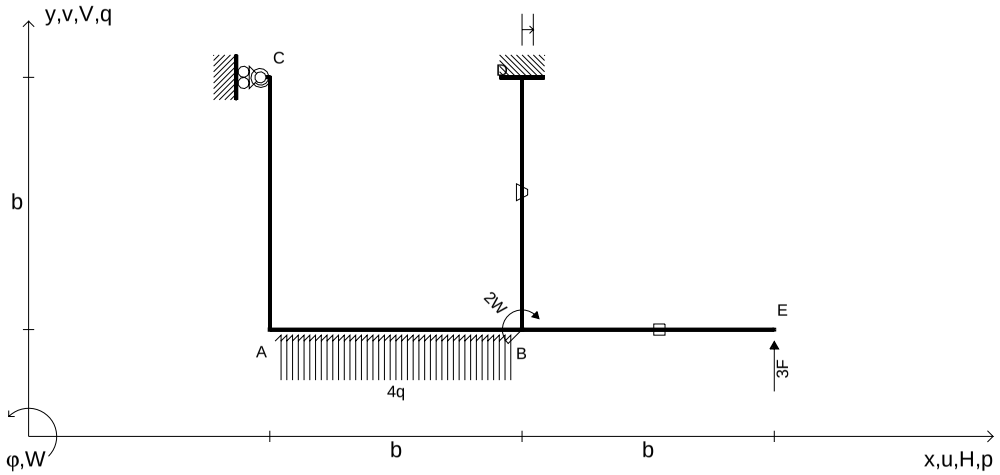
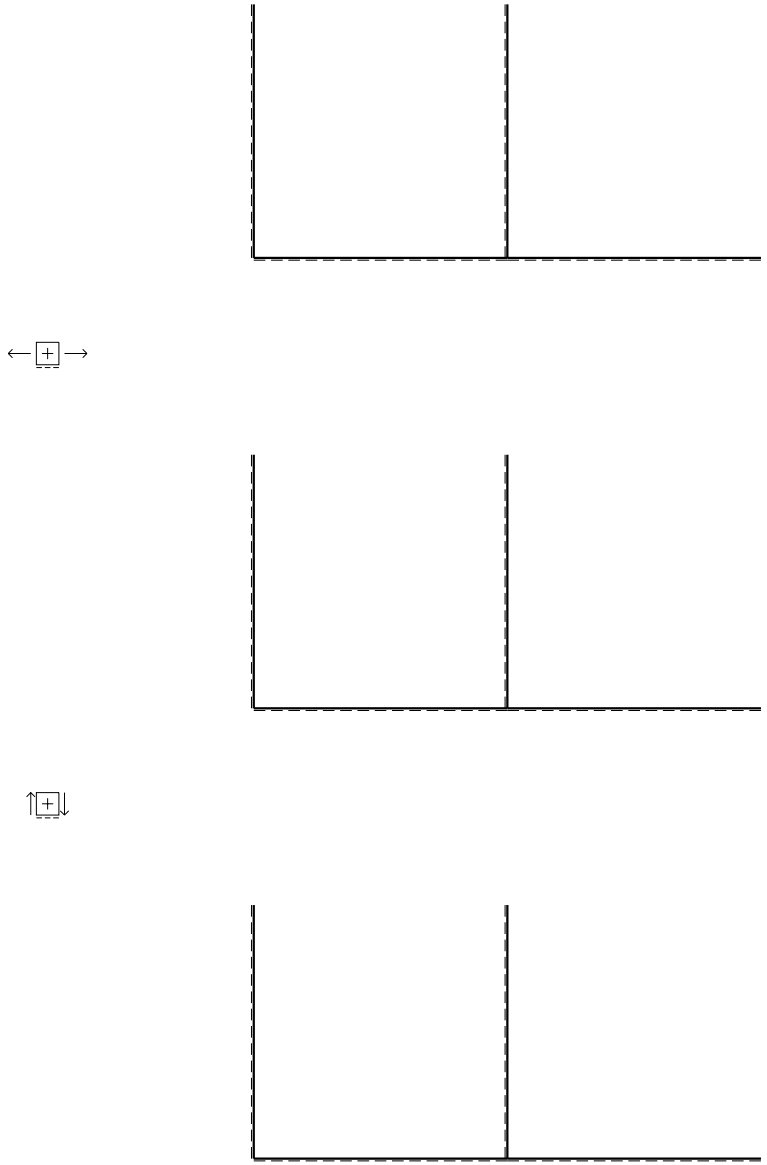


$V_E = 3F$
$W_B = -2W = -2Fb$
$q_{AB} = 4q = 4F/b$
$\varepsilon_{BE} = -3\alpha T = -3b^2F/EJ$

$\theta_{DB} = 2\theta = 2\alpha T/b = 2bF/EJ$
$u_D = \delta = b^3F/EJ$
$k_C = 2EJ/b$
$EJ_{AB} = EJ$

$EJ_{CA} = EJ$
$EJ_{DB} = 1/4EJ$
$EJ_{BE} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio (retro incluso).
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.
Curvatura θ asta DB positiva se convessa a destra con inizio D.
Spostamento orizzontale assoluto u imposto al nodo D.



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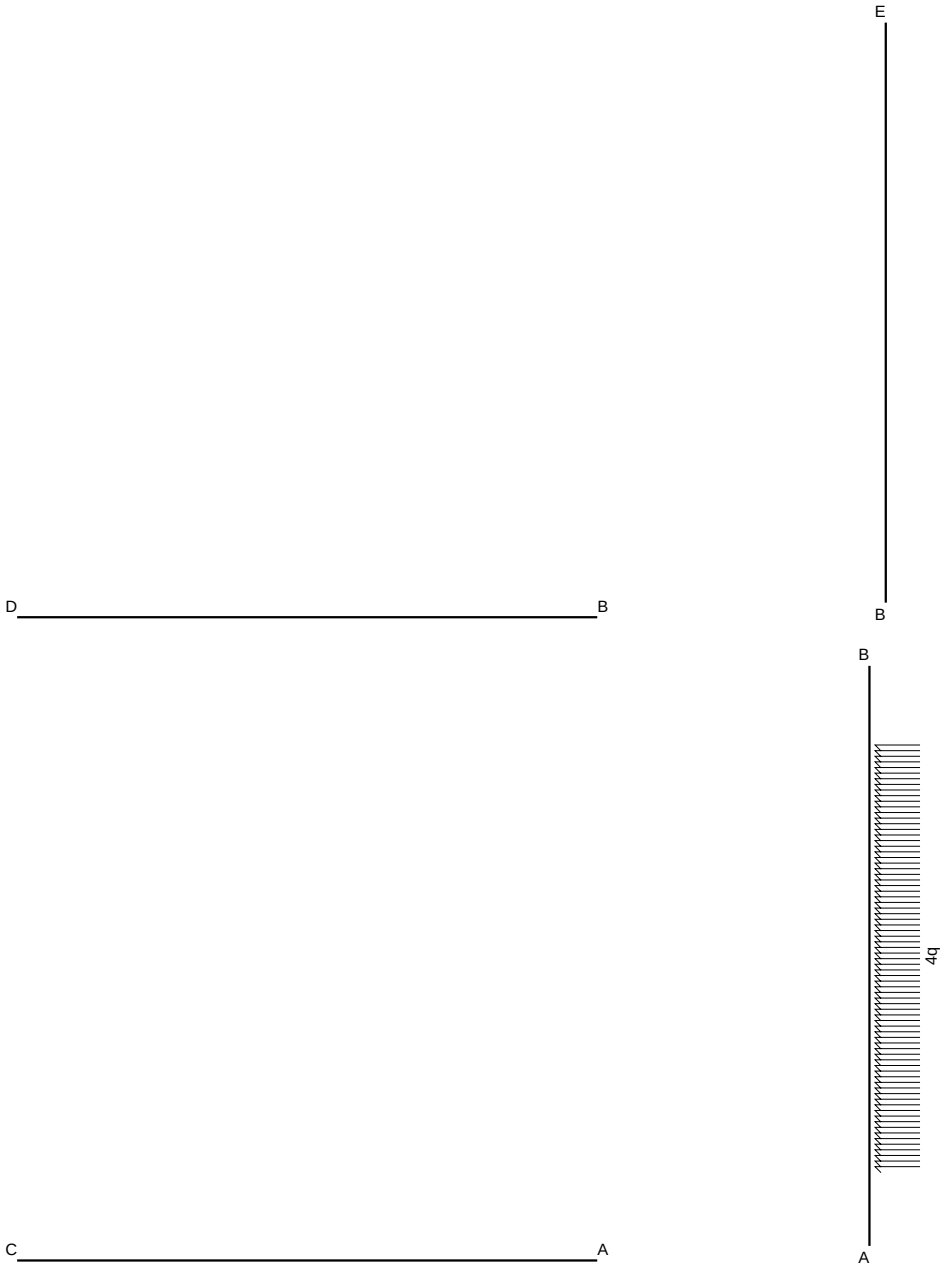

DEFORMATA (coordinate locali)
AB y(x)EJ =
CA y(x)EJ =
DB y(x)EJ =
BE y(x)EJ =

SPOSTAMENTI NODALI

u_A = u_B = u_C = u_D =
v_A = v_B = v_C = v_D =
φ_A = φ_B = φ_C = φ_D =

u_E =
v_E =
φ_E =

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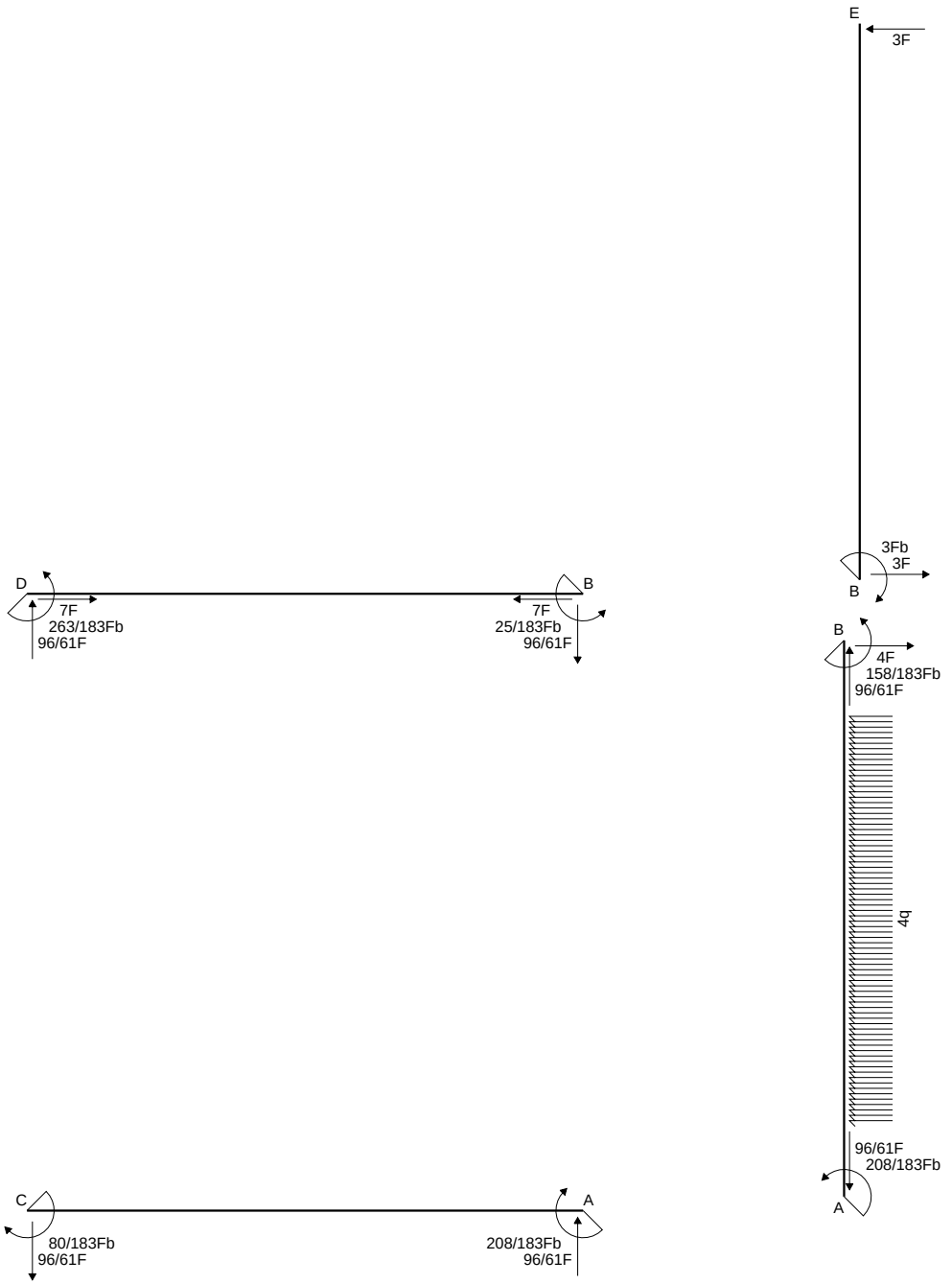

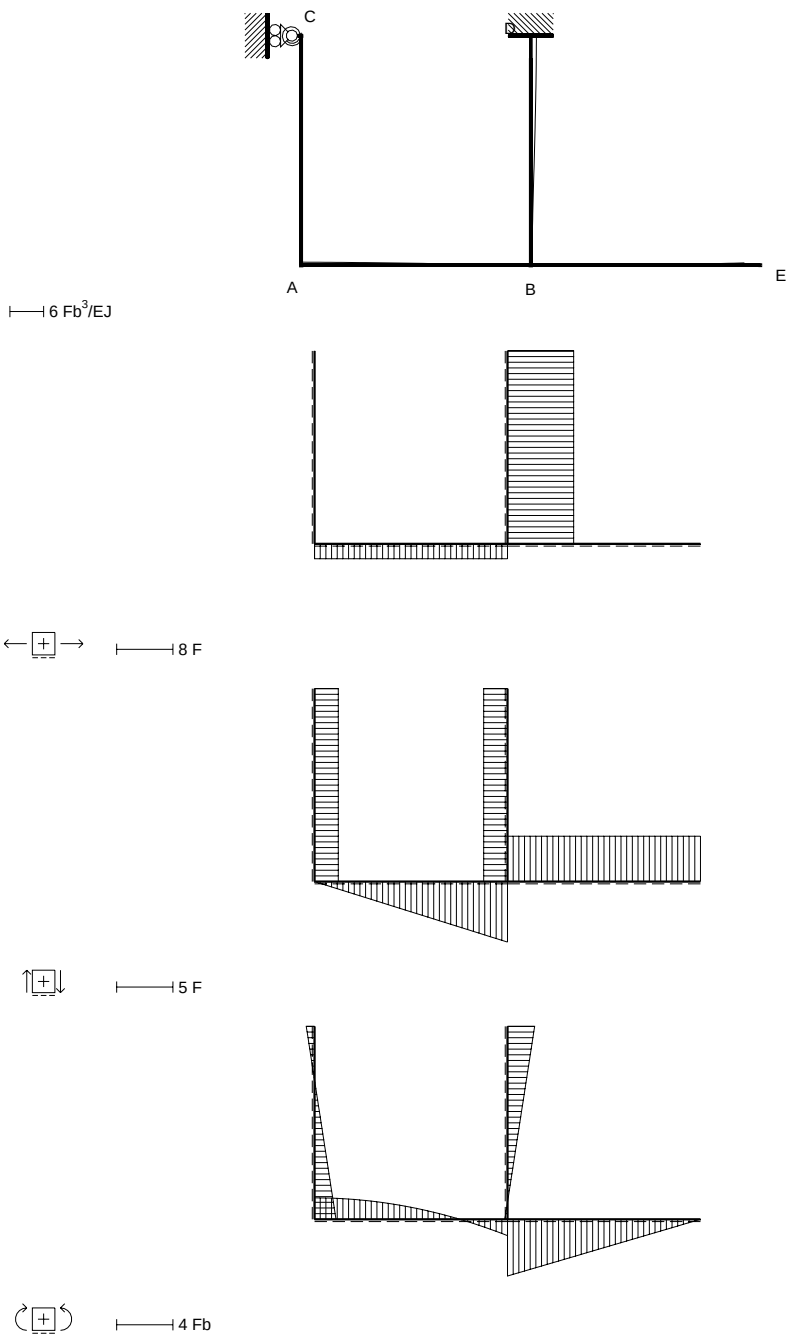

REAZIONI IPERSTATICHE

$X = W_{AB}$ $Y = W_{CA}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: $\varphi_{AB}\ K_{AB}\ \varphi_{CA}\ K_{CA}\ \varphi_{DB}\ K_{DB}\ \varphi_{BE}\ K_{BE}$

Relazioni di congruenza

$y'_{AB}(0) - y'_{CA}(b) = 0$
$y'_{AB}(b) - y'_{BE}(0) = 0$
$y'_{CA}(0) + 1/2W_C b/EJ = 0$
$y'_{DB}(0) = 0$
$y'_{DB}(b) - y'_{BE}(0) = 0$
$y_{AB}(b) = 0$
$y_{CA}(0) = 0$
$y_{DB}(0) - \delta = 0$
$y_{DB}(b) - y_{CA}(b) = 0$
$y_{BE}(0) = 0$

$M_{AB} = 2qx^2 - X$
$EJy'' = 2qx^2 - X$
$EJy' = 2/3qx^3 - Xx + EJ\varphi_{AB}$
$EJy = 1/6qx^4 - 1/2Xx^2 + EJ\varphi_{AB}x + EJK_{AB}$

$M_{CA} = -Xx/b + Yx/b - Y$
$EJy'' = -Xx/b + Yx/b - Y$
$EJy' = -1/2Xx^2/b + 1/2Yx^2/b - Yx + EJ\varphi_{CA}$
$EJy = -1/6Xx^3/b + 1/6Yx^3/b - 1/2Yx^2 + EJ\varphi_{CA}x + EJK_{CA}$

$M_{DB} = -Fb + Xx/b - Yx/b + Y$
$EJy'' = -4Fb + 2EJ\theta + 4Xx/b - 4Yx/b + 4Y$
$EJy' = -4Fbx + 2EJ\theta x + 2Xx^2/b - 2Yx^2/b + 4Yx + EJ\varphi_{DB}$
$EJy = -2Fbx^2 + EJ\theta x^2 + 2/3Xx^3/b - 2/3Yx^3/b + 2Yx^2 + EJ\varphi_{DB}x + EJK_{DB}$

$M_{BE} = -3Fx + 3Fb$
$EJy'' = -3Fx + 3Fb$
$EJy' = -3/2Fx^2 + 3Fbx + EJ\varphi_{BE}$
$EJy = -1/2Fx^3 + 3/2Fbx^2 + EJ\varphi_{BE}x + EJK_{BE}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CA}b$	K_{CA}	$\varphi_{DB}b$	K_{DB}	$\varphi_{BE}b$	K_{BE}	Xb^2/EJ	Yb^2/EJ		Fb^3/EJ
y'_{AB}	1	0	-1	0	0	0	0	0	1/2	1/2		0
y'_{BA}	1	0	0	0	0	0	-1	0	-1	0		-2/3
y'_{CA}	0	0	1	0	0	0	0	0	0	1/2		0
y'_{DB}	0	0	0	0	1	0	0	0	0	0		0
y'_{BD}	0	0	0	0	1	0	-1	0	2	2		4
y_{BA}	1	1	0	0	0	0	0	0	-1/2	0	=	-1/6
y_{CA}	0	0	0	1	0	0	0	0	0	0		0
y_{DB}	0	0	0	0	0	1	0	0	0	0		0
y_{BD}	0	0	-1	-1	1	1	0	0	5/6	5/3		2
y_{BE}	0	0	0	0	0	0	0	1	0	0		0

Condizioni al contorno

αTb	δ
0	0
0	0
0	0
0	0
-2	0
0	0
0	0
0	1
-1	0
0	0

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		-8/61
$\varphi_{CA}b$		40/183
$\varphi_{BE}b$		-110/183
$\varphi_{DB}b$		0
Xb^2/EJ	=	208/183
K_{AB}		65/122
K_{CA}		0
K_{DB}		1
Yb^2/EJ		-80/183
K_{BE}		0

DEFORMATA (coordinate locali)

AB $y(x)EJ = 65/122Fb^3 - 8/61xFb^2 - 104/183x^2Fb + 1/6x^4q$
BA $y(x)EJ = 110/183xFb^2 + 79/183x^2Fb - 2/3x^3F + 1/6x^4q$
CA $y(x)EJ = 40/183xFb^2 + 40/183x^2Fb - 16/61x^3F$
AC $y(x)EJ = 32/183Fb^3 + 8/61xFb^2 - 104/183x^2Fb + 16/61x^3F$
DB $y(x)EJ = Fb^3 - 343/183x^2Fb + 64/61x^3F$
BD $y(x)EJ = 32/183Fb^3 + 110/183xFb^2 + 233/183x^2Fb - 64/61x^3F$
BE $y(x)EJ = -110/183xFb^2 + 3/2x^2Fb - 1/2x^3F$
EB $y(x)EJ = 73/183Fb^3 - 329/366xFb^2 + 1/2x^3F$

SPOSTAMENTI NODALI

$u_A = 32/183(Fb^3/EJ)$	$u_B = 32/183(Fb^3/EJ)$	$u_C = 0$	$u_D = (Fb^3/EJ)$
$v_A = 65/122(Fb^3/EJ)$	$v_B = 0$	$v_C = 65/122(Fb^3/EJ)$	$v_D = 0$
$\varphi_A = -8/61(Fb^2/EJ)$	$\varphi_B = -110/183(Fb^2/EJ)$	$\varphi_C = 40/183(Fb^2/EJ)$	$\varphi_D = 0$

$u_E = -517/183(Fb^3/EJ)$
$v_E = 73/183(Fb^3/EJ)$
$\varphi_E = 329/366(Fb^2/EJ)$

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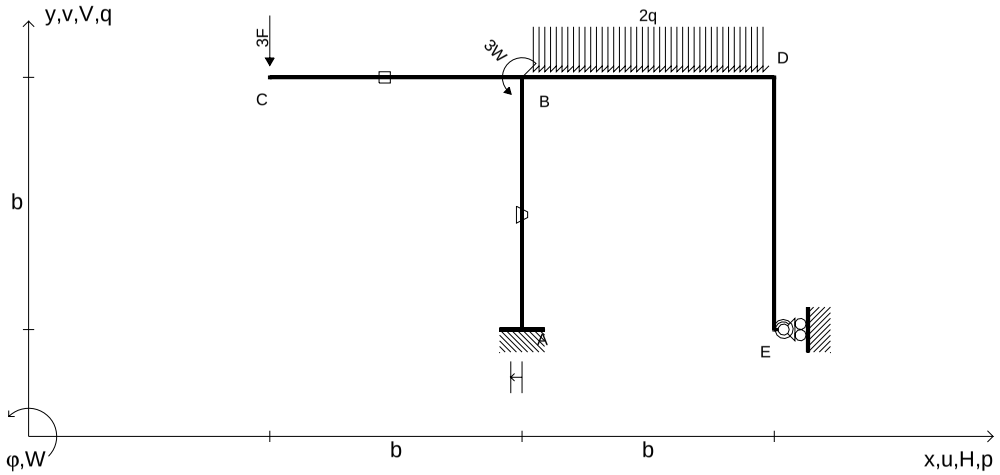
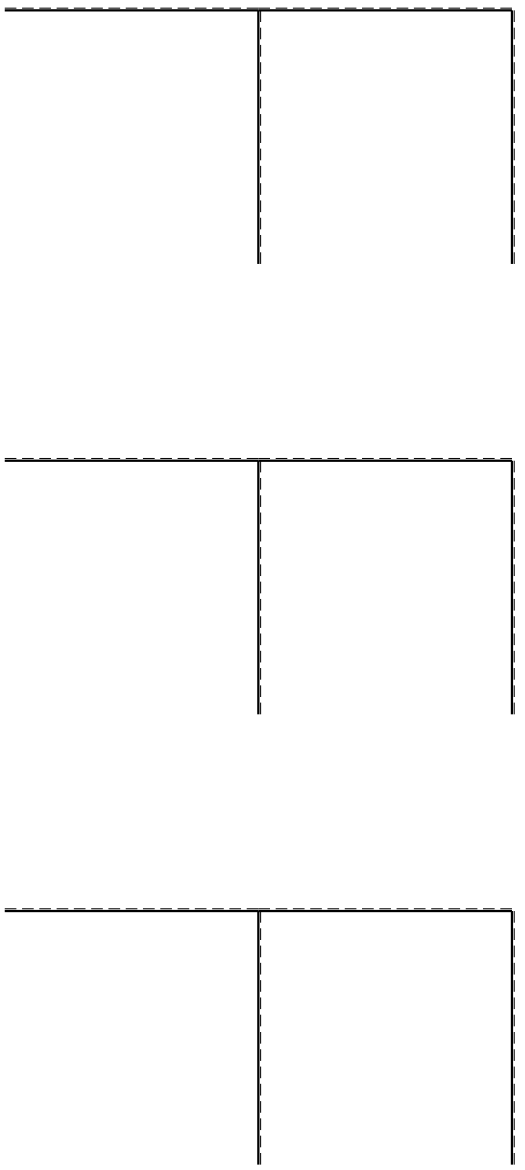


$V_C = -3F$
$W_B = 3W = 3Fb$
$q_{DB} = -2q = -2F/b$
$\varepsilon_{BC} = 2\alpha T = 2b^2F/EJ$

$\theta_{AB} = -4\theta = -4\alpha T/b = -4bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$k_E = EJ/b$
$EJ_{AB} = 1/3EJ$

$EJ_{BC} = EJ$
$EJ_{DB} = EJ$
$EJ_{ED} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio (retro incluso).
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BC.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.



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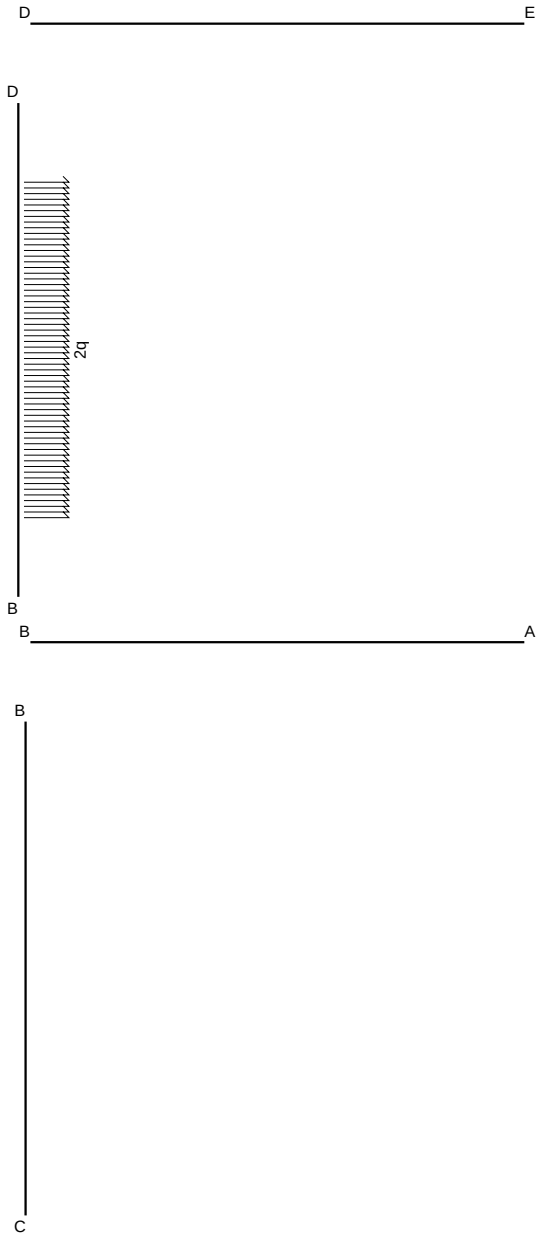


DEFORMATA (coordinate locali)
AB y(x)EJ =
BC y(x)EJ =
DB y(x)EJ =
ED y(x)EJ =

SPOSTAMENTI NODALI

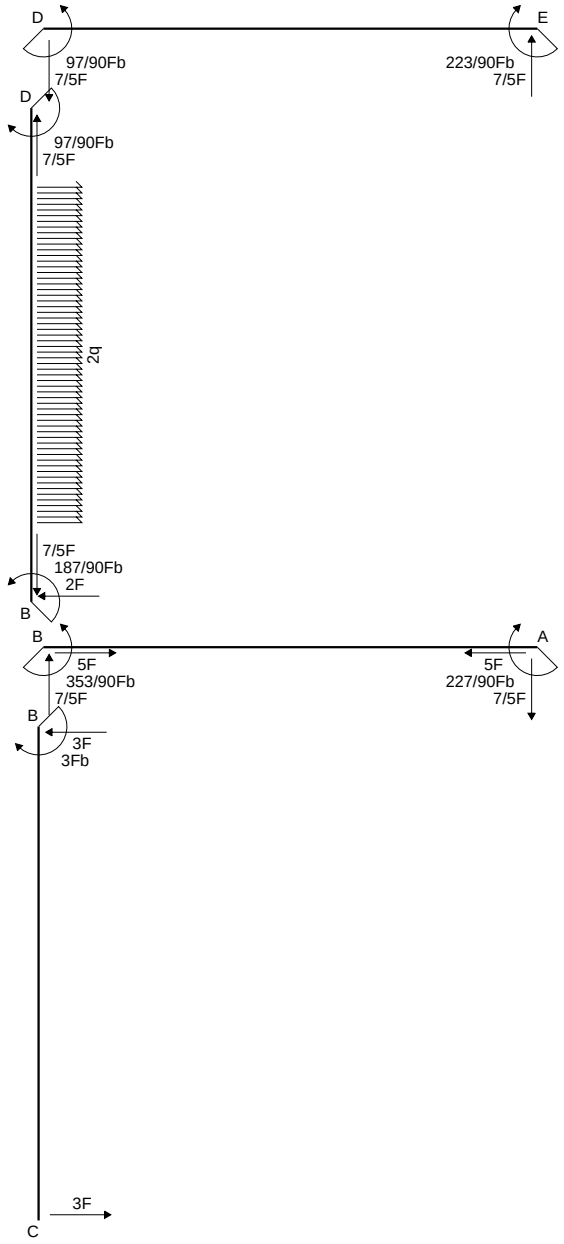
u_A =
v_A =
φ_A =

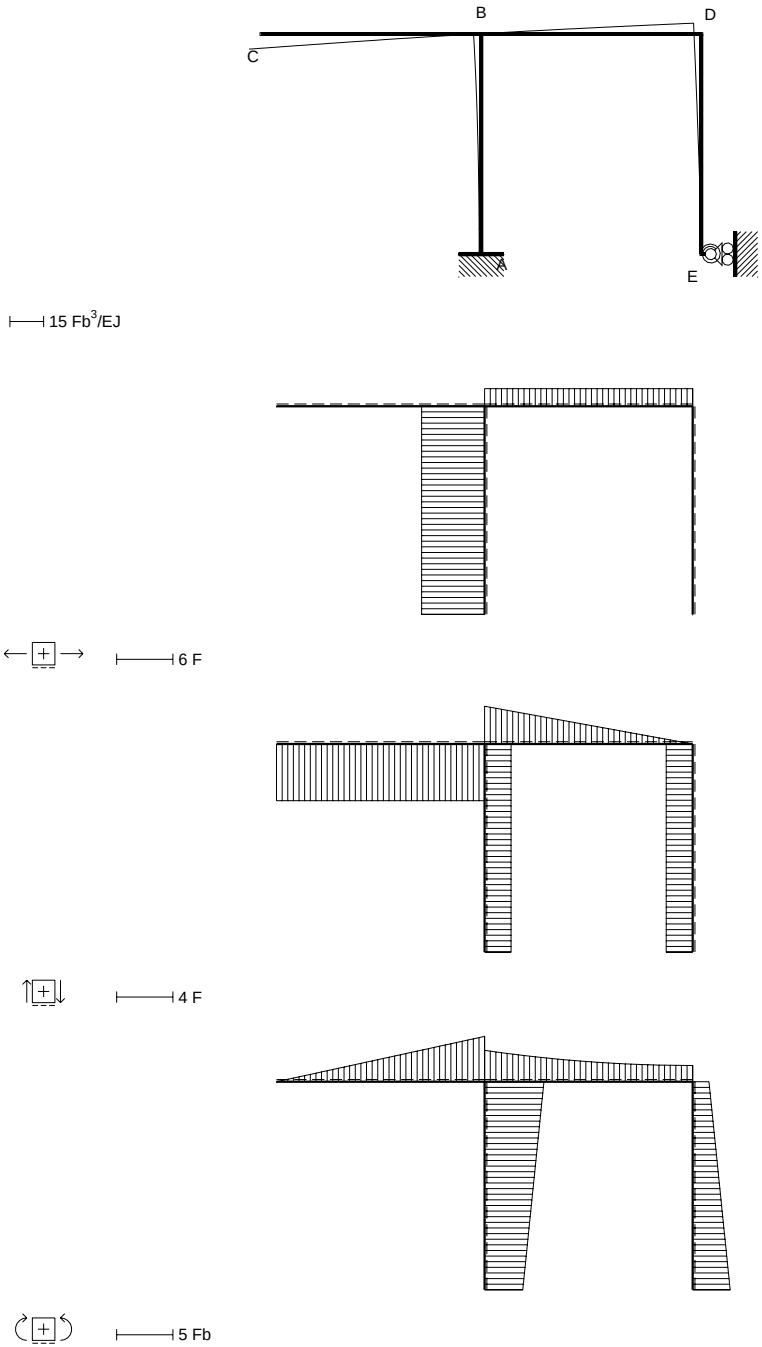
u_B =
v_B =
φ_B =

u_C =
v_C =
φ_C =

u_D =
v_D =
φ_D =

u_E =
v_E =
φ_E =





REAZIONI IPERSTATICHE

$X = W_{AB}$ $Y = W_{DB}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{BC} K_{BC} φ_{DB} K_{DB} φ_{ED} K_{ED}

Relazioni di congruenza

$y'_{AB}(0) = 0$
$y'_{AB}(b) - y'_{DB}(b) = 0$
$y'_{BC}(0) - y'_{DB}(b) = 0$
$y'_{DB}(0) - y'_{ED}(b) = 0$
$y'_{ED}(0) + W_E b/EJ = 0$
$y_{AB}(0) - \delta = 0$
$y_{BC}(0) = 0$
$y_{DB}(b) = 0$
$y_{ED}(0) = 0$
$y_{ED}(b) - y_{AB}(b) = 0$

$M_{AB} = 5Fx + Xx/b - X + Yx/b$
$EJy'' = 15Fx - 4EJ\theta + 3Xx/b - 3X + 3Yx/b$
$EJy' = 15/2Fx^2 - 4EJ\theta x + 3/2Xx^2/b - 3Xx + 3/2Yx^2/b + EJ\varphi_{AB}$
$EJy = 5/2Fx^3 - 2EJ\theta x^2 + 1/2Xx^3/b - 3/2Xx^2 + 1/2Yx^3/b + EJ\varphi_{AB}x + EJK_{AB}$

$M_{BC} = -3Fx + 3Fb$
$EJy'' = -3Fx + 3Fb$
$EJy' = -3/2Fx^2 + 3Fbx + EJ\varphi_{BC}$
$EJy = -1/2Fx^3 + 3/2Fbx^2 + EJ\varphi_{BC}x + EJK_{BC}$

$M_{DB} = qx^2 - Y$
$EJy'' = qx^2 - Y$
$EJy' = 1/3qx^3 - Yx + EJ\varphi_{DB}$
$EJy = 1/12qx^4 - 1/2Yx^2 + EJ\varphi_{DB}x + EJK_{DB}$

$M_{ED} = -5Fx + 5Fb - Xx/b + X - Yx/b$
$EJy'' = -5Fx + 5Fb - Xx/b + X - Yx/b$
$EJy' = -5/2Fx^2 + 5Fbx - 1/2Xx^2/b + Xx - 1/2Yx^2/b + EJ\varphi_{ED}$
$EJy = -5/6Fx^3 + 5/2Fbx^2 - 1/6Xx^3/b + 1/2Xx^2 - 1/6Yx^3/b + EJ\varphi_{ED}x + EJK_{ED}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{BC}b$	K_{BC}	$\varphi_{DB}b$	K_{DB}	$\varphi_{ED}b$	K_{ED}	Xb^2/EJ	Yb^2/EJ		Fb^3/EJ
y'_{AB}	1	0	0	0	0	0	0	0	0	0		0
y'_{BA}	1	0	0	0	-1	0	0	0	-3/2	5/2		-43/6
y'_{BC}	0	0	1	0	-1	0	0	0	0	1		1/3
y'_{DB}	0	0	0	0	1	0	-1	0	-1/2	1/2		5/2
y'_{ED}	0	0	0	0	0	0	1	0	-1	0		5
y_{AB}	0	1	0	0	0	0	0	0	0	0	=	0
y_{BC}	0	0	0	1	0	0	0	0	0	0		0
y_{BD}	0	0	0	0	1	1	0	0	0	-1/2		-1/12
y_{ED}	0	0	0	0	0	0	0	1	0	0		0
y_{DE}	-1	-1	0	0	0	0	1	1	4/3	-2/3		5/6

Condizioni al contorno

αTb	δ
0	0
4	0
0	0
0	0
0	0
0	1
0	0
0	0
0	0
-2	0

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		0
$\varphi_{DB}b$		383/90
$\varphi_{BC}b$		17/3
$\varphi_{ED}b$		223/90
Xb^2/EJ		-227/90
K_{AB}	=	1
K_{BC}		0
K_{DB}		-439/90
K_{ED}		0
Yb^2/EJ		-97/90

DEFORMATA (coordinate locali)

AB $y(x)EJ = Fb^3 + 107/60x^2Fb + 7/10x^3F$
BA $y(x)EJ = 209/60Fb^3 - 17/3xFb^2 + 233/60x^2Fb - 7/10x^3F$
BC $y(x)EJ = 17/3xFb^2 + 3/2x^2Fb - 1/2x^3F$
CB $y(x)EJ = 20/3Fb^3 - 43/6xFb^2 + 1/2x^3F$
DB $y(x)EJ = -439/90Fb^3 + 383/90xFb^2 + 97/180x^2Fb + 1/12x^4q$
BD $y(x)EJ = -17/3xFb^2 + 187/180x^2Fb - 1/3x^3F + 1/12x^4q$
ED $y(x)EJ = 223/90xFb^2 + 223/180x^2Fb - 7/30x^3F$
DE $y(x)EJ = 209/60Fb^3 - 383/90xFb^2 + 97/180x^2Fb + 7/30x^3F$

SPOSTAMENTI NODALI

$u_A = -(Fb^3/EJ)$	$u_B = -209/60(Fb^3/EJ)$	$u_C = -329/60(Fb^3/EJ)$	$u_D = -209/60(Fb^3/EJ)$
$v_A = 0$	$v_B = 0$	$v_C = -20/3(Fb^3/EJ)$	$v_D = 439/90(Fb^3/EJ)$
$\varphi_A = 0$	$\varphi_B = 17/3(Fb^2/EJ)$	$\varphi_C = 43/6(Fb^2/EJ)$	$\varphi_D = 383/90(Fb^2/EJ)$

$u_E = 0$
$v_E = 439/90(Fb^3/EJ)$
$\varphi_E = 223/90(Fb^2/EJ)$

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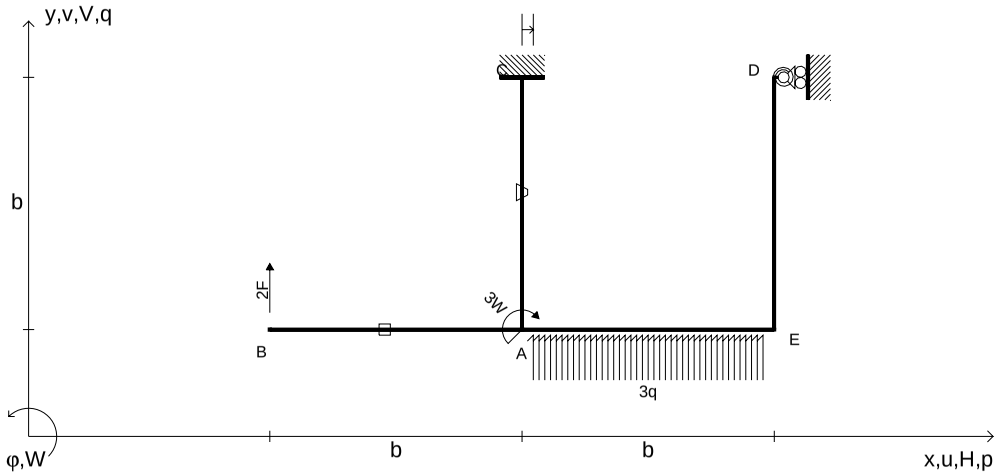
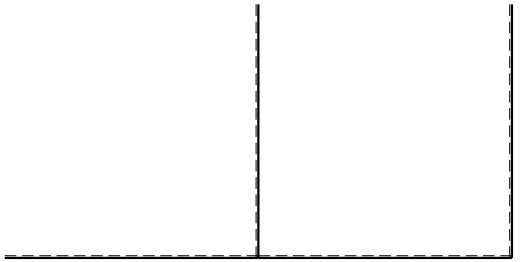


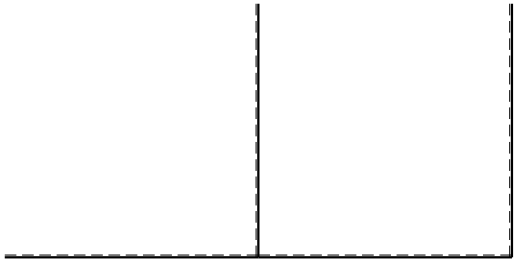
$V_B = 2F$ $\varepsilon_{AB} = -4\alpha T = -4b^2F/EJ$ $k_D = 4EJ/b$ $EJ_{DE} = EJ$

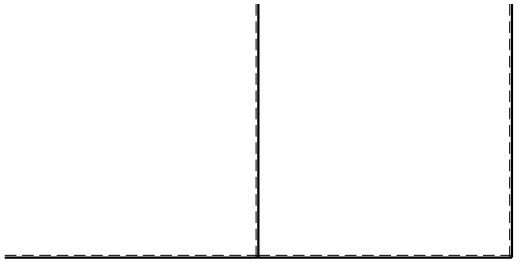
$W_A = -3W = -3Fb$ $\theta_{CA} = \theta = \alpha T/b = bF/EJ$ $EJ_{AB} = EJ$ $EJ_{EA} = EJ$

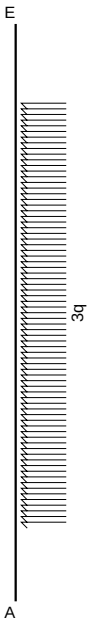
$q_{EA} = 3q = 3F/b$ $u_C = 4\delta = 4b^3F/EJ$ $EJ_{CA} = 1/2EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio (retro incluso).
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta CA positiva se convessa a destra con inizio C.
Spostamento orizzontale assoluto u imposto al nodo C.









DEFORMATA (coordinate locali)
AB y(x)EJ =
CA y(x)EJ =
DE y(x)EJ =
EA y(x)EJ =

SPOSTAMENTI NODALI

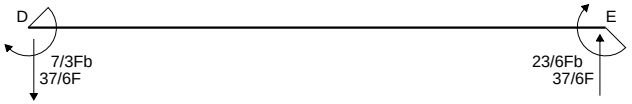
u_A =
v_A =
φ_A =

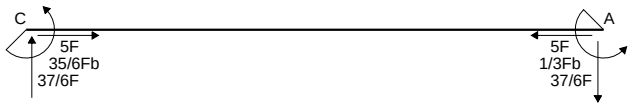
u_B =
v_B =
φ_B =

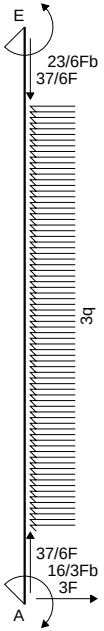
u_C =
v_C =
φ_C =

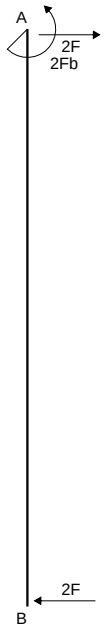
u_D =
v_D =
φ_D =

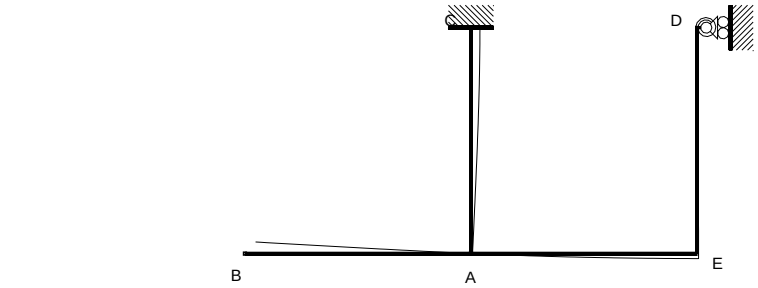
u_E =
v_E =
φ_E =



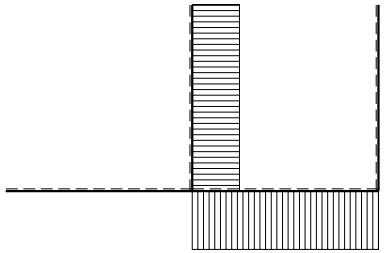




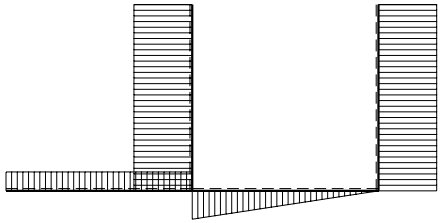




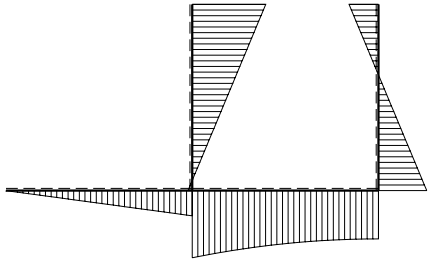
15 Fb^3/EJ



8 F



8 F



6 Fb

REAZIONI IPERSTATICHE

$X = W_{CA}$ $Y = W_{EA}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: $\varphi_{AB}\ K_{AB}\ \varphi_{CA}\ K_{CA}\ \varphi_{DE}\ K_{DE}\ \varphi_{EA}\ K_{EA}$

Relazioni di congruenza

$y'_{AB}(0) - y'_{EA}(b) = 0$
$y'_{CA}(0) = 0$
$y'_{CA}(b) - y'_{EA}(b) = 0$
$y'_{DE}(0) + 1/4W_D b/EJ = 0$
$y'_{DE}(b) - y'_{EA}(0) = 0$
$y_{AB}(0) = 0$
$y_{CA}(0) - 4\delta = 0$
$y_{DE}(0) = 0$
$y_{DE}(b) - y_{CA}(b) = 0$
$y_{EA}(b) = 0$

$M_{AB} = 2Fx - 2Fb$
$EJy'' = 2Fx - 2Fb$
$EJy' = Fx^2 - 2Fbx + EJ\varphi_{AB}$
$EJy = 1/3Fx^3 - Fbx^2 + EJ\varphi_{AB}x + EJK_{AB}$

$M_{CA} = -7/2Fx + Xx/b - X + Yx/b$
$EJy'' = -7Fx + EJ\theta + 2Xx/b - 2X + 2Yx/b$
$EJy' = -7/2Fx^2 + EJ\theta x + Xx^2/b - 2Xx + Yx^2/b + EJ\varphi_{CA}$
$EJy = -7/6Fx^3 + 1/2EJ\theta x^2 + 1/3Xx^3/b - Xx^2 + 1/3Yx^3/b + EJ\varphi_{CA}x + EJK_{CA}$

$M_{DE} = 7/2Fx - 7/2Fb - Xx/b + X - Yx/b$
$EJy'' = 7/2Fx - 7/2Fb - Xx/b + X - Yx/b$
$EJy' = 7/4Fx^2 - 7/2Fbx - 1/2Xx^2/b + Xx - 1/2Yx^2/b + EJ\varphi_{DE}$
$EJy = 7/12Fx^3 - 7/4Fbx^2 - 1/6Xx^3/b + 1/2Xx^2 - 1/6Yx^3/b + EJ\varphi_{DE}x + EJK_{DE}$

$M_{EA} = -3/2qx^2 - Y$
$EJy'' = -3/2qx^2 - Y$
$EJy' = -1/2qx^3 - Yx + EJ\varphi_{EA}$
$EJy = -1/8qx^4 - 1/2Yx^2 + EJ\varphi_{EA}x + EJK_{EA}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CA}b$	K_{CA}	$\varphi_{DE}b$	K_{DE}	$\varphi_{EA}b$	K_{EA}	Xb^2/EJ	Yb^2/EJ		Fb^3/EJ
y'_{AB}	1	0	0	0	0	0	-1	0	0	1		-1/2
y'_{CA}	0	0	1	0	0	0	0	0	0	0		0
y'_{AC}	0	0	1	0	0	0	-1	0	-1	2		3
y'_{DE}	0	0	0	0	1	0	0	0	-1/4	0		-7/8
y'_{ED}	0	0	0	0	1	0	-1	0	1/2	-1/2		7/4
y_{AB}	0	1	0	0	0	0	0	0	0	0	=	0
y_{CA}	0	0	0	1	0	0	0	0	0	0		0
y_{DE}	0	0	0	0	0	1	0	0	0	0		0
y_{ED}	0	0	-1	-1	1	1	0	0	1	-1/2		0
y_{AE}	0	0	0	0	0	0	1	1	0	-1/2		1/8

Condizioni al contorno

αTb	δ
0	0
0	0
-1	0
0	0
0	0
0	0
0	4
0	0
1/2	0
0	0

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		-9/2
$\varphi_{CA}b$		0
$\varphi_{EA}b$		-1/6
$\varphi_{DE}b$		7/12
Xb^2/EJ	=	35/6
K_{AB}		0
K_{CA}		4
K_{DE}		0
Yb^2/EJ		23/6
K_{EA}		53/24

DEFORMATA (coordinate locali)

AB $y(x)EJ = -9/2xFb^2 - x^2Fb + 1/3x^3F$
BA $y(x)EJ = -31/6Fb^3 + 11/2xFb^2 - 1/3x^3F$
CA $y(x)EJ = 4Fb^3 - 16/3x^2Fb + 37/18x^3F$
AC $y(x)EJ = 13/18Fb^3 + 9/2xFb^2 + 5/6x^2Fb - 37/18x^3F$
DE $y(x)EJ = 7/12xFb^2 + 7/6x^2Fb - 37/36x^3F$
ED $y(x)EJ = 13/18Fb^3 + 1/6xFb^2 - 23/12x^2Fb + 37/36x^3F$
EA $y(x)EJ = 53/24Fb^3 - 1/6xFb^2 - 23/12x^2Fb - 1/8x^4q$
AE $y(x)EJ = 9/2xFb^2 - 8/3x^2Fb + 1/2x^3F - 1/8x^4q$

SPOSTAMENTI NODALI

$u_A = 13/18(Fb^3/EJ)$ $u_B = 85/18(Fb^3/EJ)$ $u_C = 4(Fb^3/EJ)$ $u_D = 0$
$v_A = 0$ $v_B = 31/6(Fb^3/EJ)$ $v_C = 0$ $v_D = -53/24(Fb^3/EJ)$
$\varphi_A = -9/2(Fb^2/EJ)$ $\varphi_B = -11/2(Fb^2/EJ)$ $\varphi_C = 0$ $\varphi_D = 7/12(Fb^2/EJ)$

$u_E = 13/18(Fb^3/EJ)$
$v_E = -53/24(Fb^3/EJ)$
$\varphi_E = -1/6(Fb^2/EJ)$

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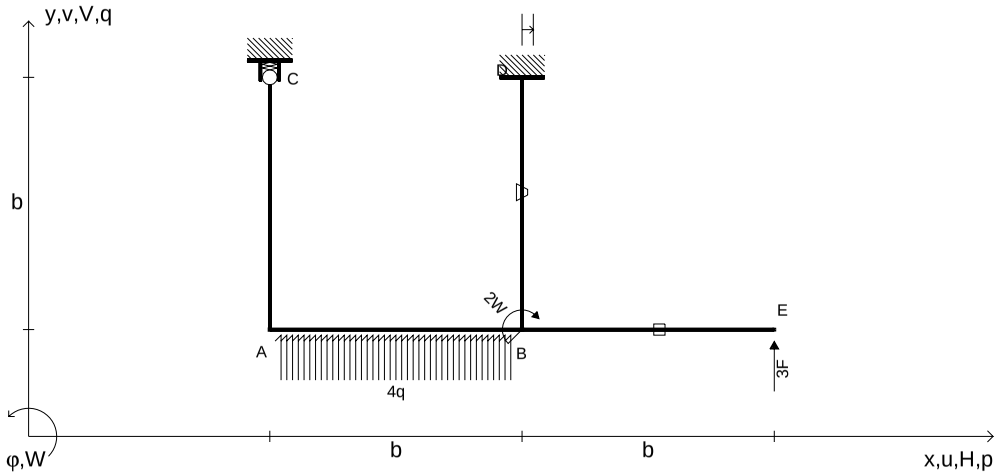
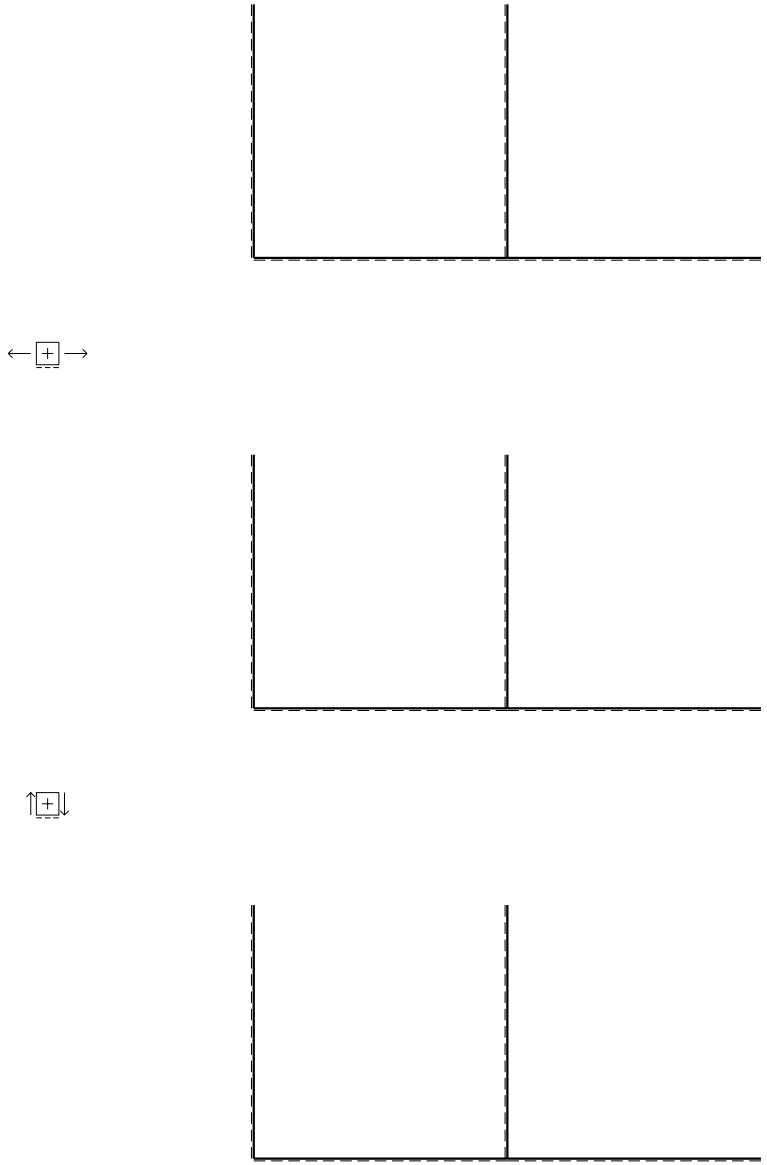


$V_E = 3F$
$W_B = -2W = -2Fb$
$q_{AB} = 4q = 4F/b$
$\varepsilon_{BE} = -3\alpha T = -3b^2F/EJ$

$\theta_{DB} = 2\theta = 2\alpha T/b = 2bF/EJ$
$u_D = \delta = b^3F/EJ$
$k_C = 2EJ/b^3$
$EJ_{AB} = EJ$

$EJ_{CA} = EJ$
$EJ_{DB} = 2/3EJ$
$EJ_{BE} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio (retro incluso).
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.
Curvatura θ asta DB positiva se convessa a destra con inizio D.
Spostamento orizzontale assoluto u imposto al nodo D.



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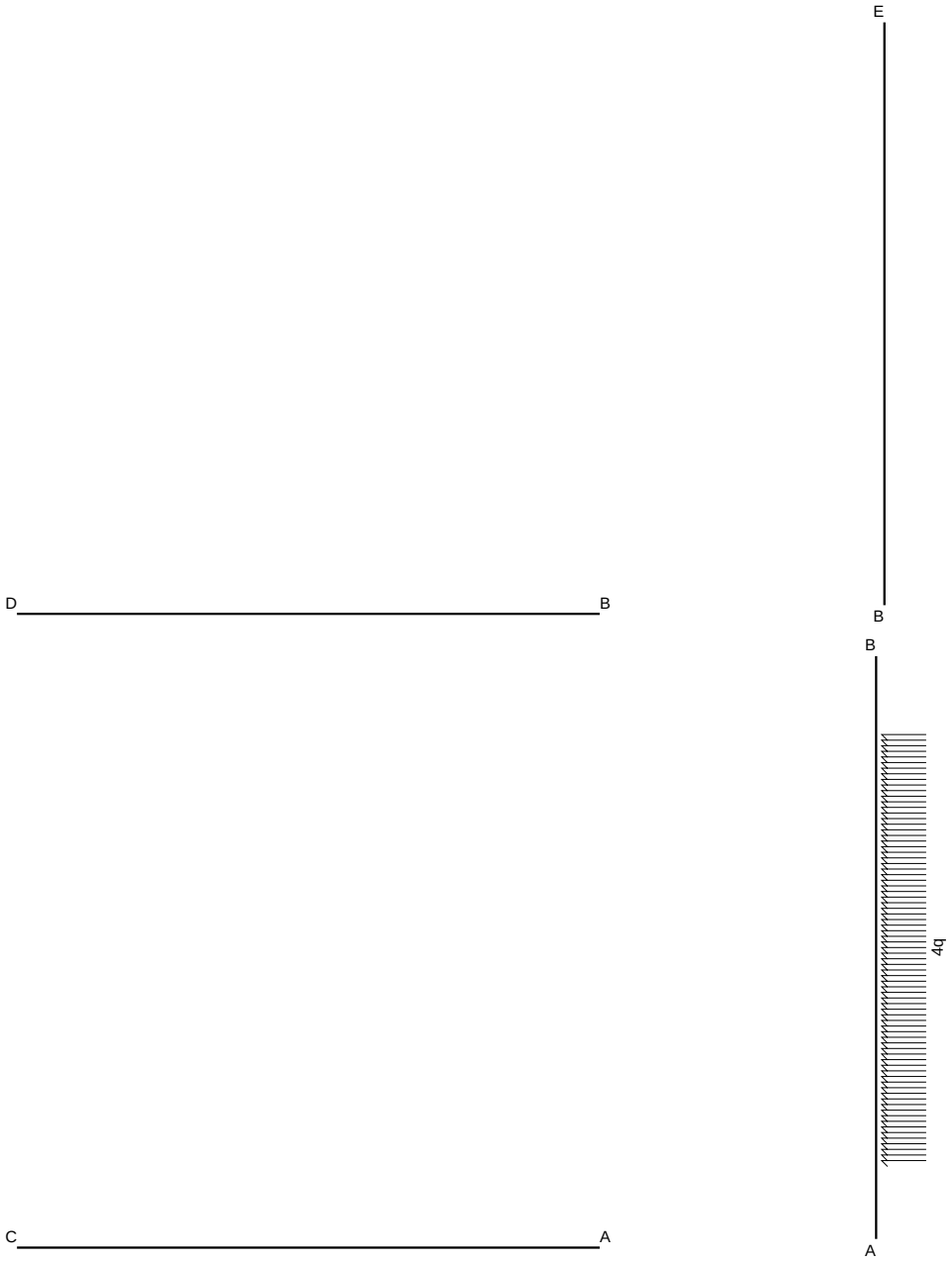


DEFORMATA (coordinate locali)
AB y(x)EJ =
CA y(x)EJ =
DB y(x)EJ =
BE y(x)EJ =

SPOSTAMENTI NODALI

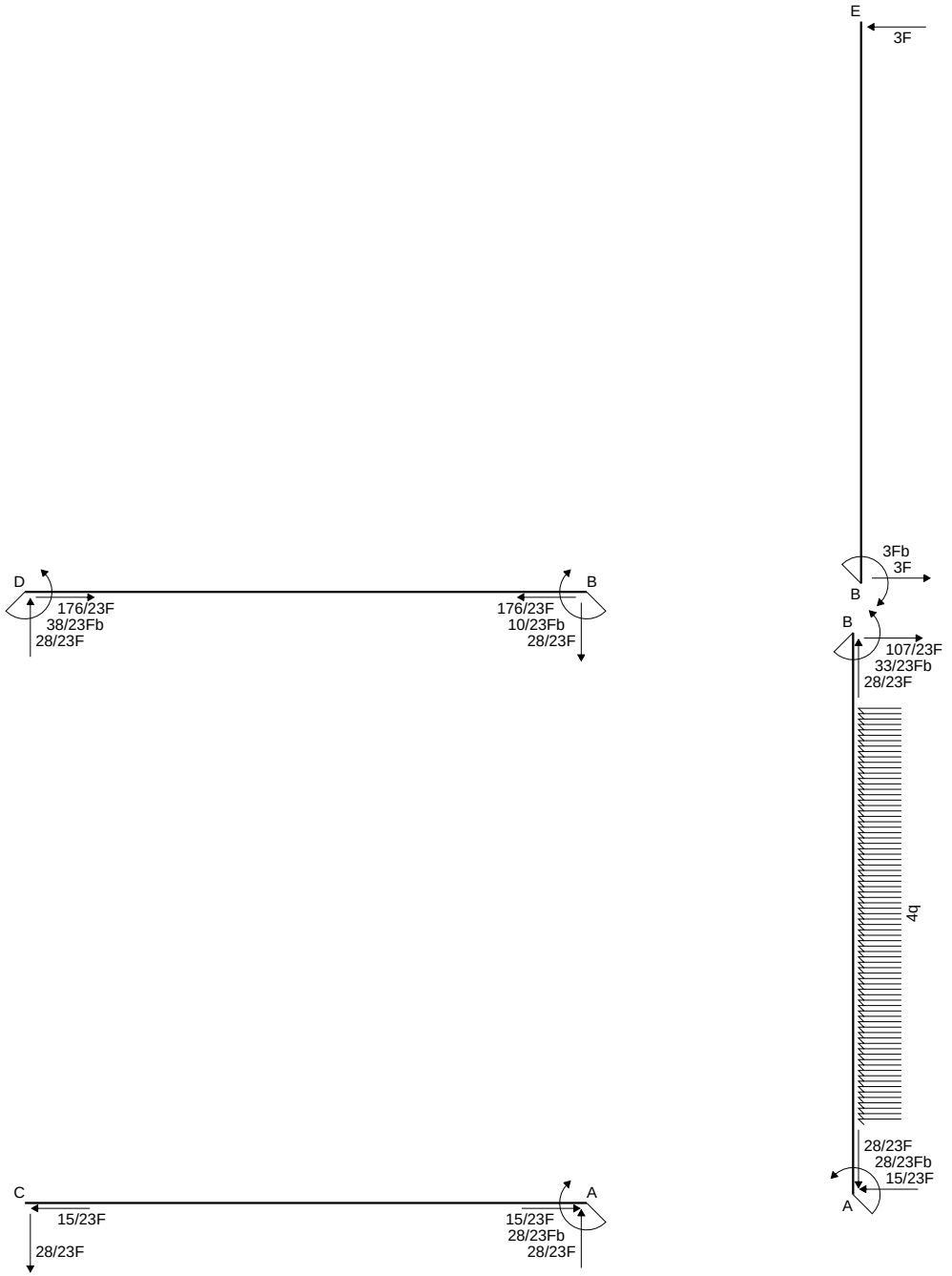
u_A =
v_A =
φ_A =

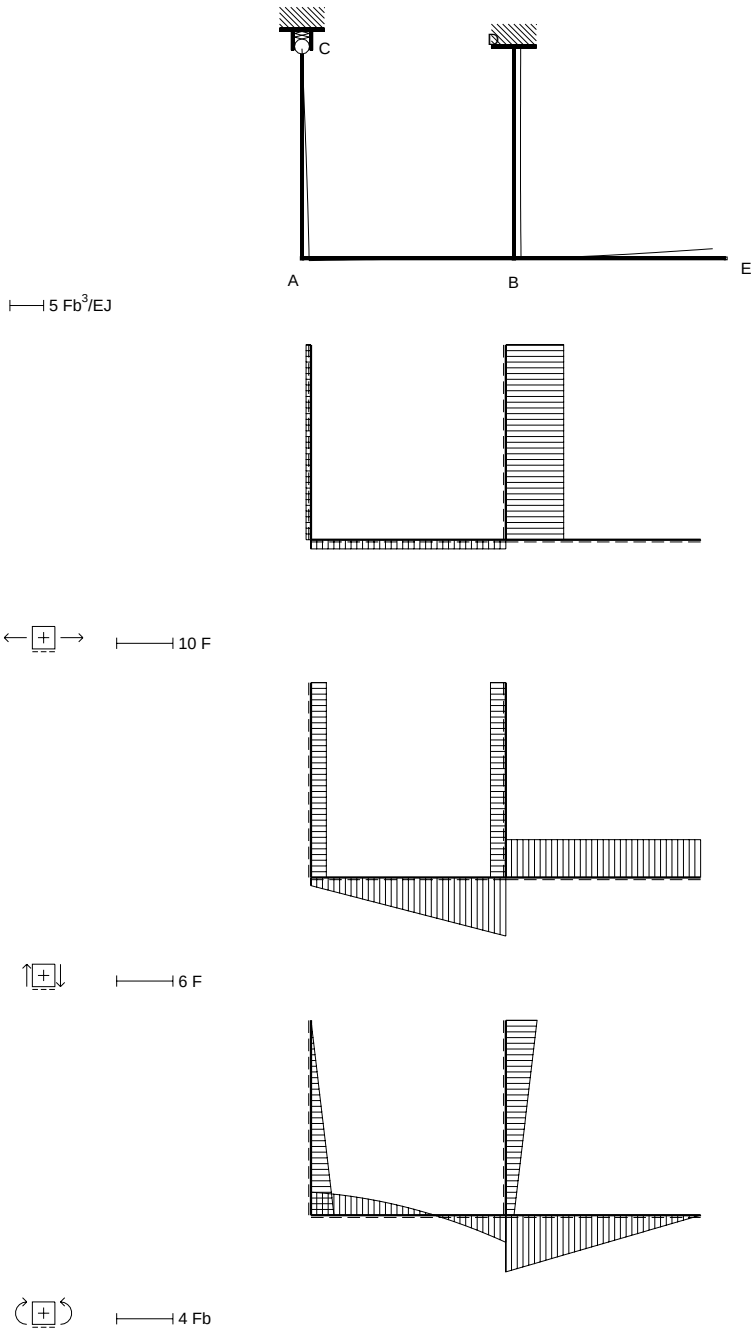
u_B =
v_B =
φ_B =

u_C =
v_C =
φ_{CCA} =

u_D =
v_D =
φ_D =

u_E =
v_E =
φ_E =





REAZIONI IPERSTATICHE

$X = W_{AB}$ $Y = W_{DB}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: $\varphi_{AB}\ K_{AB}\ \varphi_{CA}\ K_{CA}\ \varphi_{DB}\ K_{DB}\ \varphi_{BE}\ K_{BE}$

Relazioni di congruenza

$y'_{AB}(0) - y'_{CA}(b) = 0$
$y'_{AB}(b) - y'_{BE}(0) = 0$
$y'_{DB}(0) = 0$
$y'_{DB}(b) - y'_{BE}(0) = 0$
$y_{AB}(0) + 1/2V_C b^3/EJ = 0$
$y_{AB}(b) = 0$
$y_{CA}(0) = 0$
$y_{DB}(0) - \delta = 0$
$y_{DB}(b) - y_{CA}(b) = 0$
$y_{BE}(0) = 0$

$M_{AB} = -Fx +2qx^2 -X +Yx/b$
$EJy'' = -Fx +2qx^2 -X +Yx/b$
$EJy' = -1/2Fx^2 +2/3qx^3 -Xx +1/2Yx^2/b +EJ\varphi_{AB}$
$EJy = -1/6Fx^3 +1/6qx^4 -1/2Xx^2 +1/6Yx^3/b +EJ\varphi_{AB}x +EJK_{AB}$

$M_{CA} = -Xx/b$
$EJy'' = -Xx/b$
$EJy' = -1/2Xx^2/b +EJ\varphi_{CA}$
$EJy = -1/6Xx^3/b +EJ\varphi_{CA}x +EJK_{CA}$

$M_{DB} = Xx/b -Y$
$EJy'' = 2EJ\theta +3/2Xx/b -3/2Y$
$EJy' = 2EJ\theta x +3/4Xx^2/b -3/2Yx +EJ\varphi_{DB}$
$EJy = EJ\theta x^2 +1/4Xx^3/b -3/4Yx^2 +EJ\varphi_{DB}x +EJK_{DB}$

$M_{BE} = -3Fx +3Fb$
$EJy'' = -3Fx +3Fb$
$EJy' = -3/2Fx^2 +3Fbx +EJ\varphi_{BE}$
$EJy = -1/2Fx^3 +3/2Fbx^2 +EJ\varphi_{BE}x +EJK_{BE}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CA}b$	K_{CA}	$\varphi_{DB}b$	K_{DB}	$\varphi_{BE}b$	K_{BE}	Xb^2/EJ	Yb^2/EJ		Fb^3/EJ
y'_{AB}	1	0	-1	0	0	0	0	0	1/2	0		0
y'_{BA}	1	0	0	0	0	0	-1	0	-1	1/2		-1/6
y'_{DB}	0	0	0	0	1	0	0	0	0	0		0
y'_{BD}	0	0	0	0	1	0	-1	0	3/4	-3/2		0
y_{AB}	0	1	0	0	0	0	0	0	0	1/2		1/2
y_{BA}	1	1	0	0	0	0	0	0	-1/2	1/6	=	0
y_{CA}	0	0	0	1	0	0	0	0	0	0		0
y_{DB}	0	0	0	0	0	1	0	0	0	0		0
y_{BD}	0	0	-1	-1	1	1	0	0	5/12	-3/4		0
y_{BE}	0	0	0	0	0	0	0	1	0	0		0

Condizioni al contorno

αTb	δ
0	0
0	0
0	0
-2	0
0	0
0	0
0	0
0	1
-1	0
0	0

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		91/138
$\varphi_{CA}b$		175/138
$\varphi_{DB}b$		0
$\varphi_{BE}b$		10/23
K_{AB}		-15/46
Xb^2/EJ	=	28/23
K_{CA}		0
K_{DB}		1
Yb^2/EJ		38/23
K_{BE}		0

DEFORMATA (coordinate locali)

AB $y(x)EJ = -15/46Fb^3 +91/138xFb^2 -14/23x^2Fb +5/46x^3F +1/6x^4q$
BA $y(x)EJ = -10/23xFb^2 +33/46x^2Fb -107/138x^3F +1/6x^4q$
CA $y(x)EJ = 175/138xFb^2 -14/69x^3F$
AC $y(x)EJ = 49/46Fb^3 -91/138xFb^2 -14/23x^2Fb +14/69x^3F$
DB $y(x)EJ = Fb^3 -11/46x^2Fb +7/23x^3F$
BD $y(x)EJ = 49/46Fb^3 -10/23xFb^2 +31/46x^2Fb -7/23x^3F$
BE $y(x)EJ = 10/23xFb^2 +3/2x^2Fb -1/2x^3F$
EB $y(x)EJ = 33/23Fb^3 -89/46xFb^2 +1/2x^3F$

SPOSTAMENTI NODALI

$u_A = 49/46(Fb^3/EJ)$ $u_B = 49/46(Fb^3/EJ)$ $u_C = 0$ $u_D = (Fb^3/EJ)$
$v_A = -15/46(Fb^3/EJ)$ $v_B = 0$ $v_C = -15/46(Fb^3/EJ)$ $v_D = 0$
$\varphi_A = 91/138(Fb^2/EJ)$ $\varphi_B = 10/23(Fb^2/EJ)$ $\varphi_{CCA} = 175/138(Fb^2/EJ)$ $\varphi_D = 0$

$u_E = -89/46(Fb^3/EJ)$
$v_E = 33/23(Fb^3/EJ)$
$\varphi_E = 89/46(Fb^2/EJ)$

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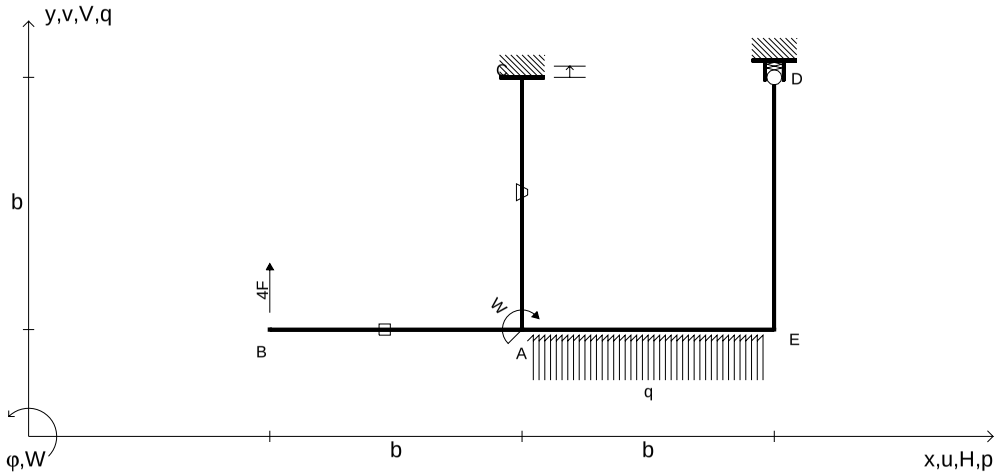
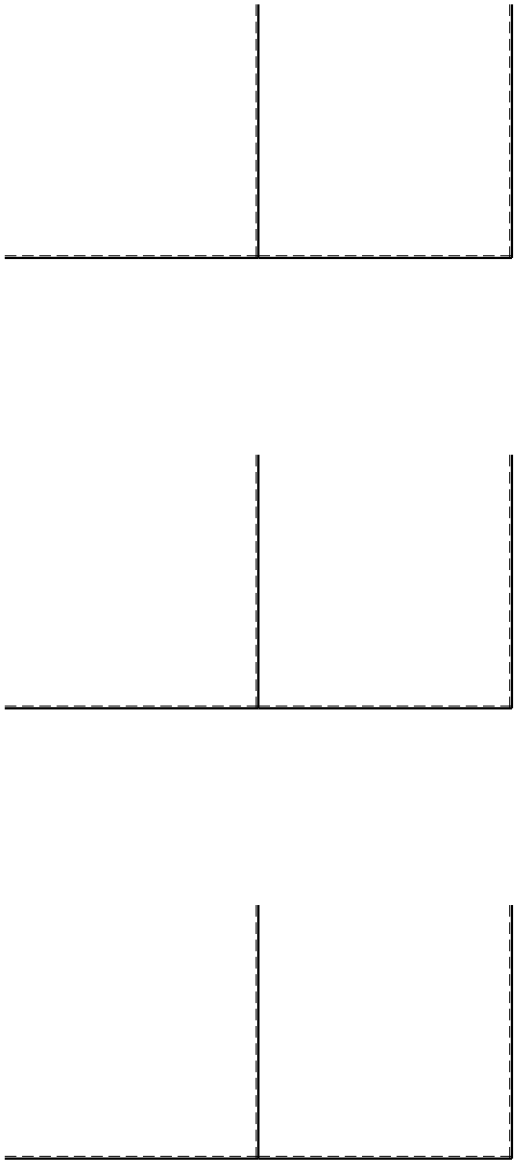


$V_B = 4F$
$W_A = -W = -Fb$
$q_{EA} = q = F/b$
$\varepsilon_{AB} = -2\alpha T = -2b^2F/EJ$

$\theta_{CA} = 3\theta = 3\alpha T/b = 3bF/EJ$
$v_C = 2\delta = 2b^3F/EJ$
$k_D = 4EJ/b^3$
$EJ_{AB} = EJ$

$EJ_{CA} = 3/4EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio (retro incluso).
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta CA positiva se convessa a destra con inizio C.
Spostamento verticale assoluto v imposto al nodo C.



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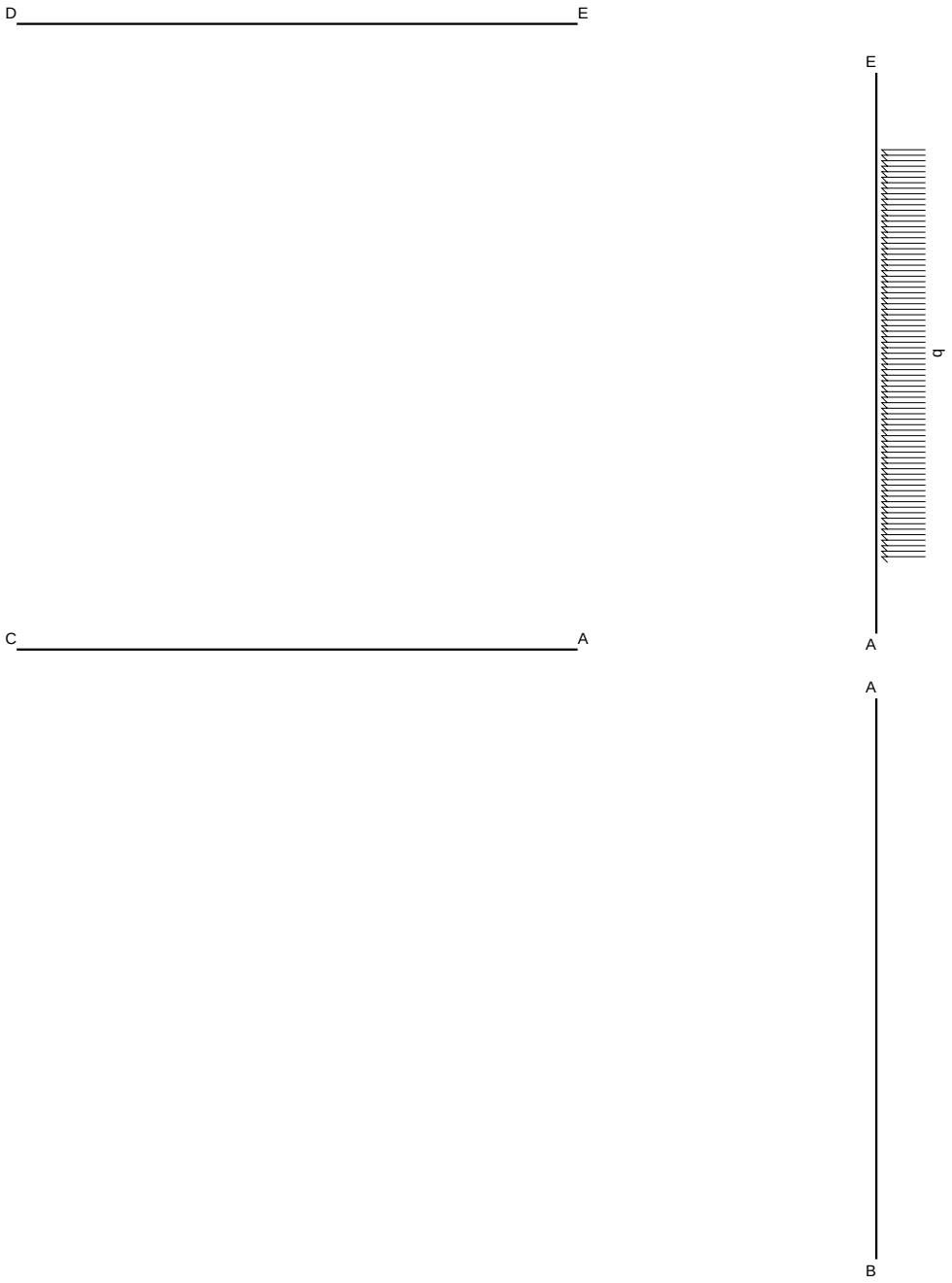


DEFORMATA (coordinate locali)
AB y(x)EJ =
CA y(x)EJ =
DE y(x)EJ =
EA y(x)EJ =

SPOSTAMENTI NODALI

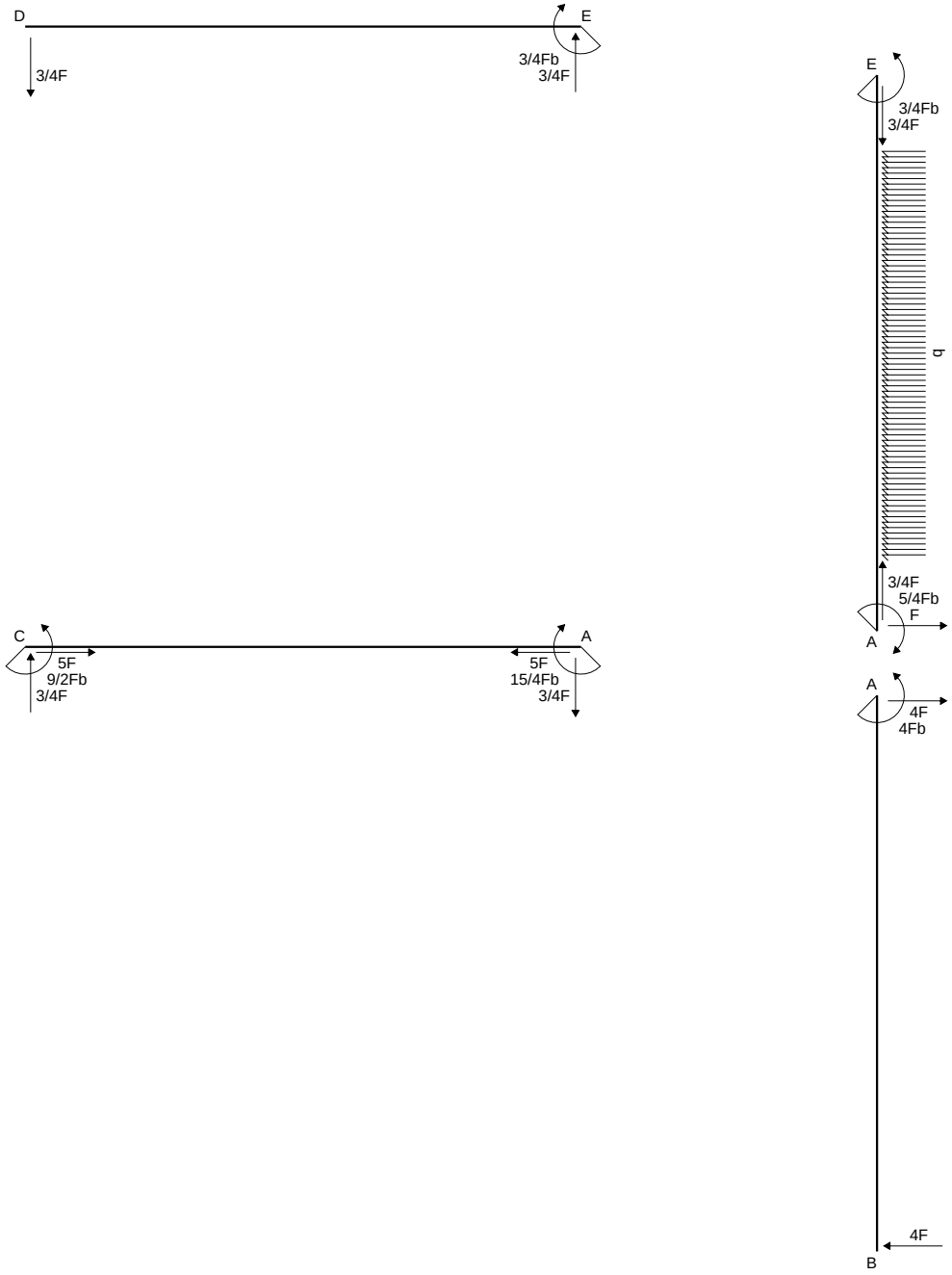
u_A =
v_A =
φ_A =

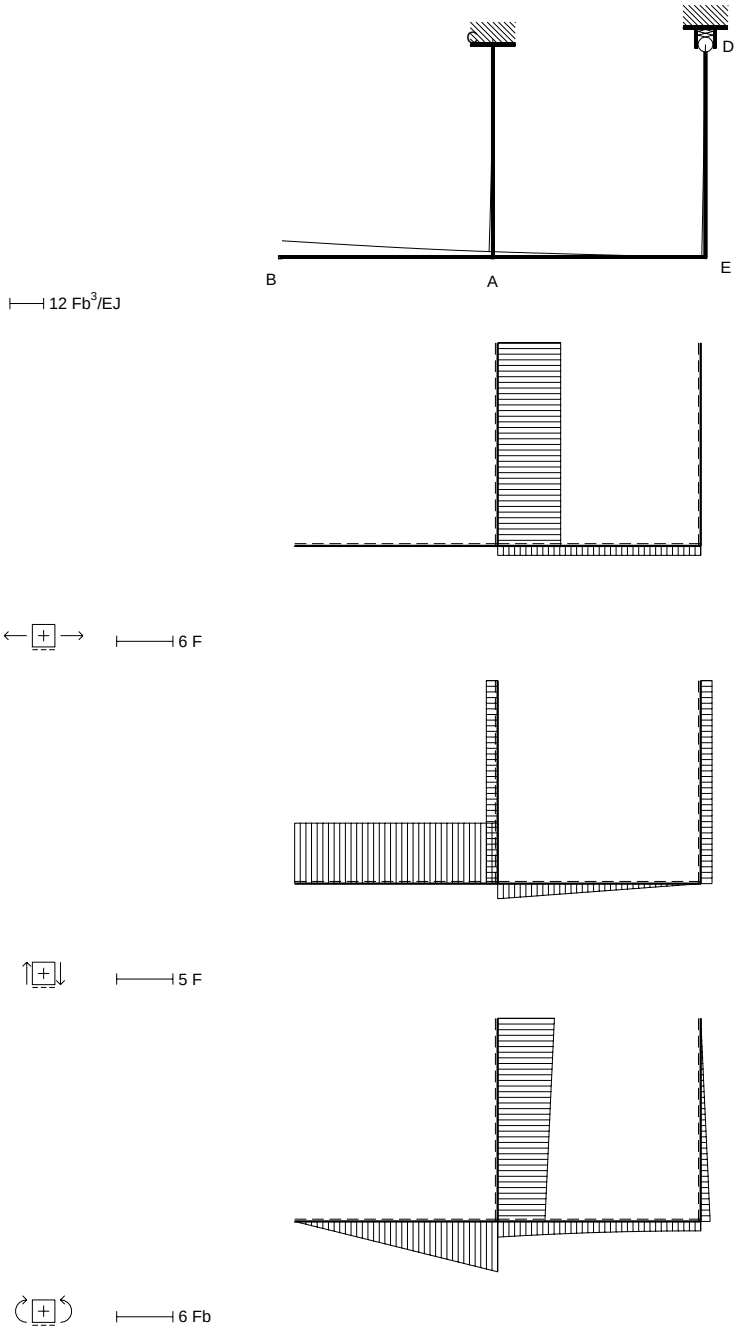
u_B =
v_B =
φ_B =

u_C =
v_C =
φ_C =

u_D =
v_D =
φ_{DDE} =

u_E =
v_E =
φ_E =





REAZIONI IPERSTATICHE

$X = W_{CA}$ $Y = W_{EA}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CA} K_{CA} φ_{DE} K_{DE} φ_{EA} K_{EA}

Relazioni di congruenza

$y'_{AB}(0) - y'_{EA}(b) = 0$
$y'_{CA}(0) = 0$
$y'_{CA}(b) - y'_{EA}(b) = 0$
$y'_{DE}(b) - y'_{EA}(0) = 0$
$y_{AB}(0) + 2\delta = 0$
$y_{CA}(0) = 0$
$y_{DE}(0) = 0$
$y_{DE}(b) - y_{CA}(b) - 4\delta = 0$
$y_{EA}(0) - 1/4V_D b^3/EJ = 0$
$y_{EA}(b) + 2\delta = 0$

$M_{AB} = 4Fx - 4Fb$
$EJy'' = 4Fx - 4Fb$
$EJy' = 2Fx^2 - 4Fbx + EJ\varphi_{AB}$
$EJy = 2/3Fx^3 - 2Fbx^2 + EJ\varphi_{AB}x + EJK_{AB}$

$M_{CA} = -X + Yx/b$
$EJy'' = 3EJ\theta - 4/3X + 4/3Yx/b$
$EJy' = 3EJ\theta x - 4/3Xx + 2/3Yx^2/b + EJ\varphi_{CA}$
$EJy = 3/2EJ\theta x^2 - 2/3Xx^2 + 2/9Yx^3/b + EJ\varphi_{CA}x + EJK_{CA}$

$M_{DE} = -Yx/b$
$EJy'' = -Yx/b$
$EJy' = -1/2Yx^2/b + EJ\varphi_{DE}$
$EJy = -1/6Yx^3/b + EJ\varphi_{DE}x + EJK_{DE}$

$M_{EA} = -9/2Fx - 1/2qx^2 + Xx/b - Y$
$EJy'' = -9/2Fx - 1/2qx^2 + Xx/b - Y$
$EJy' = -9/4Fx^2 - 1/6qx^3 + 1/2Xx^2/b - Yx + EJ\varphi_{EA}$
$EJy = -3/4Fx^3 - 1/24qx^4 + 1/6Xx^3/b - 1/2Yx^2 + EJ\varphi_{EA}x + EJK_{EA}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CA}b$	K_{CA}	$\varphi_{DE}b$	K_{DE}	$\varphi_{EA}b$	K_{EA}	Xb^2/EJ	Yb^2/EJ		Fb^3/EJ
y'_{AB}	1	0	0	0	0	0	-1	0	-1/2	1		-29/12
y'_{CA}	0	0	1	0	0	0	0	0	0	0		0
y'_{AC}	0	0	1	0	0	0	-1	0	-11/6	5/3		-29/12
y'_{ED}	0	0	0	0	1	0	-1	0	0	-1/2		0
y_{AB}	0	1	0	0	0	0	0	0	0	0		0
y_{CA}	0	0	0	1	0	0	0	0	0	0	=	0
y_{DE}	0	0	0	0	0	1	0	0	0	0		0
y_{ED}	0	0	-1	-1	1	1	0	0	2/3	-7/18		0
y_{EA}	0	0	0	0	0	0	0	1	1/4	0		9/8
y_{AE}	0	0	0	0	0	0	1	1	1/6	-1/2		19/24

Condizioni al contorno

αTb	δ
0	0
0	0
-3	0
0	0
0	-2
0	0
0	0
3/2	0
0	0
0	-2

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		-5/2
$\varphi_{CA}b$		0
$\varphi_{EA}b$		-19/12
$\varphi_{DE}b$		-29/24
K_{AB}		-2
K_{CA}	=	0
K_{DE}		0
Xb^2/EJ		9/2
K_{EA}		0
Yb^2/EJ		3/4

DEFORMATA (coordinate locali)

AB $y(x)EJ = -2Fb^3 - 5/2xFb^2 - 2x^2Fb + 2/3x^3F$
BA $y(x)EJ = -35/6Fb^3 + 9/2xFb^2 - 2/3x^3F$
CA $y(x)EJ = -3/2x^2Fb + 1/6x^3F$
AC $y(x)EJ = -4/3Fb^3 + 5/2xFb^2 - x^2Fb - 1/6x^3F$
DE $y(x)EJ = -29/24xFb^2 - 1/8x^3F$
ED $y(x)EJ = -4/3Fb^3 + 19/12xFb^2 - 3/8x^2Fb + 1/8x^3F$
EA $y(x)EJ = -19/12xFb^2 - 3/8x^2Fb - 1/24x^4q$
AE $y(x)EJ = -2Fb^3 + 5/2xFb^2 - 5/8x^2Fb + 1/6x^3F - 1/24x^4q$

SPOSTAMENTI NODALI

$u_A = -4/3(Fb^3/EJ)$ $u_B = 2/3(Fb^3/EJ)$ $u_C = 0$ $u_D = 0$
$v_A = 2(Fb^3/EJ)$ $v_B = 35/6(Fb^3/EJ)$ $v_C = 2(Fb^3/EJ)$ $v_D = 0$
$\varphi_A = -5/2(Fb^2/EJ)$ $\varphi_B = -9/2(Fb^2/EJ)$ $\varphi_C = 0$ $\varphi_{DDE} = -29/24(Fb^2/EJ)$

$u_E = -4/3(Fb^3/EJ)$
$v_E = 0$
$\varphi_E = -19/12(Fb^2/EJ)$

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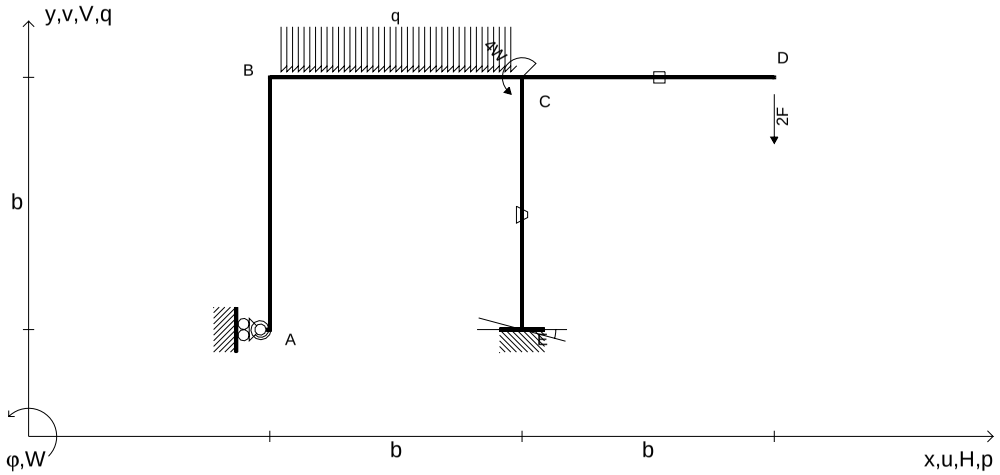
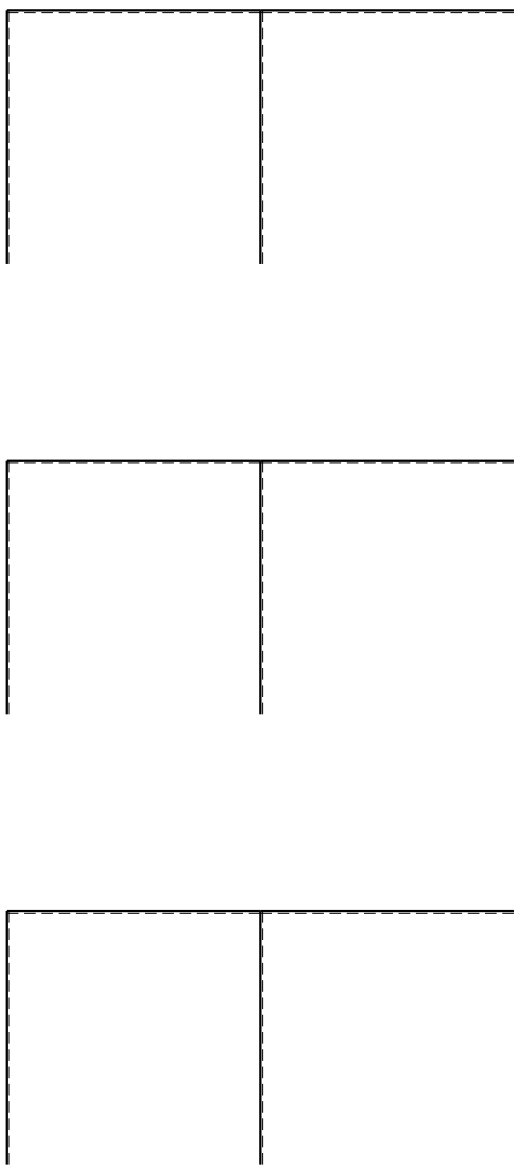


$V_D = -2F$
$W_C = 4W = 4Fb$
$q_{BC} = -q = -F/b$
$\varepsilon_{CD} = 3\alpha T = 3b^2F/EJ$

$\theta_{EC} = -3\theta = -3\alpha T/b = -3bF/EJ$
$\varphi_E = -4\delta/b = -4b^2F/EJ$
$k_A = 3EJ/b$
$EJ_{AB} = EJ$

$EJ_{BC} = EJ$
$EJ_{CD} = EJ$
$EJ_{EC} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio (retro incluso).
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta CD.
Curvatura θ asta EC positiva se convessa a destra con inizio E.
Rotazione assoluta φ imposta al nodo E.



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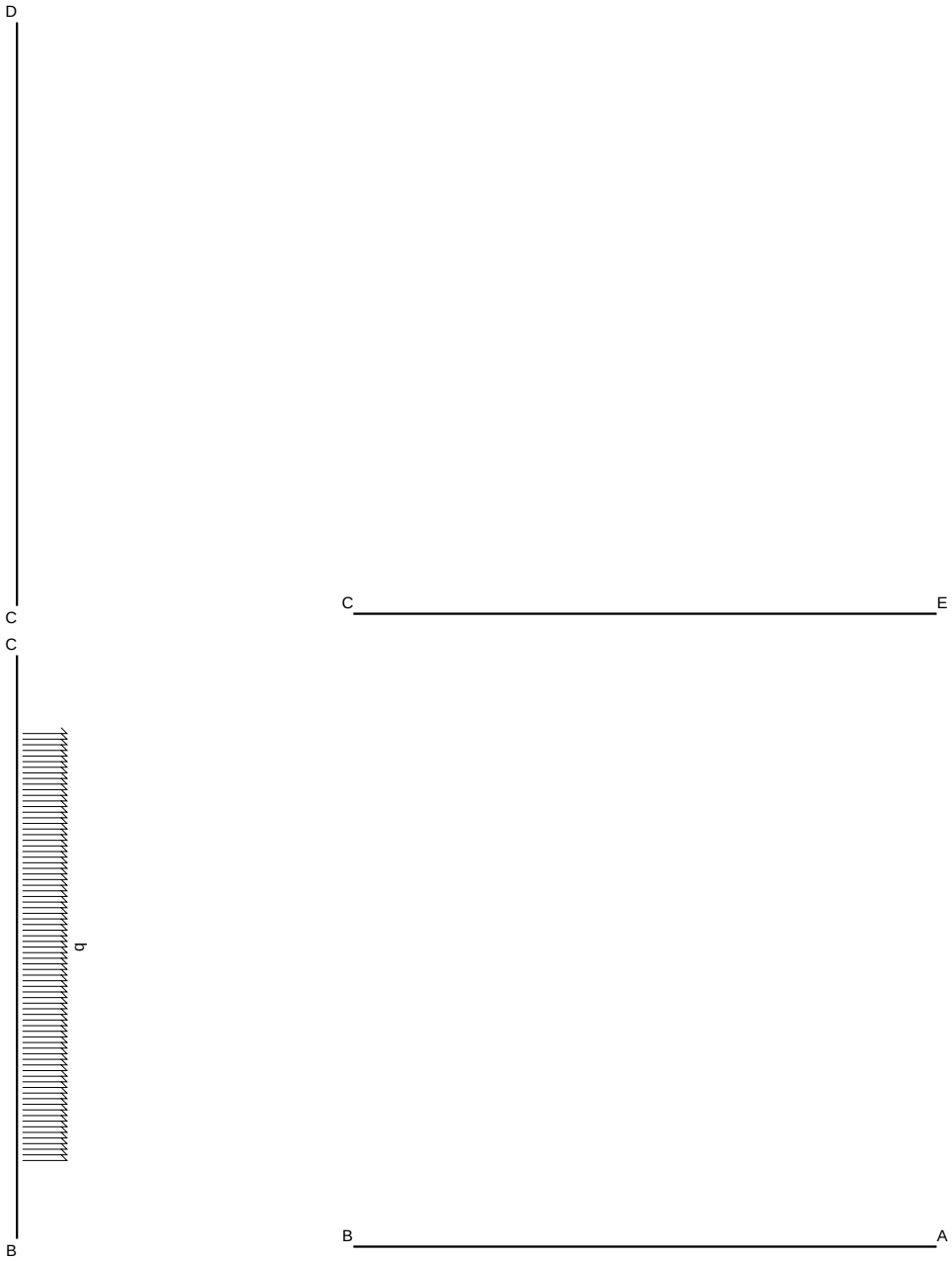

DEFORMATA (coordinate locali)
AB y(x)EJ =
BC y(x)EJ =
CD y(x)EJ =
EC y(x)EJ =

SPOSTAMENTI NODALI

u_A = u_B = u_C = u_D =
v_A = v_B = v_C = v_D =
φ_A = φ_B = φ_C = φ_D =

u_E =
v_E =
φ_E =

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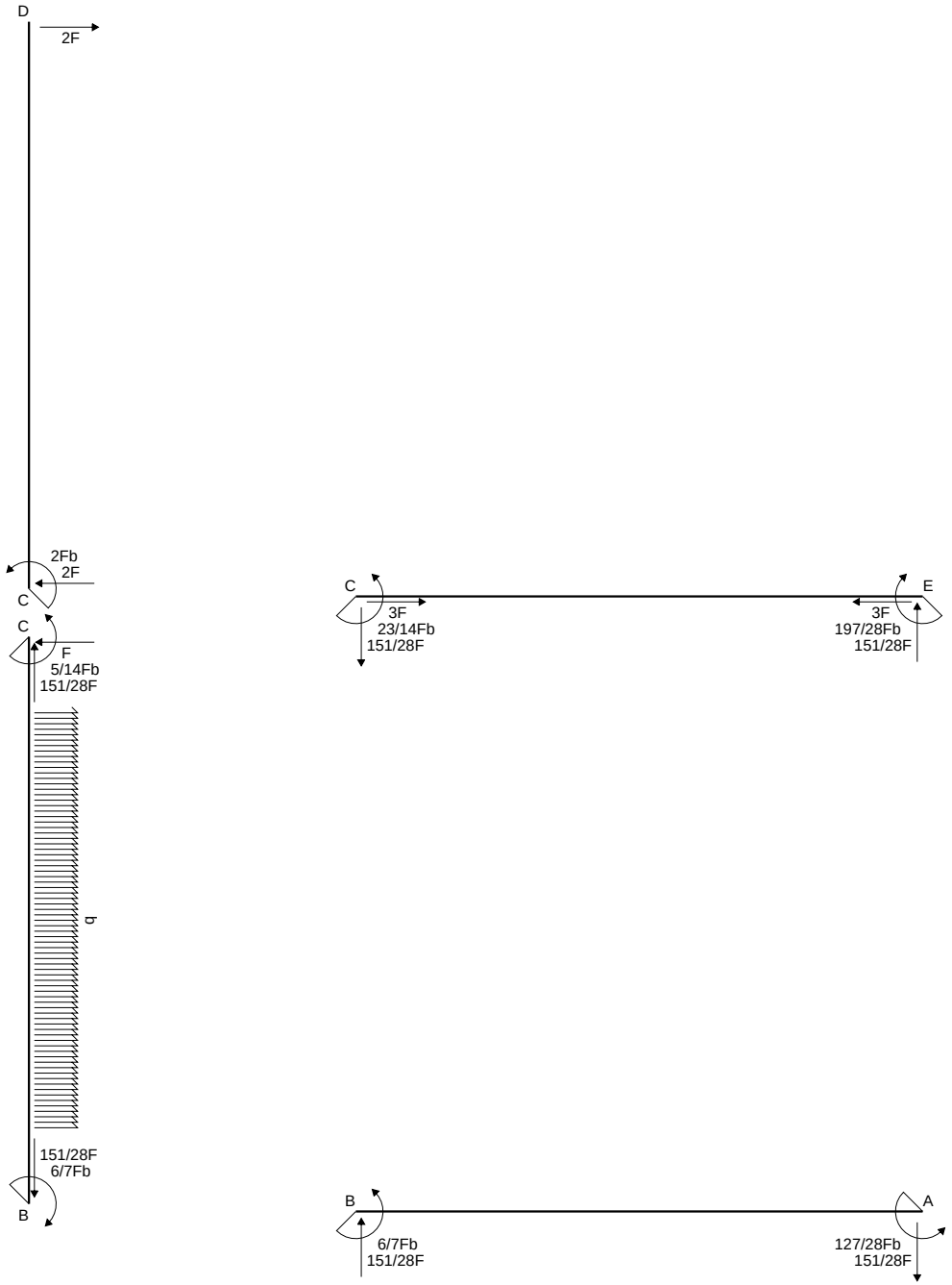

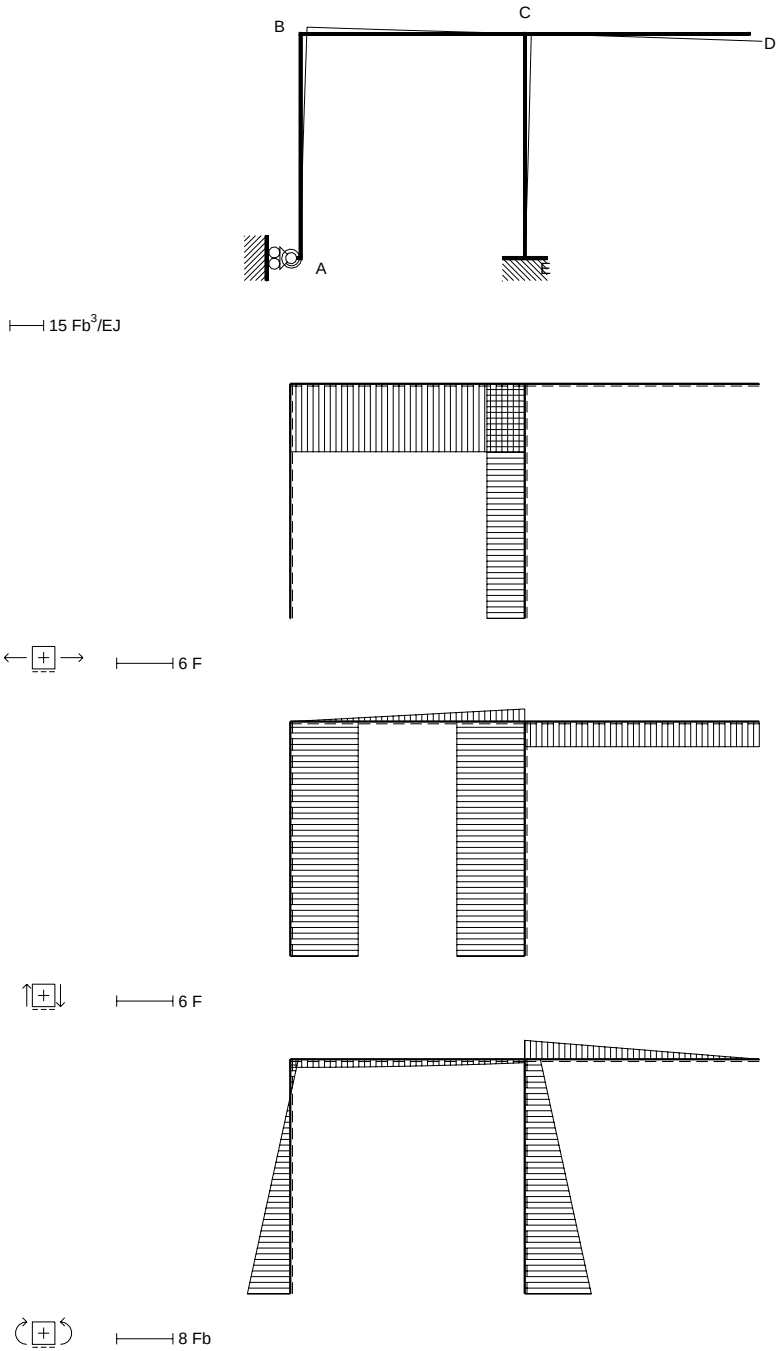

REAZIONI IPERSTATICHE

$X = W_{AB}$ $Y = W_{BC}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: $\varphi_{AB}\ K_{AB}\ \varphi_{BC}\ K_{BC}\ \varphi_{CD}\ K_{CD}\ \varphi_{EC}\ K_{EC}$

Relazioni di congruenza

$y'_{AB}(0) + 1/3W_A b/EJ = 0$
$y'_{AB}(b) - y'_{BC}(0) = 0$
$y'_{BC}(b) - y'_{EC}(b) = 0$
$y'_{CD}(0) - y'_{EC}(b) = 0$
$y'_{EC}(0) + 4\delta/b = 0$
$y_{AB}(0) = 0$
$y_{BC}(b) = 0$
$y_{CD}(0) = 0$
$y_{EC}(0) = 0$
$y_{EC}(b) - y_{AB}(b) = 0$

$M_{AB} = Xx/b - X - Yx/b$
$EJy'' = Xx/b - X - Yx/b$
$EJy' = 1/2Xx^2/b - Xx - 1/2Yx^2/b + EJ\varphi_{AB}$
$EJy = 1/6Xx^3/b - 1/2Xx^2 - 1/6Yx^3/b + EJ\varphi_{AB}x + EJK_{AB}$

$M_{BC} = -1/2qx^2 - Y$
$EJy'' = -1/2qx^2 - Y$
$EJy' = -1/6qx^3 - Yx + EJ\varphi_{BC}$
$EJy = -1/24qx^4 - 1/2Yx^2 + EJ\varphi_{BC}x + EJK_{BC}$

$M_{CD} = 2Fx - 2Fb$
$EJy'' = 2Fx - 2Fb$
$EJy' = Fx^2 - 2Fbx + EJ\varphi_{CD}$
$EJy = 1/3Fx^3 - Fbx^2 + EJ\varphi_{CD}x + EJK_{CD}$

$M_{EC} = 5/2Fb - Xx/b + X + Yx/b$
$EJy'' = 5/2Fb - 3EJ\theta - Xx/b + X + Yx/b$
$EJy' = 5/2Fbx - 3EJ\theta x - 1/2Xx^2/b + Xx + 1/2Yx^2/b + EJ\varphi_{EC}$
$EJy = 5/4Fbx^2 - 3/2EJ\theta x^2 - 1/6Xx^3/b + 1/2Xx^2 + 1/6Yx^3/b + EJ\varphi_{EC}x + EJK_{EC}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{BC}b$	K_{BC}	$\varphi_{CD}b$	K_{CD}	$\varphi_{EC}b$	K_{EC}	Xb^2/EJ	Yb^2/EJ		Fb^3/EJ
y'_{AB}	1	0	0	0	0	0	0	0	1/3	0		0
y'_{BA}	1	0	-1	0	0	0	0	0	-1/2	-1/2		0
y'_{CB}	0	0	1	0	0	0	-1	0	-1/2	-3/2		8/3
y'_{CD}	0	0	0	0	1	0	-1	0	-1/2	-1/2		5/2
y'_{EC}	0	0	0	0	0	0	1	0	0	0		0
y_{AB}	0	1	0	0	0	0	0	0	0	0	=	0
y_{CB}	0	0	1	1	0	0	0	0	0	-1/2		1/24
y_{CD}	0	0	0	0	0	1	0	0	0	0		0
y_{EC}	0	0	0	0	0	0	0	1	0	0		0
y_{CE}	-1	-1	0	0	0	0	1	1	2/3	1/3		-5/4

Condizioni al contorno

αTb	δ
0	0
0	0
-3	0
-3	0
0	-4
0	0
0	0
0	0
0	0
3/2	0

		Soluzione
		Fb^3/EJ
$\varphi_{AB}b$		-127/84
$\varphi_{BC}b$		-563/168
$\varphi_{EC}b$		-4
$\varphi_{CD}b$		-149/56
Xb^2/EJ	=	127/28
K_{AB}		0
K_{BC}		83/28
K_{CD}		0
K_{EC}		0
Yb^2/EJ		-6/7

DEFORMATA (coordinate locali)

AB $y(x)EJ = -127/84xFb^2 - 127/56x^2Fb + 151/168x^3F$
BA $y(x)EJ = -121/42Fb^3 + 563/168xFb^2 + 3/7x^2Fb - 151/168x^3F$
BC $y(x)EJ = 83/28Fb^3 - 563/168xFb^2 + 3/7x^2Fb - 1/24x^4q$
CB $y(x)EJ = 149/56xFb^2 + 5/28x^2Fb + 1/6x^3F - 1/24x^4q$
CD $y(x)EJ = -149/56xFb^2 - x^2Fb + 1/3x^3F$
DC $y(x)EJ = -559/168Fb^3 + 205/56xFb^2 - 1/3x^3F$
EC $y(x)EJ = -4xFb^2 + 113/56x^2Fb - 151/168x^3F$
CE $y(x)EJ = -121/42Fb^3 + 149/56xFb^2 - 19/28x^2Fb + 151/168x^3F$

SPOSTAMENTI NODALI

$u_A = 0$ $u_B = 121/42(Fb^3/EJ)$ $u_C = 121/42(Fb^3/EJ)$ $u_D = 247/42(Fb^3/EJ)$
$v_A = 83/28(Fb^3/EJ)$ $v_B = 83/28(Fb^3/EJ)$ $v_C = 0$ $v_D = -559/168(Fb^3/EJ)$
$\varphi_A = -127/84(Fb^2/EJ)$ $\varphi_B = -563/168(Fb^2/EJ)$ $\varphi_C = -149/56(Fb^2/EJ)$ $\varphi_D = -205/56(Fb^2/EJ)$

$u_E = 0$
$v_E = 0$
$\varphi_E = -4(Fb^2/EJ)$

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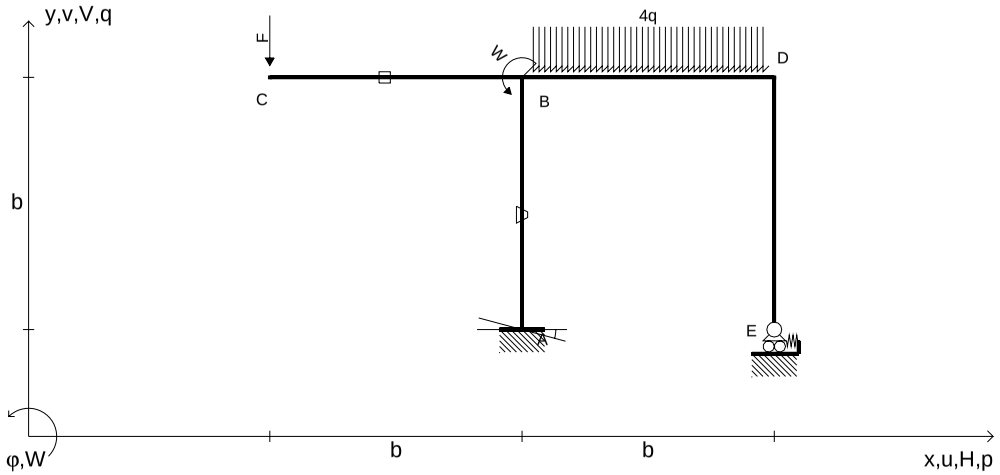
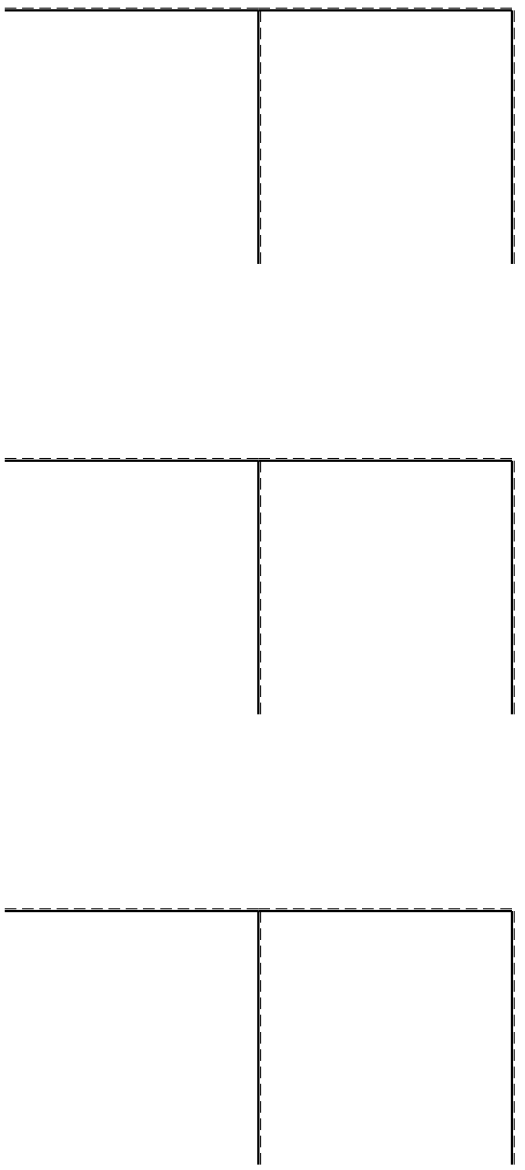


$V_C = -F$
$W_B = W = Fb$
$q_{DB} = -4q = -4F/b$
$\varepsilon_{BC} = 4\alpha T = 4b^2F/EJ$

$\theta_{AB} = -2\theta = -2\alpha T/b = -2bF/EJ$
$\varphi_A = -3\delta/b = -3b^2F/EJ$
$k_E = EJ/b^3$
$EJ_{AB} = 3/2EJ$

$EJ_{BC} = EJ$
$EJ_{DB} = EJ$
$EJ_{ED} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio (retro incluso).
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BC.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Rotazione assoluta φ imposta al nodo A.



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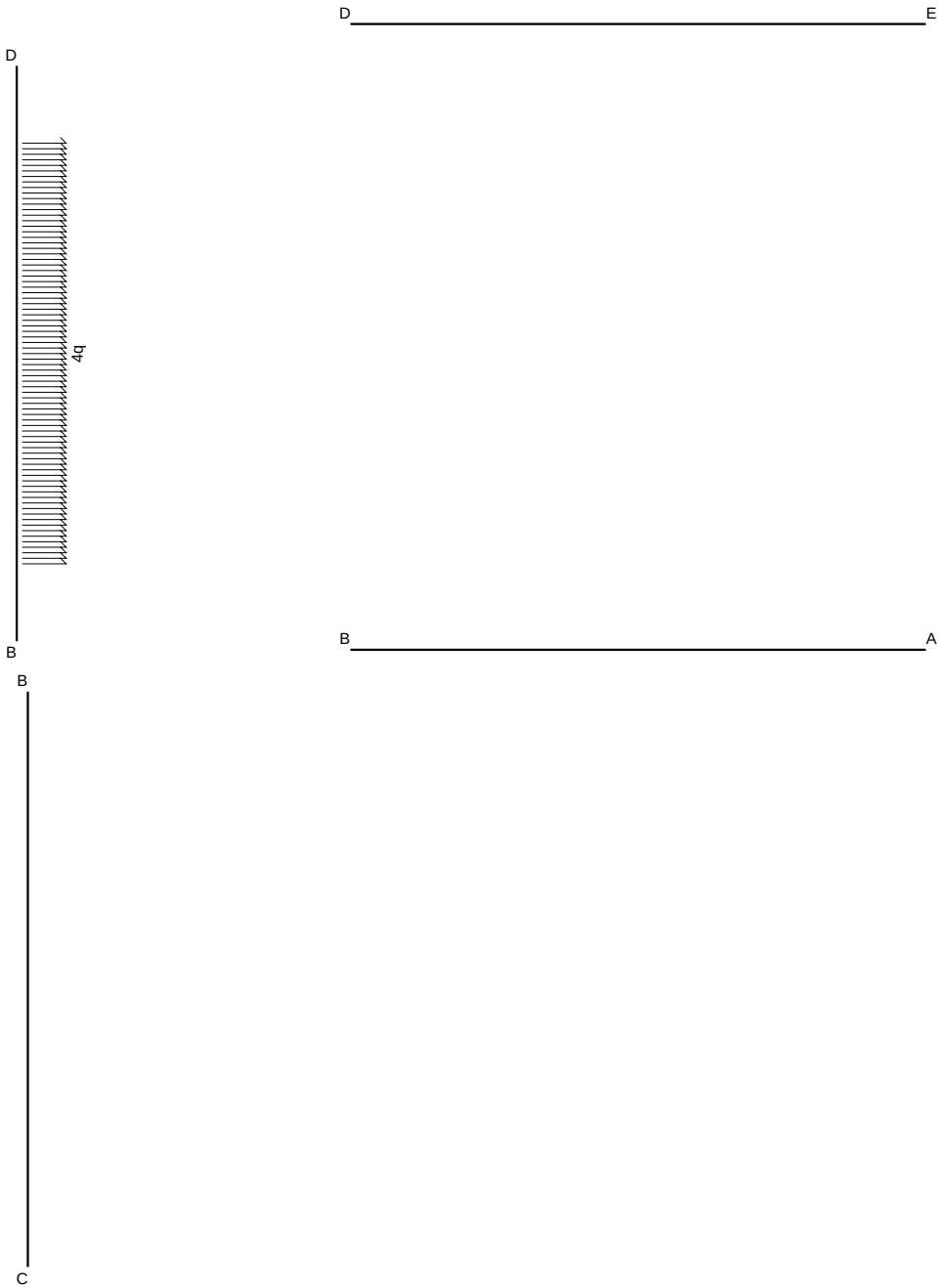

DEFORMATA (coordinate locali)
AB y(x)EJ =
BC y(x)EJ =
DB y(x)EJ =
ED y(x)EJ =

SPOSTAMENTI NODALI

u_A =
v_A =
φ_A =

u_B =
v_B =
φ_B =

u_C =
v_C =
φ_C =

u_D =
v_D =
φ_D =

u_E =
v_E =
φ_{EED} =

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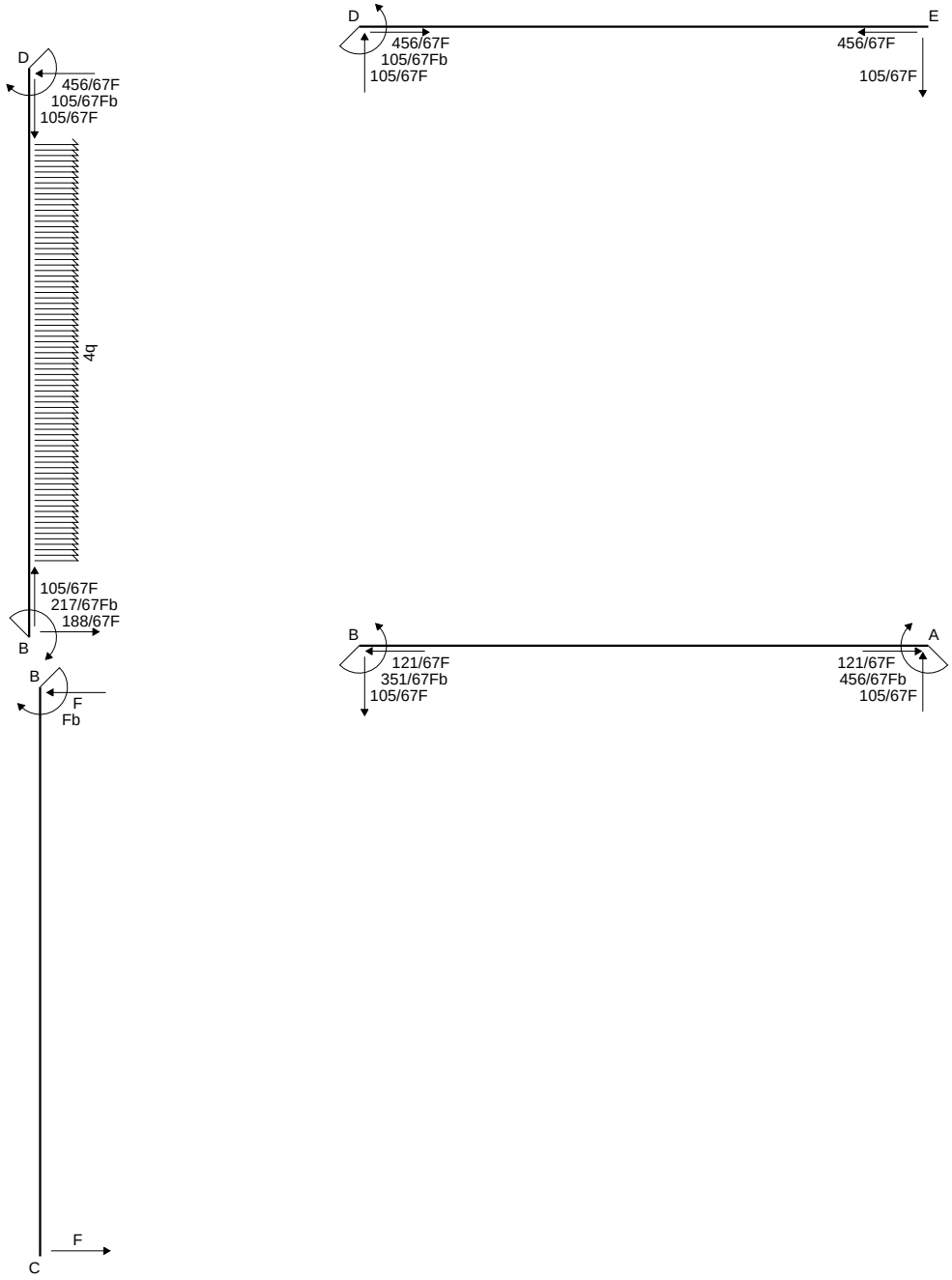

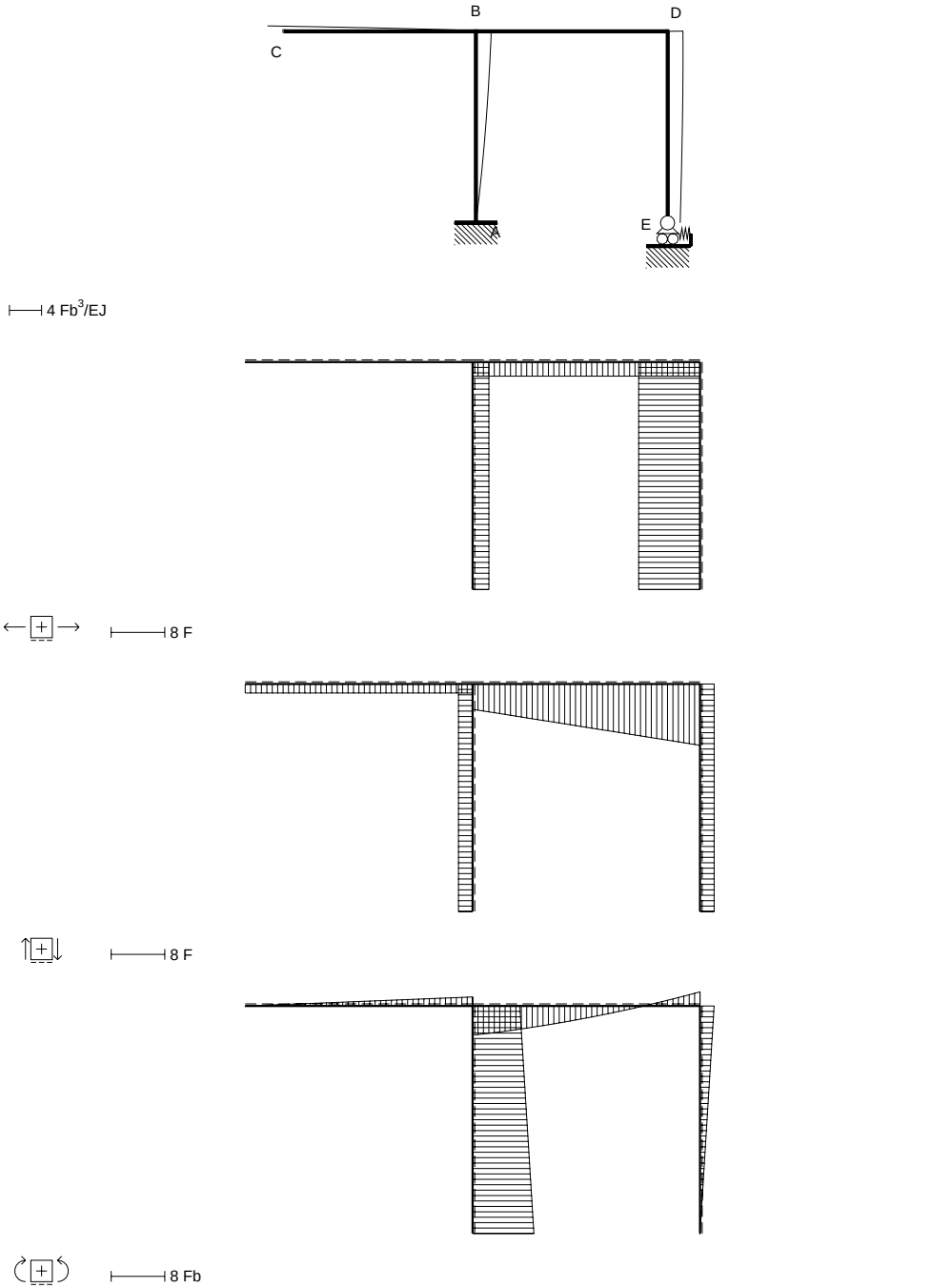

REAZIONI IPERSTATICHE

$X = W_{AB}$ $Y = W_{DB}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: $\varphi_{AB}\ K_{AB}\ \varphi_{BC}\ K_{BC}\ \varphi_{DB}\ K_{DB}\ \varphi_{ED}\ K_{ED}$

Relazioni di congruenza

$y'_{AB}(0) + 3\delta/b = 0$
$y'_{AB}(b) - y'_{DB}(b) = 0$
$y'_{BC}(0) - y'_{DB}(b) = 0$
$y'_{DB}(0) - y'_{ED}(b) = 0$
$y_{AB}(0) = 0$
$y_{BC}(0) = 0$
$y_{DB}(0) = 0$
$y_{DB}(b) = 0$
$y_{ED}(0) - H_E b^3/EJ = 0$
$y_{ED}(b) - y_{AB}(b) = 0$

$M_{AB} = -X + Yx/b$
$EJy'' = -2EJ\theta - 2/3X + 2/3Yx/b$
$EJy' = -2EJ\theta x - 2/3Xx + 1/3Yx^2/b + EJ\varphi_{AB}$
$EJy = -EJ\theta x^2 - 1/3Xx^2 + 1/9Yx^3/b + EJ\varphi_{AB}x + EJK_{AB}$

$M_{BC} = -Fx + Fb$
$EJy'' = -Fx + Fb$
$EJy' = -1/2Fx^2 + Fbx + EJ\varphi_{BC}$
$EJy = -1/6Fx^3 + 1/2Fbx^2 + EJ\varphi_{BC}x + EJK_{BC}$

$M_{DB} = 2qx^2 + Xx/b - Y$
$EJy'' = 2qx^2 + Xx/b - Y$
$EJy' = 2/3qx^3 + 1/2Xx^2/b - Yx + EJ\varphi_{DB}$
$EJy = 1/6qx^4 + 1/6Xx^3/b - 1/2Yx^2 + EJ\varphi_{DB}x + EJK_{DB}$

$M_{ED} = -Yx/b$
$EJy'' = -Yx/b$
$EJy' = -1/2Yx^2/b + EJ\varphi_{ED}$
$EJy = -1/6Yx^3/b + EJ\varphi_{ED}x + EJK_{ED}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{BC}b$	K_{BC}	$\varphi_{DB}b$	K_{DB}	$\varphi_{ED}b$	K_{ED}	Xb^2/EJ	Yb^2/EJ		Fb^3/EJ
y'_{AB}	1	0	0	0	0	0	0	0	0	0		0
y'_{BA}	1	0	0	0	-1	0	0	0	-7/6	4/3		2/3
y'_{BC}	0	0	1	0	-1	0	0	0	-1/2	1		2/3
y'_{DB}	0	0	0	0	1	0	-1	0	0	1/2		0
y_{AB}	0	1	0	0	0	0	0	0	0	0		0
y_{BC}	0	0	0	1	0	0	0	0	0	0	=	0
y_{DB}	0	0	0	0	0	1	0	0	0	0		0
y_{BD}	0	0	0	0	1	1	0	0	1/6	-1/2		-1/6
y_{ED}	0	0	0	0	0	0	0	1	0	-1		0
y_{DE}	-1	-1	0	0	0	0	1	1	1/3	-5/18		0

Condizioni al contorno

αTb	δ
0	-3
2	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
-1	0

Soluzione

	Fb^3/EJ
$\varphi_{AB}b$	-3
$\varphi_{DB}b$	37/201
$\varphi_{BC}b$	-66/67
$\varphi_{ED}b$	-241/402
K_{AB}	0
K_{BC}	0
K_{DB}	0
Xb^2/EJ	-456/67
K_{ED}	-105/67
Yb^2/EJ	-105/67

DEFORMATA (coordinate locali)

AB $y(x)EJ = -3xFb^2 + 85/67x^2Fb - 35/201x^3F$
BA $y(x)EJ = -383/201Fb^3 + 66/67xFb^2 + 50/67x^2Fb + 35/201x^3F$
BC $y(x)EJ = -66/67xFb^2 + 1/2x^2Fb - 1/6x^3F$
CB $y(x)EJ = -131/201Fb^3 + 65/134xFb^2 + 1/6x^3F$
DB $y(x)EJ = 37/201xFb^2 + 105/134x^2Fb - 76/67x^3F + 1/6x^4q$
BD $y(x)EJ = 66/67xFb^2 - 217/134x^2Fb + 94/201x^3F + 1/6x^4q$
ED $y(x)EJ = -105/67Fb^3 - 241/402xFb^2 + 35/134x^3F$
DE $y(x)EJ = -383/201Fb^3 - 37/201xFb^2 + 105/134x^2Fb - 35/134x^3F$

SPOSTAMENTI NODALI

$u_A = 0$ $u_B = 383/201(Fb^3/EJ)$ $u_C = -421/201(Fb^3/EJ)$
$v_A = 0$ $v_B = 0$ $v_C = 131/201(Fb^3/EJ)$
$\varphi_A = -3(Fb^2/EJ)$ $\varphi_B = -66/67(Fb^2/EJ)$ $\varphi_C = -65/134(Fb^2/EJ)$

$u_D = 383/201(Fb^3/EJ)$ $u_E = 105/67(Fb^3/EJ)$
$v_D = 0$ $v_E = 0$
$\varphi_D = 37/201(Fb^2/EJ)$ $\varphi_{EED} = -241/402(Fb^2/EJ)$

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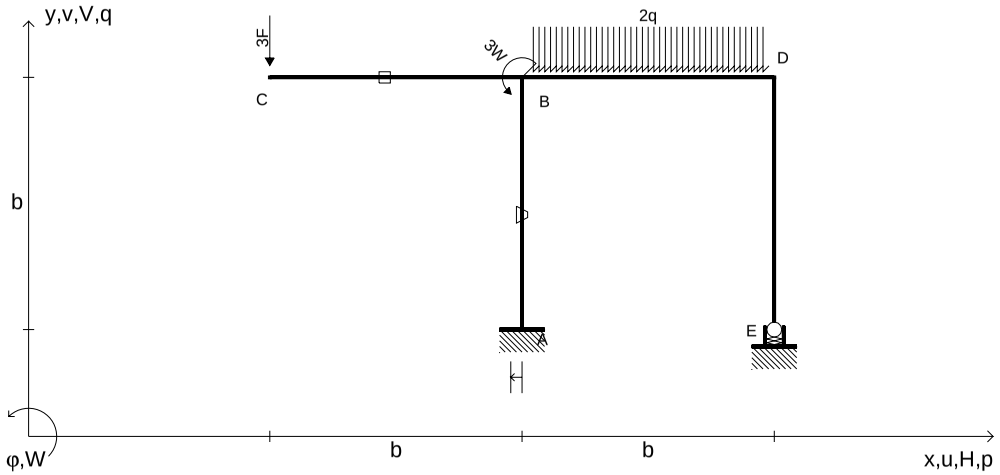
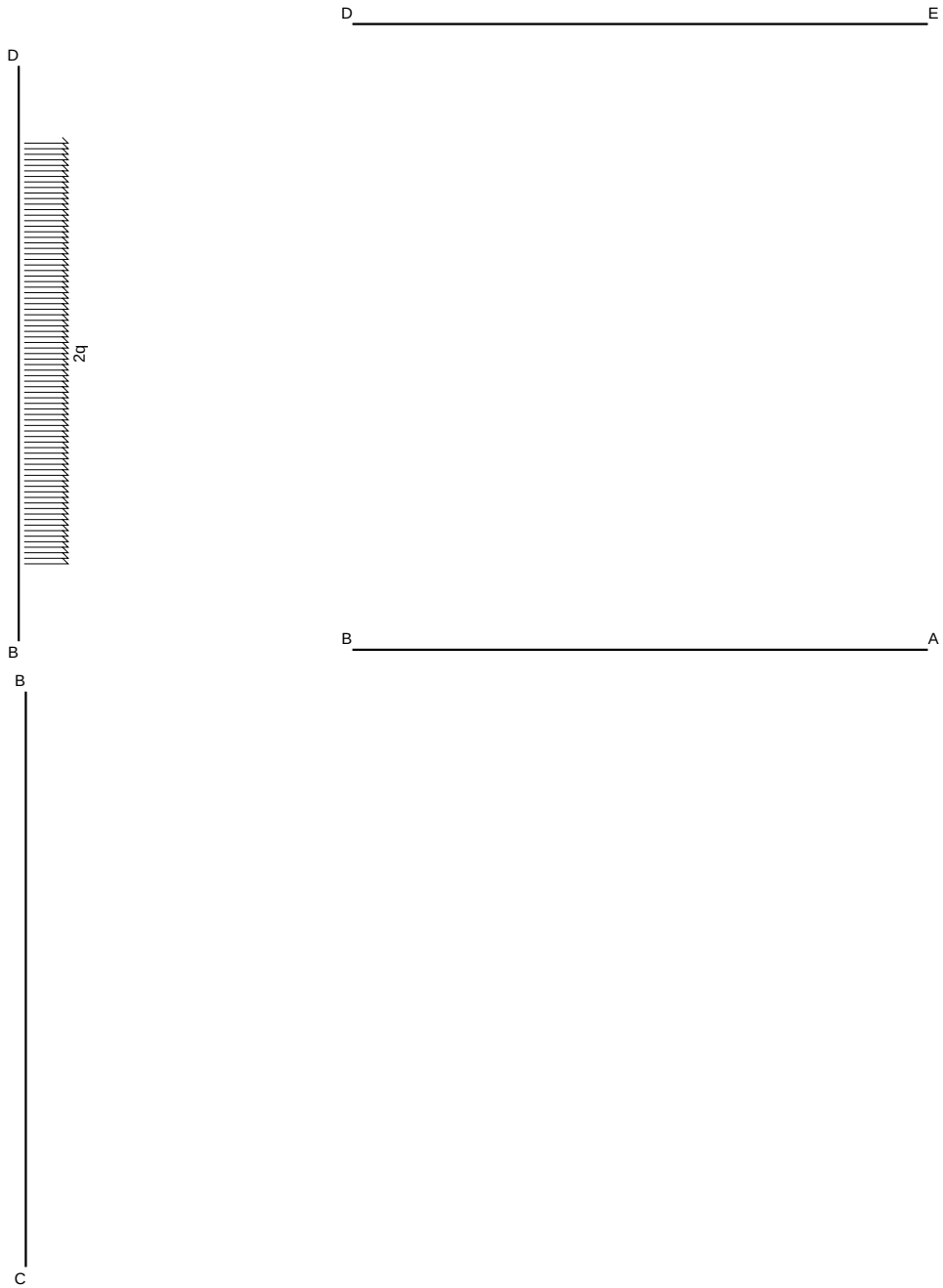


$V_C = -3F$
$W_B = 3W = 3Fb$
$q_{DB} = -2q = -2F/b$
$\varepsilon_{BC} = 2\alpha T = 2b^2F/EJ$

$\theta_{AB} = -4\theta = -4\alpha T/b = -4bF/EJ$
$u_A = -\delta = -b^3F/EJ$
$k_E = 4EJ/b^3$
$EJ_{AB} = 4/3EJ$

$EJ_{BC} = EJ$
$EJ_{DB} = EJ$
$EJ_{ED} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio (retro incluso).
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BC.
Curvatura θ asta AB positiva se convessa a destra con inizio A.
Spostamento orizzontale assoluto u imposto al nodo A.



DEFORMATA (coordinate locali)
AB y(x)EJ =
BC y(x)EJ =
DB y(x)EJ =
ED y(x)EJ =

SPOSTAMENTI NODALI

u_A =	u_B =	u_C =	u_D =
v_A =	v_B =	v_C =	v_D =
φ_A =	φ_B =	φ_C =	φ_D =

u_E =
v_E =
φ_{EED} =

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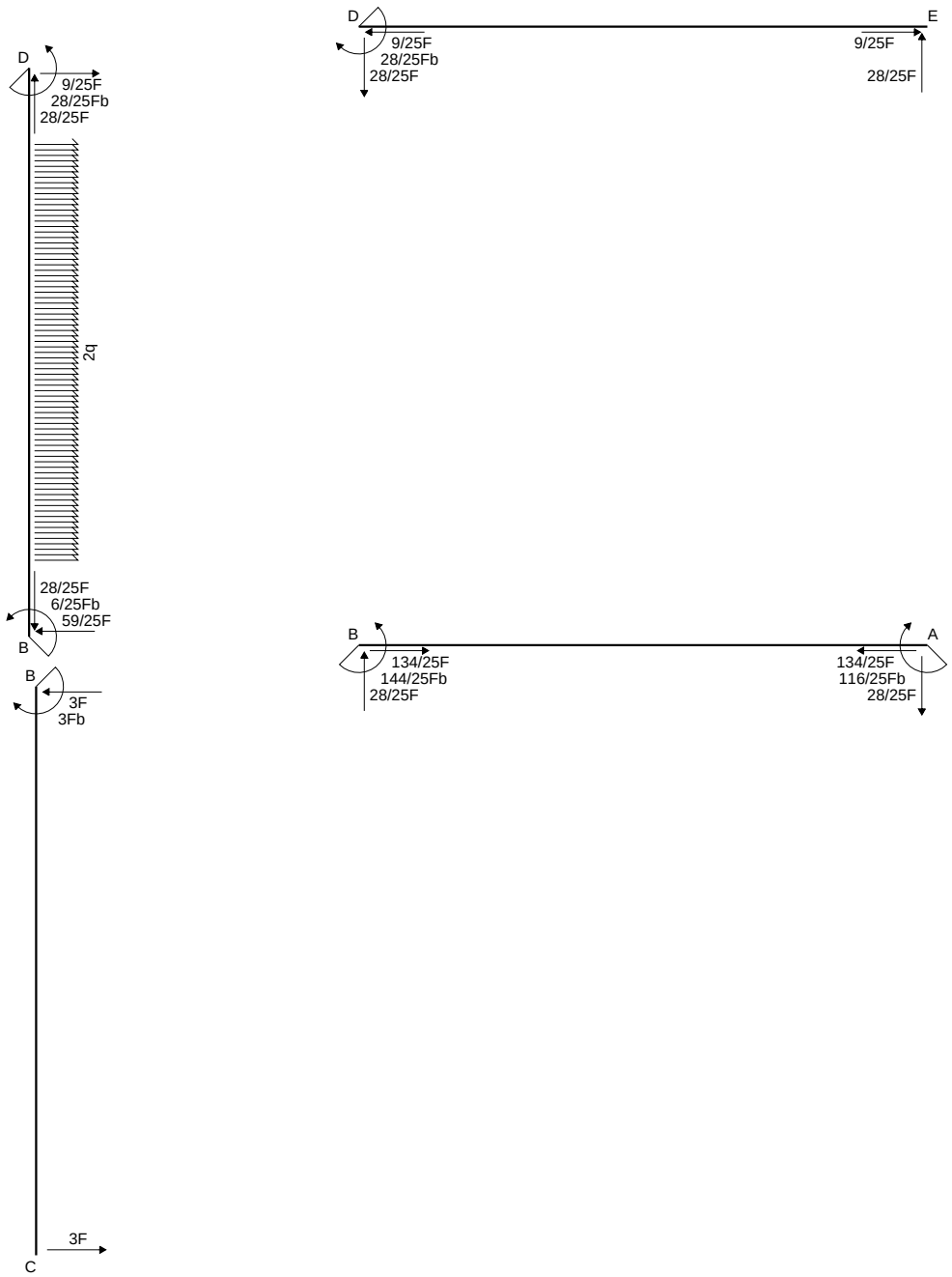

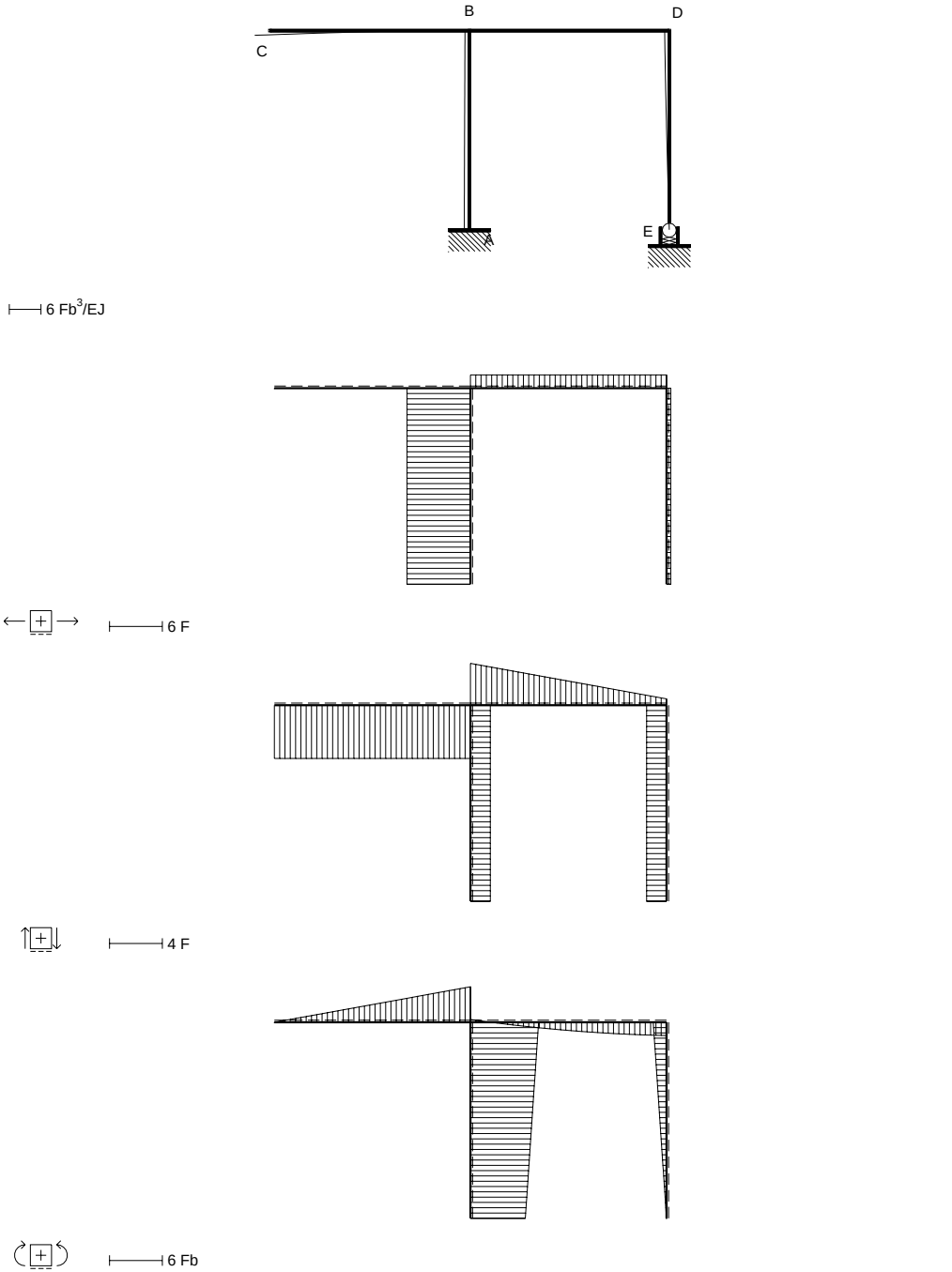

REAZIONI IPERSTATICHE

$X = W_{AB}$ $Y = W_{DB}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{BC} K_{BC} φ_{DB} K_{DB} φ_{ED} K_{ED}

Relazioni di congruenza

$y'_{AB}(0) = 0$

$y'_{AB}(b) - y'_{DB}(b) = 0$

$y'_{BC}(0) - y'_{DB}(b) = 0$

$y'_{DB}(0) - y'_{ED}(b) = 0$

$y_{AB}(0) - \delta = 0$

$y_{BC}(0) = 0$

$y_{DB}(0) - 1/4V_E b^3/EJ = 0$

$y_{DB}(b) = 0$

$y_{ED}(0) = 0$

$y_{ED}(b) - y_{AB}(b) + 2\delta = 0$

$M_{AB} = -X + Yx/b$

$EJy'' = -4EJ\theta - 3/4X + 3/4Yx/b$

$EJy' = -4EJ\theta x - 3/4Xx + 3/8Yx^2/b + EJ\varphi_{AB}$

$EJy = -2EJ\theta x^2 - 3/8Xx^2 + 1/8Yx^3/b + EJ\varphi_{AB}x + EJK_{AB}$

$M_{BC} = -3Fx + 3Fb$

$EJy'' = -3Fx + 3Fb$

$EJy' = -3/2Fx^2 + 3Fbx + EJ\varphi_{BC}$

$EJy = -1/2Fx^3 + 3/2Fbx^2 + EJ\varphi_{BC}x + EJK_{BC}$

$M_{DB} = 5Fx + qx^2 + Xx/b - Y$

$EJy'' = 5Fx + qx^2 + Xx/b - Y$

$EJy' = 5/2Fx^2 + 1/3qx^3 + 1/2Xx^2/b - Yx + EJ\varphi_{DB}$

$EJy = 5/6Fx^3 + 1/12qx^4 + 1/6Xx^3/b - 1/2Yx^2 + EJ\varphi_{DB}x + EJK_{DB}$

$M_{ED} = -Yx/b$

$EJy'' = -Yx/b$

$EJy' = -1/2Yx^2/b + EJ\varphi_{ED}$

$EJy = -1/6Yx^3/b + EJ\varphi_{ED}x + EJK_{ED}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{BC}b$	K_{BC}	$\varphi_{DB}b$	K_{DB}	$\varphi_{ED}b$	K_{ED}	Xb^2/EJ	Yb^2/EJ		Fb^3/EJ
y'_{AB}	1	0	0	0	0	0	0	0	0	0		0
y'_{BA}	1	0	0	0	-1	0	0	0	-5/4	11/8		17/6
y'_{BC}	0	0	1	0	-1	0	0	0	-1/2	1		17/6
y'_{DB}	0	0	0	0	1	0	-1	0	0	1/2		0
y_{AB}	0	1	0	0	0	0	0	0	0	0		0
y_{BC}	0	0	0	1	0	0	0	0	0	0	=	0
y_{DB}	0	0	0	0	0	1	0	0	1/4	0		-5/4
y_{BD}	0	0	0	0	1	1	0	0	1/6	-1/2		-11/12
y_{ED}	0	0	0	0	0	0	0	1	0	0		0
y_{DE}	-1	-1	0	0	0	0	1	1	3/8	-7/24		0

Condizioni al contorno

αTb	δ
0	0
4	0
0	0
0	0
0	1
0	0
0	0
0	0
0	0
-2	0

Soluzione

		Fb^3/EJ
$\varphi_{AB}b$		0
$\varphi_{DB}b$		38/75
$\varphi_{BC}b$		-1/10
$\varphi_{ED}b$		16/15
K_{AB}		1
K_{BC}	=	0
K_{DB}		-9/100
Xb^2/EJ		-116/25
K_{ED}		0
Yb^2/EJ		28/25

DEFORMATA (coordinate locali)

AB $y(x)EJ = Fb^3 - 13/50x^2Fb + 7/50x^3F$

BA $y(x)EJ = 22/25Fb^3 + 1/10xFb^2 + 4/25x^2Fb - 7/50x^3F$

BC $y(x)EJ = -1/10xFb^2 + 3/2x^2Fb - 1/2x^3F$

CB $y(x)EJ = 9/10Fb^3 - 7/5xFb^2 + 1/2x^3F$

DB $y(x)EJ = -9/100Fb^3 + 38/75xFb^2 - 14/25x^2Fb + 3/50x^3F + 1/12x^4q$

BD $y(x)EJ = 1/10xFb^2 + 3/25x^2Fb - 59/150x^3F + 1/12x^4q$

ED $y(x)EJ = 16/15xFb^2 - 14/75x^3F$

DE $y(x)EJ = 22/25Fb^3 - 38/75xFb^2 - 14/25x^2Fb + 14/75x^3F$

SPOSTAMENTI NODALI

$u_A = -(Fb^3/EJ)$ $u_B = -22/25(Fb^3/EJ)$ $u_C = -72/25(Fb^3/EJ)$ $u_D = -22/25(Fb^3/EJ)$

$v_A = 0$ $v_B = 0$ $v_C = -9/10(Fb^3/EJ)$ $v_D = 9/100(Fb^3/EJ)$

$\varphi_A = 0$ $\varphi_B = -1/10(Fb^2/EJ)$ $\varphi_C = 7/5(Fb^2/EJ)$ $\varphi_D = 38/75(Fb^2/EJ)$

$u_E = 0$
$v_E = 9/100(Fb^3/EJ)$
$\varphi_{EED} = 16/15(Fb^2/EJ)$

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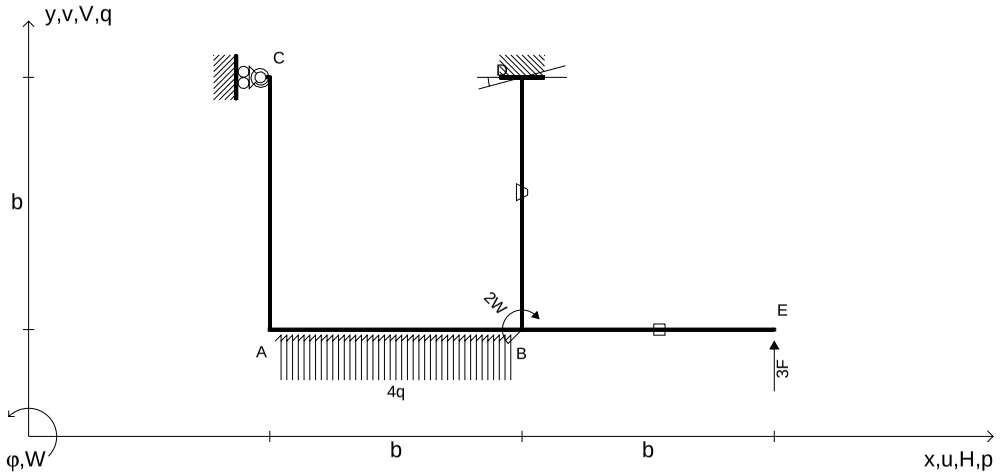
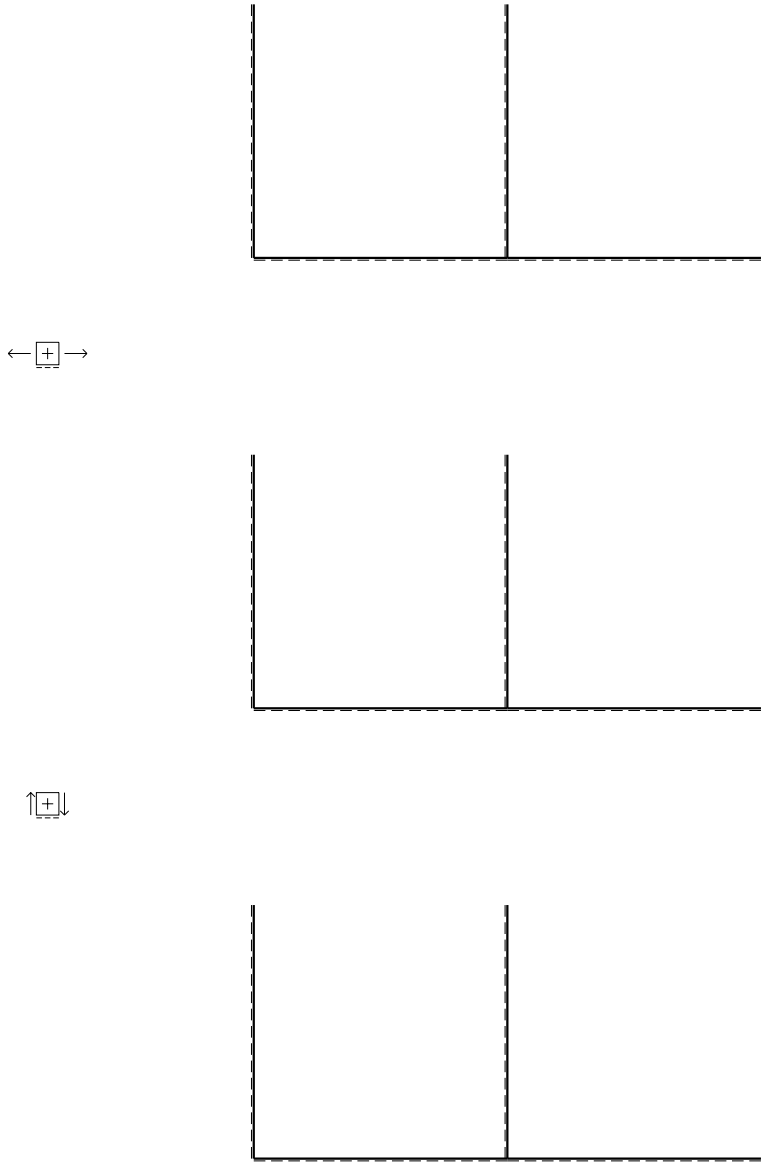


$V_E = 3F$
$W_B = -2W = -2Fb$
$q_{AB} = 4q = 4F/b$
$\varepsilon_{BE} = -3\alpha T = -3b^2F/EJ$

$\theta_{DB} = 2\theta = 2\alpha T/b = 2bF/EJ$
$\varphi_D = \delta/b = b^2F/EJ$
$k_C = EJ/b$
$EJ_{AB} = EJ$

$EJ_{CA} = EJ$
$EJ_{DB} = 2EJ$
$EJ_{BE} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio (retro incluso).
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta BE.
Curvatura θ asta DB positiva se convessa a destra con inizio D.
Rotazione assoluta φ imposta al nodo D.



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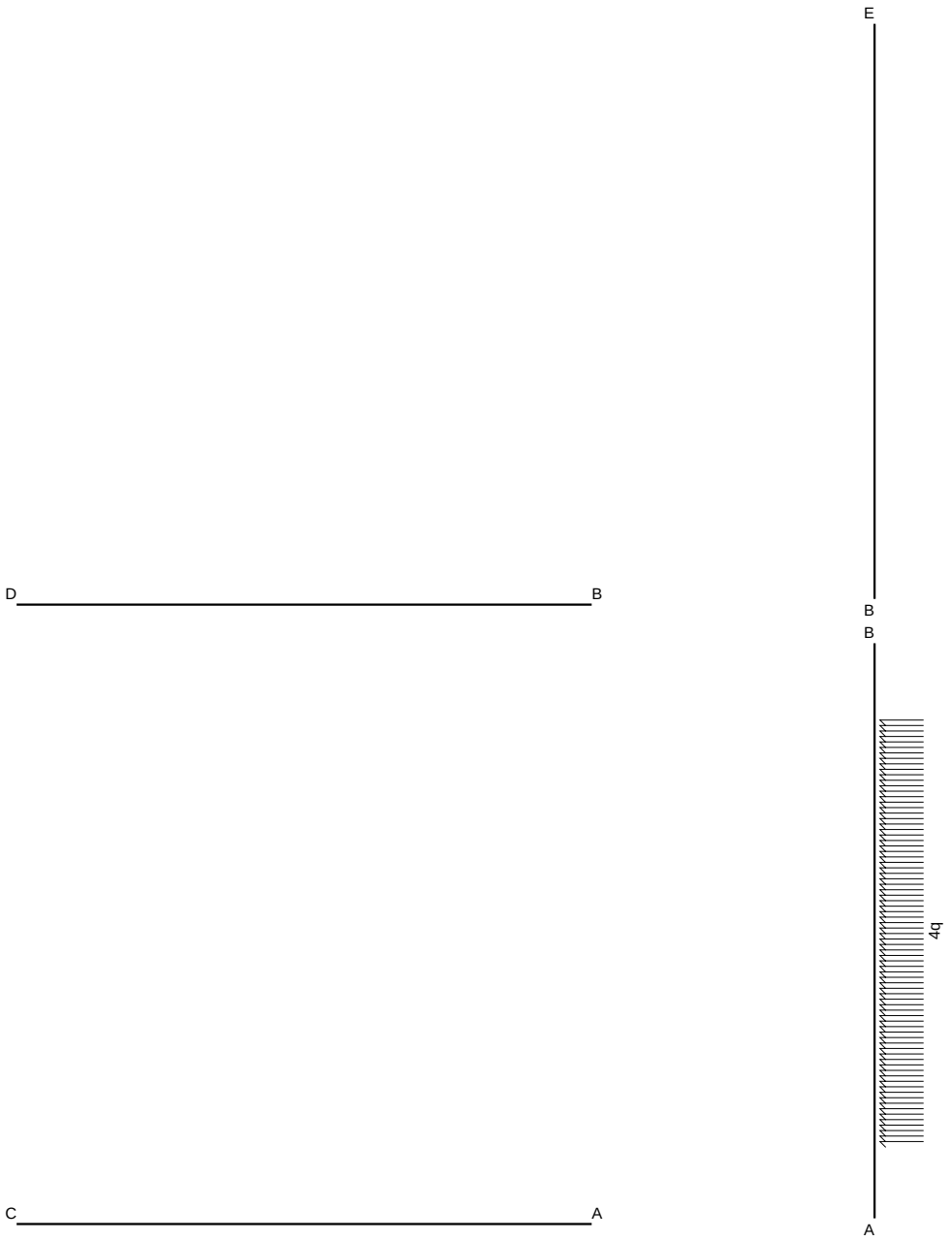


DEFORMATA (coordinate locali)
AB y(x)EJ =
CA y(x)EJ =
DB y(x)EJ =
BE y(x)EJ =

SPOSTAMENTI NODALI

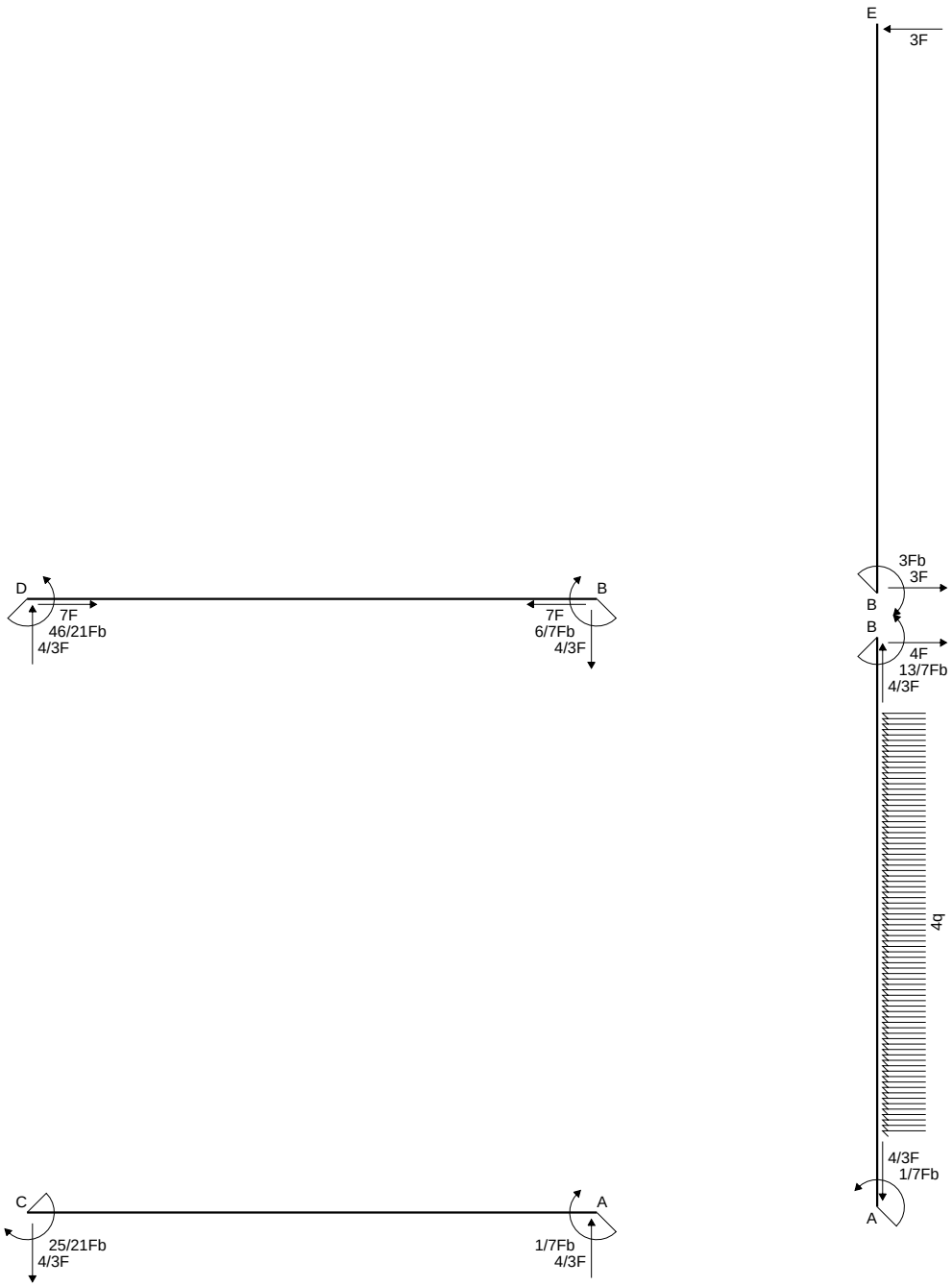
u_A =
v_A =
φ_A =

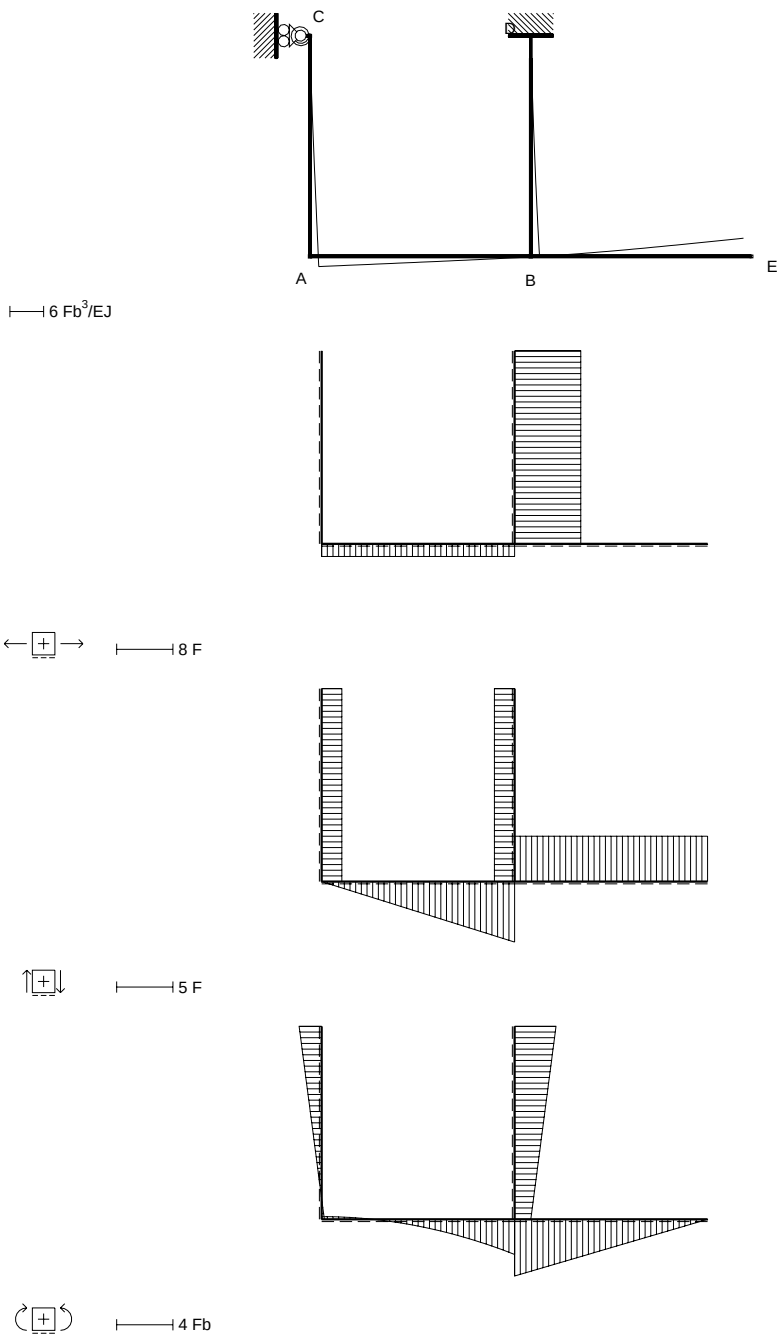
u_B =
v_B =
φ_B =

u_C =
v_C =
φ_C =

u_D =
v_D =
φ_D =

u_E =
v_E =
φ_E =





REAZIONI IPERSTATICHE

$X = W_{AB}$ $Y = W_{CA}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: $\varphi_{AB}\ K_{AB}\ \varphi_{CA}\ K_{CA}\ \varphi_{DB}\ K_{DB}\ \varphi_{BE}\ K_{BE}$

Relazioni di congruenza

$y'_{AB}(0) - y'_{CA}(b) = 0$
$y'_{AB}(b) - y'_{BE}(0) = 0$
$y'_{CA}(0) + W_C b/EJ = 0$
$y'_{DB}(0) - \delta/b = 0$
$y'_{DB}(b) - y'_{BE}(0) = 0$
$y_{AB}(b) = 0$
$y_{CA}(0) = 0$
$y_{DB}(0) = 0$
$y_{DB}(b) - y_{CA}(b) = 0$
$y_{BE}(0) = 0$

$M_{AB} = 2qx^2 - X$
$EJy'' = 2qx^2 - X$
$EJy' = 2/3qx^3 - Xx + EJ\varphi_{AB}$
$EJy = 1/6qx^4 - 1/2Xx^2 + EJ\varphi_{AB}x + EJK_{AB}$

$M_{CA} = -Xx/b + Yx/b - Y$
$EJy'' = -Xx/b + Yx/b - Y$
$EJy' = -1/2Xx^2/b + 1/2Yx^2/b - Yx + EJ\varphi_{CA}$
$EJy = -1/6Xx^3/b + 1/6Yx^3/b - 1/2Yx^2 + EJ\varphi_{CA}x + EJK_{CA}$

$M_{DB} = -Fb + Xx/b - Yx/b + Y$
$EJy'' = -1/2Fb + 2EJ\theta + 1/2Xx/b - 1/2Yx/b + 1/2Y$
$EJy' = -1/2Fbx + 2EJ\theta x + 1/4Xx^2/b - 1/4Yx^2/b + 1/2Yx + EJ\varphi_{DB}$
$EJy = -1/4Fbx^2 + EJ\theta x^2 + 1/12Xx^3/b - 1/12Yx^3/b + 1/4Yx^2 + EJ\varphi_{DB}x + EJK_{DB}$

$M_{BE} = -3Fx + 3Fb$
$EJy'' = -3Fx + 3Fb$
$EJy' = -3/2Fx^2 + 3Fbx + EJ\varphi_{BE}$
$EJy = -1/2Fx^3 + 3/2Fbx^2 + EJ\varphi_{BE}x + EJK_{BE}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CA}b$	K_{CA}	$\varphi_{DB}b$	K_{DB}	$\varphi_{BE}b$	K_{BE}	Xb^2/EJ	Yb^2/EJ		$[Fb^3/EJ]$
y'_{AB}	1	0	-1	0	0	0	0	0	1/2	1/2		0
y'_{BA}	1	0	0	0	0	0	-1	0	-1	0		-2/3
y'_{CA}	0	0	1	0	0	0	0	0	0	1		0
y'_{DB}	0	0	0	0	1	0	0	0	0	0		0
y'_{BD}	0	0	0	0	1	0	-1	0	1/4	1/4		1/2
y_{BA}	1	1	0	0	0	0	0	0	-1/2	0	=	-1/6
y_{CA}	0	0	0	1	0	0	0	0	0	0		0
y_{DB}	0	0	0	0	0	1	0	0	0	0		0
y_{BD}	0	0	-1	-1	1	1	0	0	1/4	1/2		1/4
y_{BE}	0	0	0	0	0	0	0	1	0	0		0

Condizioni al contorno

αTb	δ
0	0
0	0
0	0
0	1
-2	0
0	0
0	0
0	0
-1	0
0	0

Soluzione

		$[Fb^3/EJ]$
$\varphi_{AB}b$		12/7
$\varphi_{CA}b$		25/21
$\varphi_{BE}b$		47/21
$\varphi_{DB}b$		1
Xb^2/EJ	=	1/7
K_{AB}		-38/21
K_{CA}		0
K_{DB}		0
Yb^2/EJ		-25/21
K_{BE}		0

DEFORMATA (coordinate locali)

AB $y(x)EJ = -38/21Fb^3 + 12/7xFb^2 - 1/14x^2Fb + 1/6x^4q$
BA $y(x)EJ = -47/21xFb^2 + 13/14x^2Fb - 2/3x^3F + 1/6x^4q$
CA $y(x)EJ = 25/21xFb^2 + 25/42x^2Fb - 2/9x^3F$
AC $y(x)EJ = 197/126Fb^3 - 12/7xFb^2 - 1/14x^2Fb + 2/9x^3F$
DB $y(x)EJ = xFb^2 + 19/42x^2Fb + 1/9x^3F$
BD $y(x)EJ = 197/126Fb^3 - 47/21xFb^2 + 11/14x^2Fb - 1/9x^3F$
BE $y(x)EJ = 47/21xFb^2 + 3/2x^2Fb - 1/2x^3F$
EB $y(x)EJ = 68/21Fb^3 - 157/42xFb^2 + 1/2x^3F$

SPOSTAMENTI NODALI

$u_A = 197/126(Fb^3/EJ)$	$u_B = 197/126(Fb^3/EJ)$	$u_C = 0$	$u_D = 0$
$v_A = -38/21(Fb^3/EJ)$	$v_B = 0$	$v_C = -38/21(Fb^3/EJ)$	$v_D = 0$
$\varphi_A = 12/7(Fb^2/EJ)$	$\varphi_B = 47/21(Fb^2/EJ)$	$\varphi_C = 25/21(Fb^2/EJ)$	$\varphi_D = (Fb^2/EJ)$

$u_E = -181/126(Fb^3/EJ)$
$v_E = 68/21(Fb^3/EJ)$
$\varphi_E = 157/42(Fb^2/EJ)$

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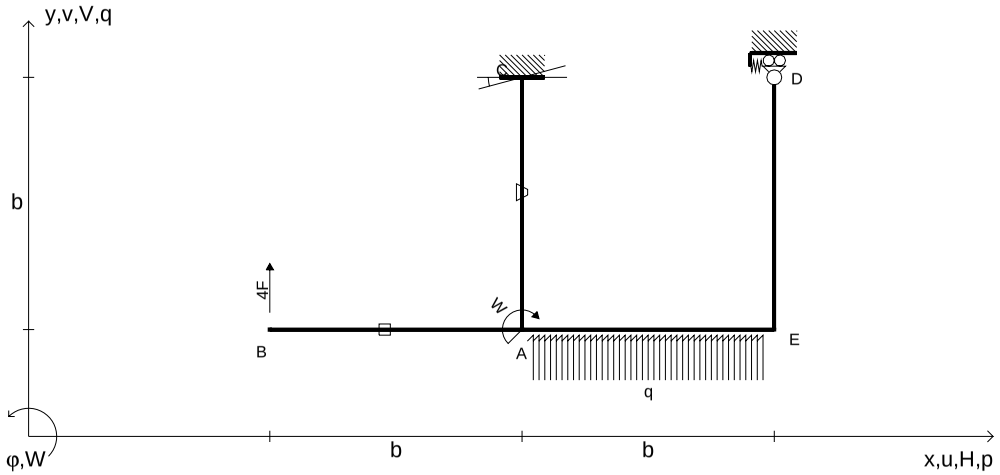
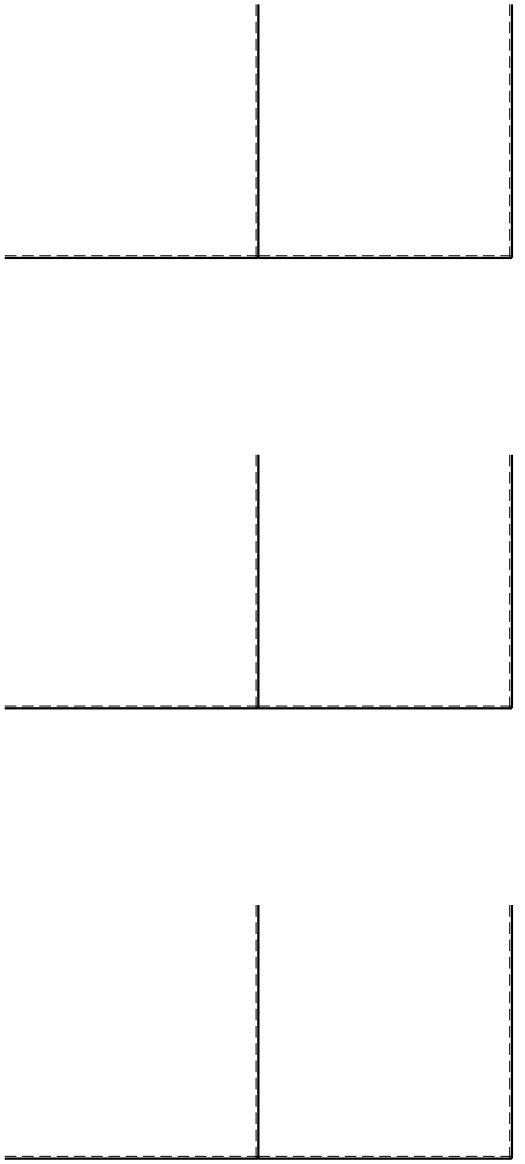


$V_B = 4F$
$W_A = -W = -Fb$
$q_{EA} = q = F/b$
$\varepsilon_{AB} = -2\alpha T = -2b^2F/EJ$

$\theta_{CA} = 3\theta = 3\alpha T/b = 3bF/EJ$
$\varphi_C = 2\delta/b = 2b^2F/EJ$
$k_D = 3EJ/b^3$
$EJ_{AB} = EJ$

$EJ_{CA} = 3EJ$
$EJ_{DE} = EJ$
$EJ_{EA} = EJ$

Svolgere l'analisi cinematica.
Risolvere con PLV e LE.
Tracciare la deformata elastica.
Riportare la soluzione su questo foglio (retro incluso).
Carichi e deformazioni date hanno verso efficace in disegno.
Calcolare reazioni vincolari della struttura e delle aste.
Tracciare i diagrammi delle azioni interne nelle aste.
Esprimere la linea elastica delle aste.
Calcolare spostamento e rotazione di tutti i nodi.
J_{YZ} - x_{YZ} - θ_{YZ} riferimento locale asta YZ con origine in Y.
Elongazione termica specifica ε assegnata su asta AB.
Curvatura θ asta CA positiva se convessa a destra con inizio C.
Rotazione assoluta φ imposta al nodo C.



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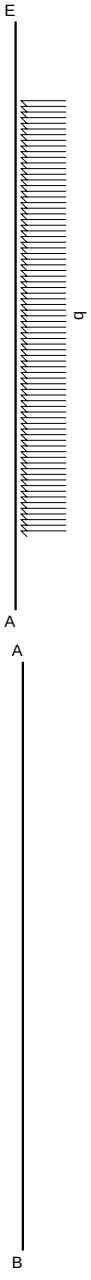


DEFORMATA (coordinate locali)
AB y(x)EJ =
CA y(x)EJ =
DE y(x)EJ =
EA y(x)EJ =

SPOSTAMENTI NODALI

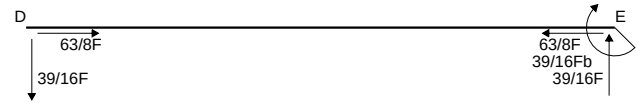
u_A =
v_A =
φ_A =

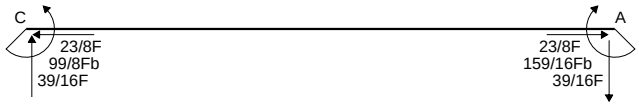
u_B =
v_B =
φ_B =

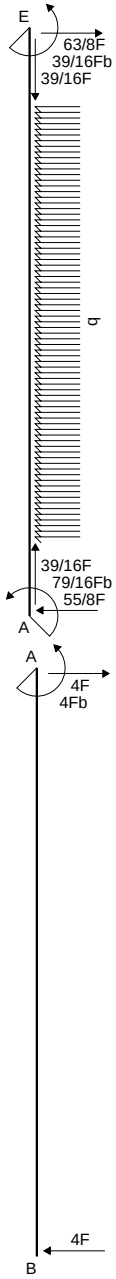
u_C =
v_C =
φ_C =

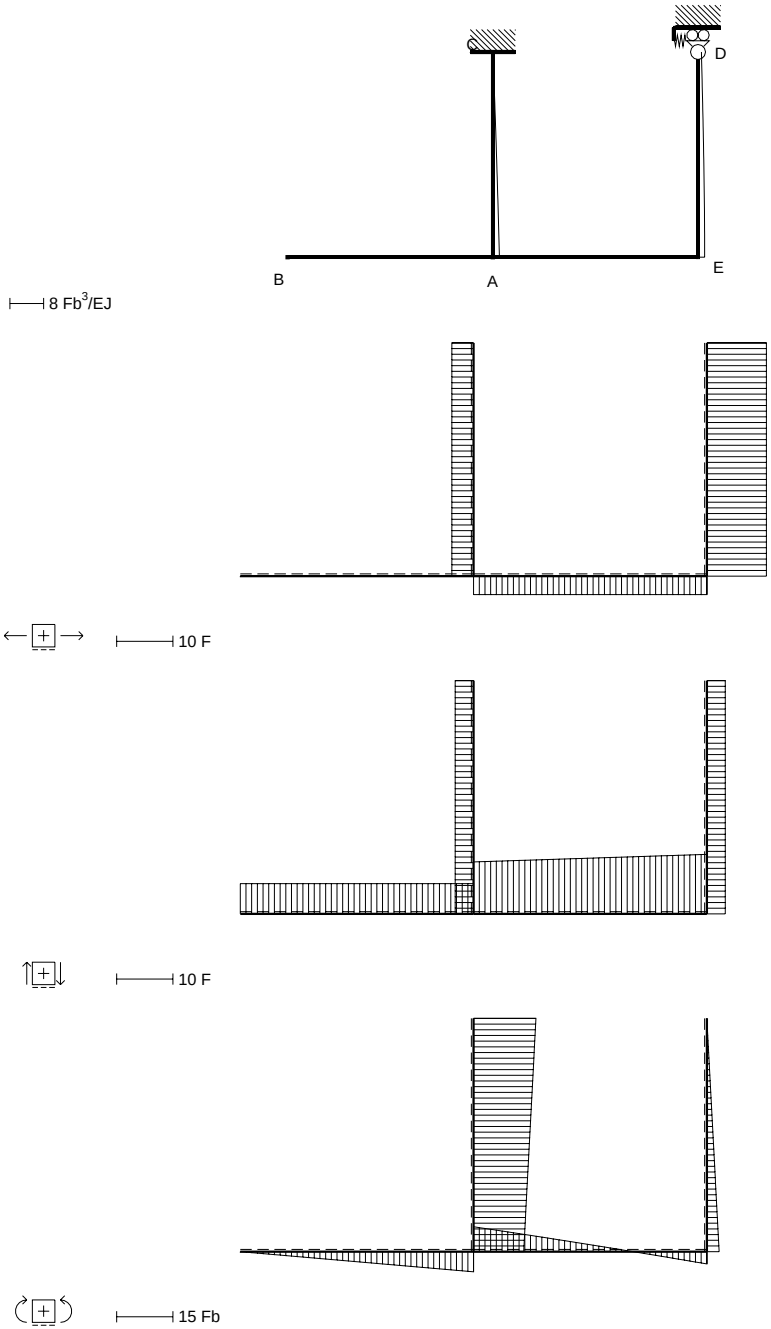
u_D =
v_D =
φ_{DDE} =

u_E =
v_E =
φ_E =


D
E
63/8F
63/8F
39/16Fb
39/16F
39/16F


C
A
23/8F
99/8Fb
39/16F
23/8F
159/16Fb
39/16F


E
63/8F
39/16Fb
39/16F
q
39/16F
79/16Fb
55/8F
A
A
4F
4Fb
4F
B


D
B
A
E
8 Fb3/EJ
10 F
10 F
15 Fb

REAZIONI IPERSTATICHE

$X = W_{CA}$ $Y = W_{EA}$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CA} K_{CA} φ_{DE} K_{DE} φ_{EA} K_{EA}

Relazioni di congruenza

$y'_{AB}(0) - y'_{EA}(b) = 0$
$y'_{CA}(0) - 2\delta/b = 0$
$y'_{CA}(b) - y'_{EA}(b) = 0$
$y'_{DE}(b) - y'_{EA}(0) = 0$
$y_{AB}(0) = 0$
$y_{CA}(0) = 0$
$y_{DE}(0) + 1/3H_D b^3/EJ = 0$
$y_{DE}(b) - y_{CA}(b) = 0$
$y_{EA}(0) = 0$
$y_{EA}(b) = 0$

$M_{AB} = 4Fx - 4Fb$
$EJy'' = 4Fx - 4Fb$
$EJy' = 2Fx^2 - 4Fbx + EJ\varphi_{AB}$
$EJy = 2/3Fx^3 - 2Fbx^2 + EJ\varphi_{AB}x + EJK_{AB}$

$M_{CA} = -X + Yx/b$
$EJy'' = 3EJ\theta - 1/3X + 1/3Yx/b$
$EJy' = 3EJ\theta x - 1/3Xx + 1/6Yx^2/b + EJ\varphi_{CA}$
$EJy = 3/2EJ\theta x^2 - 1/6Xx^2 + 1/18Yx^3/b + EJ\varphi_{CA}x + EJK_{CA}$

$M_{DE} = -Yx/b$
$EJy'' = -Yx/b$
$EJy' = -1/2Yx^2/b + EJ\varphi_{DE}$
$EJy = -1/6Yx^3/b + EJ\varphi_{DE}x + EJK_{DE}$

$M_{EA} = -9/2Fx - 1/2qx^2 + Xx/b - Y$
$EJy'' = -9/2Fx - 1/2qx^2 + Xx/b - Y$
$EJy' = -9/4Fx^2 - 1/6qx^3 + 1/2Xx^2/b - Yx + EJ\varphi_{EA}$
$EJy = -3/4Fx^3 - 1/24qx^4 + 1/6Xx^3/b - 1/2Yx^2 + EJ\varphi_{EA}x + EJK_{EA}$

Condizioni al contorno

	$\varphi_{AB}b$	K_{AB}	$\varphi_{CA}b$	K_{CA}	$\varphi_{DE}b$	K_{DE}	$\varphi_{EA}b$	K_{EA}	Xb^2/EJ	Yb^2/EJ		$[Fb^3/EJ]$
y'_{AB}	1	0	0	0	0	0	-1	0	-1/2	1		-29/12
y'_{CA}	0	0	1	0	0	0	0	0	0	0		0
y'_{AC}	0	0	1	0	0	0	-1	0	-5/6	7/6		-29/12
y'_{ED}	0	0	0	0	1	0	-1	0	0	-1/2		0
y_{AB}	0	1	0	0	0	0	0	0	0	0		0
y_{CA}	0	0	0	1	0	0	0	0	0	0	=	0
y_{DE}	0	0	0	0	0	1	0	0	0	-1/3		0
y_{ED}	0	0	-1	-1	1	1	0	0	1/6	-2/9		0
y_{EA}	0	0	0	0	0	0	0	1	0	0		0
y_{AE}	0	0	0	0	0	0	1	1	1/6	-1/2		19/24

Condizioni al contorno

αTb	δ
0	0
0	2
-3	0
0	0
0	0
0	0
0	0
3/2	0
0	0
0	0

Soluzione

		$[Fb^3/EJ]$
$\varphi_{AB}b$		41/32
$\varphi_{CA}b$		2
$\varphi_{EA}b$		-5/96
$\varphi_{DE}b$		7/6
K_{AB}		0
K_{CA}	=	0
K_{DE}		13/16
Xb^2/EJ		99/8
K_{EA}		0
Yb^2/EJ		39/16

DEFORMATA (coordinate locali)

AB $y(x)EJ = 41/32xFb^2 - 2x^2Fb + 2/3x^3F$
BA $y(x)EJ = -5/96Fb^3 + 23/32xFb^2 - 2/3x^3F$
CA $y(x)EJ = 2xFb^2 - 9/16x^2Fb + 13/96x^3F$
AC $y(x)EJ = 151/96Fb^3 - 41/32xFb^2 - 5/32x^2Fb - 13/96x^3F$
DE $y(x)EJ = 13/16Fb^3 + 7/6xFb^2 - 13/32x^3F$
ED $y(x)EJ = 151/96Fb^3 + 5/96xFb^2 - 39/32x^2Fb + 13/32x^3F$
EA $y(x)EJ = -5/96xFb^2 - 39/32x^2Fb + 21/16x^3F - 1/24x^4q$
AE $y(x)EJ = -41/32xFb^2 + 79/32x^2Fb - 55/48x^3F - 1/24x^4q$

SPOSTAMENTI NODALI

$u_A = 151/96(Fb^3/EJ)$ $u_B = 343/96(Fb^3/EJ)$ $u_C = 0$ $u_D = 13/16(Fb^3/EJ)$
$v_A = 0$ $v_B = 5/96(Fb^3/EJ)$ $v_C = 0$ $v_D = 0$
$\varphi_A = 41/32(Fb^2/EJ)$ $\varphi_B = -23/32(Fb^2/EJ)$ $\varphi_C = 2(Fb^2/EJ)$ $\varphi_{DDE} = 7/6(Fb^2/EJ)$

$u_E = 151/96(Fb^3/EJ)$
$v_E = 0$
$\varphi_E = -5/96(Fb^2/EJ)$

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