJ = 3/28xFb^2 + 1/4x^2Fb - 59/168x^3F$
ED $y(x)EJ = xFb^2 - 6/7x^2Fb + 17/84x^3F$
DE $y(x)EJ = 29/84Fb^3 + 3/28xFb^2 - 1/4x^2Fb - 17/84x^3F$

SPOSTAMENTI NODALI

$u_A = 0$	$u_B = -29/84(Fb^3/EJ)$	$u_C = -29/84(Fb^3/EJ)$	$u_D = -29/84(Fb^3/EJ)$
$v_{AAB} = -31/56(Fb^3/EJ)$	$v_B = -31/56(Fb^3/EJ)$	$v_C = 1/168(Fb^3/EJ)$	$v_D = 0$
$\varphi_{AAB} = -19/42(Fb^2/EJ)$	$\varphi_B = 79/84(Fb^2/EJ)$	$\varphi_C = 25/56(Fb^2/EJ)$	$\varphi_D = -3/28(Fb^2/EJ)$

$u_E = 0$
$v_E = 0$
$\varphi_E = (Fb^2/EJ)$

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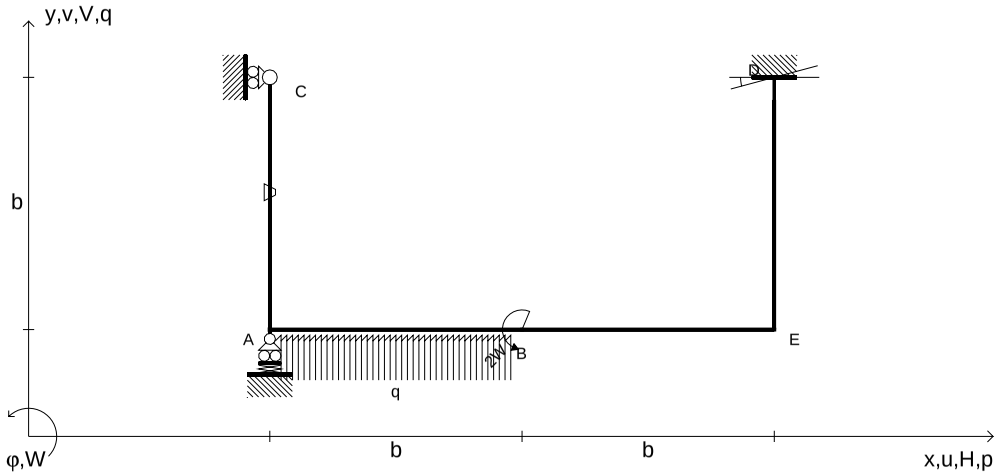
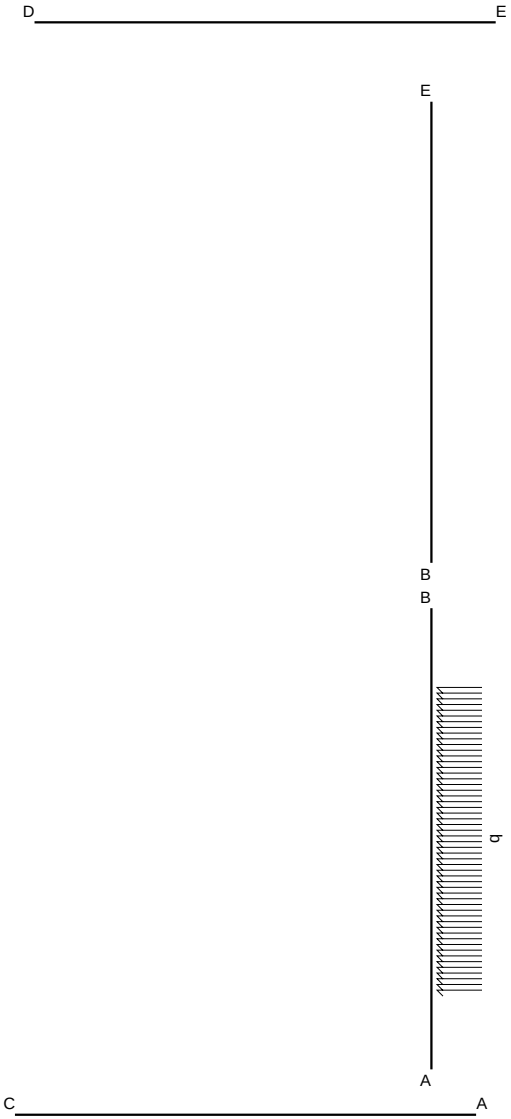


$W_B = 2W = 2Fb$ $\varphi_D = \delta/b = b^2F/EJ$ $EJ_{CA} = EJ$

$q_{AB} = q = F/b$ $k_A = 2EJ/b^3$ $EJ_{DE} = EJ$

$\theta_{CA} = 2\theta = 2\alpha T/b = 2bF/EJ$ $EJ_{AB} = EJ$ $EJ_{BE} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio.
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CA positiva se convessa a destra con inizio C.
Rotazione assoluta W imposta al nodo D.



DEFORMATA (coordinate locali)
AB y(x)EJ =
CA y(x)EJ =
DE y(x)EJ =
BE y(x)EJ =

SPOSTAMENTI NODALI

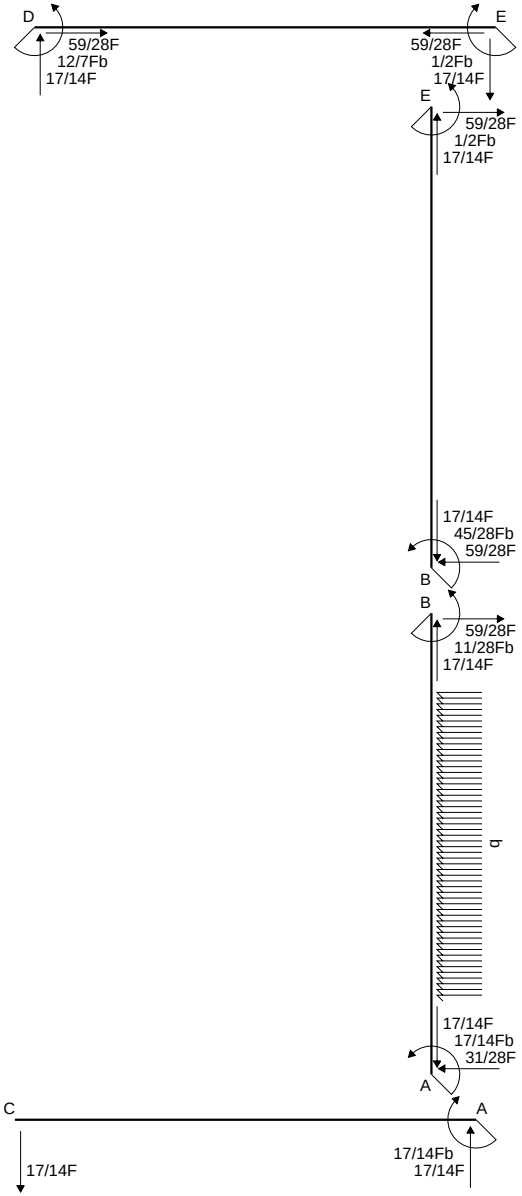
u_A =
v_A =
φ_A =

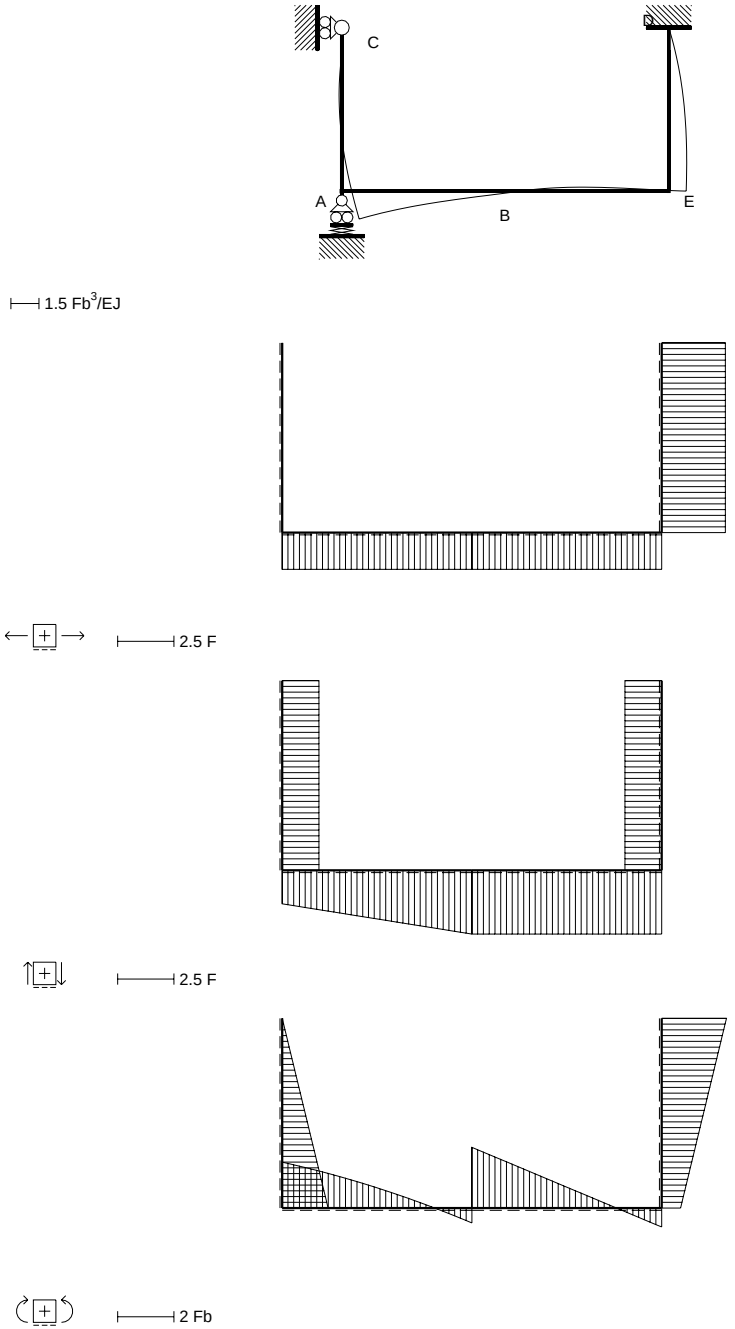
u_B =
v_B =
φ_B =

u_C =
v_{CCA} =
φ_{CCA} =

u_D =
v_D =
φ_D =

u_E =
v_E =
φ_E =


D
59/28F
12/7Fb
17/14F
E
59/28F
1/2Fb
17/14F
E
59/28F
1/2Fb
17/14F
17/14F
45/28Fb
59/28F
B
B
59/28F
11/28Fb
17/14F
q
17/14F
17/14Fb
31/28F
A
C
A
17/14F
17/14Fb
17/14F


C
D
A
B
E
1.5 Fb3/EJ
2.5 F
2.5 F
2 Fb

REAZIONI IPERSTATICHE

$X = W_{AB}$ $Y = W_{ED}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CA} K_{CA} φ_{DE} K_{DE} φ_{BE} K_{BE}

Relazioni di congruenza

$y'_{AB}(0) - y'_{CA}(b) = 0$
$y'_{AB}(b) - y'_{BE}(0) = 0$
$y'_{DE}(0) - \delta/b = 0$
$y'_{DE}(b) - y'_{BE}(b) = 0$
$y_{AB}(0) - 1/2V_A b^3/EJ = 0$
$y_{CA}(0) = 0$
$y_{DE}(0) = 0$
$y_{DE}(b) - y_{CA}(b) = 0$
$y_{BE}(0) - y_{AB}(b) = 0$
$y_{BE}(b) = 0$

$M_{AB} = 1/4Fx +1/2qx^2 +1/2Xx/b -X -1/2Yx/b$
$EJy'' = 1/4Fx +1/2qx^2 +1/2Xx/b -X -1/2Yx/b$
$EJy' = 1/8Fx^2 +1/6qx^3 +1/4Xx^2/b -Xx -1/4Yx^2/b +EJ\varphi_{AB}$
$EJy = 1/24Fx^3 +1/24qx^4 +1/12Xx^3/b -1/2Xx^2 -1/12Yx^3/b +EJ\varphi_{AB}x +EJK_{AB}$

$M_{CA} = -Xx/b$
$EJy'' = 2EJ\theta -Xx/b$
$EJy' = 2EJ\theta x -1/2Xx^2/b +EJ\varphi_{CA}$
$EJy = EJ\theta x^2 -1/6Xx^3/b +EJ\varphi_{CA}x +EJK_{CA}$

$M_{DE} = Xx/b -X +Y$
$EJy'' = Xx/b -X +Y$
$EJy' = 1/2Xx^2/b -Xx +Yx +EJ\varphi_{DE}$
$EJy = 1/6Xx^3/b -1/2Xx^2 +1/2Yx^2 +EJ\varphi_{DE}x +EJK_{DE}$

$M_{BE} = 5/4Fx -5/4Fb +1/2Xx/b -1/2X -1/2Yx/b -1/2Y$
$EJy'' = 5/4Fx -5/4Fb +1/2Xx/b -1/2X -1/2Yx/b -1/2Y$
$EJy' = 5/8Fx^2 -5/4Fbx +1/4Xx^2/b -1/2Xx -1/4Yx^2/b -1/2Yx +EJ\varphi_{BE}$
$EJy = 5/24Fx^3 -5/8Fbx^2 +1/12Xx^3/b -1/4Xx^2 -1/12Yx^3/b -1/4Yx^2 +EJ\varphi_{BE}x +EJK_{BE}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CA}b$	K_{CA}	$\varphi_{DE}b$	K_{DE}	$\varphi_{BE}b$	K_{BE}	Xb^2/EJ	Yb^2/EJ		Wb^2/EJ
y'_{AB}	1	0	-1	0	0	0	0	0	1/2	0		0
y'_{BA}	1	0	0	0	0	0	-1	0	-3/4	-1/4		-7/24
y'_{DE}	0	0	0	0	1	0	0	0	0	0		0
y'_{ED}	0	0	0	0	1	0	-1	0	-1/4	7/4		-5/8
y_{AB}	0	1	0	0	0	0	0	0	1/4	-1/4		-1/8
y_{CA}	0	0	0	1	0	0	0	0	0	0	=	0
y_{DE}	0	0	0	0	0	1	0	0	0	0		0
y_{ED}	0	0	-1	-1	1	1	0	0	-1/6	1/2		0
y_{BE}	-1	-1	0	0	0	0	0	1	5/12	1/12		1/12
y_{EB}	0	0	0	0	0	0	1	1	-1/6	-1/3		5/12

Condizioni al contorno

αTb	δ
2	0
0	0
0	1
0	0
0	0
0	0
0	0
1	0
0	0
0	0

Soluzione

		$[Fb^3/EJ]$
$\varphi_{AB}b$		79/84
$\varphi_{CA}b$		-19/42
$\varphi_{DE}b$		1
$\varphi_{BE}b$		25/56
K_{AB}		-31/56
K_{CA}	=	0
K_{DE}		0
Xb^2/EJ		17/14
K_{BE}		1/168
Yb^2/EJ		-1/2

DEFORMATA (coordinate locali)

AB $y(x)EJ = -31/56Fb^3 +79/84xFb^2 -17/28x^2Fb +31/168x^3F +1/24x^4q$
BA $y(x)EJ = 1/168Fb^3 -25/56xFb^2 +11/56x^2Fb -59/168x^3F +1/24x^4q$
CA $y(x)EJ = -19/42xFb^2 +x^2Fb -17/84x^3F$
AC $y(x)EJ = 29/84Fb^3 -79/84xFb^2 +11/28x^2Fb +17/84x^3F$
DE $y(x)EJ = xFb^2 -6/7x^2Fb +17/84x^3F$
ED $y(x)EJ = 29/84Fb^3 +3/28xFb^2 -1/4x^2Fb -17/84x^3F$
BE $y(x)EJ = 1/168Fb^3 +25/56xFb^2 -45/56x^2Fb +59/168x^3F$
EB $y(x)EJ = 3/28xFb^2 +1/4x^2Fb -59/168x^3F$

SPOSTAMENTI NODALI

$u_A = 29/84(Fb^3/EJ)$	$u_B = 29/84(Fb^3/EJ)$	$u_C = 0$	$u_D = 0$
$v_A = -31/56(Fb^3/EJ)$	$v_B = 1/168(Fb^3/EJ)$	$v_{CCA} = -31/56(Fb^3/EJ)$	$v_D = 0$
$\varphi_A = 79/84(Fb^2/EJ)$	$\varphi_B = 25/56(Fb^2/EJ)$	$\varphi_{CCA} = -19/42(Fb^2/EJ)$	$\varphi_D = (Fb^2/EJ)$

$u_E = 29/84(Fb^3/EJ)$
$v_E = 0$
$\varphi_E = -3/28(Fb^2/EJ)$

Es.N.002

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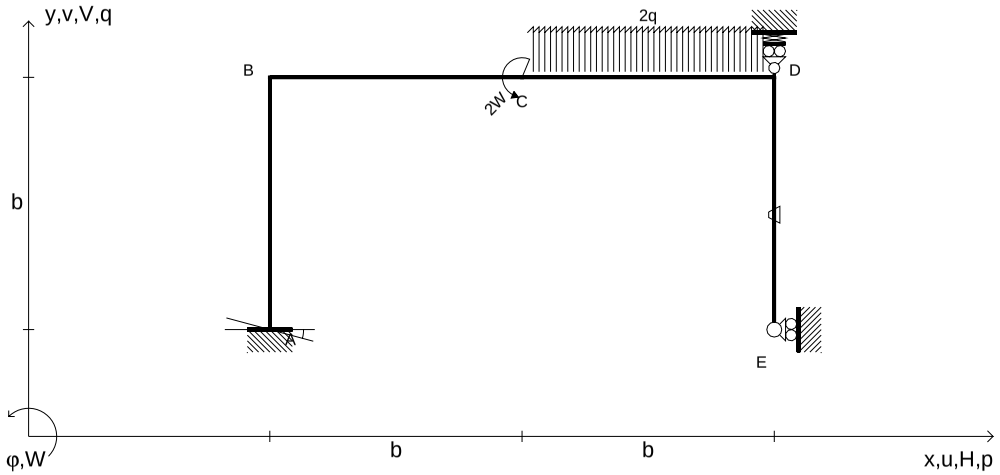
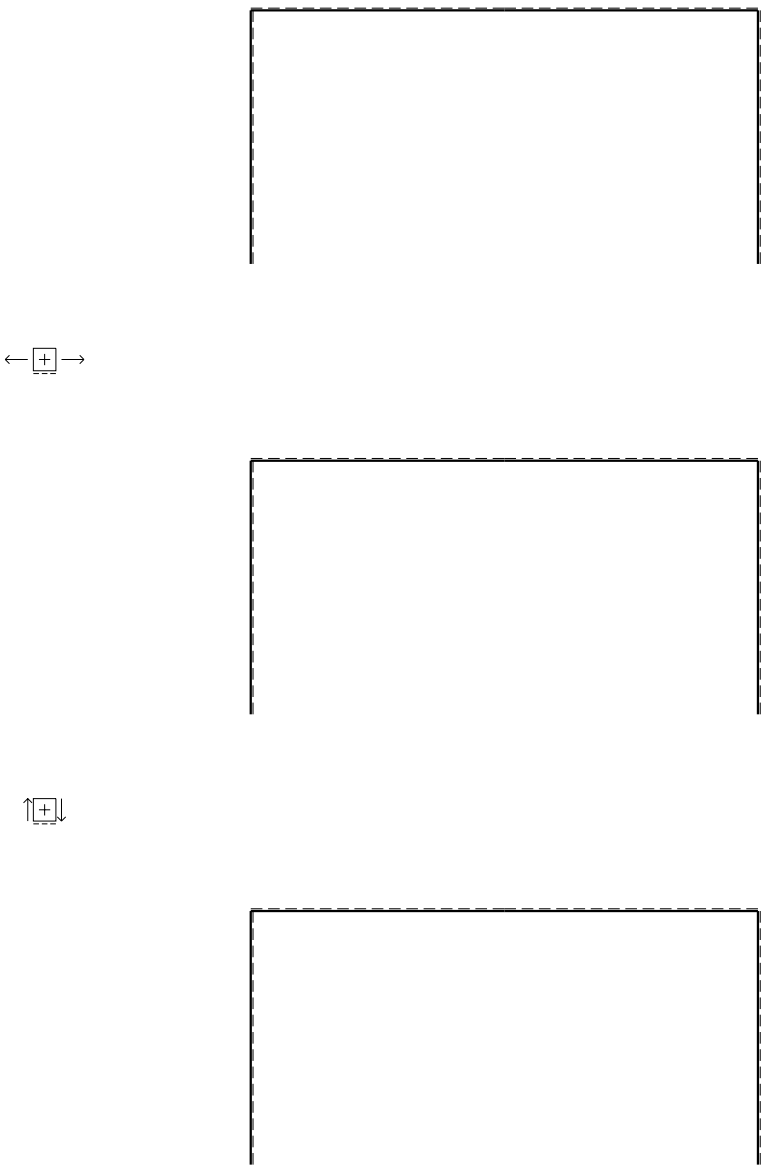


$W_C = 2W = 2Fb$
$q_{DC} = 2q = 2F/b$
$\theta_{ED} = 2\theta = 2\alpha T/b = 2bF/EJ$

$\varphi_A = -\delta/b = -b^2F/EJ$
$k_D = 2EJ/b^3$
$EJ_{AB} = EJ$

$EJ_{CB} = EJ$
$EJ_{DC} = EJ$
$EJ_{ED} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio.
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta ED positiva se convessa a destra con inizio E.
Rotazione assoluta W imposta al nodo A.



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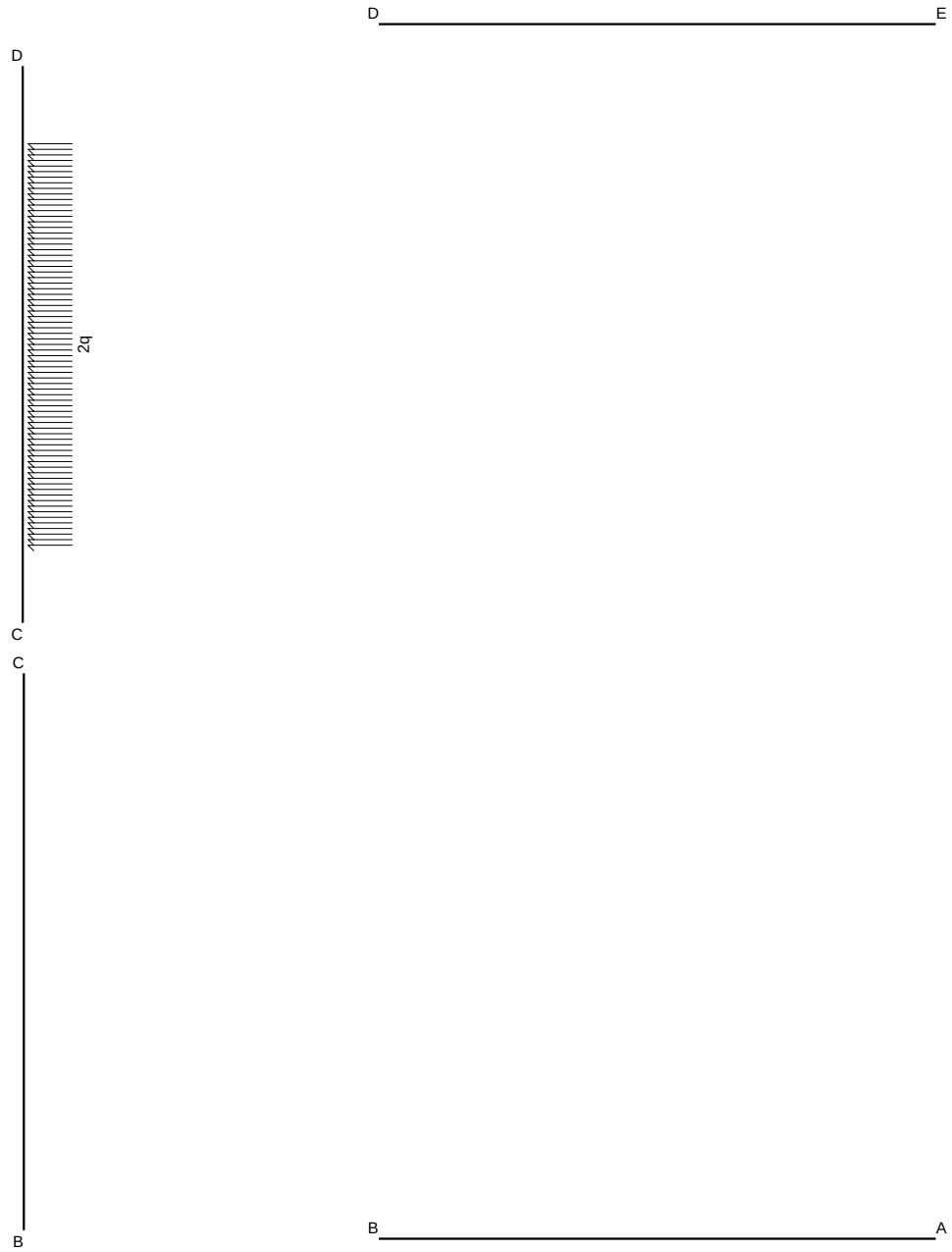

DEFORMATA (coordinate locali)
AB y(x)EJ =
CB y(x)EJ =
DC y(x)EJ =
ED y(x)EJ =

SPOSTAMENTI NODALI

u_A = u_B = u_C = u_D =
v_A = v_B = v_C = v_D =
φ_A = φ_B = φ_C = φ_D =

u_E =
v_{EED} =
φ_{EED} =

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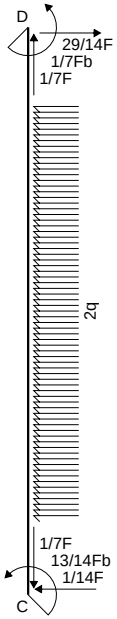
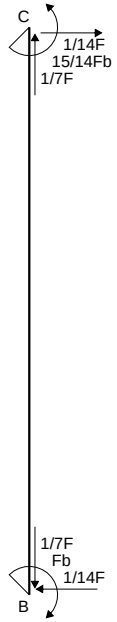
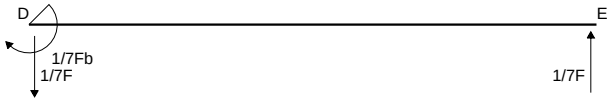
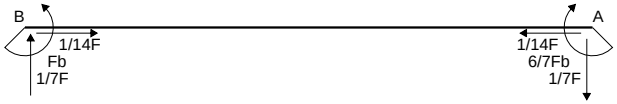
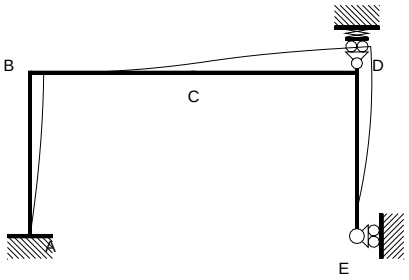


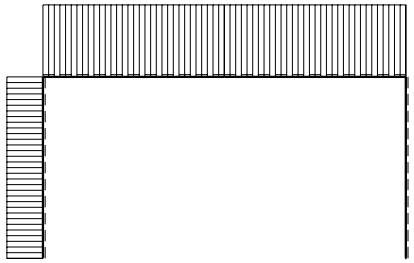




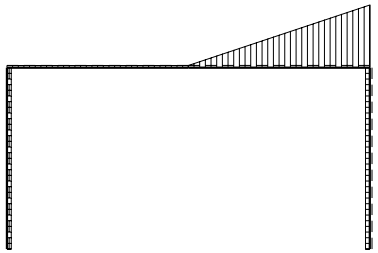




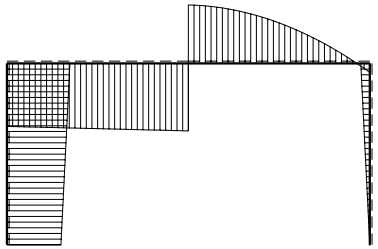
$\vdash\!\dashv$ $3\,Fb^3/EJ$



$\vdash\!\dashv$ $0.15\,F$



$\vdash\!\dashv$ $2.5\,F$



$\vdash\!\dashv$ $1.2\,Fb$

REAZIONI IPERSTATICHE

$X = W_{BA}$ $Y = W_{DC}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CB} K_{CB} φ_{DC} K_{DC} φ_{ED} K_{ED}

Relazioni di congruenza

$y'_{AB}(0) + \delta/b = 0$
$y'_{AB}(b) - y'_{CB}(b) = 0$
$y'_{CB}(0) - y'_{DC}(b) = 0$
$y'_{DC}(0) - y'_{ED}(b) = 0$
$y_{AB}(0) = 0$
$y_{CB}(b) = 0$
$y_{DC}(0) + 1/2V_D b^3/EJ = 0$
$y_{DC}(b) - y_{CB}(0) = 0$
$y_{ED}(0) = 0$
$y_{ED}(b) - y_{AB}(b) = 0$

$M_{AB} = X + Yx/b - Y$
$EJy'' = X + Yx/b - Y$
$EJy' = Xx + 1/2Yx^2/b - Yx + EJ\varphi_{AB}$
$EJy = 1/2Xx^2 + 1/6Yx^3/b - 1/2Yx^2 + EJ\varphi_{AB}x + EJK_{AB}$

$M_{CB} = 1/2Fx - 1/2Fb - 1/2Xx/b - 1/2X + 1/2Yx/b - 1/2Y$
$EJy'' = 1/2Fx - 1/2Fb - 1/2Xx/b - 1/2X + 1/2Yx/b - 1/2Y$
$EJy' = 1/4Fx^2 - 1/2Fbx - 1/4Xx^2/b - 1/2Xx + 1/4Yx^2/b - 1/2Yx + EJ\varphi_{CB}$
$EJy = 1/12Fx^3 - 1/4Fbx^2 - 1/12Xx^3/b - 1/4Xx^2 + 1/12Yx^3/b - 1/4Yx^2 + EJ\varphi_{CB}x + EJK_{CB}$

$M_{DC} = 5/2Fx - qx^2 - 1/2Xx/b + 1/2Yx/b - Y$
$EJy'' = 5/2Fx - qx^2 - 1/2Xx/b + 1/2Yx/b - Y$
$EJy' = 5/4Fx^2 - 1/3qx^3 - 1/4Xx^2/b + 1/4Yx^2/b - Yx + EJ\varphi_{DC}$
$EJy = 5/12Fx^3 - 1/12qx^4 - 1/12Xx^3/b + 1/12Yx^3/b - 1/2Yx^2 + EJ\varphi_{DC}x + EJK_{DC}$

$M_{ED} = -Yx/b$
$EJy'' = 2EJ\theta - Yx/b$
$EJy' = 2EJ\theta x - 1/2Yx^2/b + EJ\varphi_{ED}$
$EJy = EJ\theta x^2 - 1/6Yx^3/b + EJ\varphi_{ED}x + EJK_{ED}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CB}b$	K_{CB}	$\varphi_{DC}b$	K_{DC}	$\varphi_{ED}b$	K_{ED}	Xb^2/EJ	Yb^2/EJ		Wb^2/EJ
y'_{AB}	1	0	0	0	0	0	0	0	0	0		0
y'_{BA}	1	0	-1	0	0	0	0	0	7/4	-1/4		-1/4
y'_{CB}	0	0	1	0	-1	0	0	0	1/4	3/4		11/12
y'_{DC}	0	0	0	0	1	0	-1	0	0	1/2		0
y_{AB}	0	1	0	0	0	0	0	0	0	0		0
y_{BC}	0	0	1	1	0	0	0	0	-1/3	-1/6	=	1/6
y_{DC}	0	0	0	0	0	1	0	0	-1/4	1/4		-5/4
y_{CD}	0	0	0	-1	1	1	0	0	-1/12	-5/12		-1/3
y_{ED}	0	0	0	0	0	0	0	1	0	0		0
y_{DE}	-1	-1	0	0	0	0	1	1	-1/2	1/6		0

Condizioni al contorno

αTb	δ
0	-1
0	0
0	0
2	0
0	0
0	0
0	0
0	0
0	0
-1	0

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		-1
$\varphi_{CB}b$		27/28
$\varphi_{DC}b$		17/42
$\varphi_{ED}b$		-32/21
K_{AB}		0
K_{CB}	=	-37/84
K_{DC}		-29/28
Xb^2/EJ		1
K_{ED}		0
Yb^2/EJ		1/7

DEFORMATA (coordinate locali)

AB $y(x)EJ = -xFb^2 + 3/7x^2Fb + 1/42x^3F$
BA $y(x)EJ = -23/42Fb^3 + 1/14xFb^2 + 1/2x^2Fb - 1/42x^3F$
CB $y(x)EJ = -37/84Fb^3 + 27/28xFb^2 - 15/28x^2Fb + 1/84x^3F$
BC $y(x)EJ = 1/14xFb^2 - 1/2x^2Fb - 1/84x^3F$
DC $y(x)EJ = -29/28Fb^3 + 17/42xFb^2 - 1/14x^2Fb + 29/84x^3F - 1/12x^4q$
CD $y(x)EJ = -37/84Fb^3 - 27/28xFb^2 + 13/28x^2Fb - 1/84x^3F - 1/12x^4q$
ED $y(x)EJ = -32/21xFb^2 + x^2Fb - 1/42x^3F$
DE $y(x)EJ = -23/42Fb^3 - 17/42xFb^2 + 13/14x^2Fb + 1/42x^3F$

SPOSTAMENTI NODALI

$u_A = 0$	$u_B = 23/42(Fb^3/EJ)$	$u_C = 23/42(Fb^3/EJ)$	$u_D = 23/42(Fb^3/EJ)$
$v_A = 0$	$v_B = 0$	$v_C = 37/84(Fb^3/EJ)$	$v_D = 29/28(Fb^3/EJ)$
$\varphi_A = -(Fb^2/EJ)$	$\varphi_B = -1/14(Fb^2/EJ)$	$\varphi_C = 27/28(Fb^2/EJ)$	$\varphi_D = 17/42(Fb^2/EJ)$

$u_E = 0$
$v_{EED} = 29/28(Fb^3/EJ)$
$\varphi_{EED} = -32/21(Fb^2/EJ)$

Es.N.003

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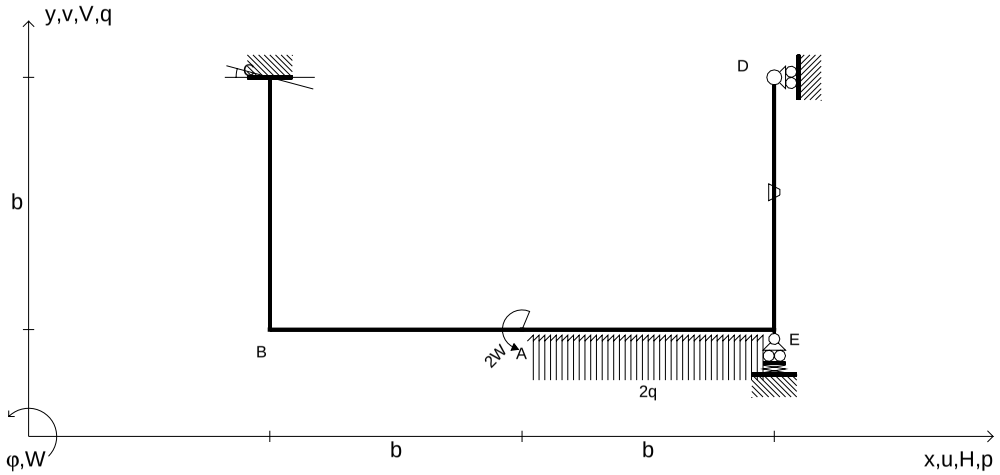

$W_A = 2W = 2Fb$
$q_{EA} = 2q = 2F/b$
$\theta_{DE} = 2\theta = 2\alpha T/b = 2bF/EJ$

$\varphi_C = -\delta/b = -b^2F/EJ$
$k_E = 2EJ/b^3$
$EJ_{AB} = EJ$

$EJ_{CB} = EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio.
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta DE positiva se convessa a destra con inizio D.
Rotazione assoluta W imposta al nodo C.

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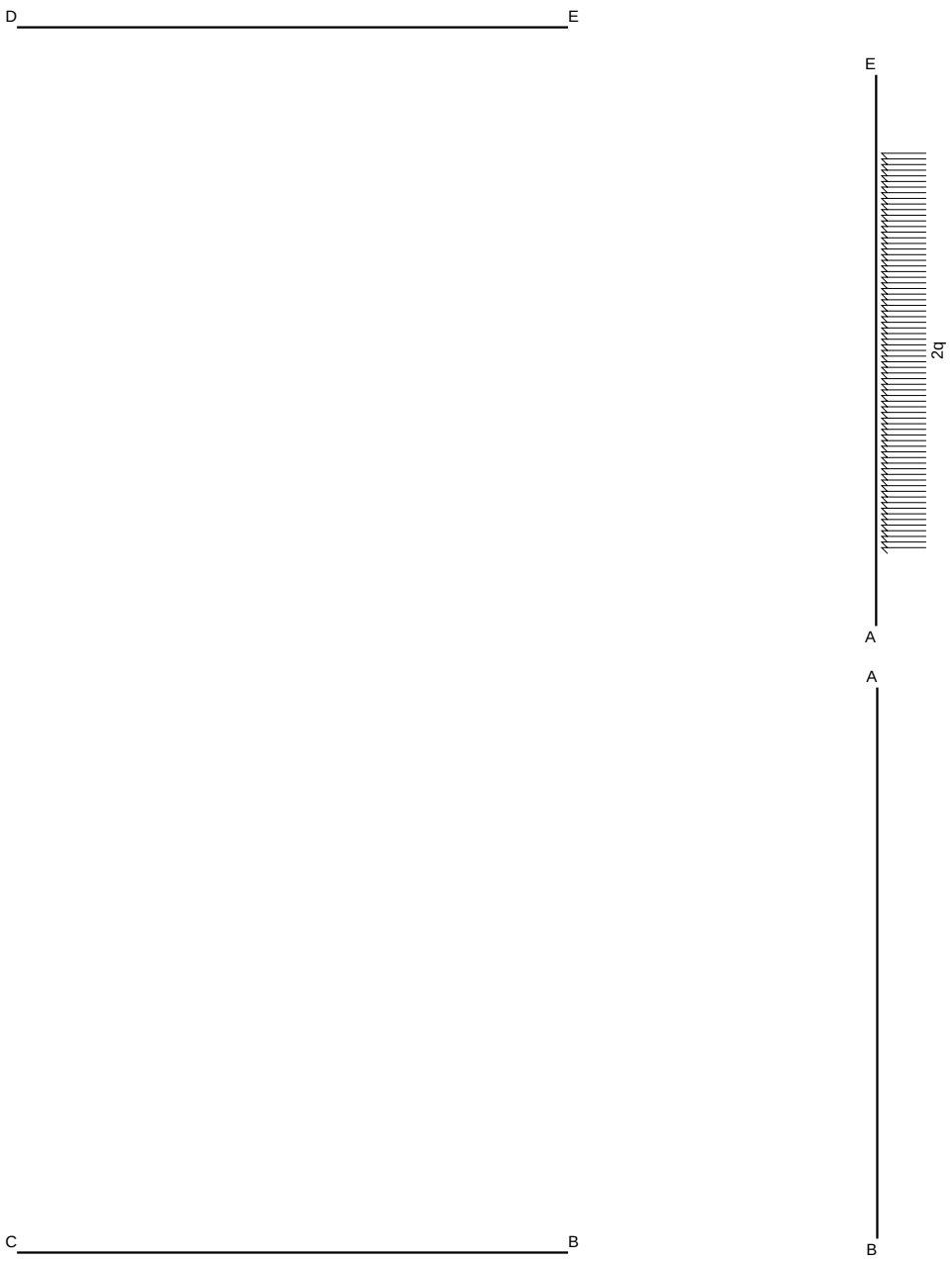

DEFORMATA (coordinate locali)
AB y(x)EJ =
CB y(x)EJ =
DE y(x)EJ =
EA y(x)EJ =

SPOSTAMENTI NODALI

u_A =
v_A =
φ_A =

u_B =
v_B =
φ_B =

u_C =
v_C =
φ_C =

u_D =
v_{DDE} =
φ_{DDE} =

u_E =
v_E =
φ_E =

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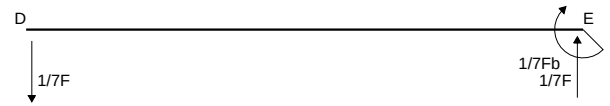
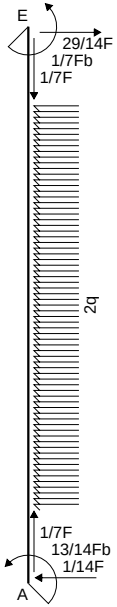
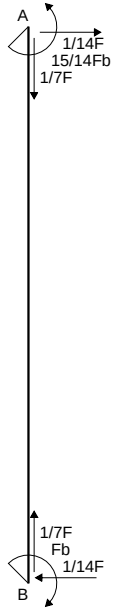
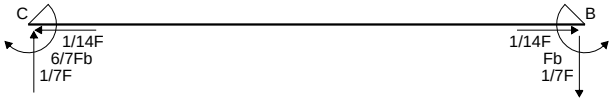
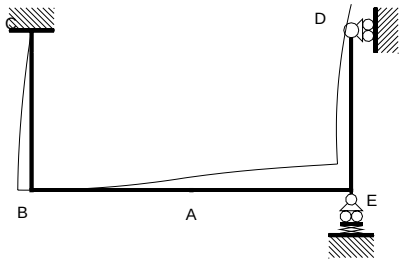


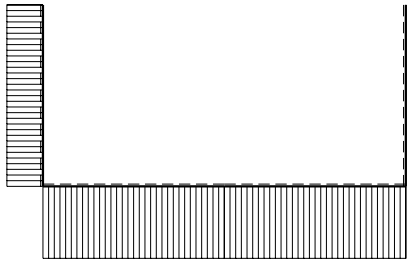




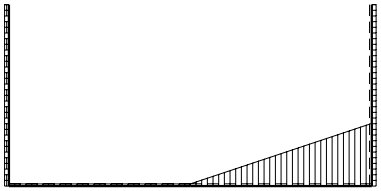




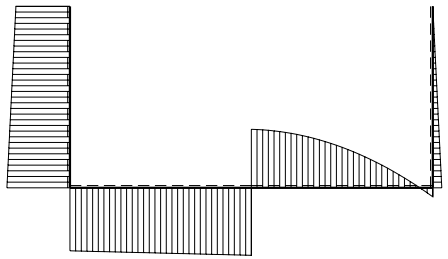
$\vdash$ 3 Fb^3/EJ



$\vdash$ 0.15 F



$\vdash$ 2.5 F



$\vdash$ 1.2 Fb

REAZIONI IPERSTATICHE

$X = W_{BC}$ $Y = W_{EA}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CB} K_{CB} φ_{DE} K_{DE} φ_{EA} K_{EA}

Relazioni di congruenza

$y'_{AB}(0) - y'_{EA}(b) = 0$
$y'_{AB}(b) - y'_{CB}(b) = 0$
$y'_{CB}(0) + \delta/b = 0$
$y'_{DE}(b) - y'_{EA}(0) = 0$
$y_{AB}(b) = 0$
$y_{CB}(0) = 0$
$y_{DE}(0) = 0$
$y_{DE}(b) - y_{CB}(b) = 0$
$y_{EA}(0) + 1/2V_E b^3/EJ = 0$
$y_{EA}(b) - y_{AB}(0) = 0$

$M_{AB} = 1/2Fx -1/2Fb -1/2Xx/b -1/2X +1/2Yx/b -1/2Y$
$EJy'' = 1/2Fx -1/2Fb -1/2Xx/b -1/2X +1/2Yx/b -1/2Y$
$EJy' = 1/4Fx^2 -1/2Fbx -1/4Xx^2/b -1/2Xx +1/4Yx^2/b -1/2Yx +EJ\varphi_{AB}$
$EJy = 1/12Fx^3 -1/4Fbx^2 -1/12Xx^3/b -1/4Xx^2 +1/12Yx^3/b -1/4Yx^2 +EJ\varphi_{AB}x +EJK_{AB}$

$M_{CB} = X +Yx/b -Y$
$EJy'' = X +Yx/b -Y$
$EJy' = Xx +1/2Yx^2/b -Yx +EJ\varphi_{CB}$
$EJy = 1/2Xx^2 +1/6Yx^3/b -1/2Yx^2 +EJ\varphi_{CB}x +EJK_{CB}$

$M_{DE} = -Yx/b$
$EJy'' = 2EJ\theta -Yx/b$
$EJy' = 2EJ\theta x -1/2Yx^2/b +EJ\varphi_{DE}$
$EJy = EJ\theta x^2 -1/6Yx^3/b +EJ\varphi_{DE}x +EJK_{DE}$

$M_{EA} = 5/2Fx -qx^2 -1/2Xx/b +1/2Yx/b -Y$
$EJy'' = 5/2Fx -qx^2 -1/2Xx/b +1/2Yx/b -Y$
$EJy' = 5/4Fx^2 -1/3qx^3 -1/4Xx^2/b +1/4Yx^2/b -Yx +EJ\varphi_{EA}$
$EJy = 5/12Fx^3 -1/12qx^4 -1/12Xx^3/b +1/12Yx^3/b -1/2Yx^2 +EJ\varphi_{EA}x +EJK_{EA}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CB}b$	K_{CB}	$\varphi_{DE}b$	K_{DE}	$\varphi_{EA}b$	K_{EA}	Xb^2/EJ	Yb^2/EJ		Wb^2/EJ
y'_{AB}	1	0	0	0	0	0	-1	0	1/4	3/4		11/12
y'_{BA}	1	0	-1	0	0	0	0	0	-7/4	1/4		1/4
y'_{CB}	0	0	1	0	0	0	0	0	0	0		0
y'_{ED}	0	0	0	0	1	0	-1	0	0	-1/2		0
y_{BA}	1	1	0	0	0	0	0	0	-1/3	-1/6	=	1/6
y_{CB}	0	0	0	1	0	0	0	0	0	0		0
y_{DE}	0	0	0	0	0	1	0	0	0	0		0
y_{ED}	0	0	-1	-1	1	1	0	0	-1/2	1/6		0
y_{EA}	0	0	0	0	0	0	0	1	-1/4	1/4		-5/4
y_{AE}	0	-1	0	0	0	0	1	1	-1/12	-5/12		-1/3

Condizioni al contorno

αTb	δ
0	0
0	0
0	-1
-2	0
0	0
0	0
0	0
-1	0
0	0
0	0

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		27/28
$\varphi_{CB}b$		-1
$\varphi_{EA}b$		17/42
$\varphi_{DE}b$		-32/21
K_{AB}		-37/84
K_{CB}	=	0
K_{DE}		0
Xb^2/EJ		1
K_{EA}		-29/28
Yb^2/EJ		1/7

DEFORMATA (coordinate locali)

AB $y(x)EJ = -37/84Fb^3 +27/28xFb^2 -15/28x^2Fb +1/84x^3F$
BA $y(x)EJ = 1/14xFb^2 -1/2x^2Fb -1/84x^3F$
CB $y(x)EJ = -xFb^2 +3/7x^2Fb +1/42x^3F$
BC $y(x)EJ = -23/42Fb^3 +1/14xFb^2 +1/2x^2Fb -1/42x^3F$
DE $y(x)EJ = -32/21xFb^2 +x^2Fb -1/42x^3F$
ED $y(x)EJ = -23/42Fb^3 -17/42xFb^2 +13/14x^2Fb +1/42x^3F$
EA $y(x)EJ = -29/28Fb^3 +17/42xFb^2 -1/14x^2Fb +29/84x^3F -1/12x^4q$
AE $y(x)EJ = -37/84Fb^3 -27/28xFb^2 +13/28x^2Fb -1/84x^3F -1/12x^4q$

SPOSTAMENTI NODALI

$u_A = -23/42(Fb^3/EJ)$ $u_B = -23/42(Fb^3/EJ)$ $u_C = 0$ $u_D = 0$
$v_A = 37/84(Fb^3/EJ)$ $v_B = 0$ $v_C = 0$ $v_{DDE} = 29/28(Fb^3/EJ)$
$\varphi_A = 27/28(Fb^2/EJ)$ $\varphi_B = -1/14(Fb^2/EJ)$ $\varphi_C = -(Fb^2/EJ)$ $\varphi_{DDE} = -32/21(Fb^2/EJ)$

$u_E = -23/42(Fb^3/EJ)$
$v_E = 29/28(Fb^3/EJ)$
$\varphi_E = 17/42(Fb^2/EJ)$

Es.N.004

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Es.N.004

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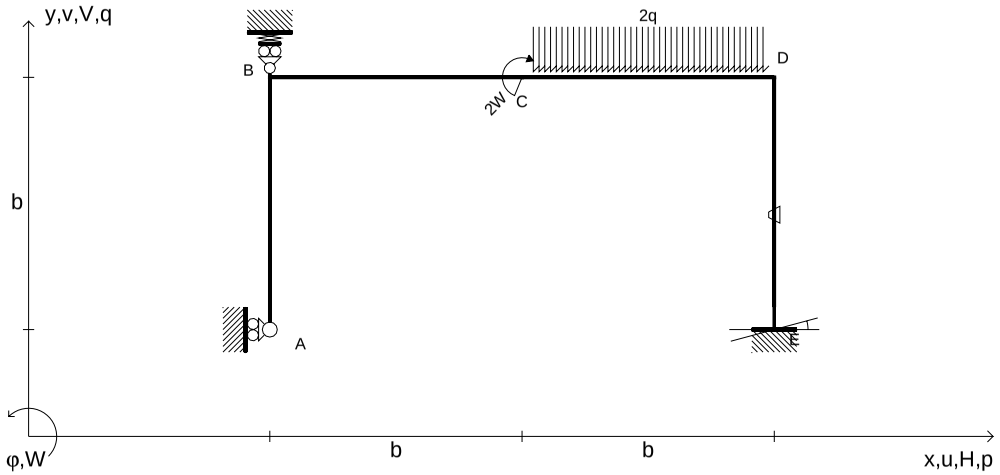
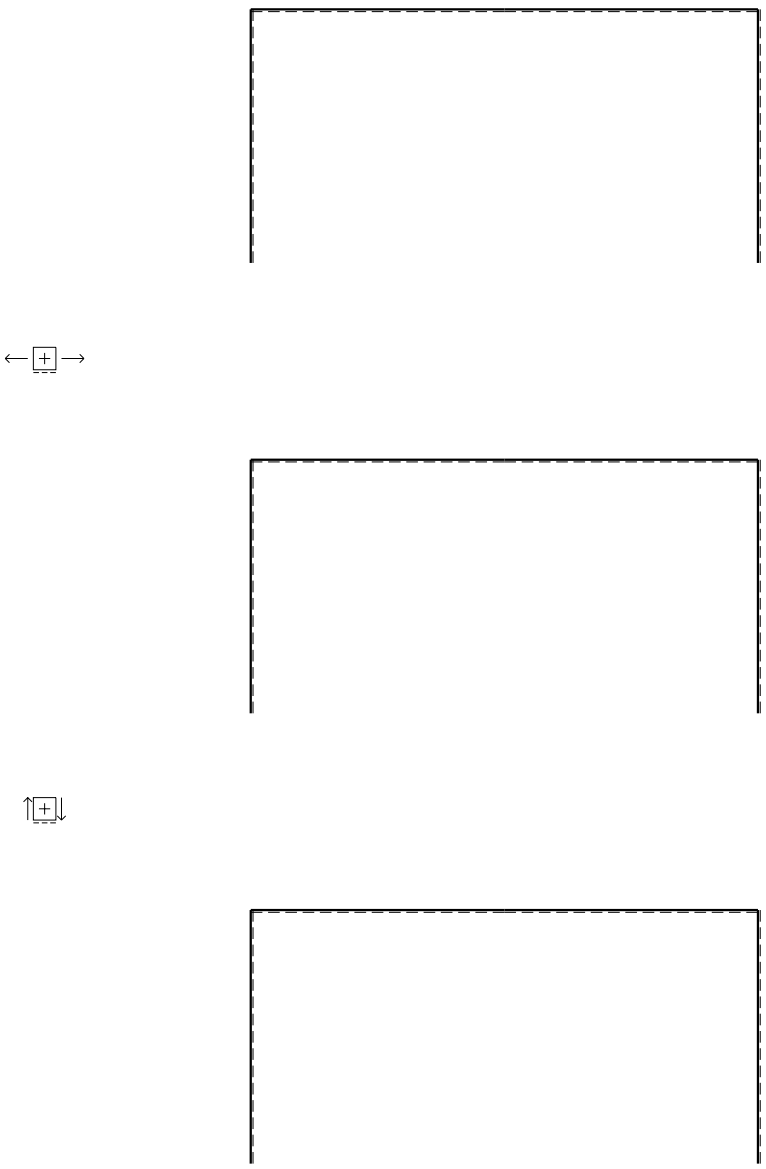


$W_C = -2W = -2Fb$ $\varphi_E = \delta/b = b^2F/EJ$ $EJ_{BC} = EJ$

$q_{CD} = -2q = -2F/b$ $k_B = 2EJ/b^3$ $EJ_{CD} = EJ$

$\theta_{ED} = \theta = \alpha T/b = bF/EJ$ $EJ_{AB} = EJ$ $EJ_{ED} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio.
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta ED positiva se convessa a destra con inizio E.
Rotazione assoluta W imposta al nodo E.



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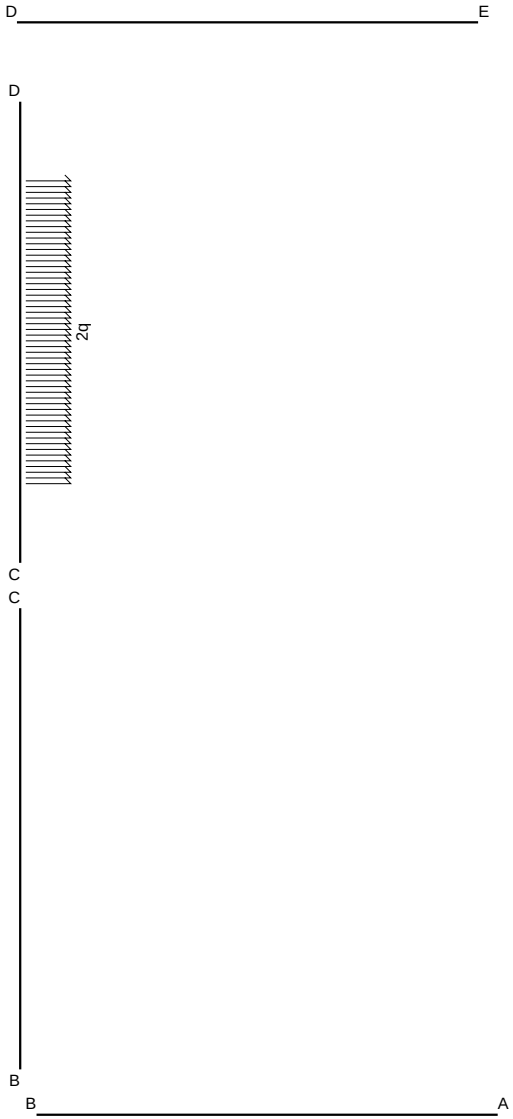
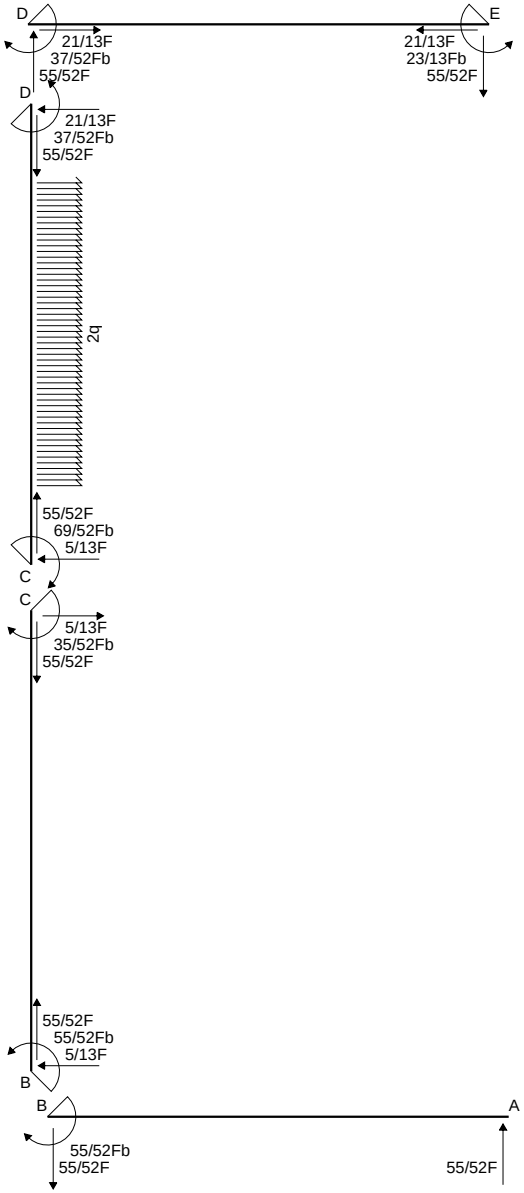


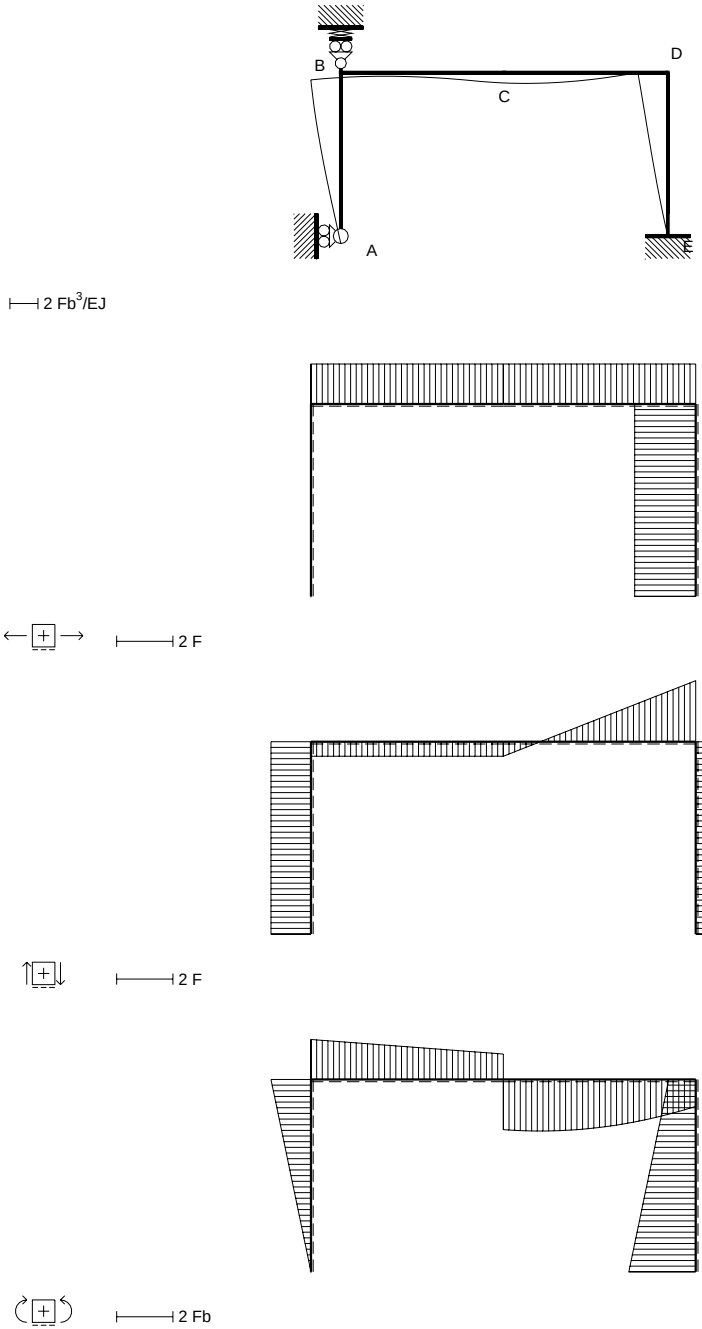
DEFORMATA (coordinate locali)
AB y(x)EJ =
BC y(x)EJ =
CD y(x)EJ =
ED y(x)EJ =

SPOSTAMENTI NODALI

u_A = u_B = u_C = u_D =
v_{AAB} = v_B = v_C = v_D =
φ_{AAB} = φ_B = φ_C = φ_D =

u_E =
v_E =
φ_E =





REAZIONI IPERSTATICHE

$X = W_{BC}$ $Y = W_{DE}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{BC} K_{BC} φ_{CD} K_{CD} φ_{ED} K_{ED}

Relazioni di congruenza

$y'_{AB}(b) - y'_{BC}(0) = 0$
$y'_{BC}(b) - y'_{CD}(0) = 0$
$y'_{CD}(b) - y'_{ED}(b) = 0$
$y'_{ED}(0) - \delta/b = 0$
$y_{AB}(0) = 0$
$y_{BC}(0) - 1/2V_B b^3/EJ = 0$
$y_{CD}(0) - y_{BC}(b) = 0$
$y_{CD}(b) = 0$
$y_{ED}(0) = 0$
$y_{ED}(b) - y_{AB}(b) = 0$

$M_{AB} = -Xx/b$
$EJy'' = -Xx/b$
$EJy' = -1/2Xx^2/b + EJ\varphi_{AB}$
$EJy = -1/6Xx^3/b + EJ\varphi_{AB}x + EJK_{AB}$

$M_{BC} = -1/2Fx + 1/2Xx/b - X - 1/2Yx/b$
$EJy'' = -1/2Fx + 1/2Xx/b - X - 1/2Yx/b$
$EJy' = -1/4Fx^2 + 1/4Xx^2/b - Xx - 1/4Yx^2/b + EJ\varphi_{BC}$
$EJy = -1/12Fx^3 + 1/12Xx^3/b - 1/2Xx^2 - 1/12Yx^3/b + EJ\varphi_{BC}x + EJK_{BC}$

$M_{CD} = -1/2Fx + 3/2Fb - qx^2 + 1/2Xx/b - 1/2X - 1/2Yx/b - 1/2Y$
$EJy'' = -1/2Fx + 3/2Fb - qx^2 + 1/2Xx/b - 1/2X - 1/2Yx/b - 1/2Y$
$EJy' = -1/4Fx^2 + 3/2Fbx - 1/3qx^3 + 1/4Xx^2/b - 1/2Xx - 1/4Yx^2/b - 1/2Yx + EJ\varphi_{CD}$
$EJy = -1/12Fx^3 + 3/4Fbx^2 - 1/12qx^4 + 1/12Xx^3/b - 1/4Xx^2 - 1/12Yx^3/b - 1/4Yx^2 + EJ\varphi_{CD}x + EJK_{CD}$

$M_{ED} = Xx/b - X + Y$
$EJy'' = EJ\theta + Xx/b - X + Y$
$EJy' = EJ\theta x + 1/2Xx^2/b - Xx + Yx + EJ\varphi_{ED}$
$EJy = 1/2EJ\theta x^2 + 1/6Xx^3/b - 1/2Xx^2 + 1/2Yx^2 + EJ\varphi_{ED}x + EJK_{ED}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{BC}b$	K_{BC}	$\varphi_{CD}b$	K_{CD}	$\varphi_{ED}b$	K_{ED}	Xb^2/EJ	Yb^2/EJ		Wb^2/EJ
y'_{BA}	1	0	-1	0	0	0	0	0	-1/2	0		0
y'_{CB}	0	0	1	0	-1	0	0	0	-3/4	-1/4		1/4
y'_{DC}	0	0	0	0	1	0	-1	0	1/4	-7/4		-11/12
y'_{ED}	0	0	0	0	0	0	1	0	0	0		0
y_{AB}	0	1	0	0	0	0	0	0	0	0		0
y_{BC}	0	0	0	1	0	0	0	0	1/4	-1/4	=	1/4
y_{CD}	0	0	-1	-1	0	1	0	0	5/12	1/12		-1/12
y_{DC}	0	0	0	0	1	1	0	0	-1/6	-1/3		-7/12
y_{ED}	0	0	0	0	0	0	0	1	0	0		0
y_{DE}	-1	-1	0	0	0	0	1	1	-1/6	1/2		0

Condizioni al contorno

αTb	δ
0	0
0	0
1	0
0	1
0	0
0	0
0	0
0	0
0	0
-1/2	0

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		151/156
$\varphi_{BC}b$		137/312
$\varphi_{CD}b$		-133/312
$\varphi_{ED}b$		1
K_{AB}		0
K_{BC}	=	-5/26
K_{CD}		-17/78
Xb^2/EJ		55/52
K_{ED}		0
Yb^2/EJ		-37/52

DEFORMATA (coordinate locali)

AB $y(x)EJ = 151/156xFb^2 - 55/312x^3F$
BA $y(x)EJ = 19/24Fb^3 - 137/312xFb^2 - 55/104x^2Fb + 55/312x^3F$
BC $y(x)EJ = -5/26Fb^3 + 137/312xFb^2 - 55/104x^2Fb + 5/78x^3F$
CB $y(x)EJ = -17/78Fb^3 + 133/312xFb^2 - 35/104x^2Fb - 5/78x^3F$
CD $y(x)EJ = -17/78Fb^3 - 133/312xFb^2 + 69/104x^2Fb + 5/78x^3F - 1/12x^4q$
DC $y(x)EJ = -79/104xFb^2 + 37/104x^2Fb + 7/26x^3F - 1/12x^4q$
ED $y(x)EJ = xFb^2 - 5/13x^2Fb + 55/312x^3F$
DE $y(x)EJ = 19/24Fb^3 - 79/104xFb^2 + 15/104x^2Fb - 55/312x^3F$

SPOSTAMENTI NODALI

$u_A = 0$	$u_B = -19/24(Fb^3/EJ)$	$u_C = -19/24(Fb^3/EJ)$	$u_D = -19/24(Fb^3/EJ)$
$v_{AAB} = -5/26(Fb^3/EJ)$	$v_B = -5/26(Fb^3/EJ)$	$v_C = -17/78(Fb^3/EJ)$	$v_D = 0$
$\varphi_{AAB} = 151/156(Fb^2/EJ)$	$\varphi_B = 137/312(Fb^2/EJ)$	$\varphi_C = -133/312(Fb^2/EJ)$	$\varphi_D = 79/104(Fb^2/EJ)$

$u_E = 0$
$v_E = 0$
$\varphi_E = (Fb^2/EJ)$

Es.N.005

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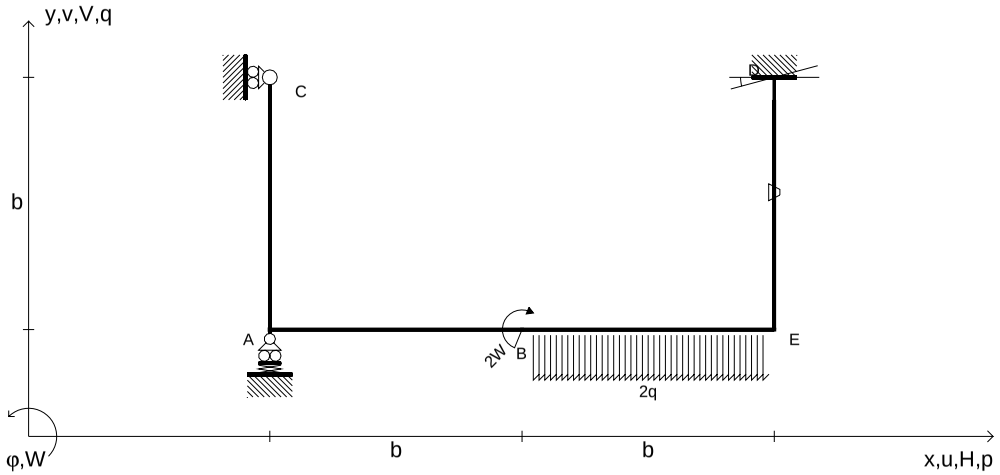

$W_B = -2W = -2Fb$ $\varphi_D = \delta/b = b^2F/EJ$ $EJ_{CA} = EJ$
$q_{BE} = -2q = -2F/b$ $k_A = 2EJ/b^3$ $EJ_{DE} = EJ$
$\theta_{DE} = \theta = \alpha T/b = bF/EJ$ $EJ_{AB} = EJ$ $EJ_{BE} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio.
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta DE positiva se convessa a destra con inizio D.
Rotazione assoluta W imposta al nodo D.

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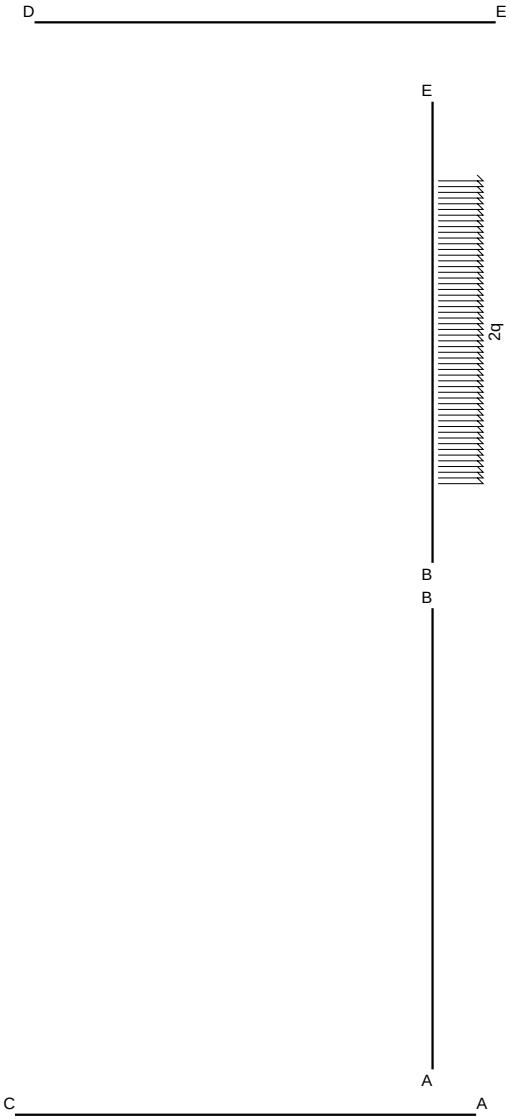

DEFORMATA (coordinate locali)
AB y(x)EJ =
CA y(x)EJ =
DE y(x)EJ =
BE y(x)EJ =

SPOSTAMENTI NODALI

u_A =
v_A =
φ_A =

u_B =
v_B =
φ_B =

u_C =
v_{CCA} =
φ_{CCA} =

u_D =
v_D =
φ_D =

u_E =
v_E =
φ_E =

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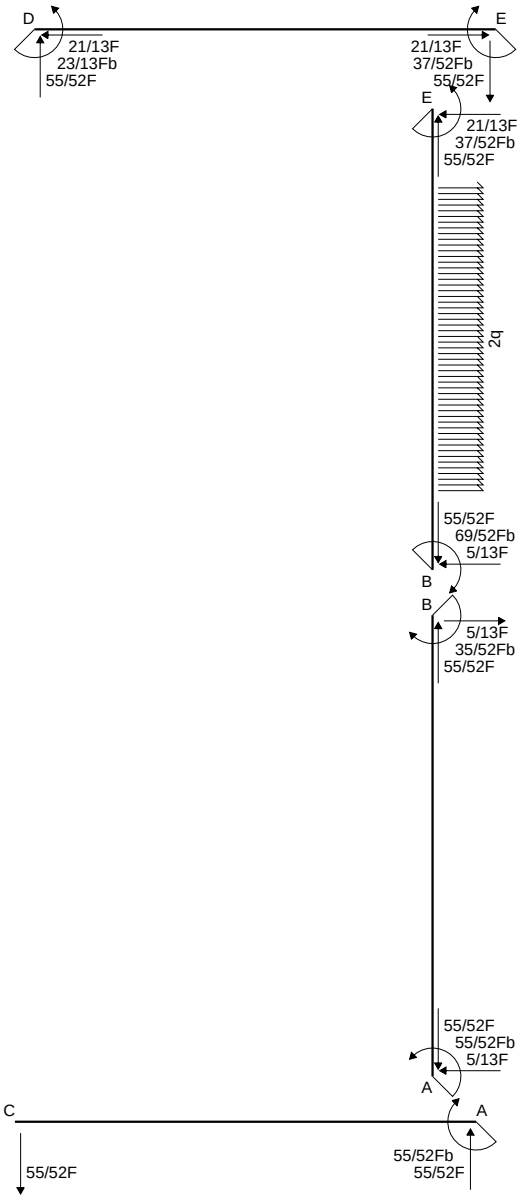
D
21/13F
23/13Fb
55/52F
E
21/13F
37/52Fb
55/52F
E
21/13F
37/52Fb
55/52F
2q
55/52F
69/52Fb
5/13F
B
B
5/13F
35/52Fb
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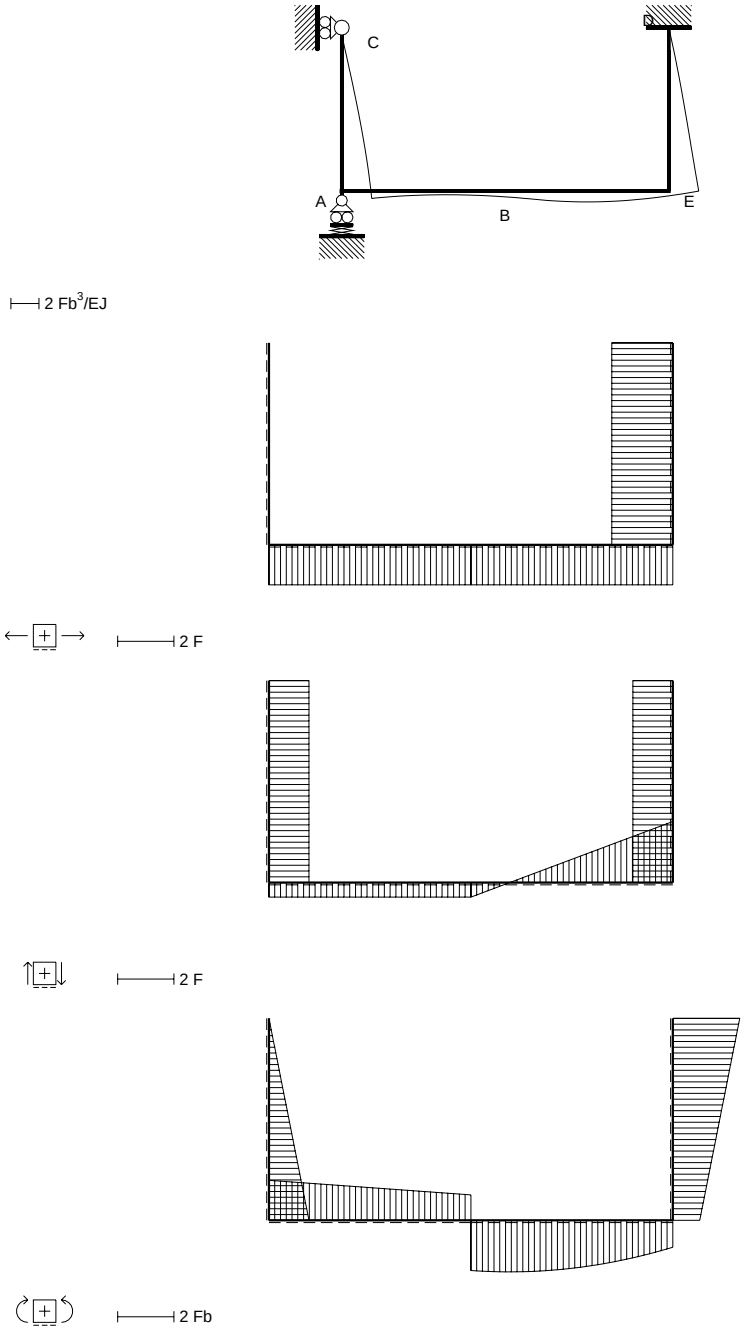
55/52Fb
5/13F
A
C
A
55/52Fb
55/52F
55/52F


C
D
A
B
E
2 Fb³/EJ
2 F
2 F
2 Fb

REAZIONI IPERSTATICHE

$X = W_{AB}$ $Y = W_{ED}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: $\varphi_{AB}\ K_{AB}\ \varphi_{CA}\ K_{CA}\ \varphi_{DE}\ K_{DE}\ \varphi_{BE}\ K_{BE}$

Relazioni di congruenza

$y'_{AB}(0) - y'_{CA}(b) = 0$
$y'_{AB}(b) - y'_{BE}(0) = 0$
$y'_{DE}(0) - \delta/b = 0$
$y'_{DE}(b) - y'_{BE}(b) = 0$
$y_{AB}(0) - 1/2V_A b^3/EJ = 0$
$y_{CA}(0) = 0$
$y_{DE}(0) = 0$
$y_{DE}(b) - y_{CA}(b) = 0$
$y_{BE}(0) - y_{AB}(b) = 0$
$y_{BE}(b) = 0$

$M_{AB} = -1/2Fx +1/2Xx/b -X -1/2Yx/b$
$EJy'' = -1/2Fx +1/2Xx/b -X -1/2Yx/b$
$EJy' = -1/4Fx^2 +1/4Xx^2/b -Xx -1/4Yx^2/b +EJ\varphi_{AB}$
$EJy = -1/12Fx^3 +1/12Xx^3/b -1/2Xx^2 -1/12Yx^3/b +EJ\varphi_{AB}x +EJK_{AB}$

$M_{CA} = -Xx/b$
$EJy'' = -Xx/b$
$EJy' = -1/2Xx^2/b +EJ\varphi_{CA}$
$EJy = -1/6Xx^3/b +EJ\varphi_{CA}x +EJK_{CA}$

$M_{DE} = Xx/b -X +Y$
$EJy'' = EJ\theta +Xx/b -X +Y$
$EJy' = EJ\theta x +1/2Xx^2/b -Xx +Yx +EJ\varphi_{DE}$
$EJy = 1/2EJ\theta x^2 +1/6Xx^3/b -1/2Xx^2 +1/2Yx^2 +EJ\varphi_{DE}x +EJK_{DE}$

$M_{BE} = -1/2Fx +3/2Fb -qx^2 +1/2Xx/b -1/2X -1/2Yx/b -1/2Y$
$EJy'' = -1/2Fx +3/2Fb -qx^2 +1/2Xx/b -1/2X -1/2Yx/b -1/2Y$
$EJy' = -1/4Fx^2 +3/2Fbx -1/3qx^3 +1/4Xx^2/b -1/2Xx -1/4Yx^2/b -1/2Yx +EJ\varphi_{BE}$
$EJy = -1/12Fx^3 +3/4Fbx^2 -1/12qx^4 +1/12Xx^3/b -1/4Xx^2 -1/12Yx^3/b -1/4Yx^2 +EJ\varphi_{BE}x +EJK_{BE}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CA}b$	K_{CA}	$\varphi_{DE}b$	K_{DE}	$\varphi_{BE}b$	K_{BE}	Xb^2/EJ	Yb^2/EJ		Wb^2/EJ
y'_{AB}	1	0	-1	0	0	0	0	0	1/2	0		0
y'_{BA}	1	0	0	0	0	0	-1	0	-3/4	-1/4		1/4
y'_{DE}	0	0	0	0	1	0	0	0	0	0		0
y'_{ED}	0	0	0	0	1	0	-1	0	-1/4	7/4		11/12
y_{AB}	0	1	0	0	0	0	0	0	1/4	-1/4		1/4
y_{CA}	0	0	0	1	0	0	0	0	0	0	=	0
y_{DE}	0	0	0	0	0	1	0	0	0	0		0
y_{ED}	0	0	-1	-1	1	1	0	0	-1/6	1/2		0
y_{BE}	-1	-1	0	0	0	0	0	1	5/12	1/12		-1/12
y_{EB}	0	0	0	0	0	0	1	1	-1/6	-1/3		-7/12

Condizioni al contorno

αTb	δ
0	0
0	0
0	1
-1	0
0	0
0	0
0	0
-1/2	0
0	0
0	0

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		137/312
$\varphi_{CA}b$		151/156
$\varphi_{DE}b$		1
$\varphi_{BE}b$		-133/312
K_{AB}		-5/26
K_{CA}	=	0
K_{DE}		0
Xb^2/EJ		55/52
K_{BE}		-17/78
Yb^2/EJ		-37/52

DEFORMATA (coordinate locali)

AB $y(x)EJ = -5/26Fb^3 +137/312xFb^2 -55/104x^2Fb +5/78x^3F$
BA $y(x)EJ = -17/78Fb^3 +133/312xFb^2 -35/104x^2Fb -5/78x^3F$
CA $y(x)EJ = 151/156xFb^2 -55/312x^3F$
AC $y(x)EJ = 19/24Fb^3 -137/312xFb^2 -55/104x^2Fb +55/312x^3F$
DE $y(x)EJ = xFb^2 -5/13x^2Fb +55/312x^3F$
ED $y(x)EJ = 19/24Fb^3 -79/104xFb^2 +15/104x^2Fb -55/312x^3F$
BE $y(x)EJ = -17/78Fb^3 -133/312xFb^2 +69/104x^2Fb +5/78x^3F -1/12x^4q$
EB $y(x)EJ = -79/104xFb^2 +37/104x^2Fb +7/26x^3F -1/12x^4q$

SPOSTAMENTI NODALI

$u_A = 19/24(Fb^3/EJ)$	$u_B = 19/24(Fb^3/EJ)$	$u_C = 0$	$u_D = 0$
$v_A = -5/26(Fb^3/EJ)$	$v_B = -17/78(Fb^3/EJ)$	$v_{CCA} = -5/26(Fb^3/EJ)$	$v_D = 0$
$\varphi_A = 137/312(Fb^2/EJ)$	$\varphi_B = -133/312(Fb^2/EJ)$	$\varphi_{CCA} = 151/156(Fb^2/EJ)$	$\varphi_D = (Fb^2/EJ)$

$u_E = 19/24(Fb^3/EJ)$
$v_E = 0$
$\varphi_E = 79/104(Fb^2/EJ)$

Es.N.006

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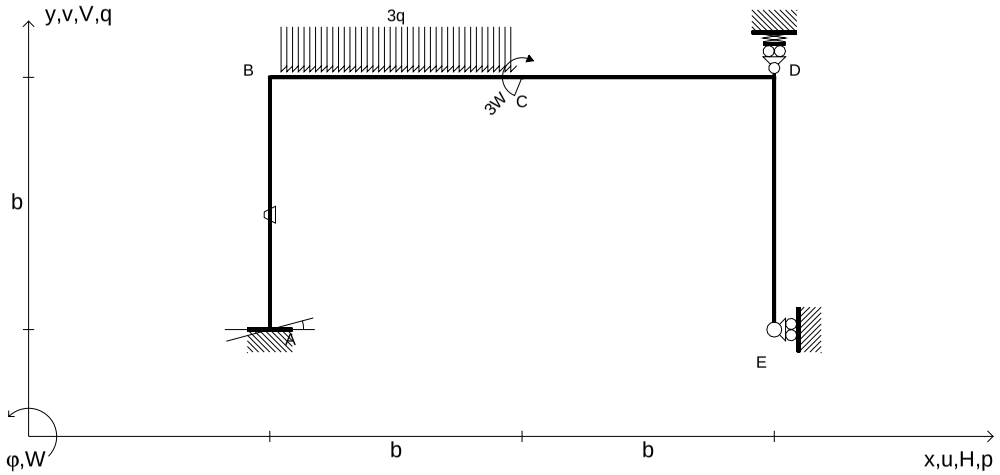

$W_C = -3W = -3Fb$ $\varphi_A = 2\delta/b = 2b^2F/EJ$ $EJ_{CB} = EJ$

$q_{CB} = -3q = -3F/b$ $k_D = 2EJ/b^3$ $EJ_{DC} = EJ$

$\theta_{AB} = \theta = \alpha T/b = bF/EJ$ $EJ_{AB} = EJ$ $EJ_{ED} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio.
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Rotazione assoluta W imposta al nodo A.

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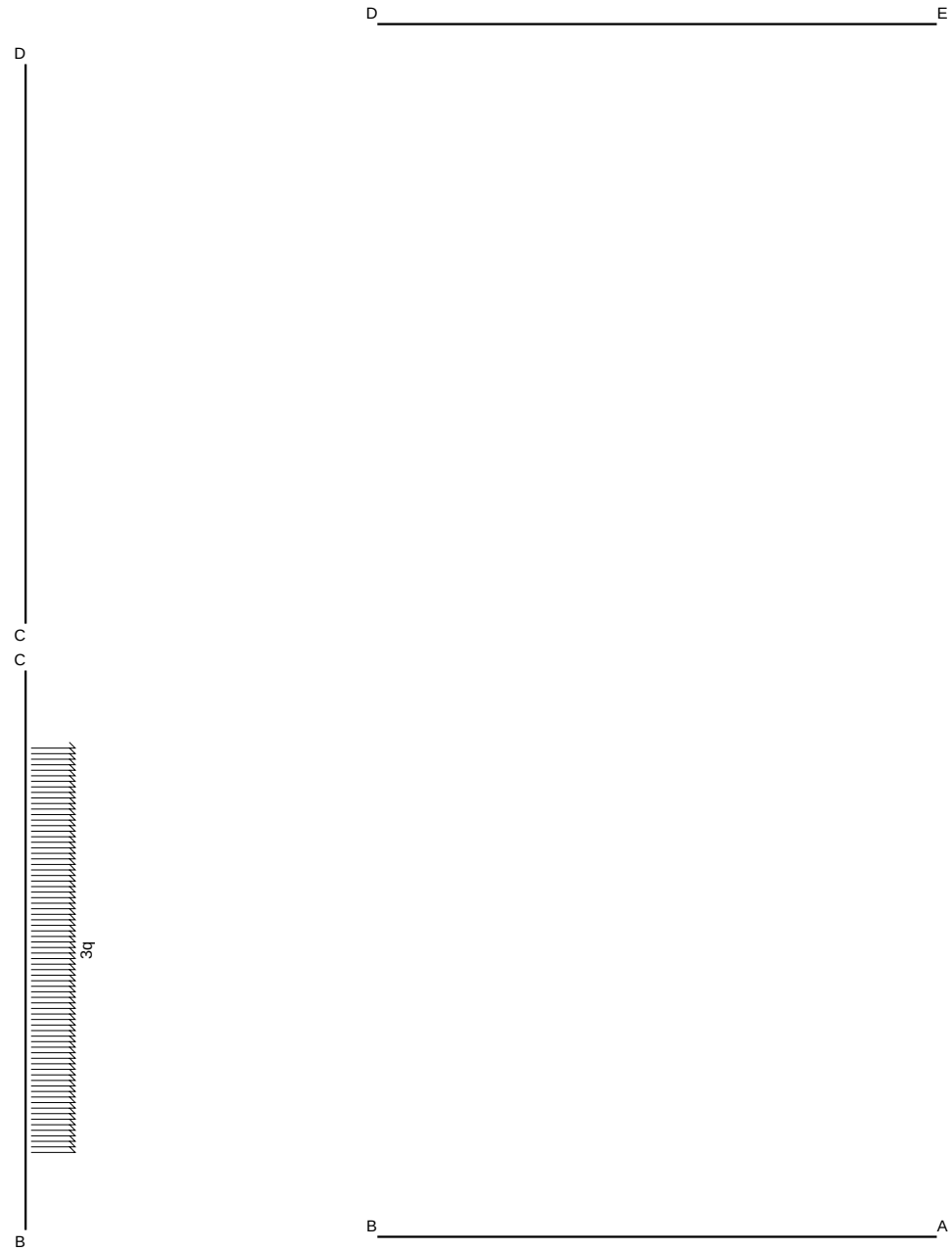


DEFORMATA (coordinate locali)
AB y(x)EJ =
CB y(x)EJ =
DC y(x)EJ =
ED y(x)EJ =

SPOSTAMENTI NODALI

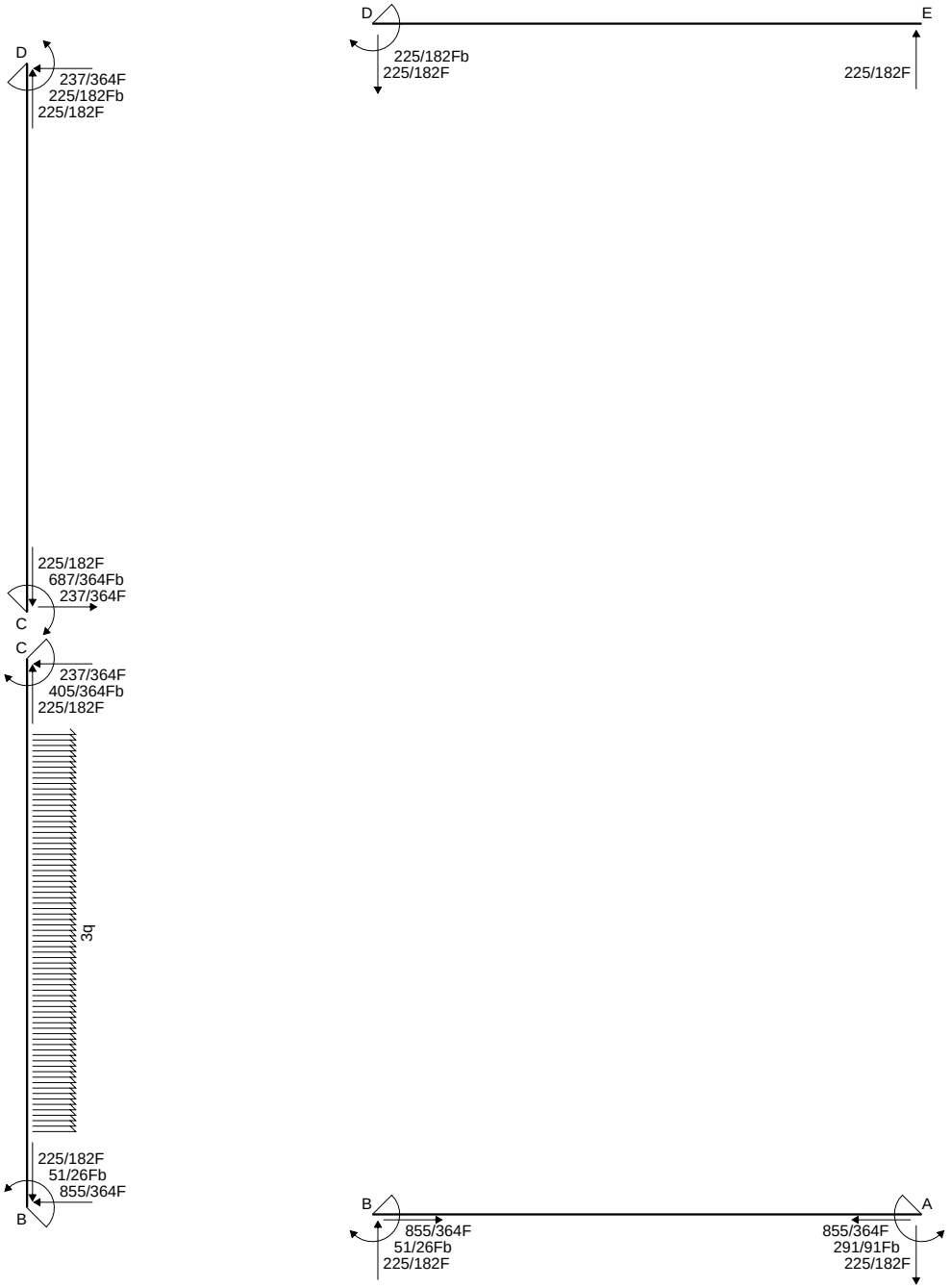
u_A =
v_A =
φ_A =

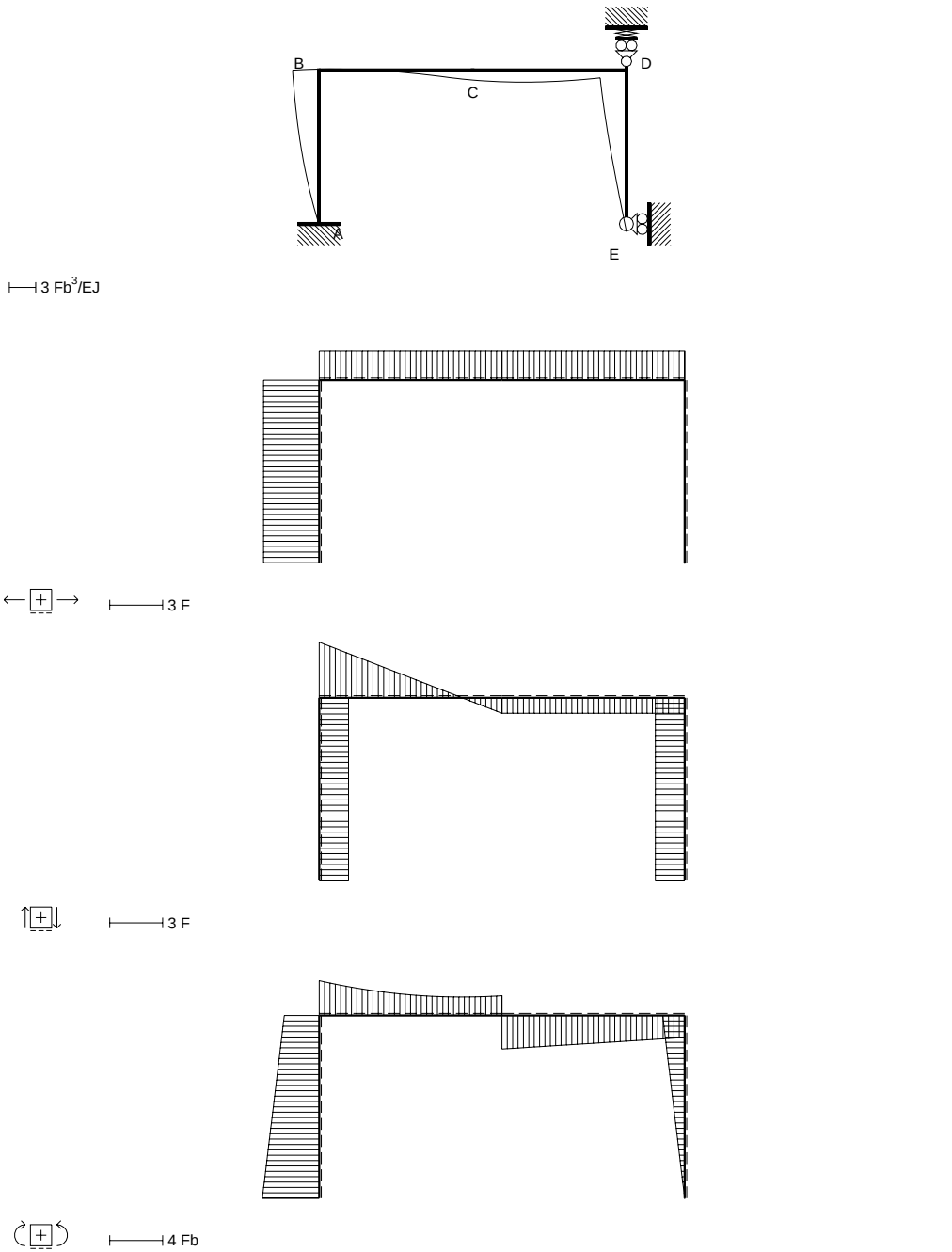
u_B =
v_B =
φ_B =

u_C =
v_C =
φ_C =

u_D =
v_D =
φ_D =

u_E =
v_{EED} =
φ_{EED} =





REAZIONI IPERSTATICHE

$X = W_{BA}$ $Y = W_{DC}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CB} K_{CB} φ_{DC} K_{DC} φ_{ED} K_{ED}

Relazioni di congruenza

$y'_{AB}(0) - 2\delta/b = 0$
$y'_{AB}(b) - y'_{CB}(b) = 0$
$y'_{CB}(0) - y'_{DC}(b) = 0$
$y'_{DC}(0) - y'_{ED}(b) = 0$
$y_{AB}(0) = 0$
$y_{CB}(b) = 0$
$y_{DC}(0) + 1/2V_D b^3/EJ = 0$
$y_{DC}(b) - y_{CB}(0) = 0$
$y_{ED}(0) = 0$
$y_{ED}(b) - y_{AB}(b) = 0$

$M_{AB} = X + Yx/b - Y$
$EJy'' = EJ\theta + X + Yx/b - Y$
$EJy' = EJ\theta x + Xx + 1/2Yx^2/b - Yx + EJ\varphi_{AB}$
$EJy = 1/2EJ\theta x^2 + 1/2Xx^2 + 1/6Yx^3/b - 1/2Yx^2 + EJ\varphi_{AB}x + EJK_{AB}$

$M_{CB} = -9/4Fx + 3/4Fb + 3/2qx^2 - 1/2Xx/b - 1/2X + 1/2Yx/b - 1/2Y$
$EJy'' = -9/4Fx + 3/4Fb + 3/2qx^2 - 1/2Xx/b - 1/2X + 1/2Yx/b - 1/2Y$
$EJy' = -9/8Fx^2 + 3/4Fbx + 1/2qx^3 - 1/4Xx^2/b - 1/2Xx + 1/4Yx^2/b - 1/2Yx + EJ\varphi_{CB}$
$EJy = -3/8Fx^3 + 3/8Fbx^2 + 1/8qx^4 - 1/12Xx^3/b - 1/4Xx^2 + 1/12Yx^3/b - 1/4Yx^2 + EJ\varphi_{CB}x + EJK_{CB}$

$M_{DC} = -9/4Fx - 1/2Xx/b + 1/2Yx/b - Y$
$EJy'' = -9/4Fx - 1/2Xx/b + 1/2Yx/b - Y$
$EJy' = -9/8Fx^2 - 1/4Xx^2/b + 1/4Yx^2/b - Yx + EJ\varphi_{DC}$
$EJy = -3/8Fx^3 - 1/12Xx^3/b + 1/12Yx^3/b - 1/2Yx^2 + EJ\varphi_{DC}x + EJK_{DC}$

$M_{ED} = -Yx/b$
$EJy'' = -Yx/b$
$EJy' = -1/2Yx^2/b + EJ\varphi_{ED}$
$EJy = -1/6Yx^3/b + EJ\varphi_{ED}x + EJK_{ED}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CB}b$	K_{CB}	$\varphi_{DC}b$	K_{DC}	$\varphi_{ED}b$	K_{ED}	Xb^2/EJ	Yb^2/EJ		Wb^2/EJ
y'_{AB}	1	0	0	0	0	0	0	0	0	0		0
y'_{BA}	1	0	-1	0	0	0	0	0	7/4	-1/4		1/8
y'_{CB}	0	0	1	0	-1	0	0	0	1/4	3/4		-9/8
y'_{DC}	0	0	0	0	1	0	-1	0	0	1/2		0
y_{AB}	0	1	0	0	0	0	0	0	0	0		0
y_{BC}	0	0	1	1	0	0	0	0	-1/3	-1/6	=	-1/8
y_{DC}	0	0	0	0	0	1	0	0	-1/4	1/4		9/8
y_{CD}	0	0	0	-1	1	1	0	0	-1/12	-5/12		3/8
y_{ED}	0	0	0	0	0	0	0	1	0	0		0
y_{DE}	-1	-1	0	0	0	0	1	1	-1/2	1/6		0

Condizioni al contorno

αTb	δ
0	2
-1	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
1/2	0

Soluzione

$\varphi_{AB}b$		2
$\varphi_{CB}b$		-631/728
$\varphi_{DC}b$		253/364
$\varphi_{ED}b$		239/182
K_{AB}		0
K_{CB}	=	107/364
K_{DC}		237/728
Xb^2/EJ		-51/26
K_{ED}		0
Yb^2/EJ		225/182

DEFORMATA (coordinate locali)

AB $y(x)EJ = 2xFb^2 - 100/91x^2Fb + 75/364x^3F$
BA $y(x)EJ = 31/28Fb^3 - 153/364xFb^2 - 25/52x^2Fb - 75/364x^3F$
CB $y(x)EJ = 107/364Fb^3 - 631/728xFb^2 + 405/728x^2Fb - 79/728x^3F + 1/8x^4q$
BC $y(x)EJ = -153/364xFb^2 + 51/52x^2Fb - 285/728x^3F + 1/8x^4q$
DC $y(x)EJ = 237/728Fb^3 + 253/364xFb^2 - 225/364x^2Fb - 79/728x^3F$
CD $y(x)EJ = 107/364Fb^3 + 631/728xFb^2 - 687/728x^2Fb + 79/728x^3F$
ED $y(x)EJ = 239/182xFb^2 - 75/364x^3F$
DE $y(x)EJ = 31/28Fb^3 - 253/364xFb^2 - 225/364x^2Fb + 75/364x^3F$

SPOSTAMENTI NODALI

$u_A = 0$ $u_B = -31/28(Fb^3/EJ)$ $u_C = -31/28(Fb^3/EJ)$
$v_A = 0$ $v_B = 0$ $v_C = -107/364(Fb^3/EJ)$
$\varphi_A = 2(Fb^2/EJ)$ $\varphi_B = 153/364(Fb^2/EJ)$ $\varphi_C = -631/728(Fb^2/EJ)$

$u_D = -31/28(Fb^3/EJ)$ $u_E = 0$

$v_D = -237/728(Fb^3/EJ)$ $v_{EED} = -237/728(Fb^3/EJ)$

$\varphi_D = 253/364(Fb^2/EJ)$ $\varphi_{EED} = 239/182(Fb^2/EJ)$

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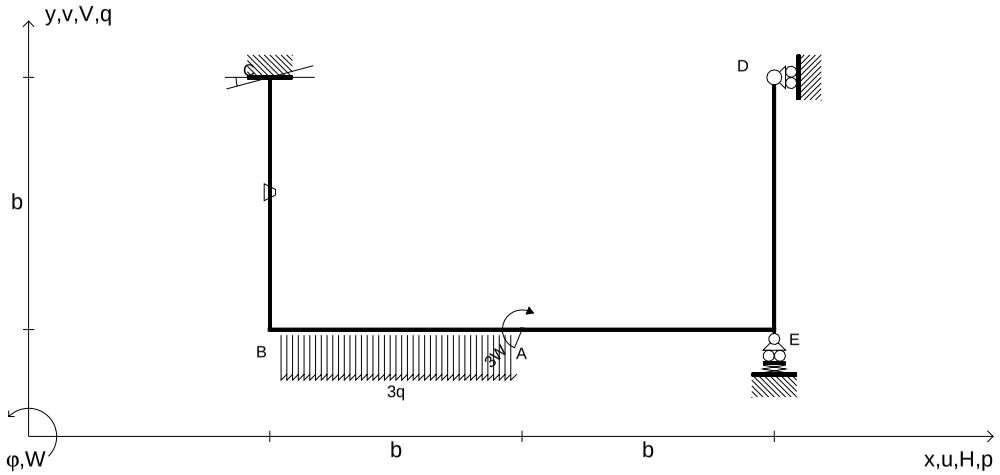

$W_A = -3W = -3Fb$ $\varphi_C = 2\delta/b = 2b^2F/EJ$ $EJ_{CB} = EJ$

$q_{AB} = -3q = -3F/b$ $k_E = 2EJ/b^3$ $EJ_{DE} = EJ$

$\theta_{CB} = \theta = \alpha T/b = bF/EJ$ $EJ_{AB} = EJ$ $EJ_{EA} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio.
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta CB positiva se convessa a destra con inizio C.
Rotazione assoluta W imposta al nodo C.

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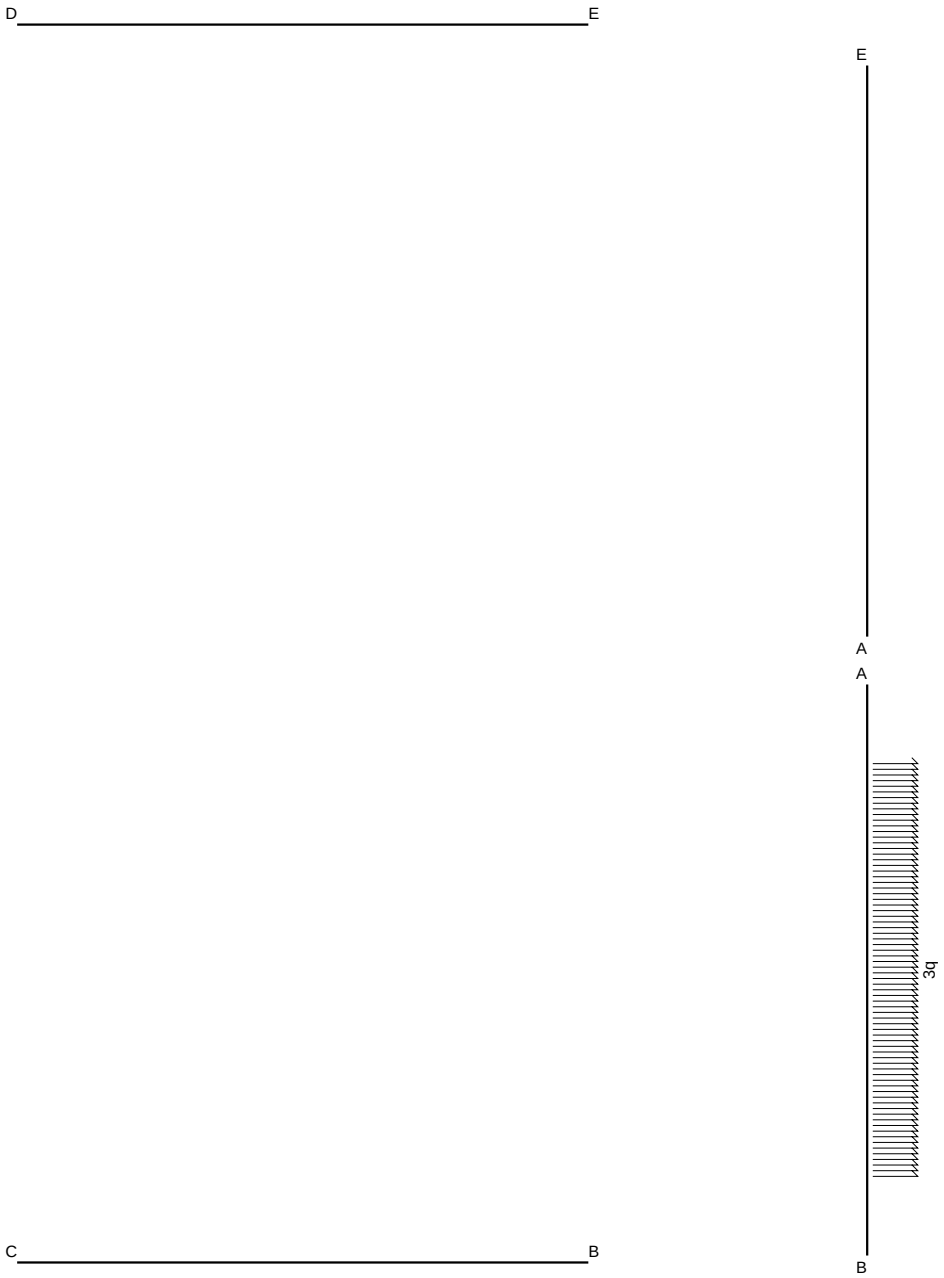

DEFORMATA (coordinate locali)
AB y(x)EJ =
CB y(x)EJ =
DE y(x)EJ =
EA y(x)EJ =

SPOSTAMENTI NODALI

u_A =
v_A =
φ_A =

u_B =
v_B =
φ_B =

u_C =
v_C =
φ_C =

u_D =
v_{DDE} =
φ_{DDE} =

u_E =
v_E =
φ_E =

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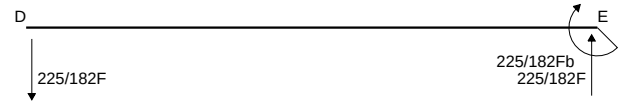

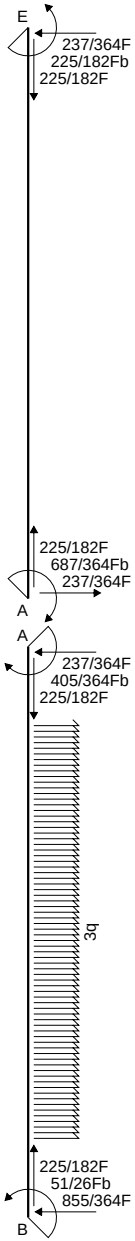

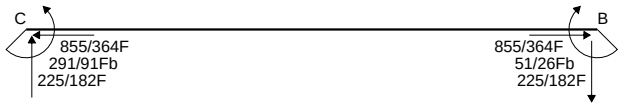

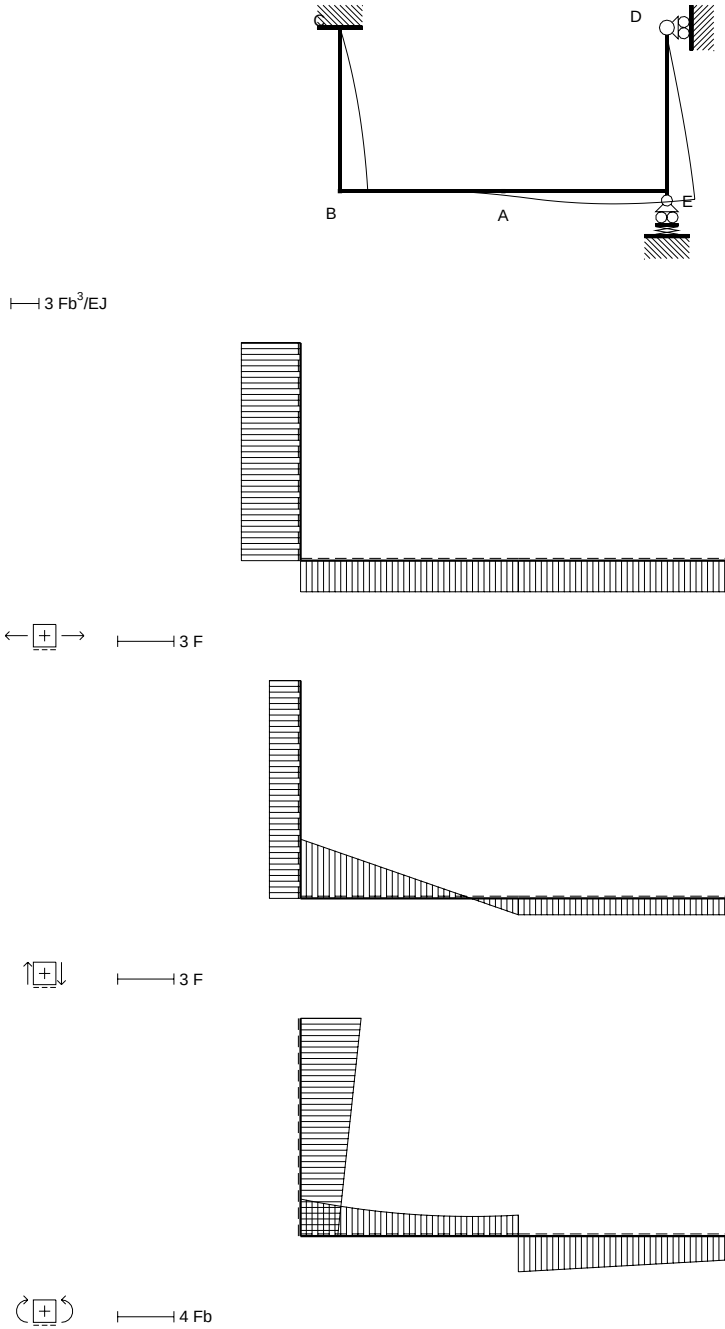

REAZIONI IPERSTATICHE

$X = W_{BC}$ $Y = W_{EA}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CB} K_{CB} φ_{DE} K_{DE} φ_{EA} K_{EA}

Relazioni di congruenza

$y'_{AB}(0) - y'_{EA}(b) = 0$
$y'_{AB}(b) - y'_{CB}(b) = 0$
$y'_{CB}(0) - 2\delta/b = 0$
$y'_{DE}(b) - y'_{EA}(0) = 0$
$y_{AB}(b) = 0$
$y_{CB}(0) = 0$
$y_{DE}(0) = 0$
$y_{DE}(b) - y_{CB}(b) = 0$
$y_{EA}(0) + 1/2V_E b^3/EJ = 0$
$y_{EA}(b) - y_{AB}(0) = 0$

$M_{AB} = -9/4Fx +3/4Fb +3/2qx^2 -1/2Xx/b -1/2X +1/2Yx/b -1/2Y$
$EJy'' = -9/4Fx +3/4Fb +3/2qx^2 -1/2Xx/b -1/2X +1/2Yx/b -1/2Y$
$EJy' = -9/8Fx^2 +3/4Fbx +1/2qx^3 -1/4Xx^2/b -1/2Xx +1/4Yx^2/b -1/2Yx +EJ\varphi_{AB}$
$EJy = -3/8Fx^3 +3/8Fbx^2 +1/8qx^4 -1/12Xx^3/b -1/4Xx^2 +1/12Yx^3/b -1/4Yx^2 +EJ\varphi_{AB}x +EJK_{AB}$

$M_{CB} = X +Yx/b -Y$
$EJy'' = EJ\theta +X +Yx/b -Y$
$EJy' = EJ\theta x +Xx +1/2Yx^2/b -Yx +EJ\varphi_{CB}$
$EJy = 1/2EJ\theta x^2 +1/2Xx^2 +1/6Yx^3/b -1/2Yx^2 +EJ\varphi_{CB}x +EJK_{CB}$

$M_{DE} = -Yx/b$
$EJy'' = -Yx/b$
$EJy' = -1/2Yx^2/b +EJ\varphi_{DE}$
$EJy = -1/6Yx^3/b +EJ\varphi_{DE}x +EJK_{DE}$

$M_{EA} = -9/4Fx -1/2Xx/b +1/2Yx/b -Y$
$EJy'' = -9/4Fx -1/2Xx/b +1/2Yx/b -Y$
$EJy' = -9/8Fx^2 -1/4Xx^2/b +1/4Yx^2/b -Yx +EJ\varphi_{EA}$
$EJy = -3/8Fx^3 -1/12Xx^3/b +1/12Yx^3/b -1/2Yx^2 +EJ\varphi_{EA}x +EJK_{EA}$

PROCEDIMENTO E RISULTATI Es.N.008

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CB}b$	K_{CB}	$\varphi_{DE}b$	K_{DE}	$\varphi_{EA}b$	K_{EA}	Xb^2/EJ	Yb^2/EJ		Wb^2/EJ
y'_{AB}	1	0	0	0	0	0	-1	0	1/4	3/4		-9/8
y'_{BA}	1	0	-1	0	0	0	0	0	-7/4	1/4		-1/8
y'_{CB}	0	0	1	0	0	0	0	0	0	0		0
y'_{ED}	0	0	0	0	1	0	-1	0	0	-1/2		0
y_{BA}	1	1	0	0	0	0	0	0	-1/3	-1/6	=	-1/8
y_{CB}	0	0	0	1	0	0	0	0	0	0		0
y_{DE}	0	0	0	0	0	1	0	0	0	0		0
y_{ED}	0	0	-1	-1	1	1	0	0	-1/2	1/6		0
y_{EA}	0	0	0	0	0	0	0	1	-1/4	1/4		9/8
y_{AE}	0	-1	0	0	0	0	1	1	-1/12	-5/12		3/8

Condizioni al contorno

αTb	δ
0	0
1	0
0	2
0	0
0	0
0	0
0	0
1/2	0
0	0
0	0

Soluzione

$\varphi_{AB}b$	=	Fb^3/EJ
$\varphi_{CB}b$		-631/728
$\varphi_{EA}b$		2
$\varphi_{DE}b$		253/364
K_{AB}		239/182
K_{CB}		107/364
K_{DE}		0
Xb^2/EJ		0
K_{EA}		-51/26
Yb^2/EJ		237/728
		225/182

DEFORMATA (coordinate locali)

AB $y(x)EJ = 107/364Fb^3 -631/728xFb^2 +405/728x^2Fb -79/728x^3F +1/8x^4q$
BA $y(x)EJ = -153/364xFb^2 +51/52x^2Fb -285/728x^3F +1/8x^4q$
CB $y(x)EJ = 2xFb^2 -100/91x^2Fb +75/364x^3F$
BC $y(x)EJ = 31/28Fb^3 -153/364xFb^2 -25/52x^2Fb -75/364x^3F$
DE $y(x)EJ = 239/182xFb^2 -75/364x^3F$
ED $y(x)EJ = 31/28Fb^3 -253/364xFb^2 -225/364x^2Fb +75/364x^3F$
EA $y(x)EJ = 237/728Fb^3 +253/364xFb^2 -225/364x^2Fb -79/728x^3F$
AE $y(x)EJ = 107/364Fb^3 +631/728xFb^2 -687/728x^2Fb +79/728x^3F$

SPOSTAMENTI NODALI

$u_A = 31/28(Fb^3/EJ)$ $u_B = 31/28(Fb^3/EJ)$ $u_C = 0$
$v_A = -107/364(Fb^3/EJ)$ $v_B = 0$ $v_C = 0$
$\varphi_A = -631/728(Fb^2/EJ)$ $\varphi_B = 153/364(Fb^2/EJ)$ $\varphi_C = 2(Fb^2/EJ)$

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$u_D = 0$ $u_E = 31/28(Fb^3/EJ)$

$v_{DDE} = -237/728(Fb^3/EJ)$ $v_E = -237/728(Fb^3/EJ)$

$\varphi_{DDE} = 239/182(Fb^2/EJ)$ $\varphi_E = 253/364(Fb^2/EJ)$

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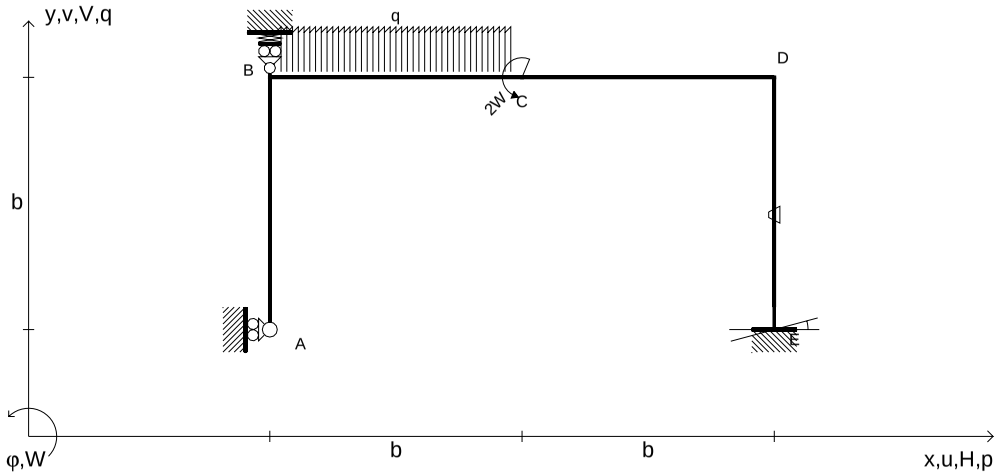

$W_C = 2W = 2Fb$ $\varphi_E = 2\delta/b = 2b^2F/EJ$ $EJ_{BC} = EJ$

$q_{BC} = q = F/b$ $k_B = 2EJ/b^3$ $EJ_{CD} = EJ$

$\theta_{ED} = 2\theta = 2\alpha T/b = 2bF/EJ$ $EJ_{AB} = EJ$ $EJ_{ED} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio.
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta ED positiva se convessa a destra con inizio E.
Rotazione assoluta W imposta al nodo E.

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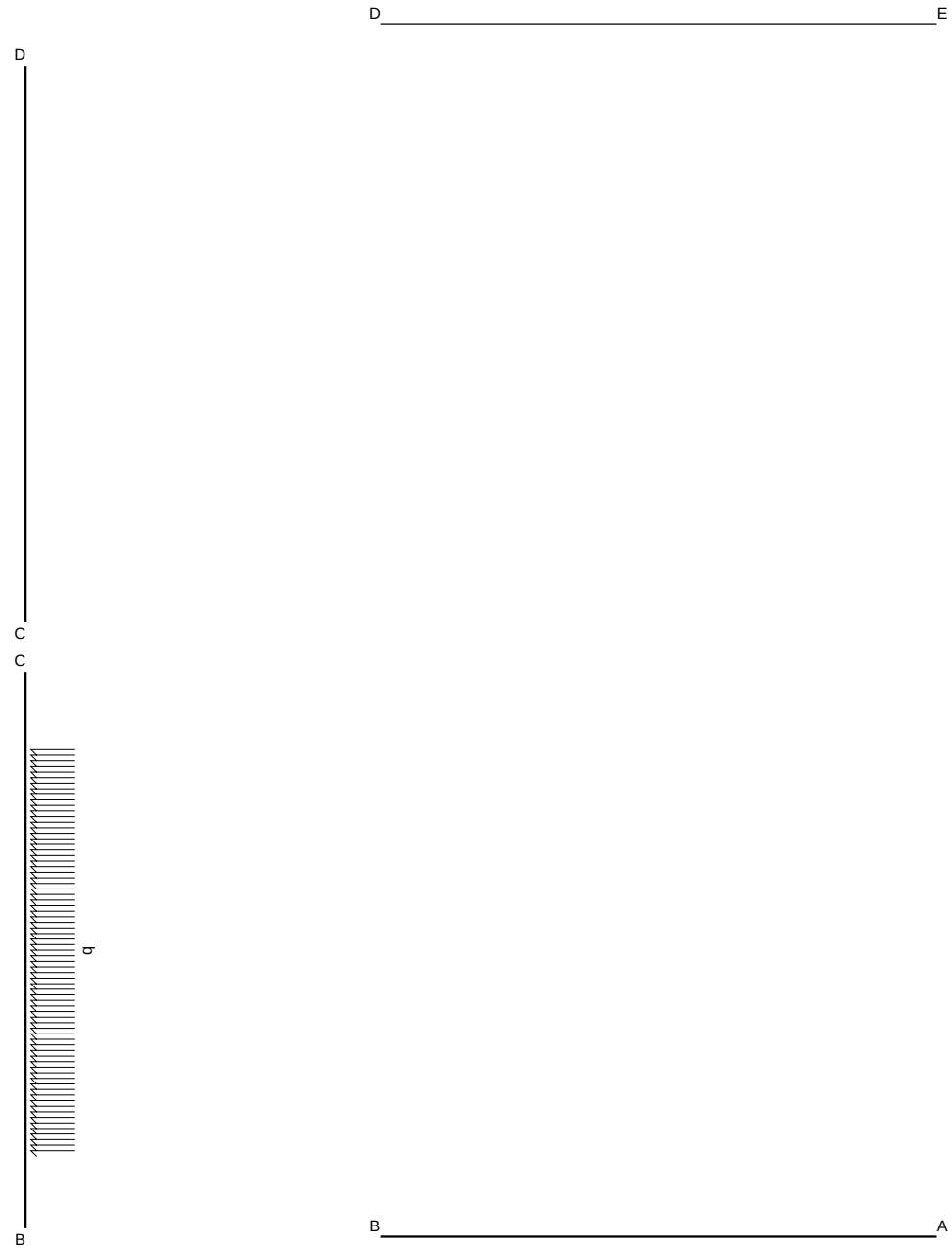

DEFORMATA (coordinate locali)
AB y(x)EJ =
BC y(x)EJ =
CD y(x)EJ =
ED y(x)EJ =

SPOSTAMENTI NODALI

u_A =
v_{AAB} =
φ_{AAB} =

u_B =
v_B =
φ_B =

u_C =
v_C =
φ_C =

u_D =
v_D =
φ_D =

u_E =
v_E =
φ_E =

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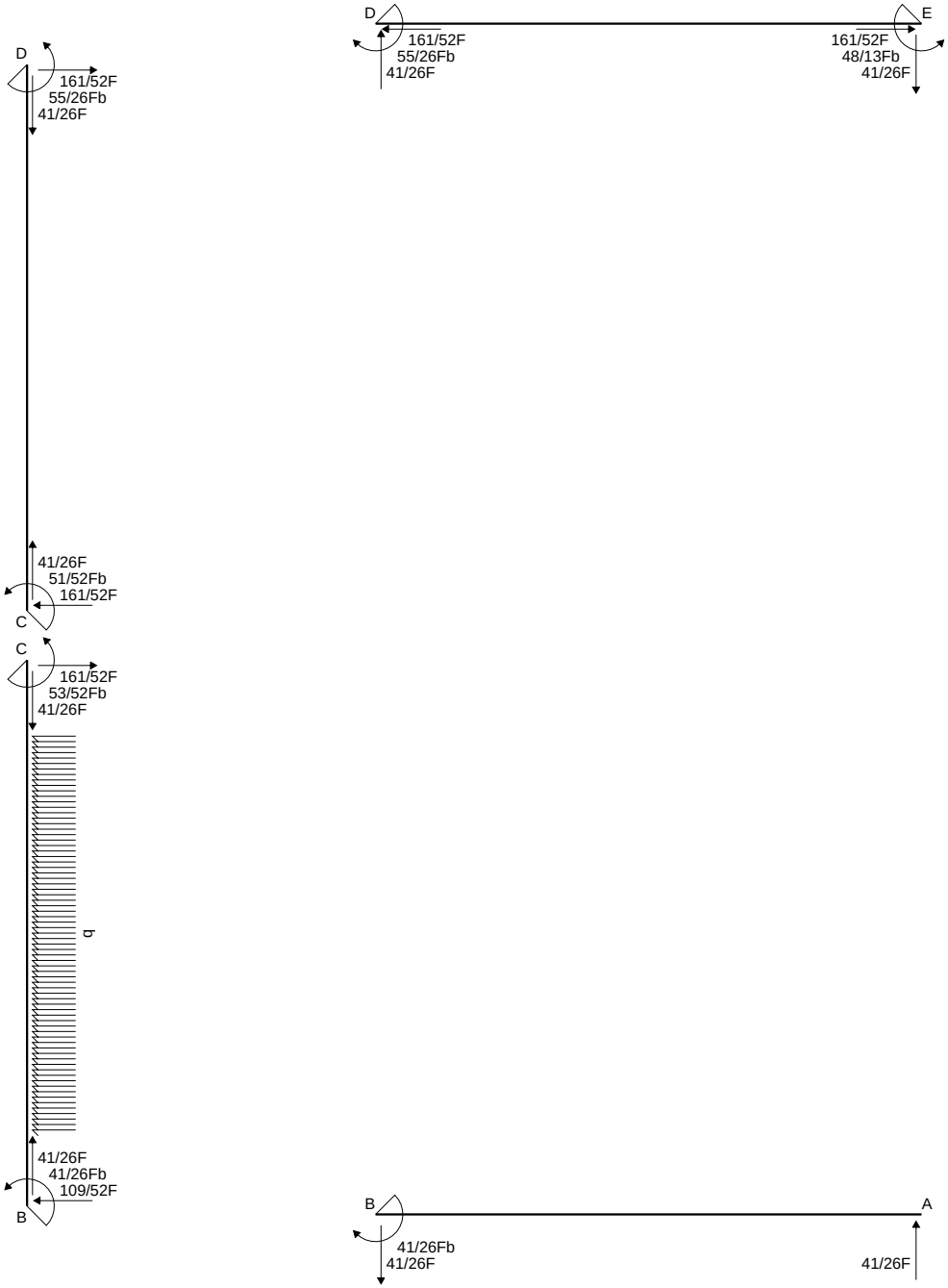

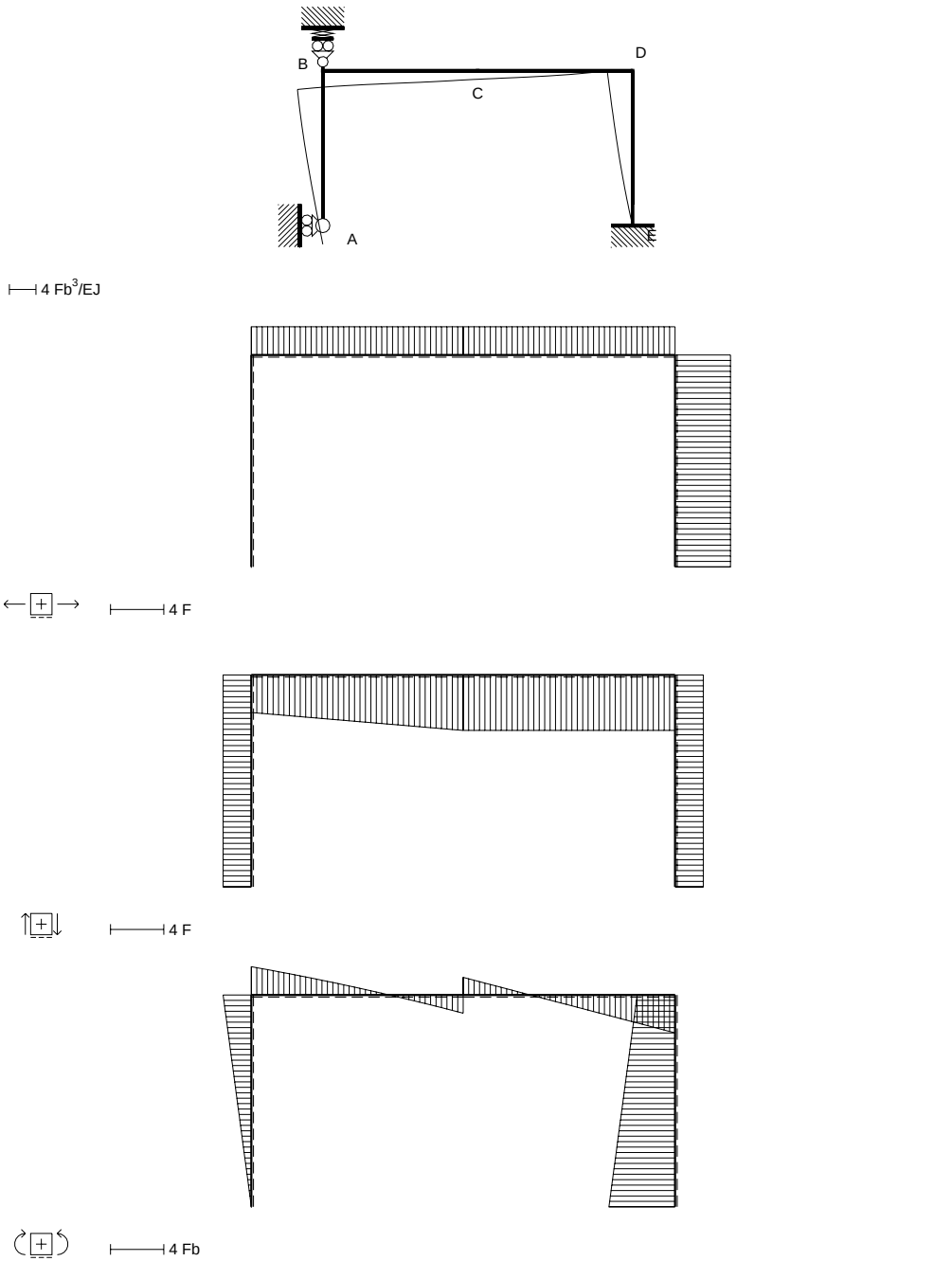

REAZIONI IPERSTATICHE

$X = W_{BC}$ $Y = W_{DE}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: $\varphi_{AB}\ K_{AB}\ \varphi_{BC}\ K_{BC}\ \varphi_{CD}\ K_{CD}\ \varphi_{ED}\ K_{ED}$

Relazioni di congruenza

$y'_{AB}(b) - y'_{BC}(0) = 0$
$y'_{BC}(b) - y'_{CD}(0) = 0$
$y'_{CD}(b) - y'_{ED}(b) = 0$
$y'_{ED}(0) - 2\delta/b = 0$
$y_{AB}(0) = 0$
$y_{BC}(0) - 1/2V_B b^3/EJ = 0$
$y_{CD}(0) - y_{BC}(b) = 0$
$y_{CD}(b) = 0$
$y_{ED}(0) = 0$
$y_{ED}(b) - y_{AB}(b) = 0$

$M_{AB} = -Xx/b$
$EJy'' = -Xx/b$
$EJy' = -1/2Xx^2/b + EJ\varphi_{AB}$
$EJy = -1/6Xx^3/b + EJ\varphi_{AB}x + EJK_{AB}$

$M_{BC} = 1/4Fx + 1/2qx^2 + 1/2Xx/b - X - 1/2Yx/b$
$EJy'' = 1/4Fx + 1/2qx^2 + 1/2Xx/b - X - 1/2Yx/b$
$EJy' = 1/8Fx^2 + 1/6qx^3 + 1/4Xx^2/b - Xx - 1/4Yx^2/b + EJ\varphi_{BC}$
$EJy = 1/24Fx^3 + 1/24qx^4 + 1/12Xx^3/b - 1/2Xx^2 - 1/12Yx^3/b + EJ\varphi_{BC}x + EJK_{BC}$

$M_{CD} = 5/4Fx - 5/4Fb + 1/2Xx/b - 1/2X - 1/2Yx/b - 1/2Y$
$EJy'' = 5/4Fx - 5/4Fb + 1/2Xx/b - 1/2X - 1/2Yx/b - 1/2Y$
$EJy' = 5/8Fx^2 - 5/4Fbx + 1/4Xx^2/b - 1/2Xx - 1/4Yx^2/b - 1/2Yx + EJ\varphi_{CD}$
$EJy = 5/24Fx^3 - 5/8Fbx^2 + 1/12Xx^3/b - 1/4Xx^2 - 1/12Yx^3/b - 1/4Yx^2 + EJ\varphi_{CD}x + EJK_{CD}$

$M_{ED} = Xx/b - X + Y$
$EJy'' = 2EJ\theta + Xx/b - X + Y$
$EJy' = 2EJ\theta x + 1/2Xx^2/b - Xx + Yx + EJ\varphi_{ED}$
$EJy = EJ\theta x^2 + 1/6Xx^3/b - 1/2Xx^2 + 1/2Yx^2 + EJ\varphi_{ED}x + EJK_{ED}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{BC}b$	K_{BC}	$\varphi_{CD}b$	K_{CD}	$\varphi_{ED}b$	K_{ED}	Xb^2/EJ	Yb^2/EJ		Wb^2/EJ
y'_{BA}	1	0	-1	0	0	0	0	0	-1/2	0		0
y'_{CB}	0	0	1	0	-1	0	0	0	-3/4	-1/4		-7/24
y'_{DC}	0	0	0	0	1	0	-1	0	1/4	-7/4		5/8
y'_{ED}	0	0	0	0	0	0	1	0	0	0		0
y_{AB}	0	1	0	0	0	0	0	0	0	0		0
y_{BC}	0	0	0	1	0	0	0	0	1/4	-1/4	=	-1/8
y_{CD}	0	0	-1	-1	0	1	0	0	5/12	1/12		1/12
y_{DC}	0	0	0	0	1	1	0	0	-1/6	-1/3		5/12
y_{ED}	0	0	0	0	0	0	0	1	0	0		0
y_{DE}	-1	-1	0	0	0	0	1	1	-1/6	1/2		0

Condizioni al contorno

αTb	δ
0	0
0	0
2	0
0	2
0	0
0	0
0	0
0	0
0	0
-1	0

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		131/78
$\varphi_{BC}b$		139/156
$\varphi_{CD}b$		55/104
$\varphi_{ED}b$		2
K_{AB}		0
K_{BC}	=	-109/104
K_{CD}		-173/312
Xb^2/EJ		41/26
K_{ED}		0
Yb^2/EJ		-55/26

DEFORMATA (coordinate locali)

AB $y(x)EJ = 131/78xFb^2 - 41/156x^3F$
BA $y(x)EJ = 17/12Fb^3 - 139/156xFb^2 - 41/52x^2Fb + 41/156x^3F$
BC $y(x)EJ = -109/104Fb^3 + 139/156xFb^2 - 41/52x^2Fb + 109/312x^3F + 1/24x^4q$
CB $y(x)EJ = -173/312Fb^3 - 55/104xFb^2 + 53/104x^2Fb - 161/312x^3F + 1/24x^4q$
CD $y(x)EJ = -173/312Fb^3 + 55/104xFb^2 - 51/104x^2Fb + 161/312x^3F$
DC $y(x)EJ = -57/52xFb^2 + 55/52x^2Fb - 161/312x^3F$
ED $y(x)EJ = 2xFb^2 - 11/13x^2Fb + 41/156x^3F$
DE $y(x)EJ = 17/12Fb^3 - 57/52xFb^2 - 3/52x^2Fb - 41/156x^3F$

SPOSTAMENTI NODALI

$u_A = 0$ $u_B = -17/12(Fb^3/EJ)$ $u_C = -17/12(Fb^3/EJ)$
$v_{AAB} = -109/104(Fb^3/EJ)$ $v_B = -109/104(Fb^3/EJ)$ $v_C = -173/312(Fb^3/EJ)$
$\varphi_{AAB} = 131/78(Fb^2/EJ)$ $\varphi_B = 139/156(Fb^2/EJ)$ $\varphi_C = 55/104(Fb^2/EJ)$

$u_D = -17/12(Fb^3/EJ)$ $u_E = 0$
$v_D = 0$ $v_E = 0$
$\varphi_D = 57/52(Fb^2/EJ)$ $\varphi_E = 2(Fb^2/EJ)$

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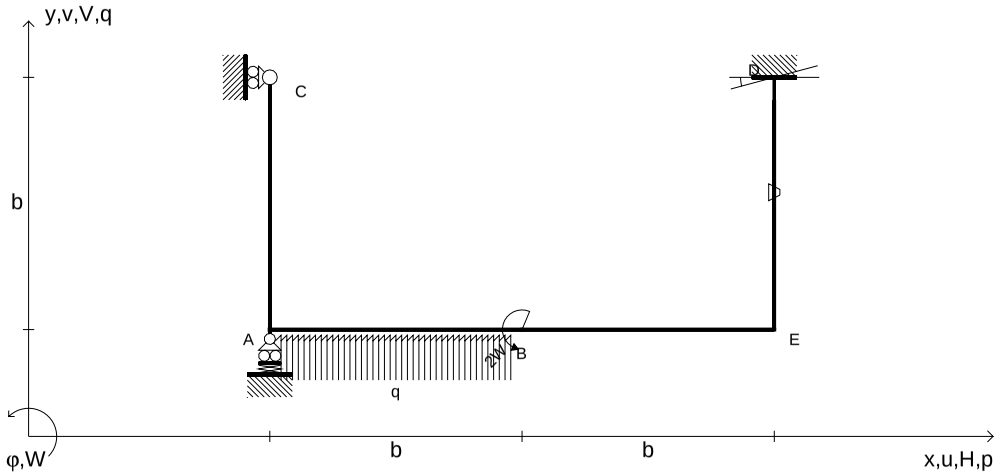
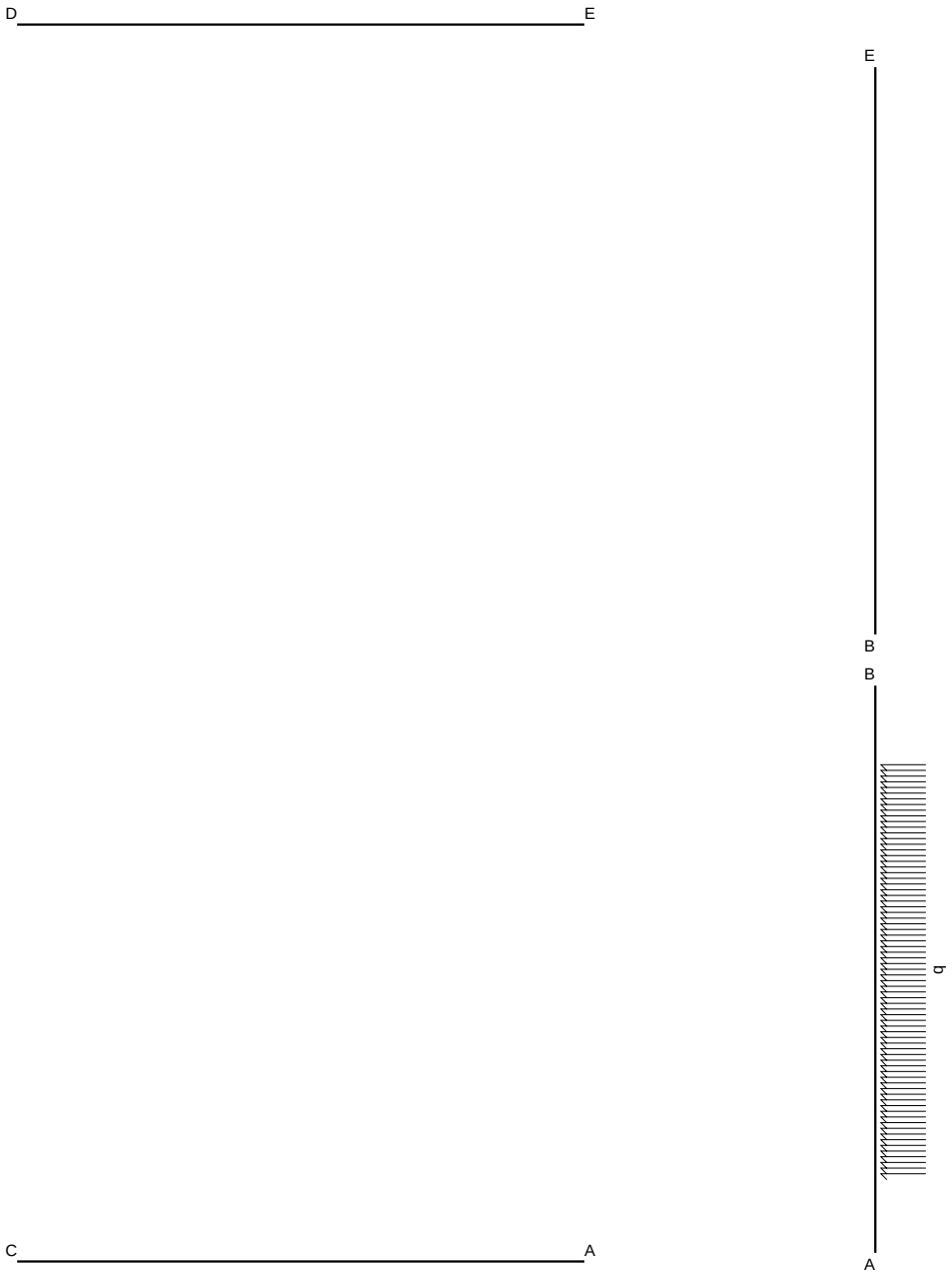


$W_B = 2W = 2Fb$ $\varphi_D = 2\delta/b = 2b^2F/EJ$ $EJ_{CA} = EJ$

$q_{AB} = q = F/b$ $k_A = 2EJ/b^3$ $EJ_{DE} = EJ$

$\theta_{DE} = 2\theta = 2\alpha T/b = 2bF/EJ$ $EJ_{AB} = EJ$ $EJ_{BE} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio.
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Curvatura θ asta DE positiva se convessa a destra con inizio D.
Rotazione assoluta W imposta al nodo D.



DEFORMATA (coordinate locali)
AB y(x)EJ =
CA y(x)EJ =
DE y(x)EJ =
BE y(x)EJ =

SPOSTAMENTI NODALI

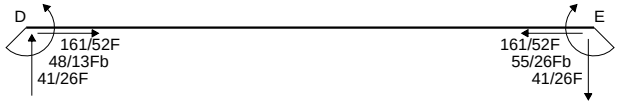
u_A =
v_A =
φ_A =

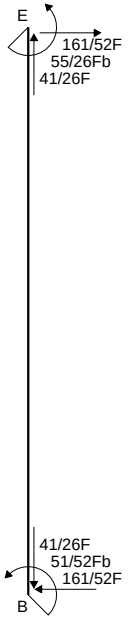
u_B =
v_B =
φ_B =

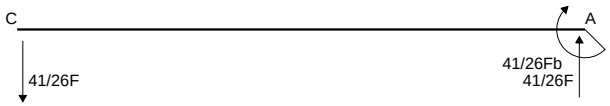
u_C =
v_{CCA} =
φ_{CCA} =

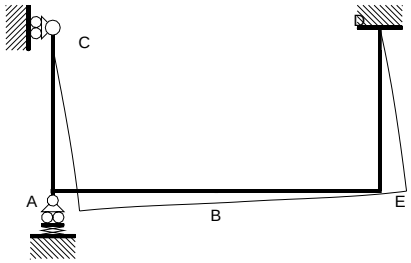
u_D =
v_D =
φ_D =

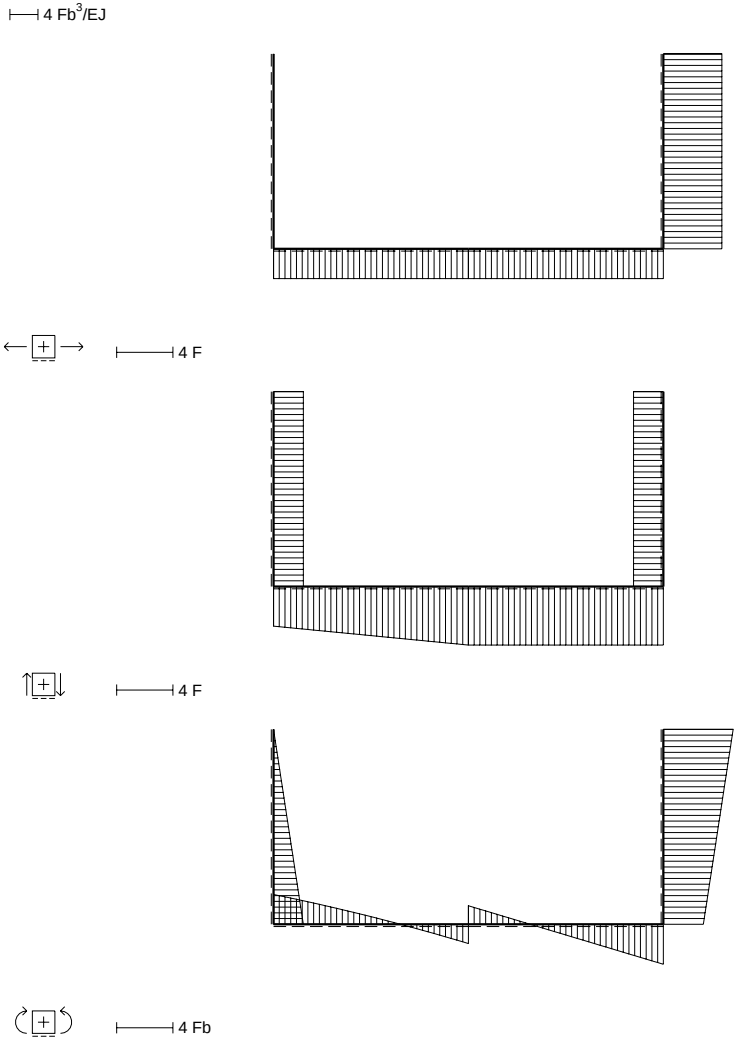
u_E =
v_E =
φ_E =











REAZIONI IPERSTATICHE

$X = W_{AB}$ $Y = W_{ED}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: $\varphi_{AB}\ K_{AB}\ \varphi_{CA}\ K_{CA}\ \varphi_{DE}\ K_{DE}\ \varphi_{BE}\ K_{BE}$

Relazioni di congruenza

$y'_{AB}(0) - y'_{CA}(b) = 0$
$y'_{AB}(b) - y'_{BE}(0) = 0$
$y'_{DE}(0) - 2\delta/b = 0$
$y'_{DE}(b) - y'_{BE}(b) = 0$
$y_{AB}(0) - 1/2V_A b^3/EJ = 0$
$y_{CA}(0) = 0$
$y_{DE}(0) = 0$
$y_{DE}(b) - y_{CA}(b) = 0$
$y_{BE}(0) - y_{AB}(b) = 0$
$y_{BE}(b) = 0$

$M_{AB} = 1/4Fx +1/2qx^2 +1/2Xx/b -X -1/2Yx/b$
$EJy'' = 1/4Fx +1/2qx^2 +1/2Xx/b -X -1/2Yx/b$
$EJy' = 1/8Fx^2 +1/6qx^3 +1/4Xx^2/b -Xx -1/4Yx^2/b +EJ\varphi_{AB}$
$EJy = 1/24Fx^3 +1/24qx^4 +1/12Xx^3/b -1/2Xx^2 -1/12Yx^3/b +EJ\varphi_{AB}x +EJK_{AB}$

$M_{CA} = -Xx/b$
$EJy'' = -Xx/b$
$EJy' = -1/2Xx^2/b +EJ\varphi_{CA}$
$EJy = -1/6Xx^3/b +EJ\varphi_{CA}x +EJK_{CA}$

$M_{DE} = Xx/b -X +Y$
$EJy'' = 2EJ\theta +Xx/b -X +Y$
$EJy' = 2EJ\theta x +1/2Xx^2/b -Xx +Yx +EJ\varphi_{DE}$
$EJy = EJ\theta x^2 +1/6Xx^3/b -1/2Xx^2 +1/2Yx^2 +EJ\varphi_{DE}x +EJK_{DE}$

$M_{BE} = 5/4Fx -5/4Fb +1/2Xx/b -1/2X -1/2Yx/b -1/2Y$
$EJy'' = 5/4Fx -5/4Fb +1/2Xx/b -1/2X -1/2Yx/b -1/2Y$
$EJy' = 5/8Fx^2 -5/4Fbx +1/4Xx^2/b -1/2Xx -1/4Yx^2/b -1/2Yx +EJ\varphi_{BE}$
$EJy = 5/24Fx^3 -5/8Fbx^2 +1/12Xx^3/b -1/4Xx^2 -1/12Yx^3/b -1/4Yx^2 +EJ\varphi_{BE}x +EJK_{BE}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CA}b$	K_{CA}	$\varphi_{DE}b$	K_{DE}	$\varphi_{BE}b$	K_{BE}	Xb^2/EJ	Yb^2/EJ		Wb^2/EJ
y'_{AB}	1	0	-1	0	0	0	0	0	1/2	0		0
y'_{BA}	1	0	0	0	0	0	-1	0	-3/4	-1/4		-7/24
y'_{DE}	0	0	0	0	1	0	0	0	0	0		0
y'_{ED}	0	0	0	0	1	0	-1	0	-1/4	7/4		-5/8
y_{AB}	0	1	0	0	0	0	0	0	1/4	-1/4		-1/8
y_{CA}	0	0	0	1	0	0	0	0	0	0	=	0
y_{DE}	0	0	0	0	0	1	0	0	0	0		0
y_{ED}	0	0	-1	-1	1	1	0	0	-1/6	1/2		0
y_{BE}	-1	-1	0	0	0	0	0	1	5/12	1/12		1/12
y_{EB}	0	0	0	0	0	0	1	1	-1/6	-1/3		5/12

Condizioni al contorno

αTb	δ
0	0
0	0
0	2
-2	0
0	0
0	0
0	0
-1	0
0	0
0	0

		Soluzione
		Fb^3/EJ
$\varphi_{AB}b$	=	139/156
$\varphi_{CA}b$		131/78
$\varphi_{DE}b$		2
$\varphi_{BE}b$		55/104
K_{AB}		-109/104
K_{CA}		0
K_{DE}		0
Xb^2/EJ		41/26
K_{BE}		-173/312
Yb^2/EJ		-55/26

DEFORMATA (coordinate locali)

AB $y(x)EJ = -109/104Fb^3 +139/156xFb^2 -41/52x^2Fb +109/312x^3F +1/24x^4q$
BA $y(x)EJ = -173/312Fb^3 -55/104xFb^2 +53/104x^2Fb -161/312x^3F +1/24x^4q$
CA $y(x)EJ = 131/78xFb^2 -41/156x^3F$
AC $y(x)EJ = 17/12Fb^3 -139/156xFb^2 -41/52x^2Fb +41/156x^3F$
DE $y(x)EJ = 2xFb^2 -11/13x^2Fb +41/156x^3F$
ED $y(x)EJ = 17/12Fb^3 -57/52xFb^2 -3/52x^2Fb -41/156x^3F$
BE $y(x)EJ = -173/312Fb^3 +55/104xFb^2 -51/104x^2Fb +161/312x^3F$
EB $y(x)EJ = -57/52xFb^2 +55/52x^2Fb -161/312x^3F$

SPOSTAMENTI NODALI

$u_A = 17/12(Fb^3/EJ)$ $u_B = 17/12(Fb^3/EJ)$ $u_C = 0$
$v_A = -109/104(Fb^3/EJ)$ $v_B = -173/312(Fb^3/EJ)$ $v_{CCA} = -109/104(Fb^3/EJ)$
$\varphi_A = 139/156(Fb^2/EJ)$ $\varphi_B = 55/104(Fb^2/EJ)$ $\varphi_{CCA} = 131/78(Fb^2/EJ)$

$u_D = 0$ $u_E = 17/12(Fb^3/EJ)$
$v_D = 0$ $v_E = 0$
$\varphi_D = 2(Fb^2/EJ)$ $\varphi_E = 57/52(Fb^2/EJ)$

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