Università degli studi di Bergamo

Scuola di Ingegneria (Dolmine)

CCS Ingegneria Edile

LM-24 Ingegneria delle Costruzioni Edili

Complementi di Scienza delle Costruzioni

(ICAR/08 - SdC ; 6 CFU)

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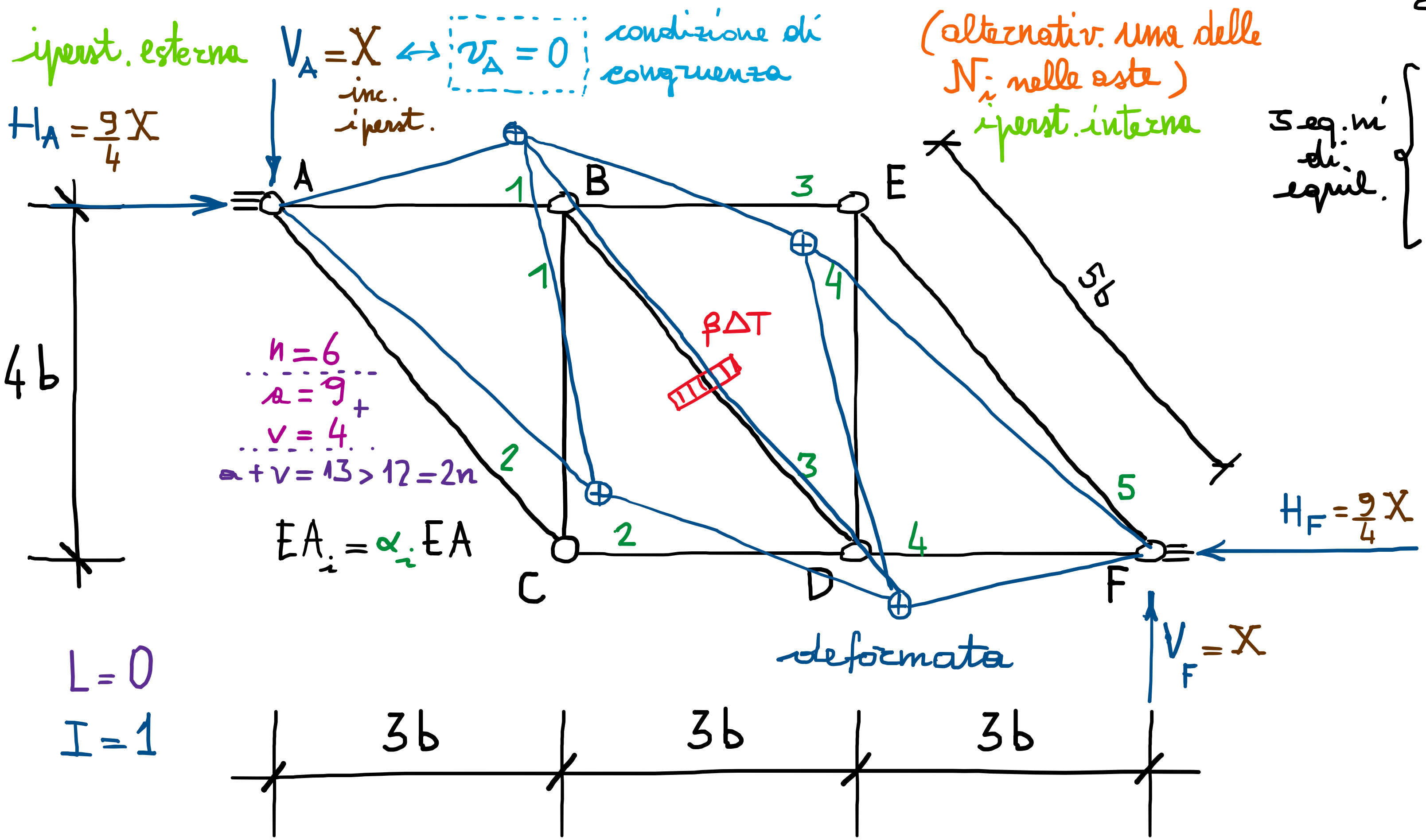
LEZIONE 11

Soluzione di travatura reticolare iperstatica



Sistema (B) cinem. amm. ⇒ sistema reale

Equilibrio globale:

3 eq.ni di equil.

$$\begin{cases} \sum F_x = 0 \Rightarrow H_A = H_F = \frac{9}{4}X \\ \sum F_y = 0 \Rightarrow V_A = V_F = X \\ \sum M_A = 0 \Rightarrow V_F \, 9\cancel{b} = H_F \, 4\cancel{b} \Rightarrow H_F = \frac{9}{4} V_F = \frac{9}{4} X \end{cases}$$

$RV = RV(X)$

Risoluzione tramite PLV (PFV).

– Struttura principale isostatica

$X = 0 \Rightarrow \cancel{N_i^0} \equiv 0$

– Struttura fittizia

$X = 1 \Rightarrow N_i' = \frac{1}{4} N_i^*$

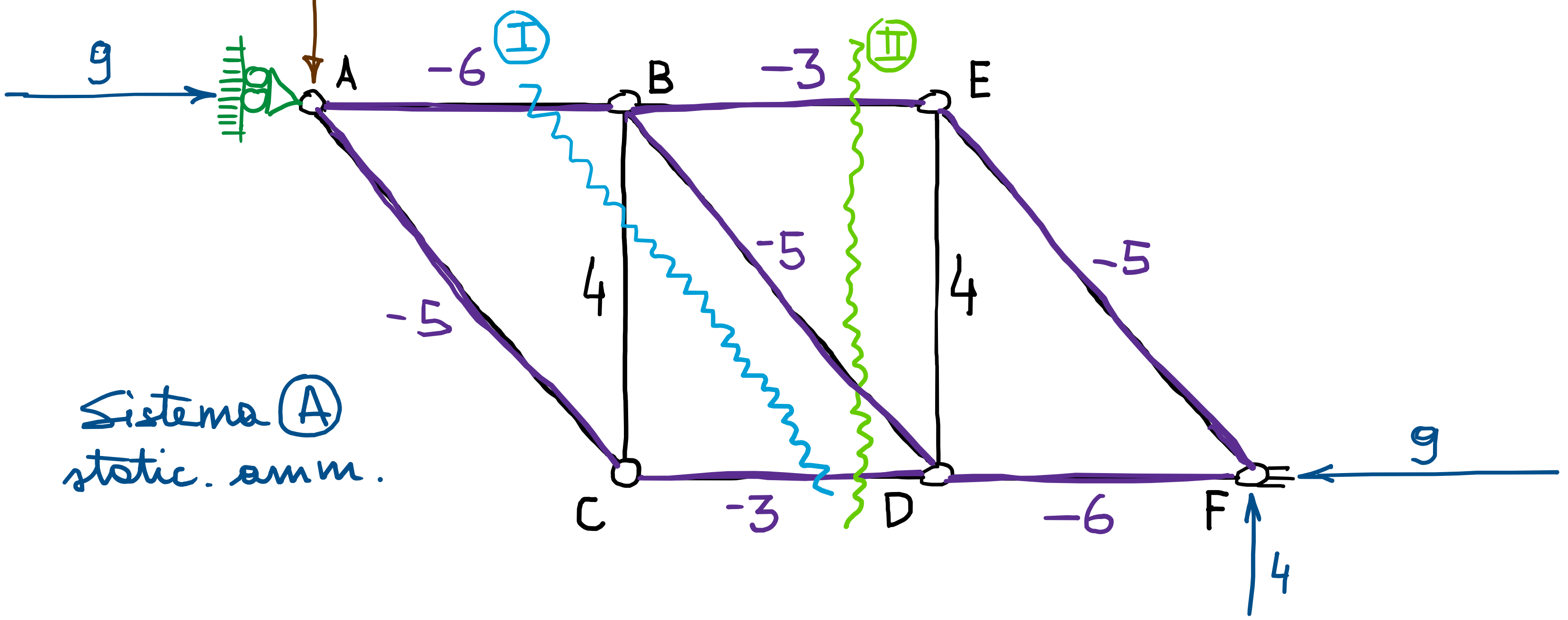
$= 4 \Rightarrow N_i^* = 4 N_i'$

$$N_i = \cancel{N_i^0} + X N_i'$$

$$= \cancel{N_i^0} + \frac{X}{4} N_i^*$$

Struttura fittizia

$X = 4 \Rightarrow N_i^*$

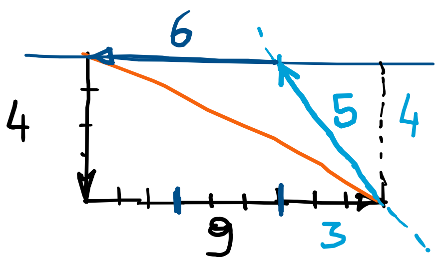


Sistema (A) static. amm.

Metodo dei nodi

Nodo A :

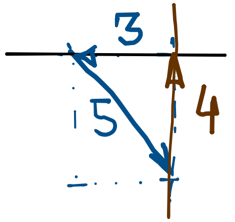
(Nodo F)



poligono delle forze chiuso (statica grafica)

Nodo C :

(Nodo E)



Metodo delle sezioni (Ritter)

(I) $\sum M_B^{sn} = 0 \Rightarrow N_{CD}^* = -3$ ✓

$\sum M_C^{sn} = 0 \Rightarrow N_{AB}^* = -6$ ✓

$\sum F_y^{sn} = 0 \Rightarrow N_{BC}^* = 4$ ✓

(v. asta tre-carrelli)

(II) $\sum M_B^{sn} = 0 \Rightarrow N_{BC} = -3$ ✓

$\sum M_D^{sn} = 0 \Rightarrow N_{BE} = -3$ ✓

$\sum F_y^{sn} = 0 \Rightarrow N_{BD} = -5 \quad (N_{BDy} = 4)$

Scrittura del PLV (PFV, CS di congruenza):

coeff. di dilataz. termica $(\sim 10^{-5} \div 10^{-6}\ °C^{-1})$

$$\mathcal{L}_e^{AB} = 1\, v_A = \int_{str} \overset{\triangledown}{N}_i \underbrace{\frac{\cancel{N_i^0} + X \overset{\triangledown}{N}_i}{EA_i}}_{dn_{e_i}} dx_i + \int_{BD} \overset{\triangledown}{N}_{BD} \underbrace{\underbrace{\alpha \beta \Delta T}_{\varepsilon_t} dx}_{dn_{tBD}} = \mathcal{L}_i^{AB}$$

$v_A = 0$

$N_i = \text{cost nelle aste}$

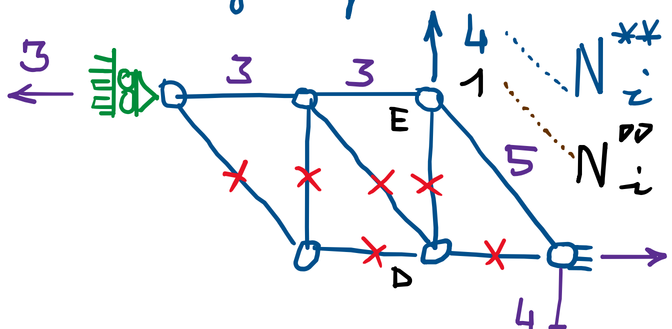
$(EA_i = \alpha_i EA \ \text{cost})$

$$X = -\frac{\sum_i \frac{\overset{\triangledown}{N}_i \cancel{N_i^0} l_i}{EA_i} + \overset{\triangledown}{N}_{BD}\, \alpha\beta\Delta T\, l_{BD}}{\sum_i \frac{\overset{\triangledown}{N}{}_i^2 l_i}{EA_i}} = -\frac{\sum_i \frac{N_i^* \cancel{N_i^0} l_i}{\alpha_i} + N_{BD}^*\, \alpha\beta\Delta T\, l_{BD}\, EA}{\sum_i \frac{N_i^{*2} l_i}{\alpha_i}} \quad \cancel{\tfrac{1}{4}} \quad \cancel{\tfrac{1}{4}}\ \tfrac{1}{4}$$

$$= -\frac{-5\alpha\beta\Delta T\, 5\cancel{b}\, EA\, 4}{\frac{1100}{3}\cancel{b}} = +\cancel{100}\, \alpha\beta\Delta T\, EA\, \frac{3}{11\cancel{00}} = \boxed{\frac{3}{11}\alpha\beta\Delta T\, EA = X}$$

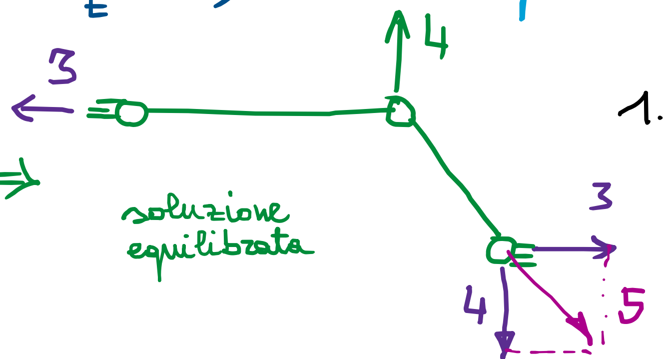
$$\overset{\triangledown}{N}_i = \frac{X}{4} N_i^*$$

Calcolo degli spost. nodali (es. $v_E = ?$) → deformata



⟺

soluzione equilibrata



$$1 \cdot v_E = \sum_i \frac{N_i^{**}}{4} \left(N_i^0 + \frac{X}{4} N_i^*\right) \frac{l_i}{\alpha_i EA}$$

$$= \boxed{-\frac{3}{2}\beta\alpha\Delta T\, b = v_E}$$

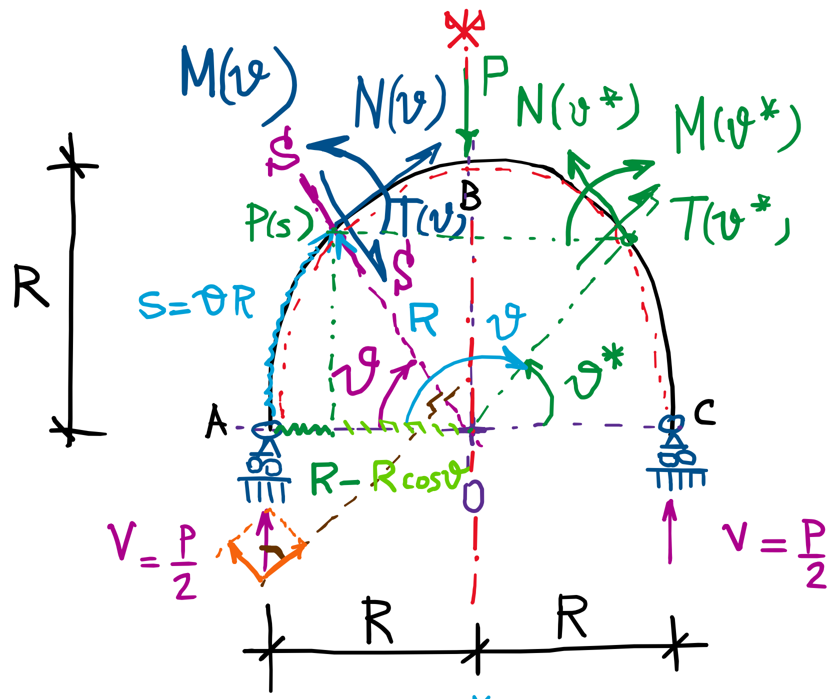
$v_E = v_D + \Delta l_{ED}$

$v_D = v_E - \frac{N_{ED}\, l_{tD}}{EA_{ED}}$

Azioni interne in aste curve

agevole rappresentazione analitica (in coordinate polari)

Arco circolare (con carico concentrato in chiave):



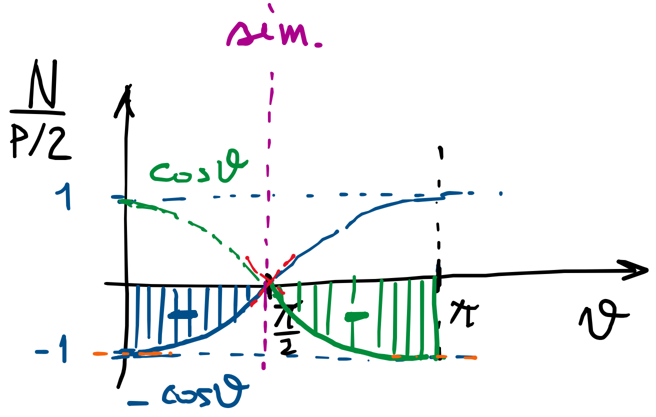
$$\vartheta + \vartheta^* = \pi$$

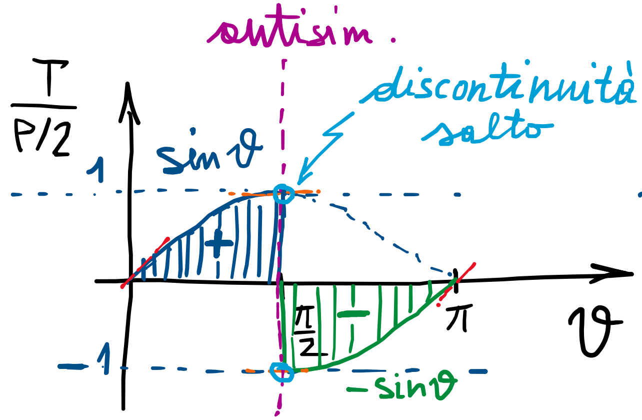
$$\vartheta^* = \pi - \vartheta$$

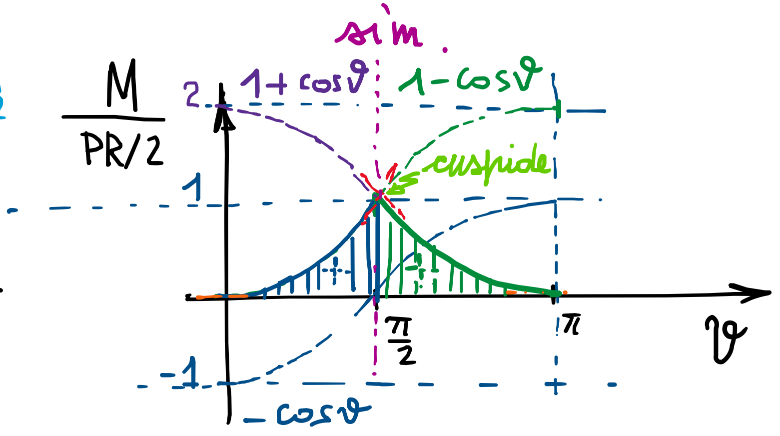
$$0 \le \vartheta < \frac{\pi}{2} \quad \begin{cases} N(\vartheta) = -\dfrac{P}{2}\cos\vartheta \\ T(\vartheta) = \dfrac{P}{2}\sin\vartheta \\ M(\vartheta) = \dfrac{P}{2}R(1-\cos\vartheta) \end{cases}$$

equilibrio locale della porzione $\widehat{AP}$

$$\begin{array}{l} 0 \le \vartheta^* < \frac{\pi}{2} \\ \frac{\pi}{2} < \vartheta \le \pi \end{array} \quad \begin{cases} N(\vartheta^*) = -\dfrac{P}{2}\cos\vartheta^* = \dfrac{P}{2}\cos\vartheta \\ T(\vartheta^*) = -\dfrac{P}{2}\sin\vartheta^* = -\dfrac{P}{2}\sin\vartheta \\ M(\vartheta^*) = \dfrac{PR}{2}(1-\cos\vartheta^*) = \dfrac{PR}{2}(1+\cos\vartheta) \end{cases}$$







dipendenza analitica delle funzioni di Azione Interna e loro rappresentazione

Diagrammi delle Azioni Interne N, T, M (andamenti funzionali rappresentati su fondamentali coincidenti con la struttura stessa)