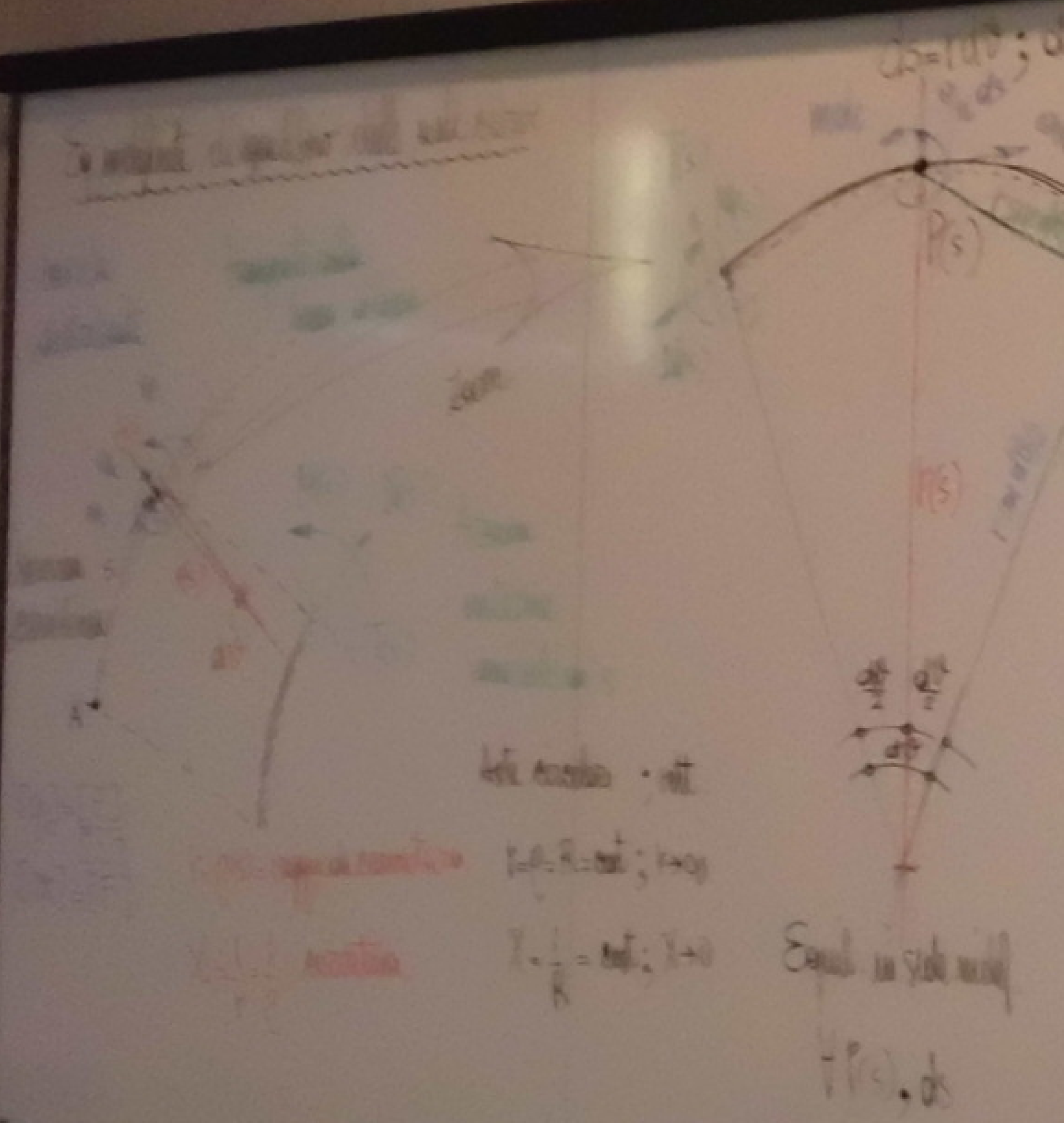

Eq. indefinite di equilibrio delle aste curve
carichi distribuiti
tangente alla linea d'asse
Zoom
ascissa s curvilinea
r(s)
dθ
M(s)
N(s)
Azioni interne variabili in s
A
[q_t] = [F]/[L]
[m] = [F]
r = ρ(s) = raggio di curvatura
χ = 1/r = 1/ρ curvatura
Aste circolari ; rett.
r = ρ = R = cost ; r→∞
χ = 1/R = cost ; χ→0
ds = r dθ ; dθ = 1/r ds
m ds
q_n ds
q_t ds
T(s)
M(s)
N(s)
P(s)
M+dM
T+dT
N+dN
Equil. in sist. indef.
∀ P(s), ds
passaggio al limite
Δs, Δθ → ds, dθ
ΔAI/Δs → dAI/ds
N'(s) = dN/ds = −q_t(s) + T(s)/r(s)
T'(s) = dT/ds = −q_n(s) − N(s)/r(s)
M'(s) = dM/ds = −m(s) + T(s)

Eq. indefinite di equilibrio delle aste curve
ds = r dϑ ; dϑ = 1/r ds
P(s)
r(s)
M+dM
N+dN
T+dT
A
Zoom
Aste circolari ; rett
r = ρ = R = cost ; r→∞
χ = 1/R = cost ; χ→0
Equil. in sole indef.
∀ P(s), ds
passaggio al limite
Δs, Δϑ → ds, dϑ
ΔAI/Δs → dAI/ds
ΣF_t^ds = 0 ⇒ (N + dN − N) cos dϑ/2 − 2T sin dϑ/2 − dT sin dϑ/2 + q_t ds = 0
scompare nel passaggio al limite
N'(s) = dN/ds = −q_t(s) + T(s)/r(s)
accoppiamento
ΣF_n^ds = 0 ⇒ (T + dT − T) cos dϑ/2 + 2N sin dϑ/2 + dN sin dϑ/2 + q_n ds = 0
T'(s) = dT/ds = −q_n(s) − N(s)/r(s)
accoppiamento T', N
ΣM_P^ds = 0 ⇒ M + dM − M + (N − N − dN) r (1 − cos dϑ/2) − 2T r sin dϑ/2 − dT r sin dϑ/2 + m ds = 0
M'(s) = dM/ds = −m(s) + T(s)
come per aste rettilinea
Aste rettilinee (r → ∞)
N'(s) = −q_t(s)
T'(s) = −q_n(s)
M'(s) = −m(s) + T(s)
Aste circolari (r(s) = cost R)
N'(s) = −q_t(s) + T(s)/R = 1/R dN/dϑ
T'(s) = −q_n(s) − N(s)/R = 1/R dT/dϑ
M'(s) = −m(s) + T(s) = 1/R dM/dϑ



passaggio al limite

$\Delta s, \Delta\vartheta \rightarrow ds, d\vartheta$

$\frac{\Delta A I}{\Delta s} \rightarrow \frac{dAI}{ds}$

- $\sum F_t^{ds} = 0 \Rightarrow \left(N + \frac{dN}{ds} - N\right)\underbrace{\cos\frac{d\vartheta}{2}}_{\sim 1} - 2T\underbrace{\sin\frac{d\vartheta}{2}}_{\sim \frac{d\vartheta}{2}} - dT\underbrace{\sin\frac{d\vartheta}{2}}_{\sim\frac{d\vartheta}{2}} + q_t\,ds = 0$

$d\vartheta \rightarrow \frac{ds}{r(s)}$; $0\,(\mathcal{O}(d^2))$ scompara nel processo al limite

$$N^{\prime}(s) = \frac{dN}{ds} = -q_t(s) + \frac{T(s)}{r(s)}$$

accoppiamento $N^{\prime}$, T

- $\sum F_n^{ds} = 0 \Rightarrow \left(T + dT - N\right)\underbrace{\cos\frac{d\vartheta}{2}}_{\sim 1} + 2N\sin\frac{d\vartheta}{2} - dN\underbrace{\sin\frac{d\vartheta}{2}}_{\sim\frac{d\vartheta}{2}} + q_n\,ds = 0$

$\frac{d\vartheta}{2}\,0$

$$T^{\prime}(s) = \frac{dT}{ds} = -q_n(s) - \frac{N(s)}{r(s)}$$

accoppiamento $T^{\prime}$, N

- $\sum M_P^{ds} = 0 \Rightarrow M + dM - M + (N - N - dN)\,r\underbrace{\left(1 - \cos\frac{d\vartheta}{2}\right)}_{\sim 0} - 2T\,r\underbrace{\sin\frac{d\vartheta}{2}}_{\sim\frac{d\vartheta}{2}} - dT\,r\underbrace{\sin\frac{d\vartheta}{2}}_{\sim\frac{d\vartheta}{2}} + m\,ds = 0$

$$M^{\prime}(s) = \frac{dM}{ds} = -m(s) + T(s)$$

come per aste rettilinee

accopp $M^{\prime\prime}$, N

$$M^{\prime\prime}(s) = \frac{d^2M}{ds^2} = -m^{\prime}(s) + \left(-q_n - \frac{N}{r}\right) = -\left(m^{\prime} + q_n\right) - \frac{N(s)}{r(s)}$$

- Aste rettilinee ($r \rightarrow \infty$)

$$\begin{cases} N^{\prime}(s) = -q_t(s) \\ T^{\prime}(s) = -q_n(s) \\ M^{\prime}(s) = -m(s) + T(s) \end{cases}$$

$\rightarrow M^{\prime\prime}(s) = -(m^{\prime}(s) + q_n)$

- Aste circolari ($r(s) = \text{cost}\ R$)

$$\begin{cases} N^{\prime}(s) = -q_t(s) + \frac{T(s)}{R} = \frac{1}{R}\frac{dN}{d\vartheta} \\ T^{\prime}(s) = -q_n(s) - \frac{N(s)}{R} = \frac{1}{R}\frac{dT}{d\vartheta} \\ M^{\prime}(s) = -m(s) + T(s) = \frac{1}{R}\frac{dM}{d\vartheta} \end{cases}$$

$$\begin{cases} N^{\prime}(\vartheta) = -q_t(\vartheta)R + T(\vartheta) \\ T^{\prime}(\vartheta) = -q_n(\vartheta)R - N(\vartheta) \\ M^{\prime}(\vartheta) = -m(\vartheta)R + R\,T(\vartheta) \end{cases}$$


Esempio
S = R θ
θ = S/R
0 ≤ θ ≤ π
P(s)
A
B
C
R cos θ
2R
V = qR
qR = V
Chiave θ = π/2
sim.
antisim.
Azioni interne in P(θ)
N(θ) = (-qR + qR(1-cosθ)) cosθ = -qR cos²θ
T(θ) = (qR - qR(1-cosθ)) sinθ = qR/2 2 sinθ cosθ
sin2θ
M(θ) = qR R(1-cosθ) - qR(1-cosθ) R (1-cosθ)/2
= qR²(1-cosθ)(1 - (1+cosθ)/2) = qR²/2 (1-cos²θ) = qR²/2 sin²θ → M'(θ) = qR²/2 2 sinθ cosθ = qR²/2 sin2θ = R T(θ)
N(θ)/qR
cosθ
cos²θ
-cos²θ
π/2
π
T(θ)/(qR/2)
sinθ
π/4
M(θ)/(qR²/2)
sin²θ
sinθ
qR
qR/2
N
T
M


Esempio
Chiave ϑ = π/2
sim
antisim
s = Rϑ
0 ≤ ϑ ≤ π
2R
N(ϑ)/qR
T(ϑ)/(qR/2)
M(ϑ)/(qR²/2)
qR
ds

dx = ds sin ϑ
ds
q ds cos ϑ
q_t = − q ds sin ϑ cos ϑ / ds
= − q/2 sin 2ϑ
q_n = q ds sin ϑ sin ϑ / ds
= q sin² ϑ
N'(ϑ) = −qR 2cos ϑ (−sin ϑ)
= qR sin 2ϑ
= q/2 sin 2ϑ R + qR/2 sin 2ϑ ✓
T'(ϑ) = qR/2 cos 2ϑ 2
= qR (cos² ϑ − sin² ϑ)
= − qR sin² ϑ + qR cos² ϑ ✓
qR
qR/2
T=0
S = p2R
M=0
T=0
q_n = p
q_t = 0
sim. → T=0
M = cost = 0
N = cost
= − pR
V = pR
N'(s) = − q_t(s) + T(s)/R = 1/R dN/dϑ
T'(s) = − q_n(s) − N(s)/R = 1/R dT/dϑ
M'(s) = − m(s) + T(s) = 1/R dM/dϑ
N'(ϑ) = − q_t(ϑ)R + T(ϑ) ✓
T'(ϑ) = − q_n(ϑ)R − N(ϑ) ✓
M'(ϑ) = − m(ϑ)R + RT(ϑ) ✓