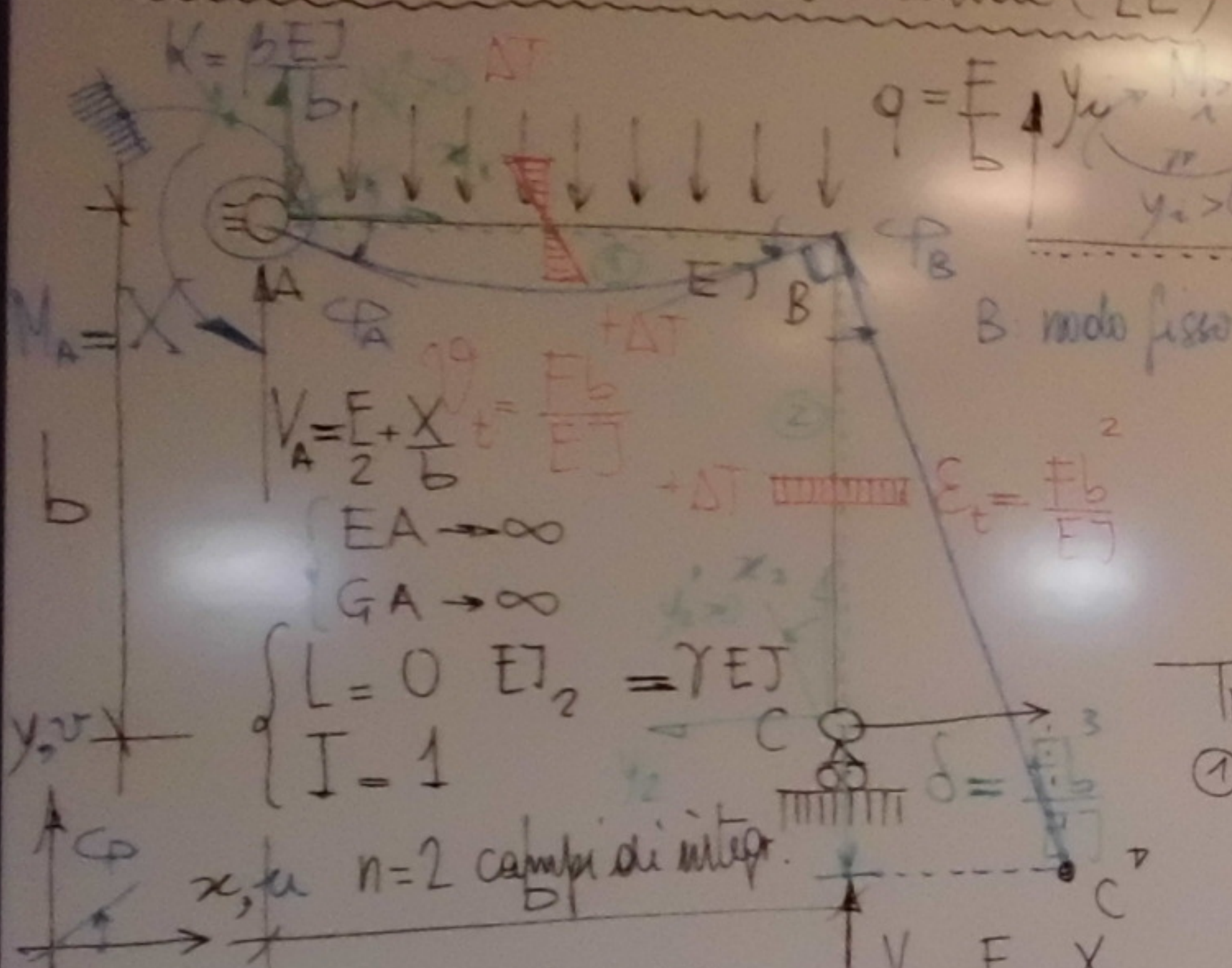


Metodo della Linea Elastica (LE)



Risoluzione del pb. static. indet. ..
 imposizione della condizione di congruenza corrispondente
 al grado di vincolo ritenuto ridondante:
 $\varphi_A = \frac{X}{K}$ $n^0 = 2n$ cost. di int. + imp. inc. = $I + 2n$

Generico tratto di trave:

$$X \approx y_i = y_{ie} + y_{it}$$

piccoli grad di spost. $\frac{M_i(x_i)}{EI_i} + \theta_i$ Legge di E-B-N
 $EI_i y_i'' = M_i(x_i) + EI_i \theta_i$
 $EI_i y_i'' = \frac{M_i(x_i)}{X} + EI_i \theta_i$

eq. ne differenziale della LE nel tratto i degli n tratti

Tratto 1: $EI y_1''(x_1) = -\frac{F}{b} \frac{x_1^2}{2} + (\frac{F}{2} + \frac{X}{b}) x_1 - X + Fb$

$$EI y_1'(x_1) = -\frac{F}{b} \frac{x_1^3}{6} + (\frac{F}{2} + \frac{X}{b}) \frac{x_1^2}{2} + (Fb - X) x_1 + A_1$$

$$EI y_1(x_1) = -\frac{F}{b} \frac{x_1^4}{24} + (\frac{F}{2} + \frac{X}{b}) \frac{x_1^3}{6} + (Fb - X) \frac{x_1^2}{2} + A_1 x_1 + A_2$$

$X, A_1, A_2; B_1, B_2$

Scrittura delle condizioni al contorno ($n_{cc} = I + 2n$) = 5

- $y_1(0) = 0$ cerniera in A
- $y_1'(0) = -\varphi_A = -\frac{X}{K}$ molla rotazionale in A
- $y_1(b) = -\delta + \epsilon_b = 0$ Ad var in C + EA $\rightarrow \infty$
- $y_1(b) = y_2(b)$ cont. della rotazione in B
- $y_2(b) = 0$ cerniera in A + EA $\rightarrow \infty$

Tratto 2: $EI y_2''(x_2) = 0 + 0$

$$EI y_2'(x_2) = B_1$$

$$EI y_2(x_2) = B_1 x_2 + B_2$$

Imposizione della r.e.

$$y_1(0) = 0 \Rightarrow A_2 = 0$$

$$EI y_1'(0) = -\frac{X}{K} \Rightarrow A_1 = -\frac{X}{K}$$

$$EI y_1(b) = 0 \Rightarrow -\frac{F}{b} \frac{b^4}{24} + (\frac{F}{2} + \frac{X}{b}) \frac{b^3}{6} + (Fb - X) \frac{b^2}{2} + A_1 b = 0$$

$$EI y_1'(b) = EI y_2'(b) \Rightarrow -\frac{F}{b} \frac{b^3}{6} + (\frac{F}{2} + \frac{X}{b}) \frac{b^2}{2} + (Fb - X) b = B_1$$

$$EI y_2(b) = 0 \Rightarrow B_1 b + B_2 = 0 \Rightarrow B_2 = -B_1 b$$

$$EI y_2(b) = 0 \Rightarrow B_1 b + B_2 = 0 \Rightarrow B_2 = -B_1 b$$

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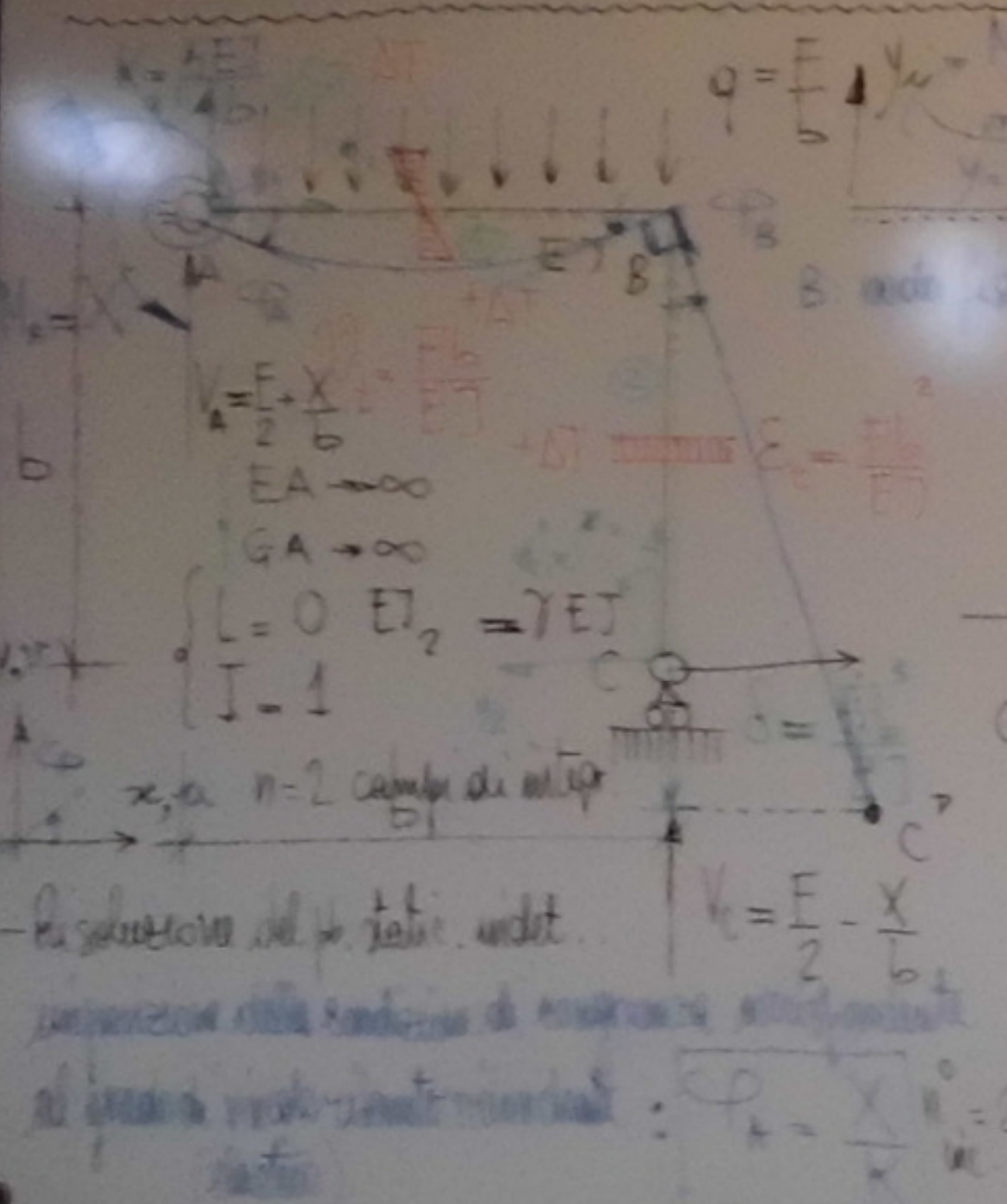
$$EI y_2(b) = 0 \Rightarrow B_1 b + B_2 = 0 \Rightarrow B_2 = -B_1 b$$

$$EI y_2(b) = 0 \Rightarrow B_1 b + B_2 = 0 \Rightarrow B_2 = -B_1 b$$

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$$EI y_2(b) = 0 \Rightarrow B_1 b + B_2 = 0 \Rightarrow B_2 = -B_1 b$$

Metodo della Linea Elastica (LE)



Generico tratto di trave

$$X = y_c = y_{c0} + y_{c1}$$

$$y_{c0} = \frac{M(x)}{EI}$$

$$y_{c1} = \frac{V(x)}{EI}$$

$$EI y'' = M(x) + EI V(x)$$

$$EI y''' = V(x)$$

Scrittura delle condizioni al contorno ($n_{cc} = I + 2n$)

$$y_1(0) = 0 \text{ (cerniera in A)}$$

$$y_1'(0) = -\varphi_A = -\frac{X}{K} \text{ (moda rotazionale in A)}$$

$$y_1(b) = -\delta + \epsilon_b = 0 \text{ (condizione di congruenza)}$$

$$y_1'(b) = y_2'(b)$$

$$y_2(b) = 0 \text{ (cerniera in A + EA \rightarrow \infty)}$$

eq m differenziale della LE nel tratto i degli n tratti

$$\text{Tratto 1: } EI y_1'' = -\frac{F}{b} \frac{x^2}{2} + \left(\frac{F}{2} + \frac{X}{b}\right) x_1 - X + Fb$$

$$\text{Tratto 2: } EI y_2'' = 0 + 0$$

$$EI y_1' = -\frac{F}{b} \frac{x^2}{2} + \left(\frac{F}{2} + \frac{X}{b}\right) x_1 + A_1$$

$$EI y_2' = B_1$$

$$EI y_1 = -\frac{F}{b} \frac{x^3}{6} + \left(\frac{F}{2} + \frac{X}{b}\right) \frac{x^2}{2} + (Fb - X) x_1 + A_1 x_1 + A_2$$

$$EI y_2 = B_1 x_2 + B_2$$

$$EI y_1 = -\frac{F}{b} \frac{x^3}{6} + \left(\frac{F}{2} + \frac{X}{b}\right) \frac{x^2}{2} + (Fb - X) x_1 + A_1 x_1 + A_2$$

$$EI y_2 = B_1 x_2 + B_2$$

Imposizione della e.e.

$$y_1(0) = 0 \Rightarrow A_2 = 0$$

$$EI y_1'(0) = -\frac{X}{K} \Rightarrow A_1 = -\frac{X}{K}$$

$$EI y_1(b) = 0 \Rightarrow -\frac{F}{b} \frac{b^3}{6} + \left(\frac{F}{2} + \frac{X}{b}\right) \frac{b^2}{2} + (Fb - X) b + A_1 b = 0$$

$$EI y_1'(b) = EI y_2'(b) \Rightarrow -\frac{F}{b} \frac{b^2}{2} + \left(\frac{F}{2} + \frac{X}{b}\right) b + (Fb - X) = B_1$$

$$EI y_2(b) = 0 \Rightarrow B_1 b + B_2 = 0 \Rightarrow B_2 = -B_1 b = -\frac{13}{48} \frac{Fb^3}{3 + \beta}$$

Linee elastiche finali (in sost. X e sost. di integr.)

X → RV(X) trova RV finali → N, T, M finali

Tracciamento della linea elastica (definita)

Calcolo di comp. di spost. di nodi caratteristici

$$\varphi_A = \frac{B_1}{EI} = \frac{A_1}{EI}$$

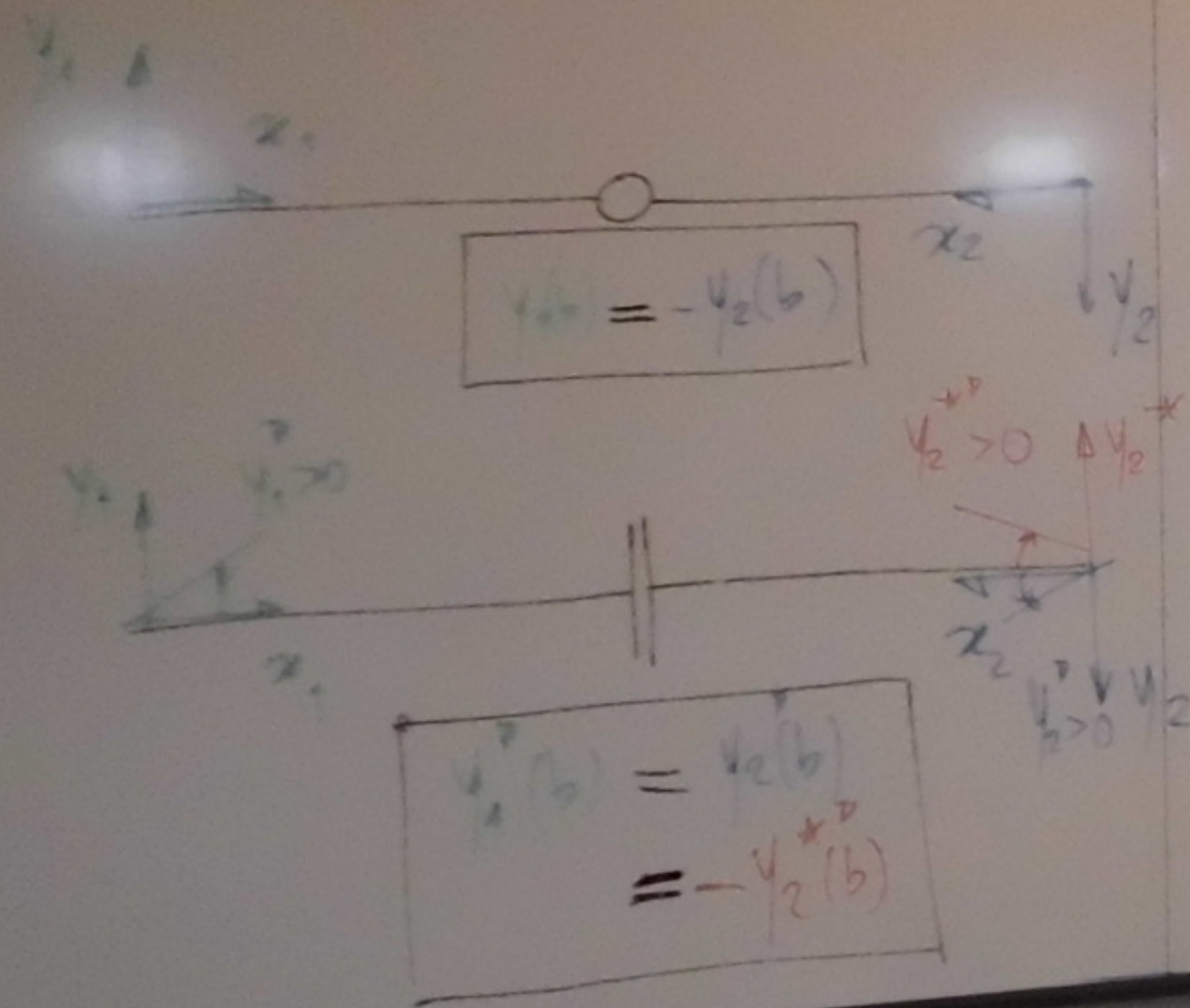
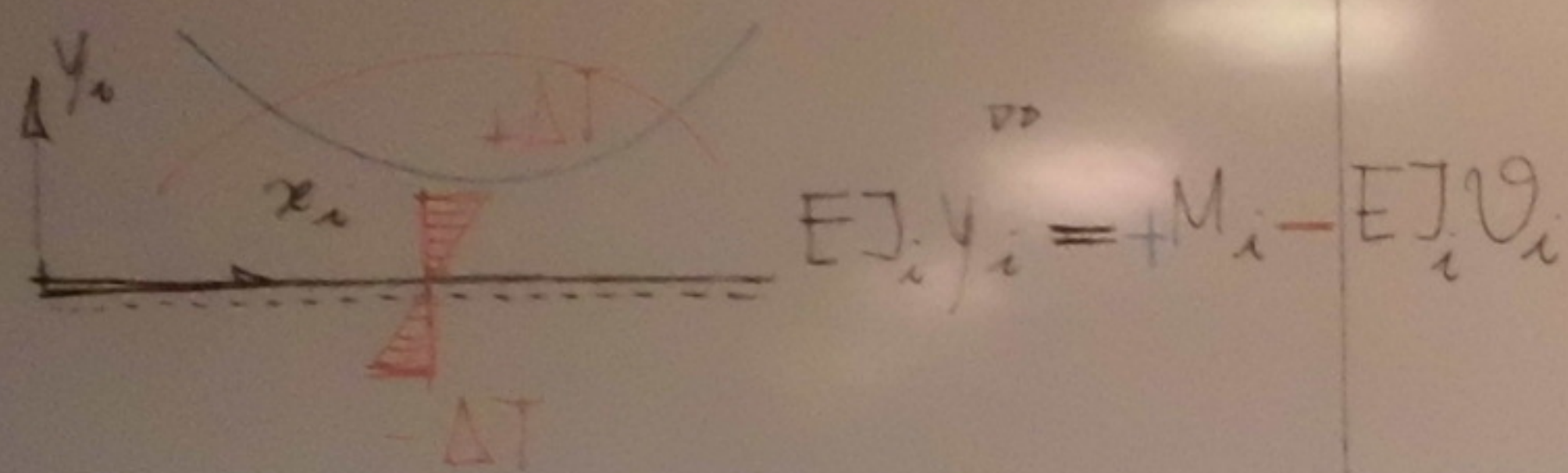
$$\varphi_A = \frac{X}{K} = \frac{13}{48} \frac{Fb^3}{3 + \beta}$$

$$A_1 = -\frac{X}{K} = -\frac{13}{48} \frac{Fb^3}{3 + \beta}$$

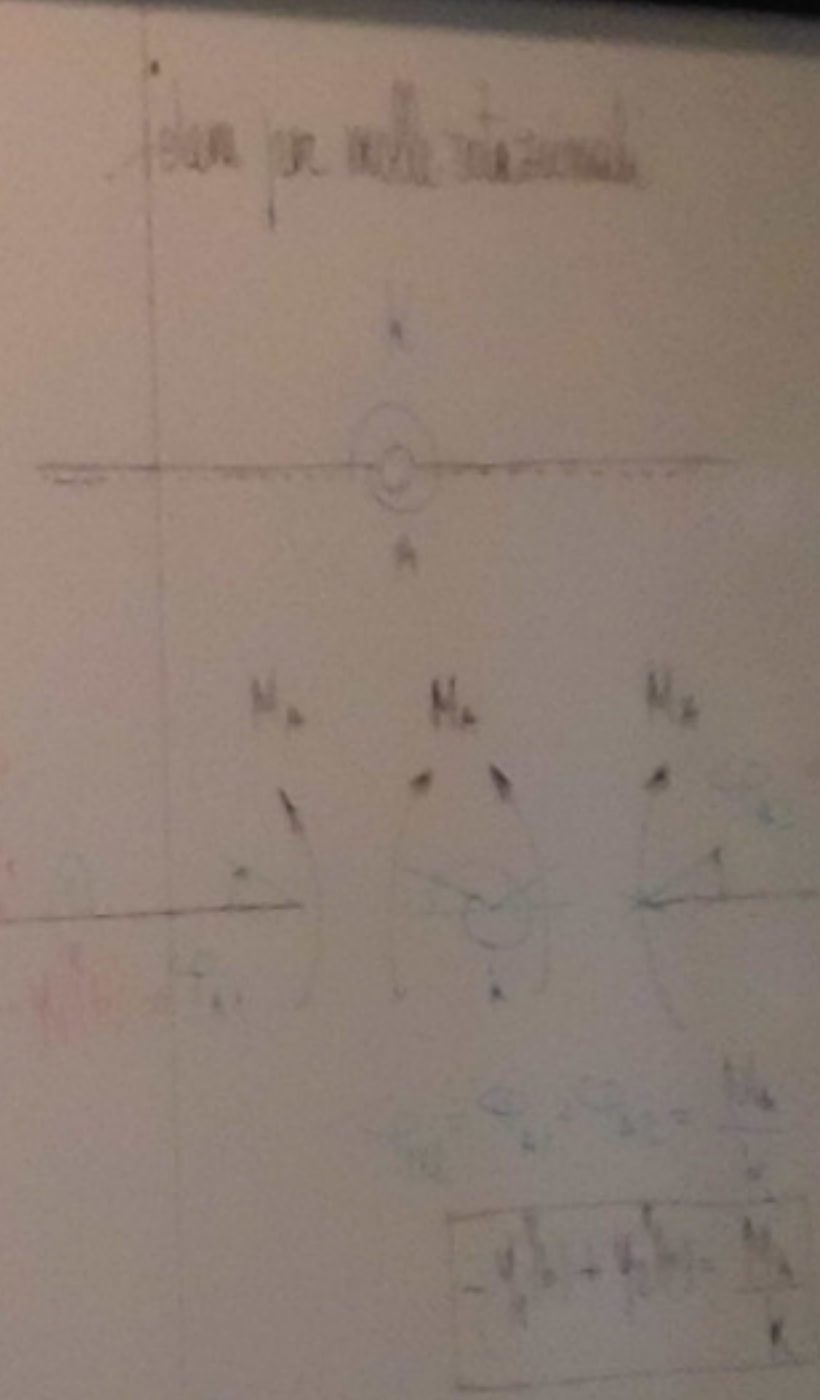
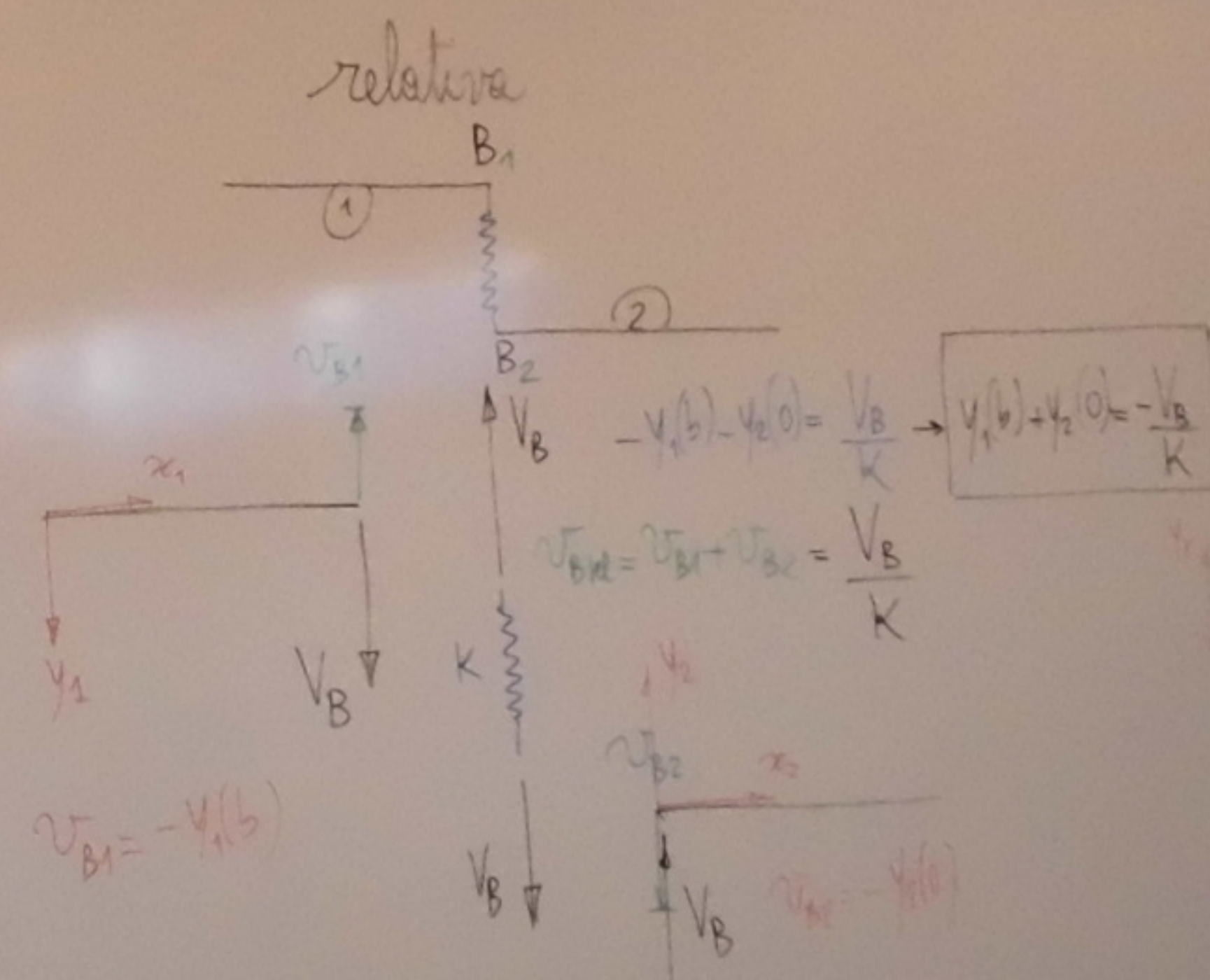
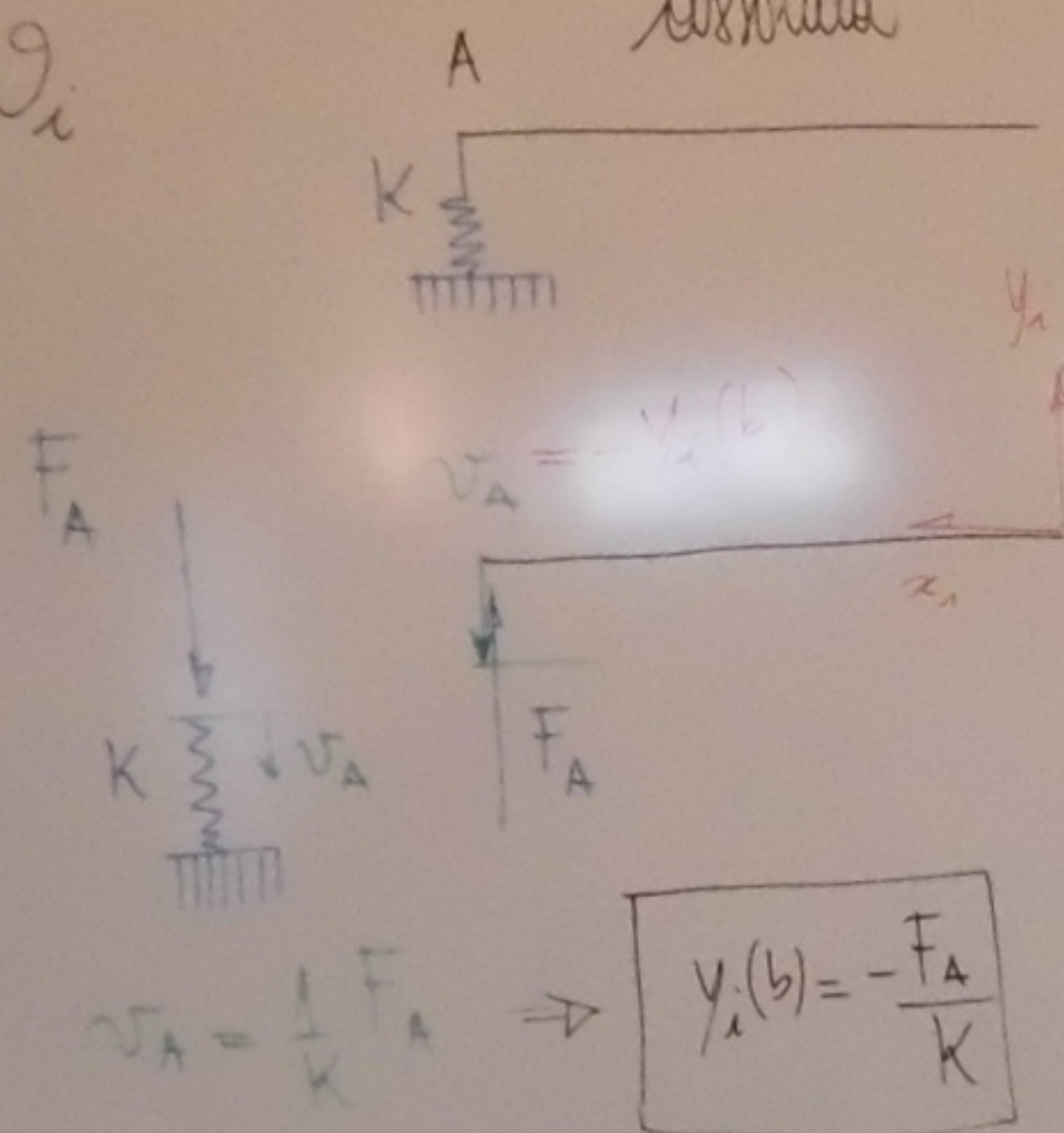
$$B_1 = \frac{13}{48} \frac{Fb^3}{3 + \beta}$$

$$B_2 = -\frac{13}{48} \frac{Fb^3}{3 + \beta}$$

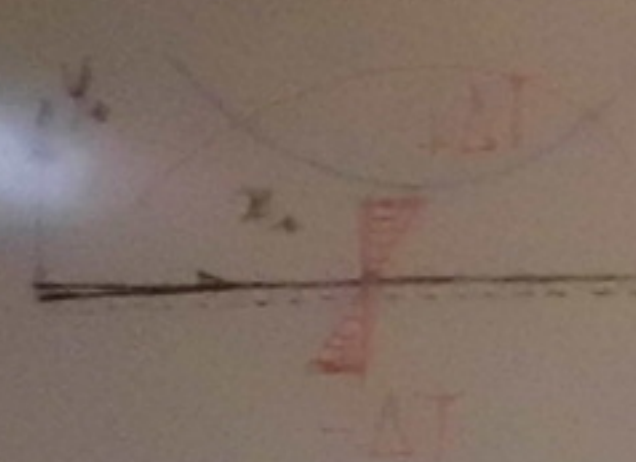
Commenti sulla scrittura delle c.c.



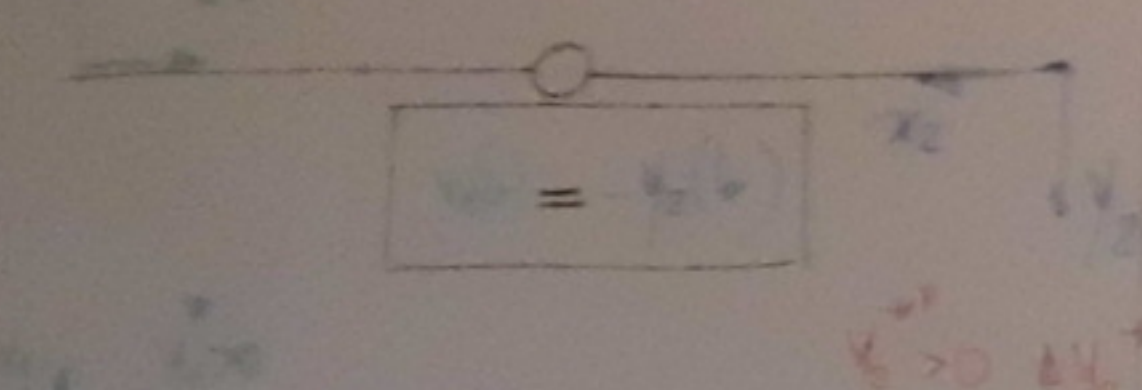
Molle elastiche:

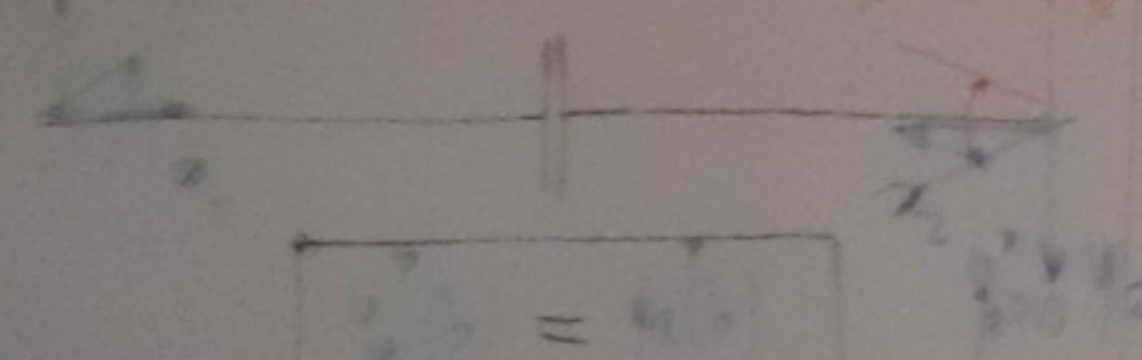


Commenti sulla scrittura delle e.e



$$EJ \frac{d^4 y}{dx^4} = -M_A - EJ \frac{d^3 y}{dx^3}$$

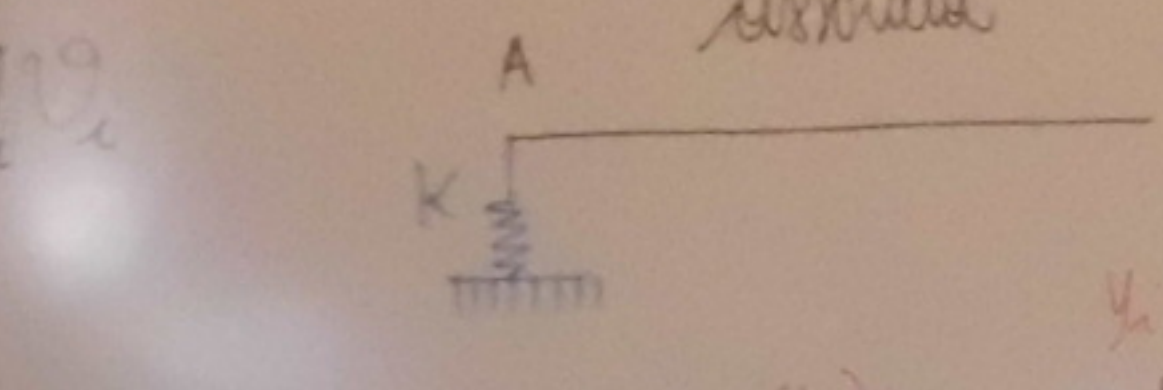


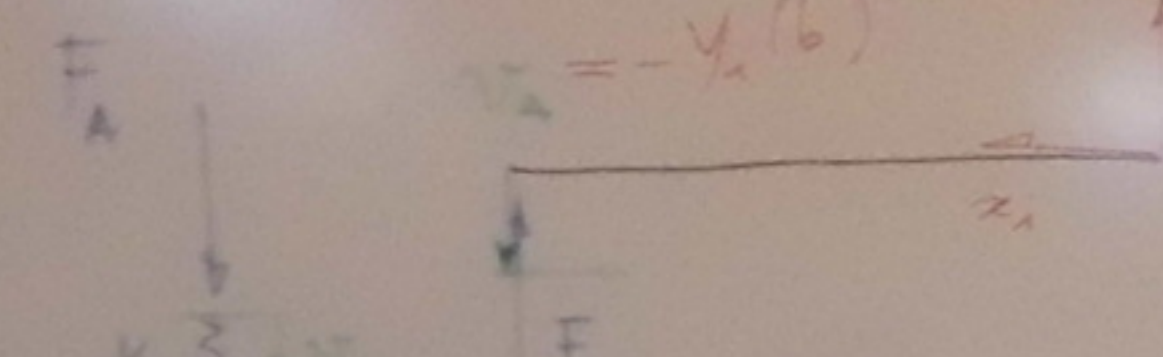
$$V_A = y_A(b)$$


$$V_A = y_A(b)$$

$$= -y_2(b)$$

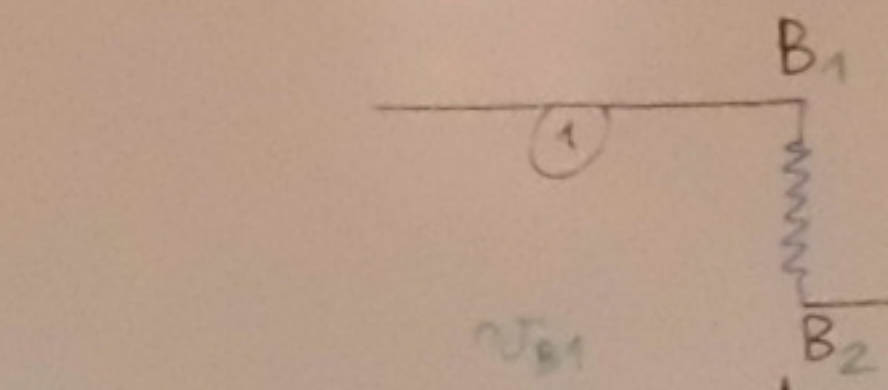
Molle elastiche:
assoluta

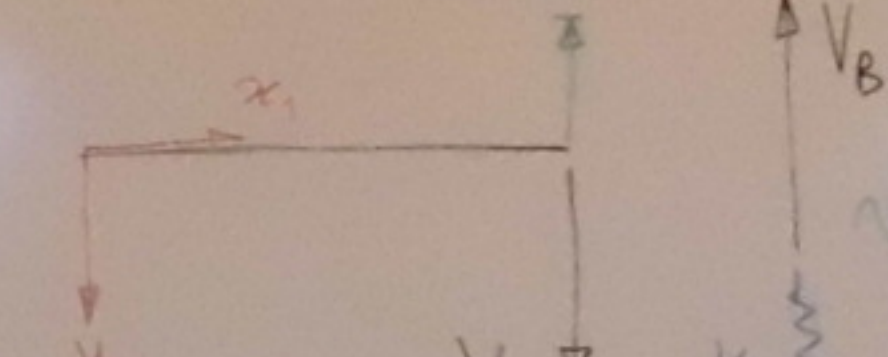


$$V_A = -y_A(b)$$


$$V_A = \frac{1}{K} F_A \Rightarrow y_A(b) = \frac{-F_A}{K}$$

relativa



$$V_{B1} = -y_1(b)$$


$$V_{B2} = -y_2(b)$$

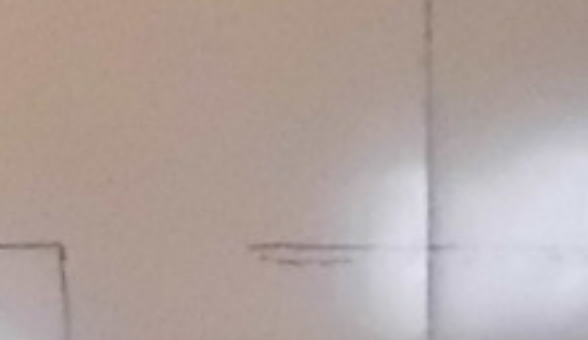
$$V_{B1} = y_1(b)$$

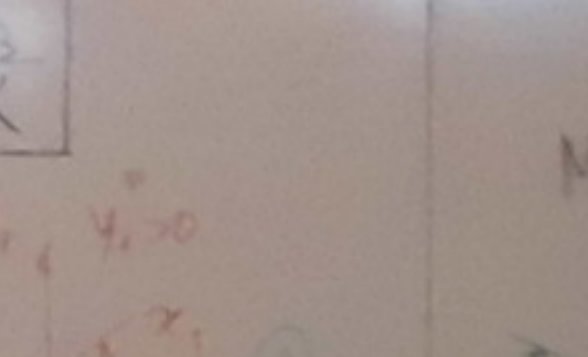
$$V_{B2} = y_2(b)$$

$$V_{B1} = V_{B2} = \frac{V_B}{K}$$

$$-y_1(b) - y_2(b) = \frac{V_B}{K} \Rightarrow y_1(b) + y_2(b) = -\frac{V_B}{K}$$

idem per molle rotazionali



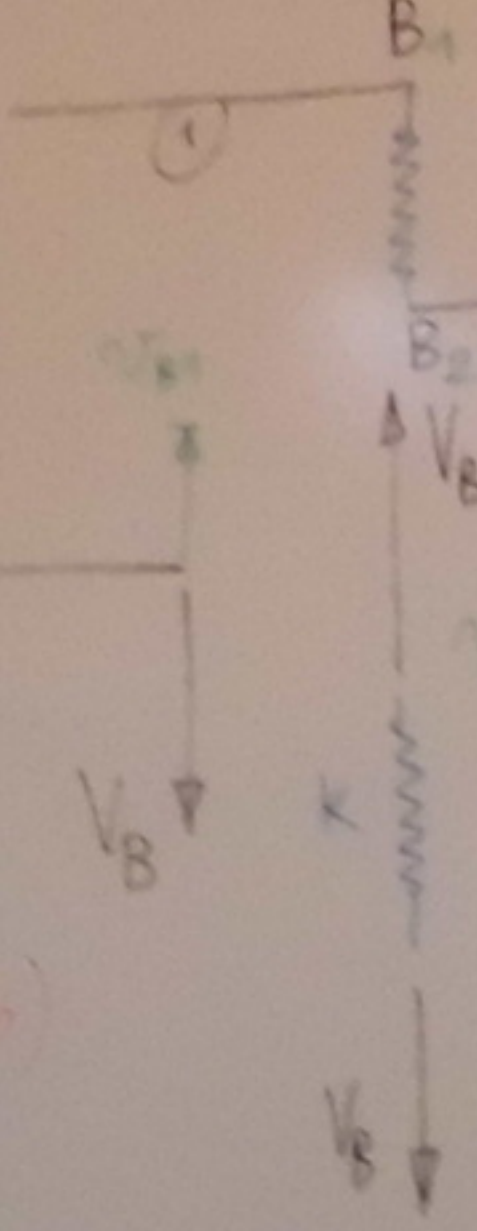
$$V_A = y_A(b)$$


$$V_A = \frac{1}{K} M_A \Rightarrow y_A(b) = \frac{-M_A}{K}$$

Molle elastiche
serie

relativa

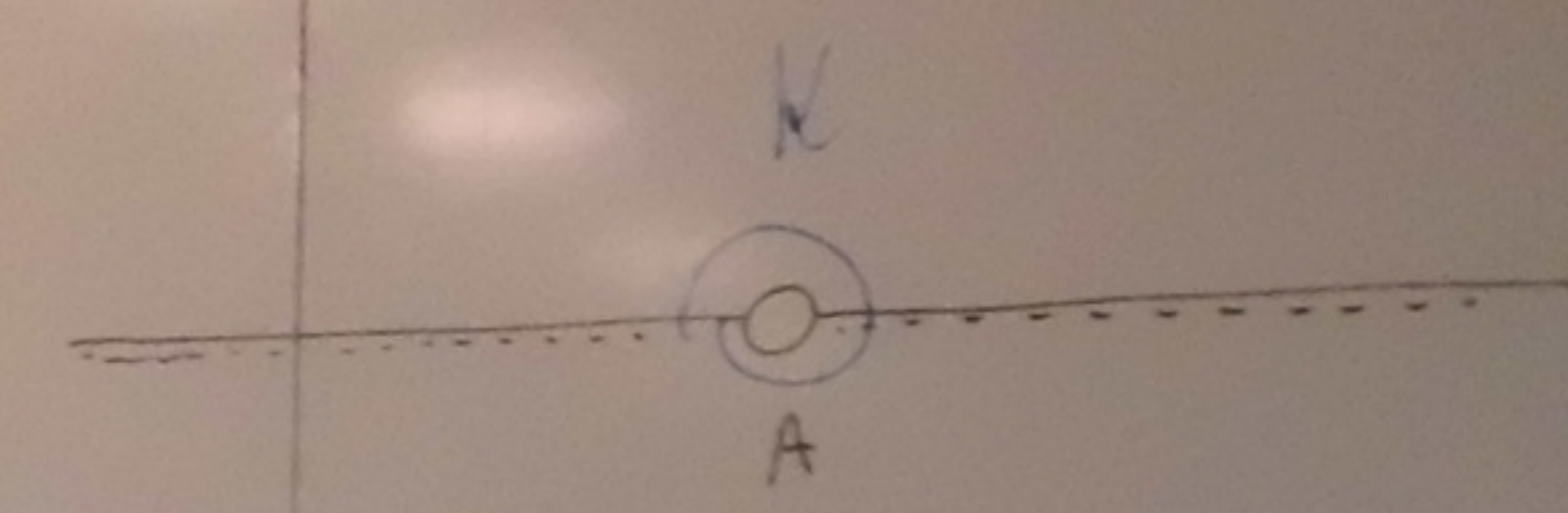
tem per molle rotazionali



$$-y_1(b) - y_2(b) = \frac{V_B}{K} \rightarrow y_1(b) + y_2(b) = -\frac{V_B}{K}$$

$$V_{Brel} = V_{B1} + V_{B2} = \frac{V_B}{K}$$

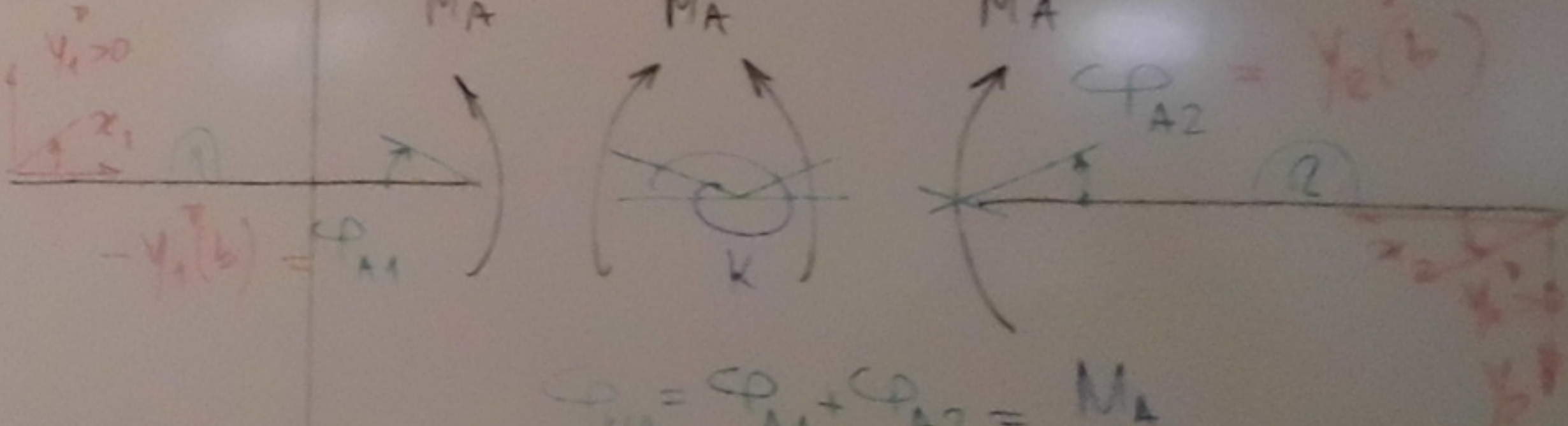
$$V_{Brel} = -y_2(b)$$



M_A

M_A

M_A



$$\phi_{rel} = \phi_{A1} + \phi_{A2} = \frac{M_A}{K}$$

$$-y_1(b) + y_2(b) = \frac{M_A}{K}$$