Università degli studi di Bergamo

Scuola di Ingegneria (Dalmine)

CCS Ingegneria Edile

LM-24 Ingegneria delle Costruzioni Edili

Complementi di Scienza delle Costruzioni

(ICAR/08 - SdC; 6 CFU)

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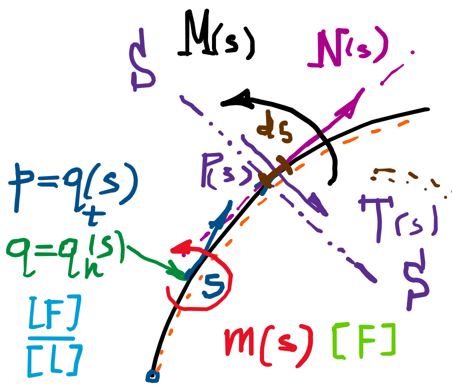
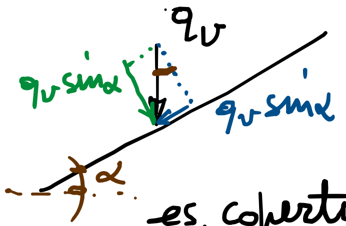
LEZIONE 12

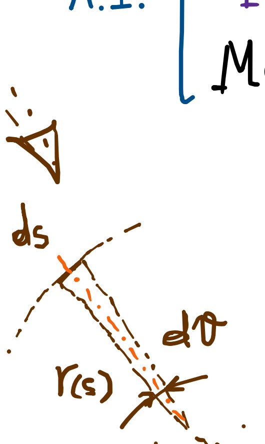
Equazioni indefinite di equilibrio delle aste curve ($\forall s, ds$)



A.I. $\begin{cases} N(s): \text{azione assiale o normale} \\ T(s): \quad \text{"} \quad \text{tagliante o taglio} \\ M(s): \quad \text{"} \quad \text{flettente o momento} \end{cases}$



es. coperture inclinate

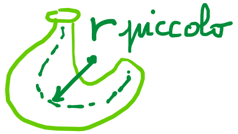


$ds = r(s)\, d\vartheta$

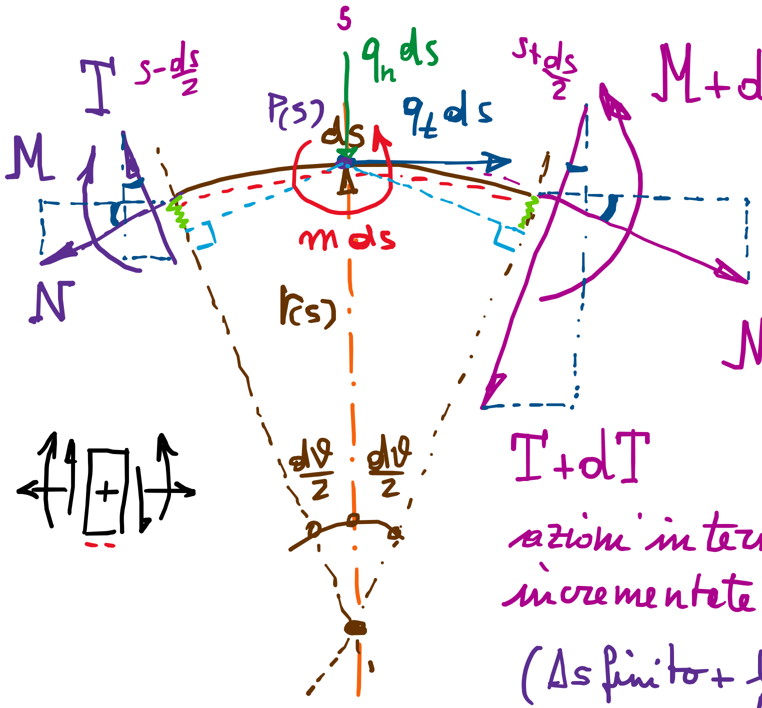
$\boxed{d\vartheta = \frac{1}{r(s)}\, ds} = \chi(s)\, ds$

raggio di curvatura locale

$r \approx \ell$ ($r \to \infty$, trave rettilinea)



elemento "tozzo" (non trave)



azioni interne incrementate in ds (Δs finito + $\lim_{\Delta s \to 0}$)

Equazioni di equilibrio:

accoppiamento N, T

$$\Sigma F_t^{ds} = 0 \Rightarrow (\cancel{N} + dN - \cancel{N}) \overset{\sim 1}{\cos\frac{d\vartheta}{2}} - (T + dT + T) \overset{\sim d\vartheta/2}{\sin\frac{d\vartheta}{2}} + q_t\, ds = 0 \Rightarrow \frac{dN}{ds} - \cancel{2}\, T \frac{\overset{ds/r(s)}{d\vartheta}}{\cancel{2}} - \cancel{dT \frac{d\vartheta}{2}} + q_t \cancel{ds} = 0 \Rightarrow \boxed{N'(s) = -q_t(s) + \frac{T(s)}{r(s)}}$$

$$\Sigma F_n^{ds} = 0 \Rightarrow (\cancel{T} + dT - \cancel{T}) \cos\frac{d\vartheta}{2} + (N + dN + N) \sin\frac{d\vartheta}{2} + q_n\, ds = 0 \Rightarrow \frac{dT}{ds} + \cancel{2}\, N \frac{d\vartheta}{\cancel{2}} + \cancel{dN \frac{d\vartheta}{2}} + q_n \cancel{ds} = 0 \Rightarrow \boxed{T'(s) = -q_n(s) - \frac{N(s)}{r(s)}}$$

$$\Sigma M_P^{ds} = 0 \Rightarrow (\cancel{M} + dM - \cancel{M}) - (T + dT + T)\, r \overset{\sim d\vartheta/2}{\sin\frac{d\vartheta}{2}} + - (\cancel{N} + dN - \cancel{N})\, r\Big(\overset{\sim 0}{1 - \cos\frac{d\vartheta}{2}}\Big) + m\, ds = 0$$

$$\Rightarrow \frac{dM}{ds} - \cancel{2}\, T \underbrace{r \frac{d\vartheta}{\cancel{2}}}_{ds} + \cancel{dT \frac{d\vartheta}{2}} + m \cancel{ds} = 0 \Rightarrow \boxed{M'(s) = -m(s) + T(s)}$$

$$\boxed{M''(s) = -m'(s) + T'(s) = -\big(m'(s) + q_n(s)\big) - \frac{N(s)}{r(s)}}$$

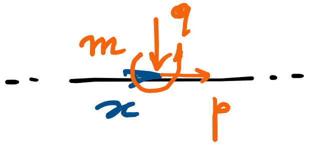
accoppiamento T, M

Note:

- nelle aste curve si registra accoppiamento N, T, in cui la variazione di ciascuna azione interna risulta accoppiata all'andamento dell'altra.

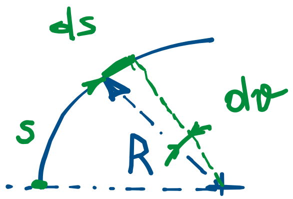
- si conferma l'accoppiamento T, M, già visto per aste rettilinee, ottenibili come segue.

- per aste rettilinee $(r \to \infty)$:



$$\begin{cases} N'(x) = -p(x) \\ T'(x) = -q(x) \\ M'(x) = -m(x) + T(x) \end{cases} \Rightarrow M''(x) = -m'(x) + T'(x) = -(m'(x) + q(x))$$

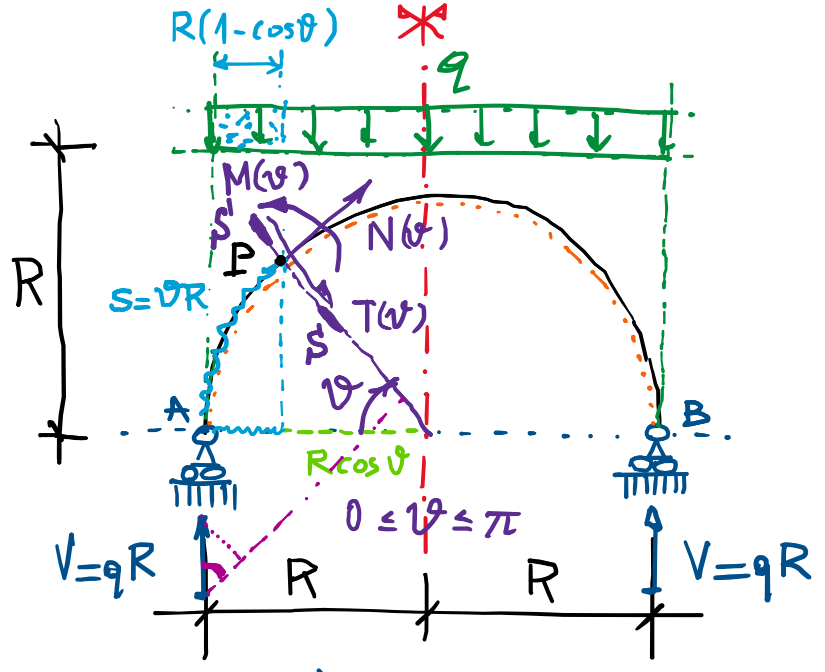
- per aste circolari $(r(s) = R = \text{cost})$



$$ds = R\,d\vartheta \qquad d\vartheta = \frac{1}{R}\,ds \rightarrow \frac{d(\,)}{ds} = \frac{1}{R}\frac{d(\,)}{d\vartheta} \Rightarrow \begin{cases} N'(\vartheta) = -q_t(\vartheta)R + T(\vartheta) \\ T'(\vartheta) = -q_n(\vartheta)R - N(\vartheta) \\ M'(\vartheta) = -m(\vartheta)R + T(\vartheta)R \end{cases} \quad (*)$$

$$\rightarrow M''(\vartheta) = -m'(\vartheta)R + T'(\vartheta)R$$
$$= -m'(\vartheta)R - q_n(\vartheta)R^2 - N(\vartheta)R$$
$$= -(m'(\vartheta) + q_n(\vartheta)R)R - N(\vartheta)R$$

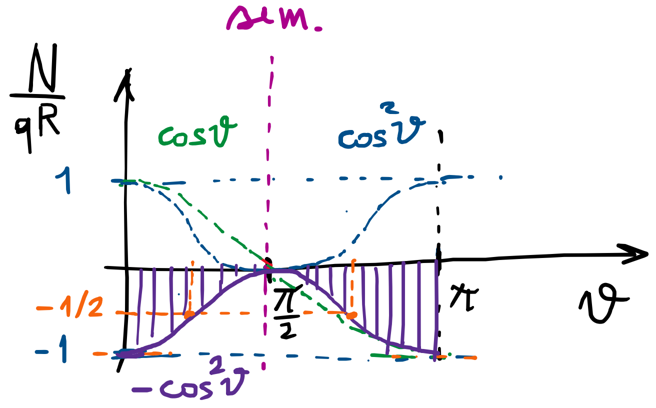
Esempio: arco semicircolare con q_v distribuito (per unità di lunghezza in direz. orizzontale)

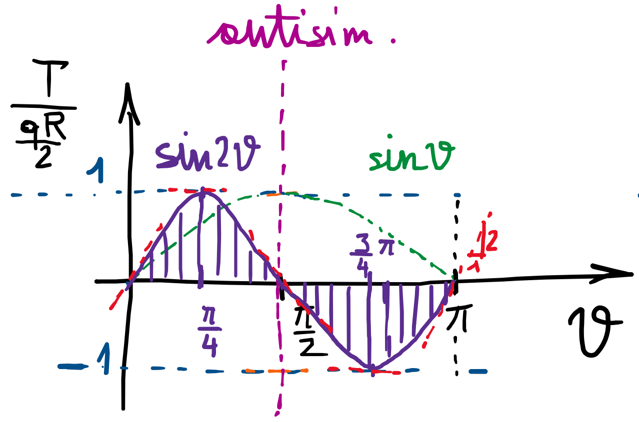


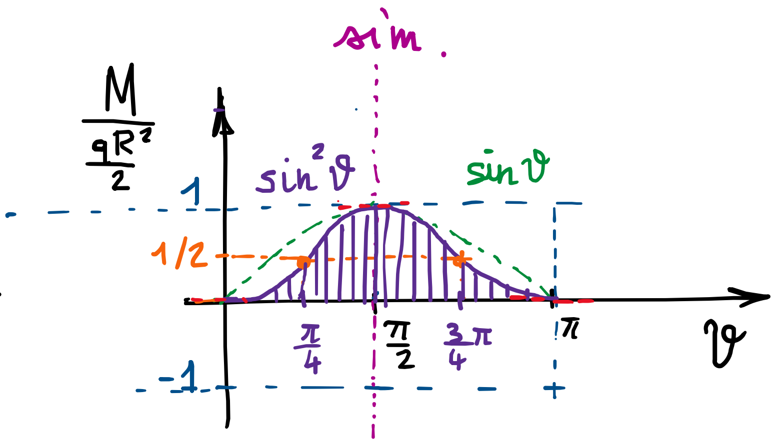
equil. globale

Equil. locale ($\widehat{AP}$):

$$\begin{cases} N(\vartheta) = -qR\cancel{\cos\vartheta} + qR(\cancel{1}-\cos\vartheta)\cos\vartheta = \boxed{-qR\cos^2\vartheta = N} \\ T(\vartheta) = qR\cancel{\sin\vartheta} - qR(\cancel{1}-\cos\vartheta)\sin\vartheta = \frac{qR}{2}\underbrace{2\sin\vartheta\cos\vartheta}_{\sin 2\vartheta} = \boxed{\frac{qR}{2}\sin 2\vartheta = T} \\ M(\vartheta) = qRR(1-\cos\vartheta) - qR(1-\cos\vartheta)\frac{R(1-\cos\vartheta)}{2} = \\ \quad = qR^2(1-\cos\vartheta)\left(1 - \frac{1-\cos\vartheta}{2}\right) = \frac{qR^2}{2}(1-\cos\vartheta)(1+\cos\vartheta) \\ \quad = \frac{qR^2}{2}(1-\cos^2\vartheta) = \boxed{\frac{qR^2}{2}\sin^2\vartheta = M} \end{cases}$$

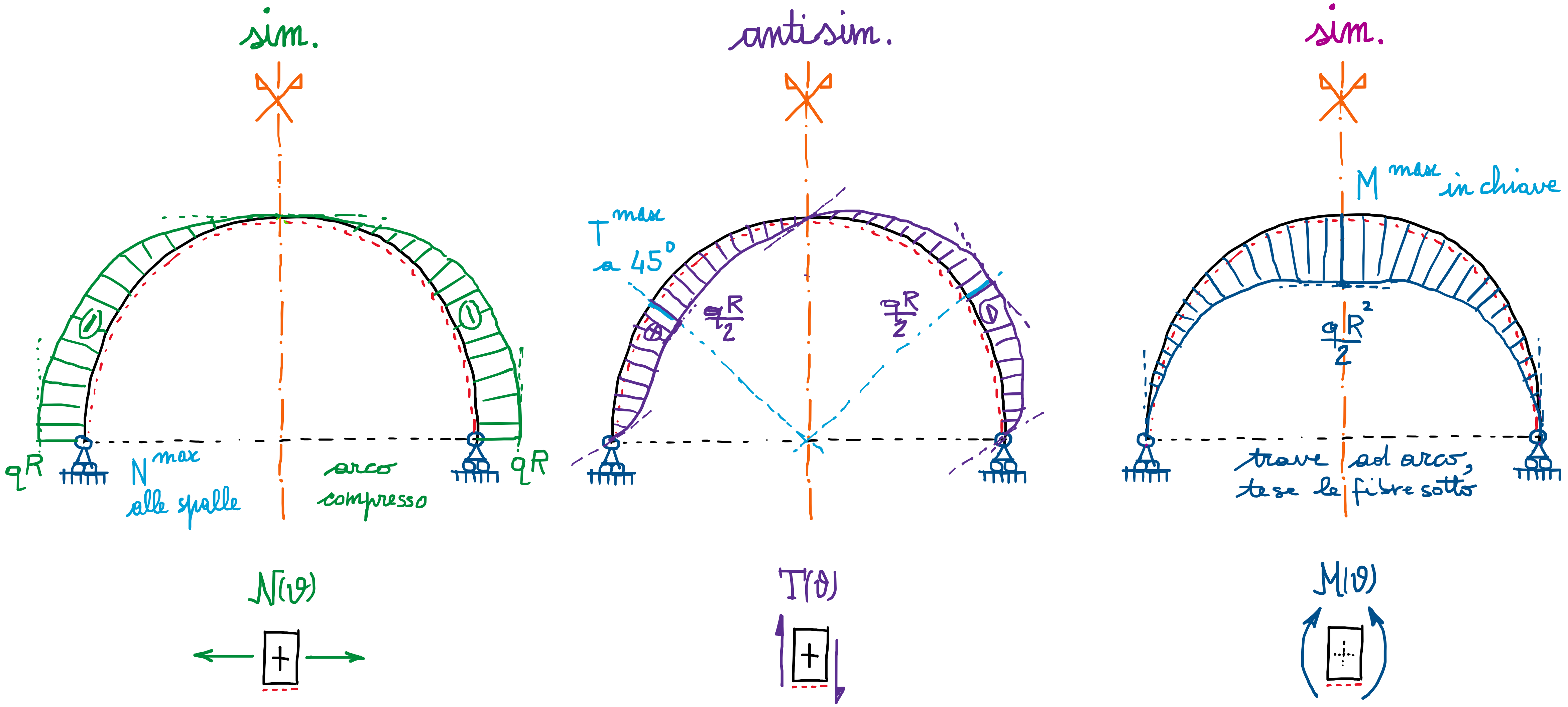




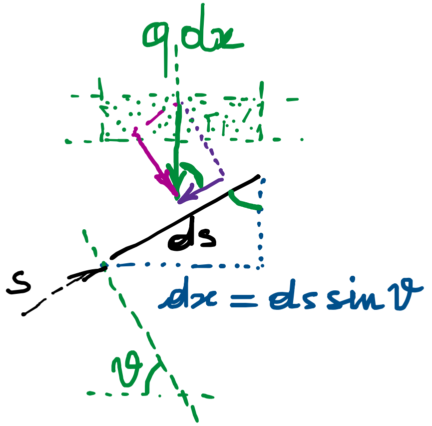


dipendenza analitica delle funzioni di Azione Interna e loro rappresentazione

Diagrammi delle Azioni Interne N, T, M (andamenti funzionali rappresentati su fondamentali coincidenti con la struttura stessa)



– Verificabili le relazioni differenziali viste (*), con q_t e q_n come segue:



$$q_t = -\frac{q\,dx\cos\vartheta}{ds} = -\frac{q\,\cancel{ds}\sin\vartheta\cos\vartheta}{\cancel{ds}}\,\frac{2}{2} = \boxed{-\frac{q}{2}\sin 2\vartheta = q_t(\vartheta)}$$

$$q_n = \frac{q\,dx\sin\vartheta}{ds} = \frac{q\,\cancel{ds}\sin\vartheta\sin\vartheta}{\cancel{ds}} = \boxed{q\sin^2\vartheta = q_n(\vartheta)}$$

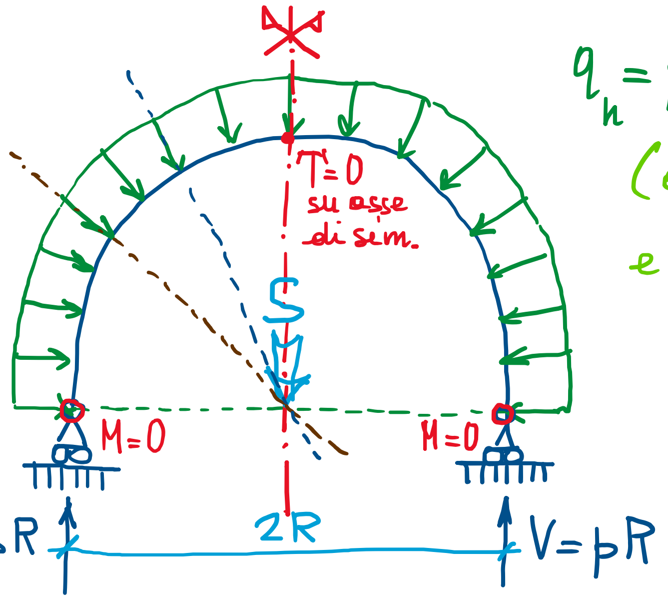
– Arco circolare soggetto a pressione uniforme esterna:

(v. Idraulica)

S: Spinta alla base

$S = p \cdot 2R$

$(S_x = 0)$



$q_n = p = \text{cost}$

$(q_t = 0$ e $m = 0)$

$$\begin{cases} T \equiv 0, \text{ per simmetria rispetto ad ogni direzione radiale (prima –·–, poi –·–, poi –·–, ecc.)} \\ N = -pR = \text{cost} \quad (N' = 0) \quad \text{compressione} \\ M \equiv 0 = \text{cost} \quad (M' = 0) \end{cases}$$

dalle (*):

$$\Leftarrow \begin{cases} N' = T \\ T' = -pR - N \\ M' = TR \end{cases}$$

come per tubo soggetto a pressione interna $\rightleftarrows$ $N = pR$, traz.