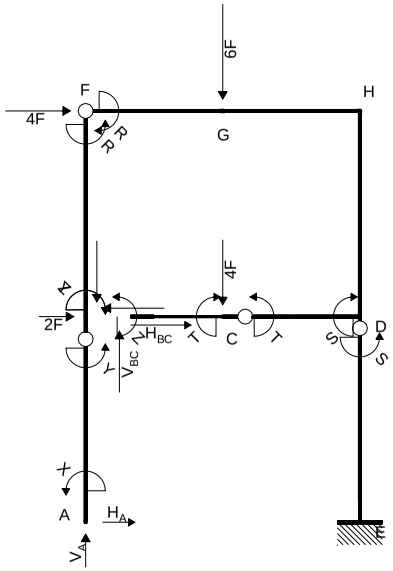


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

$3H_Ab -4V_Ab = -Xb +Sb -8Fb$

Rotazione intorno a C: aste CB

$-2V_{BC}b = -Zb +Tb$

Rotazione intorno a F: aste FB BA

$6H_Ab -3H_{BC}b = -Xb +Zb -Rb -6Fb$

Rotazione intorno a B: aste BA

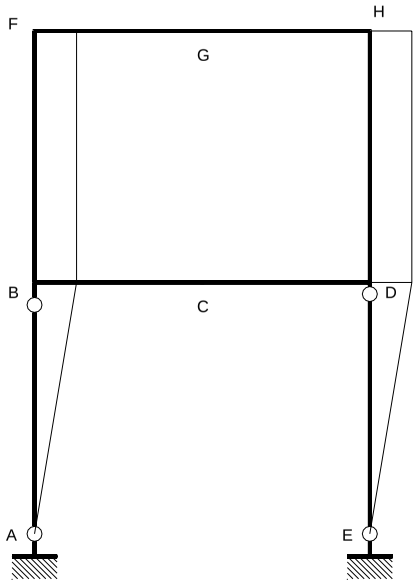
$3H_Ab = -Xb -Yb$

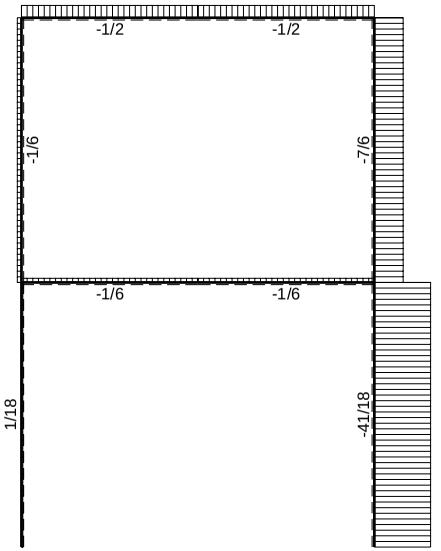
Matrice di equilibrio

	H_Ab	V_Ab	$H_{BC}b$	$V_{BC}b$		Xb	Yb	Zb	Tb	Sb	Rb	Fb
φ_{DE}	3	-4	0	0	=	-1	0	0	0	1	0	-8
φ_{CD}	0	0	0	-2		0	0	-1	1	0	0	0
φ_{FB}	6	0	-3	0		-1	0	1	0	0	-1	-6
φ_{BA}	3	0	0	0		-1	-1	0	0	0	0	0

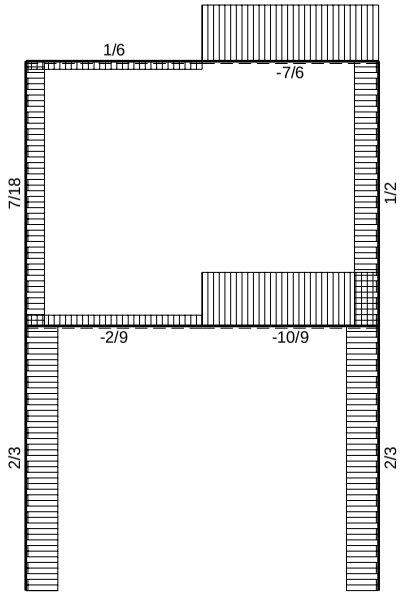
Soluzione del sistema

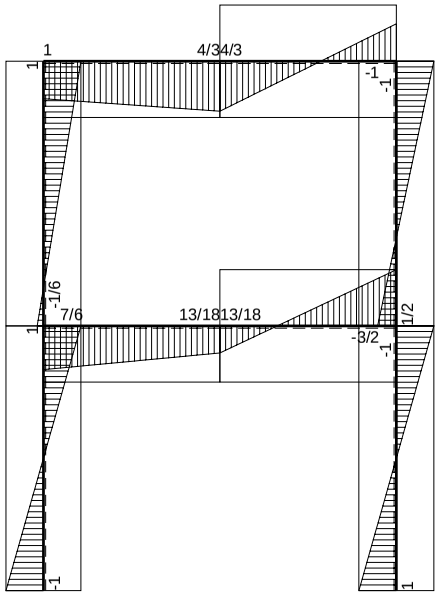
		Xb	Yb	Zb	Tb	Sb	Rb	Fb
H_Ab		-1/3	-1/3	0	0	0	0	0
$V_{BC}b$	=	0	0	1/2	-1/2	0	0	0
$H_{BC}b$		-1/3	-2/3	-1/3	0	0	1/3	2
V_Ab		0	-1/4	0	0	-1/4	0	2





F





F Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	≥	H_{14}
D_2	H_{21}	H_{22}	H_{23}	≥	H_{24}
D_3	H_{31}	H_{32}	H_{33}	≥	H_{34}
D_4	H_{41}	H_{42}	H_{43}	≥	H_{44}
D_5	H_{51}	H_{52}	H_{53}	≥	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FB}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	1	0	0	0	0	≤	3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	1	0	0	0	≤	3/2
W_{DC}-	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC}+	0	0	1	-2	0	0	-8	≤	3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-1
W_{DE}+	0	0	0	0	1	0	0	≤	1
W_{ED}-	-1	-1	0	0	-1	0	18	≥	-1
W_{ED}+	-1	-1	0	0	-1	0	18	≤	1
W_{BF}-	0	-1	-1	0	0	0	0	≥	-1
W_{BF}+	0	-1	-1	0	0	0	0	≤	1
W_{FB}-	0	0	0	0	0	1	0	≥	-1
W_{FB}+	0	0	0	0	0	1	0	≤	1
W_{GH}-	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH}+	0	1/2	1	-1	1/2	-1	-4	≤	3/2
W_{HD}-	0	1	2	-2	1	-1	4	≥	-1
W_{HD}+	0	1	2	-2	1	-1	4	≤	1
W_{DH}-	0	0	-1	2	-1	0	8	≥	-1
W_{DH}+	0	0	-1	2	-1	0	8	≤	1
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	≥	-3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	≥	-3/2
W_{DC}-	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	8	≥	-3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-1
W_{DE}+	0	0	0	0	-1	0	0	≥	-1
W_{ED}-	-1	-1	0	0	-1	0	18	≥	-1
W_{ED}+	1	1	0	0	1	0	-18	≥	-1
W_{BF}-	0	-1	-1	0	0	0	0	≥	-1
W_{BF}+	0	1	1	0	0	0	0	≥	-1
W_{FB}-	0	0	0	0	0	1	0	≥	-1
W_{FB}+	0	0	0	0	0	-1	0	≥	-1
W_{GH}-	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	1	4	≥	-3/2
W_{HD}-	0	1	2	-2	1	-1	4	≥	-1
W_{HD}+	0	-1	-2	2	-1	1	-4	≥	-1
W_{DH}-	0	0	-1	2	-1	0	8	≥	-1
W_{DH}+	0	0	1	-2	1	0	-8	≥	-1
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{BC}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC}-	0	0	1	-2	0	0	0	0	-1	2	0	0	-8	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	0	0	1	-2	0	0	8	≤	-3/2
W_{DE}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{DE}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{ED}-	-1	-1	0	0	-1	0	1	1	0	0	1	0	18	≥	-1
W_{ED}+	1	1	0	0	1	0	-1	-1	0	0	-1	0	-18	≤	-1
W_{BF}-	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-1
W_{BF}+	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-1
W_{FB}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-1
W_{FB}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-1
W_{GH}-	0	1/2	1	-1	1/2	-1	0	-1/2	-1	1	-1/2	1	-4	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	1	0	1/2	1	-1	1/2	-1	4	≤	-3/2
W_{HD}-	0	1	2	-2	1	-1	0	-1	-2	2	-1	1	4	≥	-1
W_{HD}+	0	-1	-2	2	-1	1	0	1	2	-2	1	-1	-4	≤	-1
W_{DH}-	0	0	-1	2	-1	0	0	0	1	-2	1	0	8	≥	-1
W_{DH}+	0	0	1	-2	1	0	0	0	-1	2	-1	0	-8	≤	-1
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	1	-2	0	0	-8	1	≥	-3/2
φ_{DC}+	0	0	-1	2	0	0	8	-1	≥	-3/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	-1	-1	0	0	-1	0	18	3	≥	-1
φ_{ED}+	1	1	0	0	1	0	-18	-3	≥	-1
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	0	1/2	1	-1	1/2	-1	-4	0	≥	-3/2
φ_{GH}+	0	-1/2	-1	1	-1/2	1	4	0	≥	-3/2
φ_{HD}-	0	1	2	-2	1	-1	4	-1	≥	-1
φ_{HD}+	0	-1	-2	2	-1	1	-4	1	≥	-1
φ_{DH}-	0	0	-1	2	-1	0	8	0	≥	-1
φ_{DH}+	0	0	1	-2	1	0	-8	0	≥	-1
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 14-7

	X	Y	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	-4/9	-4/9	1	-2	-4/9	0	4/9	7/3	≥	-19/18
φ_{DC}+	4/9	4/9	-1	2	4/9	0	-4/9	-7/3	≥	-35/18
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	1/18	1/18	0	0	1/18	0	-1/18	-1/6	≥	-1/18
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	-2/9	5/18	1	-1	5/18	-1	2/9	2/3	≥	-23/18
φ_{GH}+	2/9	-5/18	-1	1	-5/18	1	-2/9	-2/3	≥	-31/18
φ_{HD}-	2/9	11/9	2	-2	11/9	-1	-2/9	-5/3	≥	-11/9
φ_{HD}+	-2/9	-11/9	-2	2	-11/9	1	2/9	5/3	≥	-7/9
φ_{DH}-	4/9	4/9	-1	2	-5/9	0	-4/9	-4/3	≥	-13/9
φ_{DH}+	-4/9	-4/9	1	-2	5/9	0	4/9	4/3	≥	-5/9
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	1/18	1/18	0	0	1/18	0	-1/18	-1/6	=	-1/18

Scambio pivotale 2-1

	φ_{AB}+	Y	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	4/9	-4/9	1	-2	-4/9	0	4/9	17/9	≥	-11/18
φ_{DC}+	-4/9	4/9	-1	2	4/9	0	-4/9	-17/9	≥	-43/18
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	≥	-1/9
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	2/9	5/18	1	-1	5/18	-1	2/9	4/9	≥	-19/18
φ_{GH}+	-2/9	-5/18	-1	1	-5/18	1	-2/9	-4/9	≥	-35/18
φ_{HD}-	-2/9	11/9	2	-2	11/9	-1	-2/9	-13/9	≥	-13/9
φ_{HD}+	2/9	-11/9	-2	2	-11/9	1	2/9	13/9	≥	-5/9
φ_{DH}-	-4/9	4/9	-1	2	-5/9	0	-4/9	-8/9	≥	-17/9
φ_{DH}+	4/9	-4/9	1	-2	5/9	0	4/9	8/9	≥	-1/9
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	=	-1/9

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Scambio pivotale 24-2

	φ_{AB}+	φ_{DH}+	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	1	-9/4	9/4	-9/2	5/4	0	1	1	≥	-5/4
φ_{BA}+	-1	9/4	-9/4	9/2	-5/4	0	-1	-1	≥	-3/4
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	1	0	0	-1	0	0	1	≥	-1/2
φ_{DC}+	0	-1	0	0	1	0	0	-1	≥	-5/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	-1/8	1/8	-1/4	1/8	0	0	0	≥	-1/8
φ_{BF}-	-1	9/4	-13/4	9/2	-5/4	0	-1	0	≥	-3/4
φ_{BF}+	1	-9/4	13/4	-9/2	5/4	0	1	0	≥	-5/4
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/2	-5/8	13/8	-9/4	5/8	-1	1/2	1	≥	-9/8
φ_{GH}+	-1/2	5/8	-13/8	9/4	-5/8	1	-1/2	-1	≥	-15/8
φ_{HD}-	1	-11/4	19/4	-15/2	11/4	-1	1	1	≥	-7/4
φ_{HD}+	-1	11/4	-19/4	15/2	-11/4	1	-1	-1	≥	-1/4
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	1	-9/4	9/4	-9/2	5/4	0	1	2	≥	-1/4
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	-1/8	1/8	-1/4	1/8	0	0	0	=	-1/8

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Scambio pivotale 22-3

	φ_{AB}+	φ_{DH}+	φ_{HD}+	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	10/19	≥	-26/19
φ_{BA}+	-10/19	18/19	9/19	18/19	1/19	-9/19	-10/19	-10/19	≥	-12/19
φ_{BC}-	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-23/19	≥	-59/38
φ_{BC}+	4/19	-11/19	4/19	-30/19	11/19	-4/19	4/19	23/19	≥	-55/38
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	1	0	0	-1	0	0	1	≥	-1/2
φ_{DC}+	0	-1	0	0	1	0	0	-1	≥	-5/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	≥	-5/38
φ_{BF}-	-6/19	7/19	13/19	-12/19	12/19	-13/19	-6/19	13/19	≥	-11/19
φ_{BF}+	6/19	-7/19	-13/19	12/19	-12/19	13/19	6/19	-13/19	≥	-27/19
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	3/19	6/19	-13/38	6/19	-6/19	-25/38	3/19	25/38	≥	-23/19
φ_{GH}+	-3/19	-6/19	13/38	-6/19	6/19	25/38	-3/19	-25/38	≥	-34/19
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-4/19	≥	-1/19
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	29/19	≥	-7/19
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	=	-5/38

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Scambio pivotale 22-5

	φ_{AB}+	φ_{DH}+	φ_{HD}+	T	Z	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	6/11	≥	-15/11
φ_{BA}+	-6/11	1	5/11	12/11	-1/11	-5/11	-6/11	-6/11	≥	-7/11
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	4/11	0	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-9/22
φ_{DC}+	-4/11	0	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-57/22
φ_{DE}-	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-12/11
φ_{DE}+	4/11	-1	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-10/11
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	≥	-3/22
φ_{BF}-	-6/11	1	5/11	12/11	-12/11	-5/11	-6/11	5/11	≥	-7/11
φ_{BF}+	6/11	-1	-5/11	-12/11	12/11	5/11	6/11	-5/11	≥	-15/11
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	3/11	0	-5/22	-6/11	6/11	-17/22	3/11	17/22	≥	-13/11
φ_{GH}+	-3/11	0	5/22	6/11	-6/11	17/22	-3/11	-17/22	≥	-20/11
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-4/11	≥	-1/11
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	17/11	≥	-4/11
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	=	-3/22

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Scambio pivotale 9-4

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	0	≥	-6/5
φ_{BA}+	-2/5	1	3/5	-2/5	3/5	-3/5	-2/5	0	≥	-4/5
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	2/15	0	2/15	-11/30	19/30	-2/15	2/15	-1/2	≥	-33/20
φ_{CD}+	-2/15	0	-2/15	11/30	-19/30	2/15	-2/15	1/2	≥	-27/20
T	2/15	0	2/15	-11/30	19/30	-2/15	2/15	1/2	≥	-3/20
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	≥	-3/20
φ_{BF}-	-2/5	1	3/5	-2/5	-2/5	-3/5	-2/5	1	≥	-4/5
φ_{BF}+	2/5	-1	-3/5	2/5	2/5	3/5	2/5	-1	≥	-6/5
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/5	0	-3/10	1/5	1/5	-7/10	1/5	1/2	≥	-11/10
φ_{GH}+	-1/5	0	3/10	-1/5	-1/5	7/10	-1/5	-1/2	≥	-19/10
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	1	≥	-1/5
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	=	-3/20

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Scambio pivotale 18-6

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	3/5	≥	-9/5
φ_{BA}+	-2/5	1	3/5	-2/5	3/5	3/5	-2/5	-3/5	≥	-1/5
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	2/15	0	2/15	-11/30	19/30	2/15	2/15	-19/30	≥	-91/60
φ_{CD}+	-2/15	0	-2/15	11/30	-19/30	-2/15	-2/15	19/30	≥	-89/60
T	2/15	0	2/15	-11/30	19/30	2/15	2/15	11/30	≥	-1/60
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	≥	-11/60
φ_{BF}-	-2/5	1	3/5	-2/5	-2/5	3/5	-2/5	2/5	≥	-1/5
φ_{BF}+	2/5	-1	-3/5	2/5	2/5	-3/5	2/5	-2/5	≥	-9/5
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/5	0	-3/10	1/5	1/5	7/10	1/5	-1/5	≥	-2/5
φ_{GH}+	-1/5	0	3/10	-1/5	-1/5	-7/10	-1/5	1/5	≥	-13/5
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	8/5	≥	-4/5
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	=	-11/60

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Scambio pivotale 4-8

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-4/3
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-1/3
φ_{BC}-	2/3	-5/3	-1	2/3	0	-1	2/3	5/3	≥	-7/6
φ_{BC}+	-2/3	5/3	1	-2/3	0	1	-2/3	-5/3	≥	-11/6
φ_{CD}-	5/9	-19/18	-1/2	1/18	0	-1/2	5/9	19/18	≥	-47/36
φ_{CD}+	-5/9	19/18	1/2	-1/18	0	1/2	-5/9	-19/18	≥	-61/36
T	-1/9	11/18	1/2	-11/18	1	1/2	-1/9	-11/18	≥	-5/36
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	≥	-7/36
φ_{BF}-	-2/3	5/3	1	-2/3	0	1	-2/3	-2/3	≥	-1/3
φ_{BF}+	2/3	-5/3	-1	2/3	0	-1	2/3	2/3	≥	-5/3
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	5/3	1	-2/3	1	0	-2/3	-5/3	≥	-4/3
φ_{GH}-	1/3	-1/3	-1/2	1/3	0	1/2	1/3	1/3	≥	-1/3
φ_{GH}+	-1/3	1/3	1/2	-1/3	0	-1/2	-1/3	-1/3	≥	-8/3
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	8/3	1	-5/3	1	1	-2/3	-5/3	≥	-5/6
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	-2/3	5/3	1	-2/3	1	1	-2/3	-8/3	≥	-4/3
L_X	2/3	-5/3	-1	2/3	-1	-1	2/3	5/3	≥	-7/6
Max	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	=	-7/36

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Scambio pivotale 12-2

	φ_{AB}+	φ_{DE}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
φ_{BC}-	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
φ_{BC}+	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
φ_{CD}-	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
φ_{CD}+	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
T	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	-1	0	0	0	0	0	0	≥	-2
φ_{DH}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
φ_{BF}-	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
φ_{BF}+	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
φ_{GH}-	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
φ_{GH}+	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{DH}-	0	1	0	-1	0	0	0	0	≥	-3/2
Y	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_X	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

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Tableau finale

	φ_{AB}+	φ_{DE}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
φ_{BC}-	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
φ_{BC}+	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
φ_{CD}-	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
φ_{CD}+	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
T	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	-1	0	0	0	0	0	0	≥	-2
φ_{DH}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
φ_{BF}-	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
φ_{BF}+	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
φ_{GH}-	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
φ_{GH}+	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{DH}-	0	1	0	-1	0	0	0	0	≥	-3/2
Y	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_X	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BC}-	0	0	0	0	0	0	0	0	≥	0
φ_{BC}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	0
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	0
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{DE}-	0	0	0	0	0	0	0	0	≥	0
φ_{DE}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{ED}-	0	0	0	0	0	0	0	0	≥	0
φ_{ED}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BF}-	0	0	0	0	0	0	0	0	≥	0
φ_{BF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FB}-	0	0	0	0	0	0	0	0	≥	0
φ_{FB}+	0	0	0	0	0	0	0	0	≥	0
φ_{GH}-	0	0	0	0	0	0	0	0	≥	0
φ_{GH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HD}-	0	0	0	0	0	0	0	0	≥	0
φ_{HD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	13/6	13/6	0	4/9	13/6	13/6	2/9	7/6	=	-2/9

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
1	1	-7/6	-13/18	1	1

Variabili soluzione differenza tra rotazioni

$$\begin{matrix}\varphi_{AB}\\ \varphi_{BA}\\ \varphi_{BC}\\ \varphi_{CD}\\ \varphi_{DC}\\ \varphi_{DE}\\ \varphi_{ED}\\ \varphi_{BF}\\ \varphi_{FB}\\ \varphi_{GH}\\ \varphi_{HD}\\ \varphi_{DH}\end{matrix}\begin{bmatrix}1/18\\ 1/18\\ 0\\ 0\\ 0\\ 1/18\\ 1/18\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

REAZIONI Fattore di collasso = 2/9

$H_A = -2/3F$
$V_A = -1/18F$
$W_A = Fb$
$H_E = -2/3F$
$V_E = 41/18F$
$W_E = Fb$

$H_{AB} = -2/3F$	$H_{BC} = 1/6F$	$H_{CD} = 1/6F$	$H_{DE} = 2/3F$	$H_{BF} = -7/18F$
$V_{AB} = -1/18F$	$V_{BC} = -2/9F$	$V_{CD} = -10/9F$	$V_{DE} = -41/18F$	$V_{BF} = 1/6F$
$W_{AB} = Fb$	$W_{BC} = -7/6Fb$	$W_{CD} = -13/18Fb$	$W_{DE} = Fb$	$W_{BF} = 1/6Fb$
$H_{BA} = 2/3F$	$H_{CB} = -1/6F$	$H_{DC} = -1/6F$	$H_{ED} = -2/3F$	$H_{FB} = 7/18F$
$V_{BA} = 1/18F$	$V_{CB} = 2/9F$	$V_{DC} = 10/9F$	$V_{ED} = 41/18F$	$V_{FB} = -1/6F$
$W_{BA} = Fb$	$W_{CB} = 13/18Fb$	$W_{DC} = -3/2Fb$	$W_{ED} = Fb$	$W_{FB} = Fb$

$H_{FG} = 1/2F$	$H_{GH} = 1/2F$	$H_{HD} = 1/2F$
$V_{FG} = 1/6F$	$V_{GH} = -7/6F$	$V_{HD} = -7/6F$
$W_{FG} = -Fb$	$W_{GH} = -4/3Fb$	$W_{HD} = Fb$
$H_{GF} = -1/2F$	$H_{HG} = -1/2F$	$H_{DH} = -1/2F$
$V_{GF} = -1/6F$	$V_{HG} = 7/6F$	$V_{DH} = 7/6F$
$W_{GF} = 4/3Fb$	$W_{HG} = -Fb$	$W_{DH} = 1/2Fb$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 1/6\delta$	$u_C = 1/6\delta$	$u_D = 1/6\delta$	$u_{EED} = 0$
$v_{AAB} = 0$	$v_B = 0$	$v_C = 0$	$v_D = 0$	$v_{EED} = 0$
$\varphi_{AAB} = -1/18\delta/b$	$\varphi_B = -1/18\delta/b$	$\varphi_C = 0$	$\varphi_D = 0$	$\varphi_E = -1/18\delta/b$

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$u_F = 1/6\delta$	$u_G = 1/6\delta$	$u_H = 1/6\delta$
$v_F = 0$	$v_G = 0$	$v_H = 0$
$\varphi_F = 0$	$\varphi_G = 0$	$\varphi_H = 0$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{BBC} = 1/6\delta$	$u_{CCD} = 1/6\delta$	$u_{DDE} = 1/6\delta$	$u_{BBF} = 1/6\delta$
$v_{AAB} = 0$	$v_{BBC} = 0$	$v_{CCD} = 0$	$v_{DDE} = 0$	$v_{BBF} = 0$
$\varphi_{AAB} = -1/18\delta/b$	$\varphi_{BBC} = 0$	$\varphi_{CCD} = 0$	$\varphi_{DDE} = -1/18\delta/b$	$\varphi_{BBF} = 0$

$u_{FFG} = 1/6\delta$	$u_{GGH} = 1/6\delta$	$u_{HHD} = 1/6\delta$
$v_{FFG} = 0$	$v_{GGH} = 0$	$v_{HHD} = 0$
$\varphi_{FFG} = 0$	$\varphi_{GGH} = 0$	$\varphi_{HHD} = 0$

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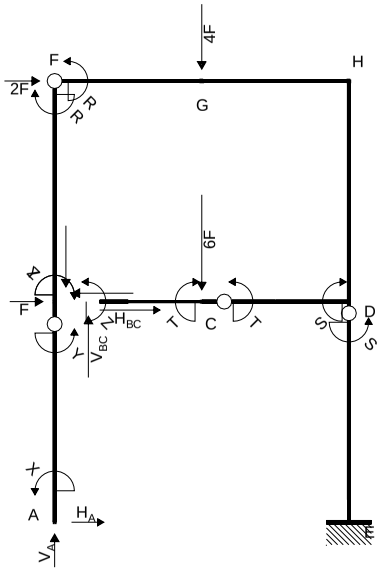


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

$3H_Ab - 4V_Ab = -Xb + Sb - 14Fb$

Rotazione intorno a C: aste CB

$-2V_{BC}b = -Zb + Tb$

Rotazione intorno a F: aste FB BA

$6H_Ab - 3H_{BC}b = -Xb + Zb + Rb - 3Fb$

Rotazione intorno a B: aste BA

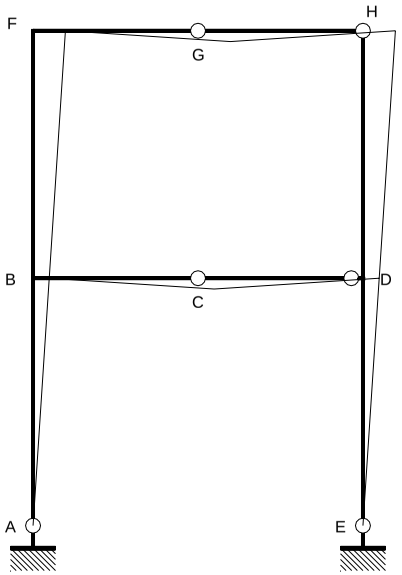
$3H_Ab = -Xb - Yb$

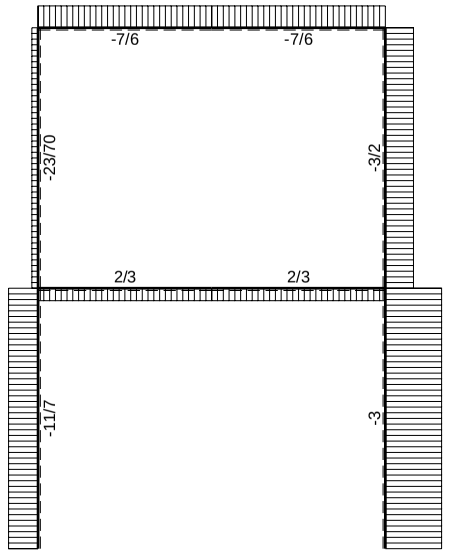
Matrice di equilibrio

$$\begin{matrix} & [H_Ab & V_Ab & H_{BC}b & V_{BC}b] \\ \varphi_{DE} & 3 & -4 & 0 & 0 \\ \varphi_{CD} & 0 & 0 & 0 & -2 \\ \varphi_{FG} & 6 & 0 & -3 & 0 \\ \varphi_{BA} & 3 & 0 & 0 & 0 \end{matrix} = \begin{matrix} [Xb & Yb & Zb & Tb & Sb & Rb & Fb] \\ -1 & 0 & 0 & 0 & 1 & 0 & -14 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & -3 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

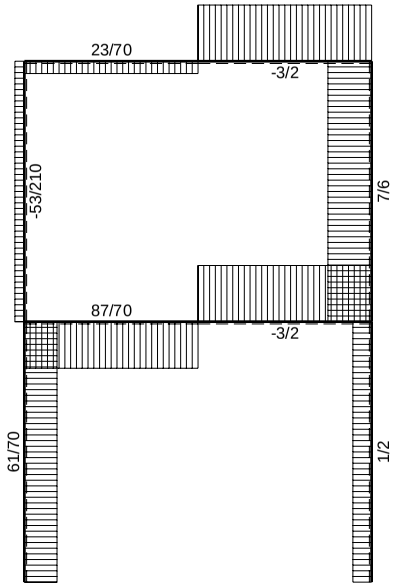
Soluzione del sistema

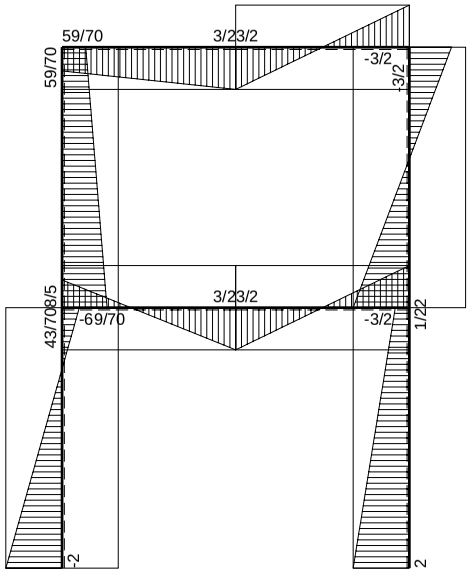
$$\begin{matrix} \\ H_Ab \\ V_{BC}b \\ H_{BC}b \\ V_Ab \end{matrix} = \begin{matrix} [Xb & Yb & Zb & Tb & Sb & Rb & Fb] \\ -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ -1/3 & -2/3 & -1/3 & 0 & 0 & -1/3 & 1 \\ 0 & -1/4 & 0 & 0 & -1/4 & 0 & 7/2 \end{matrix}$$





F





F Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	≥	H_{14}
D_2	H_{21}	H_{22}	H_{23}	≥	H_{24}
D_3	H_{31}	H_{32}	H_{33}	≥	H_{34}
D_4	H_{41}	H_{42}	H_{43}	≥	H_{44}
D_5	H_{51}	H_{52}	H_{53}	≥	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-2
W_{AB}+	1	0	0	0	0	0	0	≤	2
W_{BA}-	0	1	0	0	0	0	0	≥	-2
W_{BA}+	0	1	0	0	0	0	0	≤	2
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	1	0	0	0	0	≤	3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	1	0	0	0	≤	3/2
W_{DC}-	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC}+	0	0	1	-2	0	0	-12	≤	3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-2
W_{DE}+	0	0	0	0	1	0	0	≤	2
W_{ED}-	-1	-1	0	0	-1	0	9	≥	-2
W_{ED}+	-1	-1	0	0	-1	0	9	≤	2
W_{BF}-	0	-1	-1	0	0	0	0	≥	-2
W_{BF}+	0	-1	-1	0	0	0	0	≤	2
W_{FG}-	0	0	0	0	0	1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	1	0	≤	3/2
W_{GH}-	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH}+	0	1/2	1	-1	1/2	1	-7	≤	3/2
W_{HG}-	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG}+	0	-1	-2	2	-1	-1	6	≤	3/2
W_{DH}-	0	0	-1	2	-1	0	12	≥	-2
W_{DH}+	0	0	-1	2	-1	0	12	≤	2
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-2
W_{AB}+	-1	0	0	0	0	0	0	≥	-2
W_{BA}-	0	1	0	0	0	0	0	≥	-2
W_{BA}+	0	-1	0	0	0	0	0	≥	-2
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	≥	-3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	≥	-3/2
W_{DC}-	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	12	≥	-3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-2
W_{DE}+	0	0	0	0	-1	0	0	≥	-2
W_{ED}-	-1	-1	0	0	-1	0	9	≥	-2
W_{ED}+	1	1	0	0	1	0	-9	≥	-2
W_{BF}-	0	-1	-1	0	0	0	0	≥	-2
W_{BF}+	0	1	1	0	0	0	0	≥	-2
W_{FG}-	0	0	0	0	0	1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	-1	0	≥	-3/2
W_{GH}-	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	-1	7	≥	-3/2
W_{HG}-	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG}+	0	1	2	-2	1	1	-6	≥	-3/2
W_{DH}-	0	0	-1	2	-1	0	12	≥	-2
W_{DH}+	0	0	1	-2	1	0	-12	≥	-2
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-2
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-2
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-2
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-2
W_{BC}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC}-	0	0	1	-2	0	0	0	0	-1	2	0	0	-12	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	0	0	1	-2	0	0	12	≤	-3/2
W_{DE}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-2
W_{DE}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-2
W_{ED}-	-1	-1	0	0	-1	0	1	1	0	0	1	0	9	≥	-2
W_{ED}+	1	1	0	0	1	0	-1	-1	0	0	-1	0	-9	≤	-2
W_{BF}-	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-2
W_{BF}+	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-2
W_{FG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-3/2
W_{GH}-	0	1/2	1	-1	1/2	1	0	-1/2	-1	1	-1/2	-1	-7	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	-1	0	1/2	1	-1	1/2	1	7	≤	-3/2
W_{HG}-	0	-1	-2	2	-1	-1	0	1	2	-2	1	1	6	≥	-3/2
W_{HG}+	0	1	2	-2	1	1	0	-1	-2	2	-1	-1	-6	≤	-3/2
W_{DH}-	0	0	-1	2	-1	0	0	0	1	-2	1	0	12	≥	-2
W_{DH}+	0	0	1	-2	1	0	0	0	-1	2	-1	0	-12	≤	-2
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	1	-2	0	0	-12	1	≥	-3/2
φ_{DC}+	0	0	-1	2	0	0	12	-1	≥	-3/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	-1	-1	0	0	-1	0	9	3	≥	-2
φ_{ED}+	1	1	0	0	1	0	-9	-3	≥	-2
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-2
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	0	1/2	1	-1	1/2	1	-7	-2	≥	-3/2
φ_{GH}+	0	-1/2	-1	1	-1/2	-1	7	2	≥	-3/2
φ_{HG}-	0	-1	-2	2	-1	-1	6	3	≥	-3/2
φ_{HG}+	0	1	2	-2	1	1	-6	-3	≥	-3/2
φ_{DH}-	0	0	-1	2	-1	0	12	0	≥	-2
φ_{DH}+	0	0	1	-2	1	0	-12	0	≥	-2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 9-7

	X	Y	Z	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	0	0	1/12	-1/6	0	0	-1/12	1/12	≥	-1/8
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	-1	-1	3/4	-3/2	-1	0	-3/4	15/4	≥	-25/8
φ_{ED}+	1	1	-3/4	3/2	1	0	3/4	-15/4	≥	-7/8
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-2
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	0	1/2	5/12	1/6	1/2	1	7/12	-31/12	≥	-5/8
φ_{GH}+	0	-1/2	-5/12	-1/6	-1/2	-1	-7/12	31/12	≥	-19/8
φ_{HG}-	0	-1	-3/2	1	-1	-1	-1/2	7/2	≥	-9/4
φ_{HG}+	0	1	3/2	-1	1	1	1/2	-7/2	≥	-3/4
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	1/12	-1/6	0	0	-1/12	1/12	=	-1/8

Scambio pivotale 14-3

	X	Y	φ_{ED}+	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	4/3	4/3	-4/3	2	4/3	0	1	-6	≥	-8/3
φ_{BC}+	-4/3	-4/3	4/3	-2	-4/3	0	-1	6	≥	-1/3
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	1/9	1/9	-1/9	0	1/9	0	0	-1/3	≥	-2/9
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	4/3	4/3	-4/3	2	4/3	0	1	-5	≥	-7/6
φ_{BF}-	-4/3	-7/3	4/3	-2	-4/3	0	-1	7	≥	-5/6
φ_{BF}+	4/3	7/3	-4/3	2	4/3	0	1	-7	≥	-19/6
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	5/9	19/18	-5/9	1	19/18	1	1	-14/3	≥	-10/9
φ_{GH}+	-5/9	-19/18	5/9	-1	-19/18	-1	-1	14/3	≥	-17/9
φ_{HG}-	-2	-3	2	-2	-3	-1	-2	11	≥	-1/2
φ_{HG}+	2	3	-2	2	3	1	2	-11	≥	-5/2
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	1/9	1/9	-1/9	0	1/9	0	0	-1/3	=	-2/9

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Scambio pivotale 6-1

	φ_{BC}+	Y	φ_{ED}+	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	-3/4	-1	1	-3/2	-1	0	-3/4	7/2	≥	-9/4
φ_{AB}+	3/4	1	-1	3/2	1	0	3/4	-7/2	≥	-7/4
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	-3/4	-1	1	-3/2	-1	0	-3/4	9/2	≥	-1/4
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	-1/12	0	0	-1/6	0	0	-1/12	1/6	≥	-1/4
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	0	0	0	0	0	0	1	≥	-3/2
φ_{BF}-	1	-1	0	0	0	0	0	1	≥	-1/2
φ_{BF}+	-1	1	0	0	0	0	0	-1	≥	-7/2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	-5/12	1/2	0	1/6	1/2	1	7/12	-13/6	≥	-5/4
φ_{GH}+	5/12	-1/2	0	-1/6	-1/2	-1	-7/12	13/6	≥	-7/4
φ_{HG}-	3/2	-1	0	1	-1	-1	-1/2	2	≥	0
φ_{HG}+	-3/2	1	0	-1	1	1	1/2	-2	≥	-3
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	-1/12	0	0	-1/6	0	0	-1/12	1/6	=	-1/4

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Scambio pivotale 2-8

	φ_{BC}+	Y	φ_{ED}+	T	S	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	3/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1/2
φ_{BA}-	-3/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
φ_{BA}+	3/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-5/2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	3/14	2/7	-2/7	3/7	2/7	0	3/14	-9/7	≥	-5/2
φ_{CD}-	-3/14	-2/7	2/7	4/7	-2/7	0	-3/14	2/7	≥	-1
φ_{CD}+	3/14	2/7	-2/7	-4/7	2/7	0	3/14	-2/7	≥	-2
αbF	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	≥	-1/3
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	-3/14	-2/7	2/7	-3/7	5/7	0	-3/14	2/7	≥	-3/2
φ_{DE}+	3/14	2/7	-2/7	3/7	-5/7	0	3/14	-2/7	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-2
φ_{BF}-	17/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1
φ_{BF}+	-17/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3
φ_{FG}-	-3/14	-2/7	2/7	-3/7	-2/7	1	-3/14	2/7	≥	-1
φ_{FG}+	3/14	2/7	-2/7	3/7	2/7	-1	3/14	-2/7	≥	-2
φ_{GH}-	-37/42	-5/42	13/21	-16/21	-5/42	1	5/42	13/21	≥	-1/6
φ_{GH}+	37/42	5/42	-13/21	16/21	5/42	-1	-5/42	-13/21	≥	-17/6
φ_{HG}-	27/14	-3/7	-4/7	13/7	-3/7	-1	-1/14	-4/7	≥	-1
φ_{HG}+	-27/14	3/7	4/7	-13/7	3/7	1	1/14	4/7	≥	-2
φ_{DH}-	3/14	2/7	-2/7	3/7	-5/7	0	-11/14	-2/7	≥	-4
φ_{DH}+	-3/14	-2/7	2/7	-3/7	5/7	0	11/14	2/7	≥	0
L_X	-3/14	-2/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
Max	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	=	-1/3

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Scambio pivotale 24-2

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	S	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	0	-1	0	0	1	0	1	0	≥	-1/2
φ_{BA}-	-3/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3/2
φ_{BA}+	3/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-5/2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	0	-1	0	0	1	0	1	-1	≥	-5/2
φ_{CD}-	0	1	0	1	-1	0	-1	0	≥	-1
φ_{CD}+	0	-1	0	-1	1	0	1	0	≥	-2
αbF	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	≥	-1/3
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	-1	0	0	1	0	1	0	≥	-2
φ_{BF}-	7/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-1
φ_{BF}+	-7/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3
φ_{FG}-	0	1	0	0	-1	1	-1	0	≥	-1
φ_{FG}+	0	-1	0	0	1	-1	1	0	≥	-2
φ_{GH}-	-19/24	5/12	1/2	-7/12	-5/12	1	-5/24	1/2	≥	-1/6
φ_{GH}+	19/24	-5/12	-1/2	7/12	5/12	-1	5/24	-1/2	≥	-17/6
φ_{HG}-	9/4	3/2	-1	5/2	-3/2	-1	-5/4	-1	≥	-1
φ_{HG}+	-9/4	-3/2	1	-5/2	3/2	1	5/4	1	≥	-2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	-3/4	-7/2	1	-3/2	5/2	0	11/4	1	≥	0
L_X	0	1	0	0	-1	0	-1	0	≥	-3/2
Max	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	=	-1/3

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Scambio pivotale 19-5

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	φ_{GH}-	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-9/10
φ_{BA}-	-18/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-21/10
φ_{BA}+	18/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-19/10
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	1/5	≥	-29/10
φ_{CD}-	19/10	0	-6/5	12/5	12/5	-12/5	-1/2	-6/5	≥	-3/5
φ_{CD}+	-19/10	0	6/5	-12/5	-12/5	12/5	1/2	6/5	≥	-12/5
αbF	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	≥	-2/5
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-29/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-12/5
φ_{BF}-	23/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-2/5
φ_{BF}+	-23/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-18/5
φ_{FG}-	19/10	0	-6/5	7/5	12/5	-7/5	-1/2	-6/5	≥	-3/5
φ_{FG}+	-19/10	0	6/5	-7/5	-12/5	7/5	1/2	6/5	≥	-12/5
S	-19/10	1	6/5	-7/5	-12/5	12/5	-1/2	6/5	≥	-2/5
φ_{GH}+	0	0	0	0	-1	0	0	0	≥	-3
φ_{HG}-	51/10	0	-14/5	23/5	18/5	-23/5	-1/2	-14/5	≥	-2/5
φ_{HG}+	-51/10	0	14/5	-23/5	-18/5	23/5	1/2	14/5	≥	-13/5
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	-11/2	-1	4	-5	-6	6	3/2	4	≥	-1
L_X	19/10	0	-6/5	7/5	12/5	-12/5	-1/2	-6/5	≥	-11/10
Max	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	=	-2/5

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Scambio pivotale 21-6

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	φ_{GH}-	φ_{HG}-	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	35/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-51/46
φ_{BA}-	9/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-111/46
φ_{BA}+	-9/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-73/46
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	35/46	0	-6/23	1	-12/23	-12/23	11/46	-29/23	≥	-143/46
φ_{CD}-	-35/46	0	6/23	0	12/23	12/23	-11/46	6/23	≥	-9/23
φ_{CD}+	35/46	0	-6/23	0	-12/23	-12/23	11/46	-6/23	≥	-60/23
αbF	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	≥	-10/23
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-60/23
φ_{BF}-	14/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-2/23
φ_{BF}+	-14/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-90/23
φ_{FG}-	8/23	0	-8/23	0	30/23	7/23	-8/23	-8/23	≥	-11/23
φ_{FG}+	-8/23	0	8/23	0	-30/23	-7/23	8/23	8/23	≥	-58/23
S	35/46	1	-6/23	1	-12/23	-12/23	-35/46	-6/23	≥	-14/23
φ_{GH}+	0	0	0	0	-1	0	0	0	≥	-3
R	51/46	0	-14/23	1	18/23	-5/23	-5/46	-14/23	≥	-2/23
φ_{HG}+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	53/46	-1	8/23	1	-30/23	-30/23	39/46	8/23	≥	-35/23
L_X	-35/46	0	6/23	-1	12/23	12/23	-11/46	6/23	≥	-41/46
Max	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	=	-10/23

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Scambio pivotale 7-1

	φ_{CD}^-	φ_{DH}^+	φ_{ED}^+	T	φ_{GH}^-	φ_{HG}^-	φ_{DC}^-	φ_{AB}^+		Fb
φ_{AB}^-	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
φ_{BA}^-	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
φ_{BA}^+	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
φ_{BC}^-	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
X	-1	0	0	1	0	0	0	-1	≥	-7/2
φ_{BC}^+	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
φ_{CD}^+	-1	0	0	0	0	0	0	0	≥	-3
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
φ_{DC}^+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}^-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}^+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}^-	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
φ_{BF}^-	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
φ_{BF}^+	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
φ_{FG}^-	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{FG}^+	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
φ_{GH}^+	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{HG}^+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}^-	0	-1	0	0	0	0	0	0	≥	-4
Y	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_X	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35

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Tableau finale

	φ_{CD}^-	φ_{DH}^+	φ_{ED}^+	T	φ_{GH}^-	φ_{HG}^-	φ_{DC}^-	φ_{AB}^+		Fb
φ_{AB}^-	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
φ_{BA}^-	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
φ_{BA}^+	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
φ_{BC}^-	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
X	-1	0	0	1	0	0	0	-1	≥	-7/2
φ_{BC}^+	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
φ_{CD}^+	-1	0	0	0	0	0	0	0	≥	-3
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
φ_{DC}^+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}^-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}^+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}^-	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
φ_{BF}^-	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
φ_{BF}^+	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
φ_{FG}^-	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{FG}^+	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
φ_{GH}^+	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{HG}^+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}^-	0	-1	0	0	0	0	0	0	≥	-4
Y	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_X	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/35
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	0
φ_{BC}-	0	0	0	0	0	0	0	0	≥	0
φ_{BC}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{DE}-	0	0	0	0	0	0	0	0	≥	0
φ_{DE}+	0	0	0	0	0	0	0	0	≥	0
φ_{ED}-	0	0	0	0	0	0	0	0	≥	0
φ_{ED}+	0	0	0	0	0	0	0	0	≥	1/35
φ_{BF}-	0	0	0	0	0	0	0	0	≥	0
φ_{BF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FG}-	0	0	0	0	0	0	0	0	≥	0
φ_{FG}+	0	0	0	0	0	0	0	0	≥	0
φ_{GH}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{GH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HG}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{HG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	7/2	74/35	87/35	0	1	23/35	16/35	3/2	=	-16/35

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
2	43/70	69/70	-3/2	-1/2	-59/70

Variabili soluzione differenza tra rotazioni

φ_{AB}	1/35
φ_{BA}	0
φ_{BC}	0
φ_{CD}	-2/35
φ_{DC}	-2/35
φ_{DE}	0
φ_{ED}	1/35
φ_{BF}	0
φ_{FG}	0
φ_{GH}	-2/35
φ_{HG}	-2/35
φ_{DH}	0

REAZIONI Fattore di collasso = 16/35

H_A = -61/70F
V_A = 11/7F
W_A = 2Fb
H_E = -1/2F
V_E = 3F
W_E = 2Fb

H_{AB} = -61/70F	H_{BC} = -2/3F	H_{CD} = -2/3F	H_{DE} = 1/2F	H_{BF} = 53/210F
V_{AB} = 11/7F	V_{BC} = 87/70F	V_{CD} = -3/2F	V_{DE} = -3F	V_{BF} = 23/70F
W_{AB} = 2Fb	W_{BC} = 69/70Fb	W_{CD} = -3/2Fb	W_{DE} = -1/2Fb	W_{BF} = -8/5Fb
H_{BA} = 61/70F	H_{CB} = 2/3F	H_{DC} = 2/3F	H_{ED} = -1/2F	H_{FB} = -53/210F
V_{BA} = -11/7F	V_{CB} = -87/70F	V_{DC} = 3/2F	V_{ED} = 3F	V_{FB} = -23/70F
W_{BA} = 43/70Fb	W_{CB} = 3/2Fb	W_{DC} = -3/2Fb	W_{ED} = 2Fb	W_{FB} = 59/70Fb

H_{FG} = 7/6F	H_{GH} = 7/6F	H_{HD} = 7/6F
V_{FG} = 23/70F	V_{GH} = -3/2F	V_{HD} = -3/2F
W_{FG} = -59/70Fb	W_{GH} = -3/2Fb	W_{HD} = 3/2Fb
H_{GF} = -7/6F	H_{HG} = -7/6F	H_{DH} = -7/6F
V_{GF} = -23/70F	V_{HG} = 3/2F	V_{DH} = 3/2F
W_{GF} = 3/2Fb	W_{HG} = -3/2Fb	W_{DH} = 2Fb

SPOSTAMENTI NODALI

u_{AAB} = 0	u_B = 3/35δ	u_{CCB} = 3/35δ	u_D = 3/35δ	u_{EED} = 0
v_{AAB} = 0	v_B = 0	v_{CCB} = -2/35δ	v_D = 0	v_{EED} = 0
φ_{AAB} = -1/35δ/b	φ_B = -1/35δ/b	φ_{CCB} = -1/35δ/b	φ_D = 1/35δ/b	φ_E = -1/35δ/b

$u_F = 6/35\delta$ $u_{GGF} = 6/35\delta$ $u_{HHG} = 6/35\delta$
$v_F = 0$ $v_{GGF} = -2/35\delta$ $v_{HHG} = 0$
$\varphi_F = -1/35\delta/b$ $\varphi_{GGF} = -1/35\delta/b$ $\varphi_{HHD} = 1/35\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$ $u_{BBC} = 3/35\delta$ $u_{CCD} = 3/35\delta$ $u_{DDE} = 3/35\delta$ $u_{BBF} = 3/35\delta$
$v_{AAB} = 0$ $v_{BBC} = 0$ $v_{CCD} = -2/35\delta$ $v_{DDE} = 0$ $v_{BBF} = 0$
$\varphi_{AAB} = -1/35\delta/b$ $\varphi_{BBC} = -1/35\delta/b$ $\varphi_{CCD} = 1/35\delta/b$ $\varphi_{DDE} = -1/35\delta/b$ $\varphi_{BBF} = -1/35\delta/b$

$u_{FFG} = 6/35\delta$ $u_{GGH} = 6/35\delta$ $u_{HHD} = 6/35\delta$
$v_{FFG} = 0$ $v_{GGH} = -2/35\delta$ $v_{HHD} = 0$
$\varphi_{FFG} = -1/35\delta/b$ $\varphi_{GGH} = 1/35\delta/b$ $\varphi_{HHD} = -1/35\delta/b$

AL4.002

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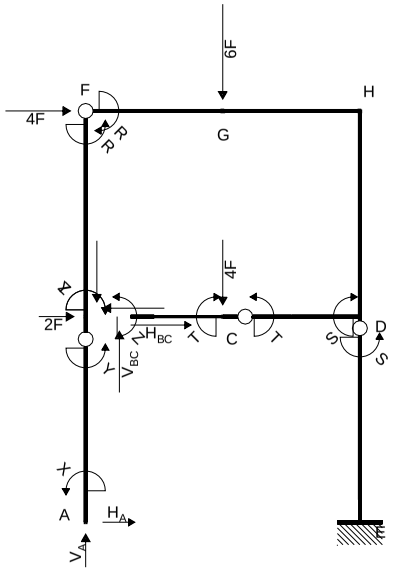


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

$3H_Ab - 4V_Ab = -Xb + Sb - 8Fb$

Rotazione intorno a C: aste CB

$-2V_{BC}b = -Zb + Tb$

Rotazione intorno a F: aste FB BA

$6H_Ab - 3H_{BC}b = -Xb + Zb - Rb - 6Fb$

Rotazione intorno a B: aste BA

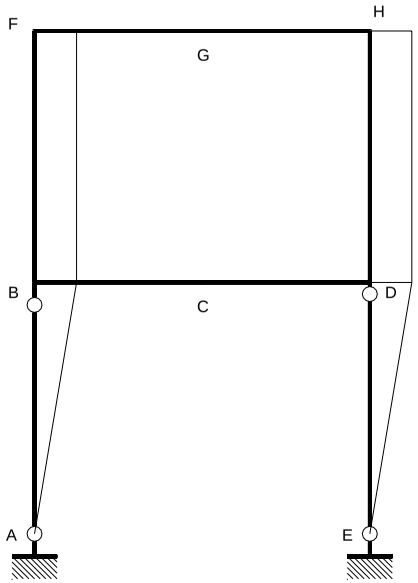
$3H_Ab = -Xb - Yb$

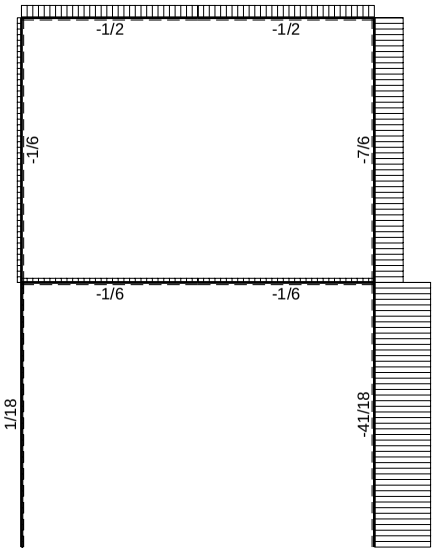
Matrice di equilibrio

$$\begin{array}{c} \\ \varphi_{DE} \\ \varphi_{CD} \\ \varphi_{FB} \\ \varphi_{BA} \end{array} \begin{bmatrix} H_Ab & V_Ab & H_{BC}b & V_{BC}b \\ 3 & -4 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 6 & 0 & -3 & 0 \\ 3 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1 & 0 & 0 & 0 & 1 & 0 & -8 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & -1 & -6 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

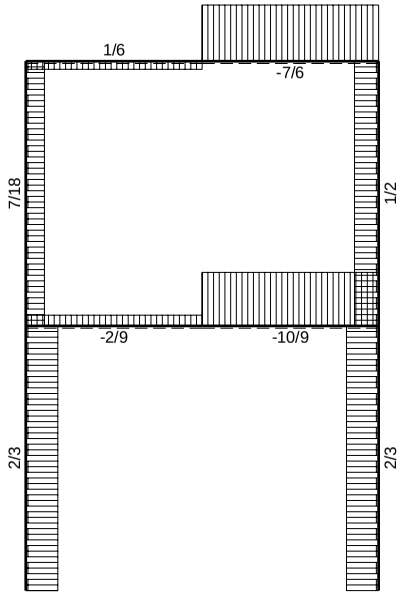
Soluzione del sistema

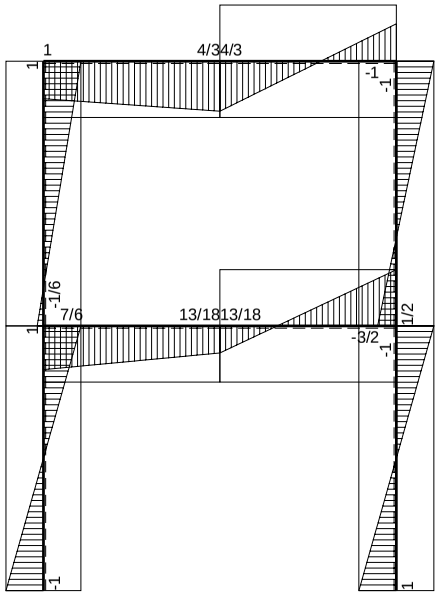
$$\begin{bmatrix} H_Ab \\ V_{BC}b \\ H_{BC}b \\ V_Ab \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ -1/3 & -2/3 & -1/3 & 0 & 0 & 1/3 & 2 \\ 0 & -1/4 & 0 & 0 & -1/4 & 0 & 2 \end{bmatrix}$$





F





F Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq} = 1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	≥	H_{14}
D_2	H_{21}	H_{22}	H_{23}	≥	H_{24}
D_3	H_{31}	H_{32}	H_{33}	≥	H_{34}
D_4	H_{41}	H_{42}	H_{43}	≥	H_{44}
D_5	H_{51}	H_{52}	H_{53}	≥	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FB}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	1	0	0	0	0	≤	3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	1	0	0	0	≤	3/2
W_{DC}-	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC}+	0	0	1	-2	0	0	-8	≤	3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-1
W_{DE}+	0	0	0	0	1	0	0	≤	1
W_{ED}-	-1	-1	0	0	-1	0	18	≥	-1
W_{ED}+	-1	-1	0	0	-1	0	18	≤	1
W_{BF}-	0	-1	-1	0	0	0	0	≥	-1
W_{BF}+	0	-1	-1	0	0	0	0	≤	1
W_{FB}-	0	0	0	0	0	1	0	≥	-1
W_{FB}+	0	0	0	0	0	1	0	≤	1
W_{GH}-	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH}+	0	1/2	1	-1	1/2	-1	-4	≤	3/2
W_{HD}-	0	1	2	-2	1	-1	4	≥	-1
W_{HD}+	0	1	2	-2	1	-1	4	≤	1
W_{DH}-	0	0	-1	2	-1	0	8	≥	-1
W_{DH}+	0	0	-1	2	-1	0	8	≤	1
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	≥	-3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	≥	-3/2
W_{DC}-	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	8	≥	-3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-1
W_{DE}+	0	0	0	0	-1	0	0	≥	-1
W_{ED}-	-1	-1	0	0	-1	0	18	≥	-1
W_{ED}+	1	1	0	0	1	0	-18	≥	-1
W_{BF}-	0	-1	-1	0	0	0	0	≥	-1
W_{BF}+	0	1	1	0	0	0	0	≥	-1
W_{FB}-	0	0	0	0	0	1	0	≥	-1
W_{FB}+	0	0	0	0	0	-1	0	≥	-1
W_{GH}-	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	1	4	≥	-3/2
W_{HD}-	0	1	2	-2	1	-1	4	≥	-1
W_{HD}+	0	-1	-2	2	-1	1	-4	≥	-1
W_{DH}-	0	0	-1	2	-1	0	8	≥	-1
W_{DH}+	0	0	1	-2	1	0	-8	≥	-1
Max	0	0	0	0	0	0	1	=	0

PROCEDIMENTO E RISULTATI Nome: AL4.003

Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{BC}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC}-	0	0	1	-2	0	0	0	0	-1	2	0	0	-8	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	0	0	1	-2	0	0	8	≤	-3/2
W_{DE}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{DE}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{ED}-	-1	-1	0	0	-1	0	1	1	0	0	1	0	18	≥	-1
W_{ED}+	1	1	0	0	1	0	-1	-1	0	0	-1	0	-18	≤	-1
W_{BF}-	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-1
W_{BF}+	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-1
W_{FB}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-1
W_{FB}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-1
W_{GH}-	0	1/2	1	-1	1/2	-1	0	-1/2	-1	1	-1/2	1	-4	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	1	0	1/2	1	-1	1/2	-1	4	≤	-3/2
W_{HD}-	0	1	2	-2	1	-1	0	-1	-2	2	-1	1	4	≥	-1
W_{HD}+	0	-1	-2	2	-1	1	0	1	2	-2	1	-1	-4	≤	-1
W_{DH}-	0	0	-1	2	-1	0	0	0	1	-2	1	0	8	≥	-1
W_{DH}+	0	0	1	-2	1	0	0	0	-1	2	-1	0	-8	≤	-1
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	1	-2	0	0	-8	1	≥	-3/2
φ_{DC}+	0	0	-1	2	0	0	8	-1	≥	-3/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	-1	-1	0	0	-1	0	18	3	≥	-1
φ_{ED}+	1	1	0	0	1	0	-18	-3	≥	-1
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	0	1/2	1	-1	1/2	-1	-4	0	≥	-3/2
φ_{GH}+	0	-1/2	-1	1	-1/2	1	4	0	≥	-3/2
φ_{HD}-	0	1	2	-2	1	-1	4	-1	≥	-1
φ_{HD}+	0	-1	-2	2	-1	1	-4	1	≥	-1
φ_{DH}-	0	0	-1	2	-1	0	8	0	≥	-1
φ_{DH}+	0	0	1	-2	1	0	-8	0	≥	-1
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 14-7

	X	Y	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	-4/9	-4/9	1	-2	-4/9	0	4/9	7/3	≥	-19/18
φ_{DC}+	4/9	4/9	-1	2	4/9	0	-4/9	-7/3	≥	-35/18
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	1/18	1/18	0	0	1/18	0	-1/18	-1/6	≥	-1/18
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	-2/9	5/18	1	-1	5/18	-1	2/9	2/3	≥	-23/18
φ_{GH}+	2/9	-5/18	-1	1	-5/18	1	-2/9	-2/3	≥	-31/18
φ_{HD}-	2/9	11/9	2	-2	11/9	-1	-2/9	-5/3	≥	-11/9
φ_{HD}+	-2/9	-11/9	-2	2	-11/9	1	2/9	5/3	≥	-7/9
φ_{DH}-	4/9	4/9	-1	2	-5/9	0	-4/9	-4/3	≥	-13/9
φ_{DH}+	-4/9	-4/9	1	-2	5/9	0	4/9	4/3	≥	-5/9
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	1/18	1/18	0	0	1/18	0	-1/18	-1/6	=	-1/18

Scambio pivotale 2-1

	φ_{AB}+	Y	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	4/9	-4/9	1	-2	-4/9	0	4/9	17/9	≥	-11/18
φ_{DC}+	-4/9	4/9	-1	2	4/9	0	-4/9	-17/9	≥	-43/18
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	≥	-1/9
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	2/9	5/18	1	-1	5/18	-1	2/9	4/9	≥	-19/18
φ_{GH}+	-2/9	-5/18	-1	1	-5/18	1	-2/9	-4/9	≥	-35/18
φ_{HD}-	-2/9	11/9	2	-2	11/9	-1	-2/9	-13/9	≥	-13/9
φ_{HD}+	2/9	-11/9	-2	2	-11/9	1	2/9	13/9	≥	-5/9
φ_{DH}-	-4/9	4/9	-1	2	-5/9	0	-4/9	-8/9	≥	-17/9
φ_{DH}+	4/9	-4/9	1	-2	5/9	0	4/9	8/9	≥	-1/9
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	=	-1/9

Scambio pivotale 24-2

	φ_{AB}+	φ_{DH}+	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	1	-9/4	9/4	-9/2	5/4	0	1	1	≥	-5/4
φ_{BA}+	-1	9/4	-9/4	9/2	-5/4	0	-1	-1	≥	-3/4
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	1	0	0	-1	0	0	1	≥	-1/2
φ_{DC}+	0	-1	0	0	1	0	0	-1	≥	-5/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	-1/8	1/8	-1/4	1/8	0	0	0	≥	-1/8
φ_{BF}-	-1	9/4	-13/4	9/2	-5/4	0	-1	0	≥	-3/4
φ_{BF}+	1	-9/4	13/4	-9/2	5/4	0	1	0	≥	-5/4
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/2	-5/8	13/8	-9/4	5/8	-1	1/2	1	≥	-9/8
φ_{GH}+	-1/2	5/8	-13/8	9/4	-5/8	1	-1/2	-1	≥	-15/8
φ_{HD}-	1	-11/4	19/4	-15/2	11/4	-1	1	1	≥	-7/4
φ_{HD}+	-1	11/4	-19/4	15/2	-11/4	1	-1	-1	≥	-1/4
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	1	-9/4	9/4	-9/2	5/4	0	1	2	≥	-1/4
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	-1/8	1/8	-1/4	1/8	0	0	0	=	-1/8

Scambio pivotale 22-3

	φ_{AB}+	φ_{DH}+	φ_{HD}+	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	10/19	≥	-26/19
φ_{BA}+	-10/19	18/19	9/19	18/19	1/19	-9/19	-10/19	-10/19	≥	-12/19
φ_{BC}-	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-23/19	≥	-59/38
φ_{BC}+	4/19	-11/19	4/19	-30/19	11/19	-4/19	4/19	23/19	≥	-55/38
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	1	0	0	-1	0	0	1	≥	-1/2
φ_{DC}+	0	-1	0	0	1	0	0	-1	≥	-5/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	≥	-5/38
φ_{BF}-	-6/19	7/19	13/19	-12/19	12/19	-13/19	-6/19	13/19	≥	-11/19
φ_{BF}+	6/19	-7/19	-13/19	12/19	-12/19	13/19	6/19	-13/19	≥	-27/19
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	3/19	6/19	-13/38	6/19	-6/19	-25/38	3/19	25/38	≥	-23/19
φ_{GH}+	-3/19	-6/19	13/38	-6/19	6/19	25/38	-3/19	-25/38	≥	-34/19
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-4/19	≥	-1/19
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	29/19	≥	-7/19
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	=	-5/38

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Scambio pivotale 22-5

	φ_{AB}+	φ_{DH}+	φ_{HD}+	T	Z	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	6/11	≥	-15/11
φ_{BA}+	-6/11	1	5/11	12/11	-1/11	-5/11	-6/11	-6/11	≥	-7/11
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	4/11	0	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-9/22
φ_{DC}+	-4/11	0	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-57/22
φ_{DE}-	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-12/11
φ_{DE}+	4/11	-1	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-10/11
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	≥	-3/22
φ_{BF}-	-6/11	1	5/11	12/11	-12/11	-5/11	-6/11	5/11	≥	-7/11
φ_{BF}+	6/11	-1	-5/11	-12/11	12/11	5/11	6/11	-5/11	≥	-15/11
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	3/11	0	-5/22	-6/11	6/11	-17/22	3/11	17/22	≥	-13/11
φ_{GH}+	-3/11	0	5/22	6/11	-6/11	17/22	-3/11	-17/22	≥	-20/11
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-4/11	≥	-1/11
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	17/11	≥	-4/11
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	=	-3/22

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Scambio pivotale 9-4

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	0	≥	-6/5
φ_{BA}+	-2/5	1	3/5	-2/5	3/5	-3/5	-2/5	0	≥	-4/5
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	2/15	0	2/15	-11/30	19/30	-2/15	2/15	-1/2	≥	-33/20
φ_{CD}+	-2/15	0	-2/15	11/30	-19/30	2/15	-2/15	1/2	≥	-27/20
T	2/15	0	2/15	-11/30	19/30	-2/15	2/15	1/2	≥	-3/20
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	≥	-3/20
φ_{BF}-	-2/5	1	3/5	-2/5	-2/5	-3/5	-2/5	1	≥	-4/5
φ_{BF}+	2/5	-1	-3/5	2/5	2/5	3/5	2/5	-1	≥	-6/5
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/5	0	-3/10	1/5	1/5	-7/10	1/5	1/2	≥	-11/10
φ_{GH}+	-1/5	0	3/10	-1/5	-1/5	7/10	-1/5	-1/2	≥	-19/10
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	1	≥	-1/5
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	=	-3/20

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Scambio pivotale 18-6

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	3/5	≥	-9/5
φ_{BA}+	-2/5	1	3/5	-2/5	3/5	3/5	-2/5	-3/5	≥	-1/5
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	2/15	0	2/15	-11/30	19/30	2/15	2/15	-19/30	≥	-91/60
φ_{CD}+	-2/15	0	-2/15	11/30	-19/30	-2/15	-2/15	19/30	≥	-89/60
T	2/15	0	2/15	-11/30	19/30	2/15	2/15	11/30	≥	-1/60
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	≥	-11/60
φ_{BF}-	-2/5	1	3/5	-2/5	-2/5	3/5	-2/5	2/5	≥	-1/5
φ_{BF}+	2/5	-1	-3/5	2/5	2/5	-3/5	2/5	-2/5	≥	-9/5
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/5	0	-3/10	1/5	1/5	7/10	1/5	-1/5	≥	-2/5
φ_{GH}+	-1/5	0	3/10	-1/5	-1/5	-7/10	-1/5	1/5	≥	-13/5
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	8/5	≥	-4/5
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	=	-11/60

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Scambio pivotale 4-8

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-4/3
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-1/3
φ_{BC}-	2/3	-5/3	-1	2/3	0	-1	2/3	5/3	≥	-7/6
φ_{BC}+	-2/3	5/3	1	-2/3	0	1	-2/3	-5/3	≥	-11/6
φ_{CD}-	5/9	-19/18	-1/2	1/18	0	-1/2	5/9	19/18	≥	-47/36
φ_{CD}+	-5/9	19/18	1/2	-1/18	0	1/2	-5/9	-19/18	≥	-61/36
T	-1/9	11/18	1/2	-11/18	1	1/2	-1/9	-11/18	≥	-5/36
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	≥	-7/36
φ_{BF}-	-2/3	5/3	1	-2/3	0	1	-2/3	-2/3	≥	-1/3
φ_{BF}+	2/3	-5/3	-1	2/3	0	-1	2/3	2/3	≥	-5/3
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	5/3	1	-2/3	1	0	-2/3	-5/3	≥	-4/3
φ_{GH}-	1/3	-1/3	-1/2	1/3	0	1/2	1/3	1/3	≥	-1/3
φ_{GH}+	-1/3	1/3	1/2	-1/3	0	-1/2	-1/3	-1/3	≥	-8/3
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	8/3	1	-5/3	1	1	-2/3	-5/3	≥	-5/6
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	-2/3	5/3	1	-2/3	1	1	-2/3	-8/3	≥	-4/3
L_X	2/3	-5/3	-1	2/3	-1	-1	2/3	5/3	≥	-7/6
Max	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	=	-7/36

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Scambio pivotale 12-2

	φ_{AB}+	φ_{DE}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
φ_{BC}-	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
φ_{BC}+	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
φ_{CD}-	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
φ_{CD}+	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
T	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	-1	0	0	0	0	0	0	≥	-2
φ_{DH}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
φ_{BF}-	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
φ_{BF}+	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
φ_{GH}-	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
φ_{GH}+	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{DH}-	0	1	0	-1	0	0	0	0	≥	-3/2
Y	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_X	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

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Tableau finale

	φ_{AB}+	φ_{DE}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
φ_{BC}-	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
φ_{BC}+	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
φ_{CD}-	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
φ_{CD}+	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
T	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	-1	0	0	0	0	0	0	≥	-2
φ_{DH}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
φ_{BF}-	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
φ_{BF}+	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
φ_{GH}-	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
φ_{GH}+	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{DH}-	0	1	0	-1	0	0	0	0	≥	-3/2
Y	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_X	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BC}-	0	0	0	0	0	0	0	0	≥	0
φ_{BC}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	0
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	0
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{DE}-	0	0	0	0	0	0	0	0	≥	0
φ_{DE}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{ED}-	0	0	0	0	0	0	0	0	≥	0
φ_{ED}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BF}-	0	0	0	0	0	0	0	0	≥	0
φ_{BF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FB}-	0	0	0	0	0	0	0	0	≥	0
φ_{FB}+	0	0	0	0	0	0	0	0	≥	0
φ_{GH}-	0	0	0	0	0	0	0	0	≥	0
φ_{GH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HD}-	0	0	0	0	0	0	0	0	≥	0
φ_{HD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	13/6	13/6	0	4/9	13/6	13/6	2/9	7/6	=	-2/9

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
1	1	-7/6	-13/18	1	1

Variabili soluzione differenza tra rotazioni

$$\begin{matrix}\varphi_{AB}\\ \varphi_{BA}\\ \varphi_{BC}\\ \varphi_{CD}\\ \varphi_{DC}\\ \varphi_{DE}\\ \varphi_{ED}\\ \varphi_{BF}\\ \varphi_{FB}\\ \varphi_{GH}\\ \varphi_{HD}\\ \varphi_{DH}\end{matrix}\begin{bmatrix}1/18\\ 1/18\\ 0\\ 0\\ 0\\ 1/18\\ 1/18\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

REAZIONI Fattore di collasso = 2/9

$H_A = -2/3F$
$V_A = -1/18F$
$W_A = Fb$
$H_E = -2/3F$
$V_E = 41/18F$
$W_E = Fb$

$H_{AB} = -2/3F$	$H_{BC} = 1/6F$	$H_{CD} = 1/6F$	$H_{DE} = 2/3F$	$H_{BF} = -7/18F$
$V_{AB} = -1/18F$	$V_{BC} = -2/9F$	$V_{CD} = -10/9F$	$V_{DE} = -41/18F$	$V_{BF} = 1/6F$
$W_{AB} = Fb$	$W_{BC} = -7/6Fb$	$W_{CD} = -13/18Fb$	$W_{DE} = Fb$	$W_{BF} = 1/6Fb$
$H_{BA} = 2/3F$	$H_{CB} = -1/6F$	$H_{DC} = -1/6F$	$H_{ED} = -2/3F$	$H_{FB} = 7/18F$
$V_{BA} = 1/18F$	$V_{CB} = 2/9F$	$V_{DC} = 10/9F$	$V_{ED} = 41/18F$	$V_{FB} = -1/6F$
$W_{BA} = Fb$	$W_{CB} = 13/18Fb$	$W_{DC} = -3/2Fb$	$W_{ED} = Fb$	$W_{FB} = Fb$

$H_{FG} = 1/2F$	$H_{GH} = 1/2F$	$H_{HD} = 1/2F$
$V_{FG} = 1/6F$	$V_{GH} = -7/6F$	$V_{HD} = -7/6F$
$W_{FG} = -Fb$	$W_{GH} = -4/3Fb$	$W_{HD} = Fb$
$H_{GF} = -1/2F$	$H_{HG} = -1/2F$	$H_{DH} = -1/2F$
$V_{GF} = -1/6F$	$V_{HG} = 7/6F$	$V_{DH} = 7/6F$
$W_{GF} = 4/3Fb$	$W_{HG} = -Fb$	$W_{DH} = 1/2Fb$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 1/6\delta$	$u_C = 1/6\delta$	$u_D = 1/6\delta$	$u_{EED} = 0$
$v_{AAB} = 0$	$v_B = 0$	$v_C = 0$	$v_D = 0$	$v_{EED} = 0$
$\varphi_{AAB} = -1/18\delta/b$	$\varphi_B = -1/18\delta/b$	$\varphi_C = 0$	$\varphi_D = 0$	$\varphi_E = -1/18\delta/b$

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$u_F = 1/6\delta$	$u_G = 1/6\delta$	$u_H = 1/6\delta$
$v_F = 0$	$v_G = 0$	$v_H = 0$
$\varphi_F = 0$	$\varphi_G = 0$	$\varphi_H = 0$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{BBC} = 1/6\delta$	$u_{CCD} = 1/6\delta$	$u_{DDE} = 1/6\delta$	$u_{BBF} = 1/6\delta$
$v_{AAB} = 0$	$v_{BBC} = 0$	$v_{CCD} = 0$	$v_{DDE} = 0$	$v_{BBF} = 0$
$\varphi_{AAB} = -1/18\delta/b$	$\varphi_{BBC} = 0$	$\varphi_{CCD} = 0$	$\varphi_{DDE} = -1/18\delta/b$	$\varphi_{BBF} = 0$

$u_{FFG} = 1/6\delta$	$u_{GGH} = 1/6\delta$	$u_{HHD} = 1/6\delta$
$v_{FFG} = 0$	$v_{GGH} = 0$	$v_{HHD} = 0$
$\varphi_{FFG} = 0$	$\varphi_{GGH} = 0$	$\varphi_{HHD} = 0$

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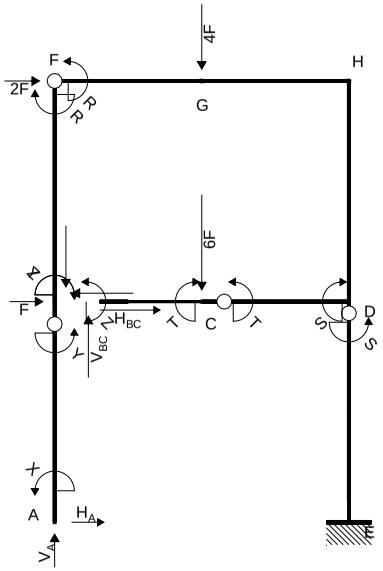


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

$3H_Ab - 4V_Ab = -Xb + Sb - 14Fb$

Rotazione intorno a C: aste CB

$-2V_{BC}b = -Zb + Tb$

Rotazione intorno a F: aste FB BA

$6H_Ab - 3H_{BC}b = -Xb + Zb + Rb - 3Fb$

Rotazione intorno a B: aste BA

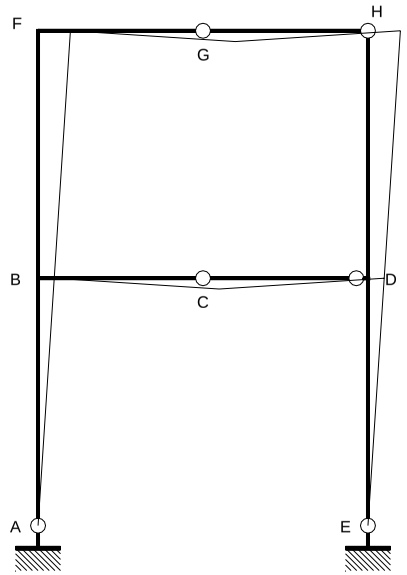
$3H_Ab = -Xb - Yb$

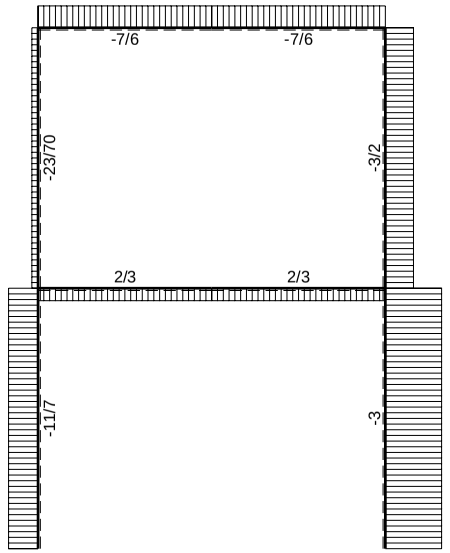
Matrice di equilibrio

$$\begin{matrix} & [H_Ab & V_Ab & H_{BC}b & V_{BC}b] \\ \varphi_{DE} & \begin{bmatrix} 3 & -4 & 0 & 0 \end{bmatrix} \\ \varphi_{CD} & \begin{bmatrix} 0 & 0 & 0 & -2 \end{bmatrix} \\ \varphi_{FG} & \begin{bmatrix} 6 & 0 & -3 & 0 \end{bmatrix} \\ \varphi_{BA} & \begin{bmatrix} 3 & 0 & 0 & 0 \end{bmatrix} \end{matrix} = \begin{matrix} [Xb & Yb & Zb & Tb & Sb & Rb & Fb] \\ \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 & -14 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & -3 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

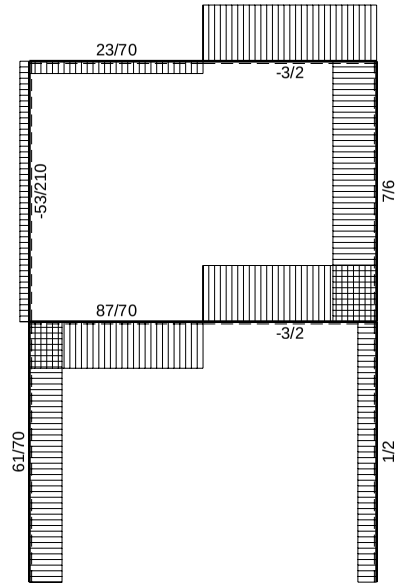
Soluzione del sistema

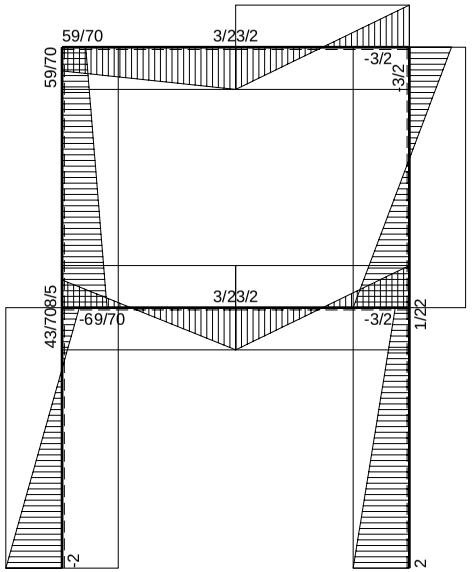
$$\begin{bmatrix} H_Ab \\ V_{BC}b \\ H_{BC}b \\ V_Ab \end{bmatrix} = \begin{matrix} [Xb & Yb & Zb & Tb & Sb & Rb & Fb] \\ \begin{bmatrix} -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ -1/3 & -2/3 & -1/3 & 0 & 0 & -1/3 & 1 \\ 0 & -1/4 & 0 & 0 & -1/4 & 0 & 7/2 \end{bmatrix} \end{matrix}$$





F





F Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	≥	H_{14}
D_2	H_{21}	H_{22}	H_{23}	≥	H_{24}
D_3	H_{31}	H_{32}	H_{33}	≥	H_{34}
D_4	H_{41}	H_{42}	H_{43}	≥	H_{44}
D_5	H_{51}	H_{52}	H_{53}	≥	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-2
W_{AB}+	1	0	0	0	0	0	0	≤	2
W_{BA}-	0	1	0	0	0	0	0	≥	-2
W_{BA}+	0	1	0	0	0	0	0	≤	2
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	1	0	0	0	0	≤	3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	1	0	0	0	≤	3/2
W_{DC}-	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC}+	0	0	1	-2	0	0	-12	≤	3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-2
W_{DE}+	0	0	0	0	1	0	0	≤	2
W_{ED}-	-1	-1	0	0	-1	0	9	≥	-2
W_{ED}+	-1	-1	0	0	-1	0	9	≤	2
W_{BF}-	0	-1	-1	0	0	0	0	≥	-2
W_{BF}+	0	-1	-1	0	0	0	0	≤	2
W_{FG}-	0	0	0	0	0	1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	1	0	≤	3/2
W_{GH}-	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH}+	0	1/2	1	-1	1/2	1	-7	≤	3/2
W_{HG}-	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG}+	0	-1	-2	2	-1	-1	6	≤	3/2
W_{DH}-	0	0	-1	2	-1	0	12	≥	-2
W_{DH}+	0	0	-1	2	-1	0	12	≤	2
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-2
W_{AB}+	-1	0	0	0	0	0	0	≥	-2
W_{BA}-	0	1	0	0	0	0	0	≥	-2
W_{BA}+	0	-1	0	0	0	0	0	≥	-2
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	≥	-3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	≥	-3/2
W_{DC}-	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	12	≥	-3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-2
W_{DE}+	0	0	0	0	-1	0	0	≥	-2
W_{ED}-	-1	-1	0	0	-1	0	9	≥	-2
W_{ED}+	1	1	0	0	1	0	-9	≥	-2
W_{BF}-	0	-1	-1	0	0	0	0	≥	-2
W_{BF}+	0	1	1	0	0	0	0	≥	-2
W_{FG}-	0	0	0	0	0	1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	-1	0	≥	-3/2
W_{GH}-	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	-1	7	≥	-3/2
W_{HG}-	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG}+	0	1	2	-2	1	1	-6	≥	-3/2
W_{DH}-	0	0	-1	2	-1	0	12	≥	-2
W_{DH}+	0	0	1	-2	1	0	-12	≥	-2
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-2
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-2
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-2
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-2
W_{BC}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC}-	0	0	1	-2	0	0	0	0	-1	2	0	0	-12	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	0	0	1	-2	0	0	12	≤	-3/2
W_{DE}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-2
W_{DE}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-2
W_{ED}-	-1	-1	0	0	-1	0	1	1	0	0	1	0	9	≥	-2
W_{ED}+	1	1	0	0	1	0	-1	-1	0	0	-1	0	-9	≤	-2
W_{BF}-	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-2
W_{BF}+	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-2
W_{FG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-3/2
W_{GH}-	0	1/2	1	-1	1/2	1	0	-1/2	-1	1	-1/2	-1	-7	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	-1	0	1/2	1	-1	1/2	1	7	≤	-3/2
W_{HG}-	0	-1	-2	2	-1	-1	0	1	2	-2	1	1	6	≥	-3/2
W_{HG}+	0	1	2	-2	1	1	0	-1	-2	2	-1	-1	-6	≤	-3/2
W_{DH}-	0	0	-1	2	-1	0	0	0	1	-2	1	0	12	≥	-2
W_{DH}+	0	0	1	-2	1	0	0	0	-1	2	-1	0	-12	≤	-2
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	1	-2	0	0	-12	1	≥	-3/2
φ_{DC}+	0	0	-1	2	0	0	12	-1	≥	-3/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	-1	-1	0	0	-1	0	9	3	≥	-2
φ_{ED}+	1	1	0	0	1	0	-9	-3	≥	-2
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-2
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	0	1/2	1	-1	1/2	1	-7	-2	≥	-3/2
φ_{GH}+	0	-1/2	-1	1	-1/2	-1	7	2	≥	-3/2
φ_{HG}-	0	-1	-2	2	-1	-1	6	3	≥	-3/2
φ_{HG}+	0	1	2	-2	1	1	-6	-3	≥	-3/2
φ_{DH}-	0	0	-1	2	-1	0	12	0	≥	-2
φ_{DH}+	0	0	1	-2	1	0	-12	0	≥	-2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 9-7

	X	Y	Z	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	0	0	1/12	-1/6	0	0	-1/12	1/12	≥	-1/8
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	-1	-1	3/4	-3/2	-1	0	-3/4	15/4	≥	-25/8
φ_{ED}+	1	1	-3/4	3/2	1	0	3/4	-15/4	≥	-7/8
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-2
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	0	1/2	5/12	1/6	1/2	1	7/12	-31/12	≥	-5/8
φ_{GH}+	0	-1/2	-5/12	-1/6	-1/2	-1	-7/12	31/12	≥	-19/8
φ_{HG}-	0	-1	-3/2	1	-1	-1	-1/2	7/2	≥	-9/4
φ_{HG}+	0	1	3/2	-1	1	1	1/2	-7/2	≥	-3/4
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	1/12	-1/6	0	0	-1/12	1/12	=	-1/8

Scambio pivotale 14-3

	X	Y	φ_{ED}+	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	4/3	4/3	-4/3	2	4/3	0	1	-6	≥	-8/3
φ_{BC}+	-4/3	-4/3	4/3	-2	-4/3	0	-1	6	≥	-1/3
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	1/9	1/9	-1/9	0	1/9	0	0	-1/3	≥	-2/9
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	4/3	4/3	-4/3	2	4/3	0	1	-5	≥	-7/6
φ_{BF}-	-4/3	-7/3	4/3	-2	-4/3	0	-1	7	≥	-5/6
φ_{BF}+	4/3	7/3	-4/3	2	4/3	0	1	-7	≥	-19/6
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	5/9	19/18	-5/9	1	19/18	1	1	-14/3	≥	-10/9
φ_{GH}+	-5/9	-19/18	5/9	-1	-19/18	-1	-1	14/3	≥	-17/9
φ_{HG}-	-2	-3	2	-2	-3	-1	-2	11	≥	-1/2
φ_{HG}+	2	3	-2	2	3	1	2	-11	≥	-5/2
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	1/9	1/9	-1/9	0	1/9	0	0	-1/3	=	-2/9

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Scambio pivotale 6-1

	φ_{BC}+	Y	φ_{ED}+	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	-3/4	-1	1	-3/2	-1	0	-3/4	7/2	≥	-9/4
φ_{AB}+	3/4	1	-1	3/2	1	0	3/4	-7/2	≥	-7/4
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	-3/4	-1	1	-3/2	-1	0	-3/4	9/2	≥	-1/4
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	-1/12	0	0	-1/6	0	0	-1/12	1/6	≥	-1/4
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	0	0	0	0	0	0	1	≥	-3/2
φ_{BF}-	1	-1	0	0	0	0	0	1	≥	-1/2
φ_{BF}+	-1	1	0	0	0	0	0	-1	≥	-7/2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	-5/12	1/2	0	1/6	1/2	1	7/12	-13/6	≥	-5/4
φ_{GH}+	5/12	-1/2	0	-1/6	-1/2	-1	-7/12	13/6	≥	-7/4
φ_{HG}-	3/2	-1	0	1	-1	-1	-1/2	2	≥	0
φ_{HG}+	-3/2	1	0	-1	1	1	1/2	-2	≥	-3
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	-1/12	0	0	-1/6	0	0	-1/12	1/6	=	-1/4

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Scambio pivotale 2-8

	φ_{BC}+	Y	φ_{ED}+	T	S	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	3/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1/2
φ_{BA}-	-3/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
φ_{BA}+	3/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-5/2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	3/14	2/7	-2/7	3/7	2/7	0	3/14	-9/7	≥	-5/2
φ_{CD}-	-3/14	-2/7	2/7	4/7	-2/7	0	-3/14	2/7	≥	-1
φ_{CD}+	3/14	2/7	-2/7	-4/7	2/7	0	3/14	-2/7	≥	-2
αbF	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	≥	-1/3
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	-3/14	-2/7	2/7	-3/7	5/7	0	-3/14	2/7	≥	-3/2
φ_{DE}+	3/14	2/7	-2/7	3/7	-5/7	0	3/14	-2/7	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-2
φ_{BF}-	17/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1
φ_{BF}+	-17/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3
φ_{FG}-	-3/14	-2/7	2/7	-3/7	-2/7	1	-3/14	2/7	≥	-1
φ_{FG}+	3/14	2/7	-2/7	3/7	2/7	-1	3/14	-2/7	≥	-2
φ_{GH}-	-37/42	-5/42	13/21	-16/21	-5/42	1	5/42	13/21	≥	-1/6
φ_{GH}+	37/42	5/42	-13/21	16/21	5/42	-1	-5/42	-13/21	≥	-17/6
φ_{HG}-	27/14	-3/7	-4/7	13/7	-3/7	-1	-1/14	-4/7	≥	-1
φ_{HG}+	-27/14	3/7	4/7	-13/7	3/7	1	1/14	4/7	≥	-2
φ_{DH}-	3/14	2/7	-2/7	3/7	-5/7	0	-11/14	-2/7	≥	-4
φ_{DH}+	-3/14	-2/7	2/7	-3/7	5/7	0	11/14	2/7	≥	0
L_X	-3/14	-2/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
Max	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	=	-1/3

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Scambio pivotale 24-2

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	S	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	0	-1	0	0	1	0	1	0	≥	-1/2
φ_{BA}-	-3/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3/2
φ_{BA}+	3/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-5/2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	0	-1	0	0	1	0	1	-1	≥	-5/2
φ_{CD}-	0	1	0	1	-1	0	-1	0	≥	-1
φ_{CD}+	0	-1	0	-1	1	0	1	0	≥	-2
αbF	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	≥	-1/3
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	-1	0	0	1	0	1	0	≥	-2
φ_{BF}-	7/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-1
φ_{BF}+	-7/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3
φ_{FG}-	0	1	0	0	-1	1	-1	0	≥	-1
φ_{FG}+	0	-1	0	0	1	-1	1	0	≥	-2
φ_{GH}-	-19/24	5/12	1/2	-7/12	-5/12	1	-5/24	1/2	≥	-1/6
φ_{GH}+	19/24	-5/12	-1/2	7/12	5/12	-1	5/24	-1/2	≥	-17/6
φ_{HG}-	9/4	3/2	-1	5/2	-3/2	-1	-5/4	-1	≥	-1
φ_{HG}+	-9/4	-3/2	1	-5/2	3/2	1	5/4	1	≥	-2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	-3/4	-7/2	1	-3/2	5/2	0	11/4	1	≥	0
L_X	0	1	0	0	-1	0	-1	0	≥	-3/2
Max	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	=	-1/3

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Scambio pivotale 19-5

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	φ_{GH}-	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-9/10
φ_{BA}-	-18/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-21/10
φ_{BA}+	18/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-19/10
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	1/5	≥	-29/10
φ_{CD}-	19/10	0	-6/5	12/5	12/5	-12/5	-1/2	-6/5	≥	-3/5
φ_{CD}+	-19/10	0	6/5	-12/5	-12/5	12/5	1/2	6/5	≥	-12/5
αbF	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	≥	-2/5
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-29/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-12/5
φ_{BF}-	23/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-2/5
φ_{BF}+	-23/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-18/5
φ_{FG}-	19/10	0	-6/5	7/5	12/5	-7/5	-1/2	-6/5	≥	-3/5
φ_{FG}+	-19/10	0	6/5	-7/5	-12/5	7/5	1/2	6/5	≥	-12/5
S	-19/10	1	6/5	-7/5	-12/5	12/5	-1/2	6/5	≥	-2/5
φ_{GH}+	0	0	0	0	-1	0	0	0	≥	-3
φ_{HG}-	51/10	0	-14/5	23/5	18/5	-23/5	-1/2	-14/5	≥	-2/5
φ_{HG}+	-51/10	0	14/5	-23/5	-18/5	23/5	1/2	14/5	≥	-13/5
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	-11/2	-1	4	-5	-6	6	3/2	4	≥	-1
L_X	19/10	0	-6/5	7/5	12/5	-12/5	-1/2	-6/5	≥	-11/10
Max	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	=	-2/5

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Scambio pivotale 21-6

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	φ_{GH}-	φ_{HG}-	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	35/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-51/46
φ_{BA}-	9/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-111/46
φ_{BA}+	-9/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-73/46
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	35/46	0	-6/23	1	-12/23	-12/23	11/46	-29/23	≥	-143/46
φ_{CD}-	-35/46	0	6/23	0	12/23	12/23	-11/46	6/23	≥	-9/23
φ_{CD}+	35/46	0	-6/23	0	-12/23	-12/23	11/46	-6/23	≥	-60/23
αbF	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	≥	-10/23
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-60/23
φ_{BF}-	14/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-2/23
φ_{BF}+	-14/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-90/23
φ_{FG}-	8/23	0	-8/23	0	30/23	7/23	-8/23	-8/23	≥	-11/23
φ_{FG}+	-8/23	0	8/23	0	-30/23	-7/23	8/23	8/23	≥	-58/23
S	35/46	1	-6/23	1	-12/23	-12/23	-35/46	-6/23	≥	-14/23
φ_{GH}+	0	0	0	0	-1	0	0	0	≥	-3
R	51/46	0	-14/23	1	18/23	-5/23	-5/46	-14/23	≥	-2/23
φ_{HG}+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	53/46	-1	8/23	1	-30/23	-30/23	39/46	8/23	≥	-35/23
L_X	-35/46	0	6/23	-1	12/23	12/23	-11/46	6/23	≥	-41/46
Max	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	=	-10/23

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Scambio pivotale 7-1

	φ_{CD}^-	φ_{DH}^+	φ_{ED}^+	T	φ_{GH}^-	φ_{HG}^-	φ_{DC}^-	φ_{AB}^+		Fb
φ_{AB}^-	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
φ_{BA}^-	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
φ_{BA}^+	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
φ_{BC}^-	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
X	-1	0	0	1	0	0	0	-1	≥	-7/2
φ_{BC}^+	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
φ_{CD}^+	-1	0	0	0	0	0	0	0	≥	-3
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
φ_{DC}^+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}^-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}^+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}^-	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
φ_{BF}^-	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
φ_{BF}^+	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
φ_{FG}^-	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{FG}^+	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
φ_{GH}^+	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{HG}^+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}^-	0	-1	0	0	0	0	0	0	≥	-4
Y	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_X	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35

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Tableau finale

	φ_{CD}^-	φ_{DH}^+	φ_{ED}^+	T	φ_{GH}^-	φ_{HG}^-	φ_{DC}^-	φ_{AB}^+		Fb
φ_{AB}^-	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
φ_{BA}^-	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
φ_{BA}^+	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
φ_{BC}^-	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
X	-1	0	0	1	0	0	0	-1	≥	-7/2
φ_{BC}^+	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
φ_{CD}^+	-1	0	0	0	0	0	0	0	≥	-3
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
φ_{DC}^+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}^-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}^+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}^-	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
φ_{BF}^-	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
φ_{BF}^+	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
φ_{FG}^-	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{FG}^+	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
φ_{GH}^+	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{HG}^+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}^-	0	-1	0	0	0	0	0	0	≥	-4
Y	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_X	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/35
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	0
φ_{BC}-	0	0	0	0	0	0	0	0	≥	0
φ_{BC}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{DE}-	0	0	0	0	0	0	0	0	≥	0
φ_{DE}+	0	0	0	0	0	0	0	0	≥	0
φ_{ED}-	0	0	0	0	0	0	0	0	≥	0
φ_{ED}+	0	0	0	0	0	0	0	0	≥	1/35
φ_{BF}-	0	0	0	0	0	0	0	0	≥	0
φ_{BF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FG}-	0	0	0	0	0	0	0	0	≥	0
φ_{FG}+	0	0	0	0	0	0	0	0	≥	0
φ_{GH}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{GH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HG}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{HG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	7/2	74/35	87/35	0	1	23/35	16/35	3/2	=	-16/35

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
2	43/70	69/70	-3/2	-1/2	-59/70

Variabili soluzione differenza tra rotazioni

φ_{AB}	1/35
φ_{BA}	0
φ_{BC}	0
φ_{CD}	-2/35
φ_{DC}	-2/35
φ_{DE}	0
φ_{ED}	1/35
φ_{BF}	0
φ_{FG}	0
φ_{GH}	-2/35
φ_{HG}	-2/35
φ_{DH}	0

REAZIONI Fattore di collasso = 16/35

H_A = -61/70F
V_A = 11/7F
W_A = 2Fb
H_E = -1/2F
V_E = 3F
W_E = 2Fb

H_{AB} = -61/70F	H_{BC} = -2/3F	H_{CD} = -2/3F	H_{DE} = 1/2F	H_{BF} = 53/210F
V_{AB} = 11/7F	V_{BC} = 87/70F	V_{CD} = -3/2F	V_{DE} = -3F	V_{BF} = 23/70F
W_{AB} = 2Fb	W_{BC} = 69/70Fb	W_{CD} = -3/2Fb	W_{DE} = -1/2Fb	W_{BF} = -8/5Fb
H_{BA} = 61/70F	H_{CB} = 2/3F	H_{DC} = 2/3F	H_{ED} = -1/2F	H_{FB} = -53/210F
V_{BA} = -11/7F	V_{CB} = -87/70F	V_{DC} = 3/2F	V_{ED} = 3F	V_{FB} = -23/70F
W_{BA} = 43/70Fb	W_{CB} = 3/2Fb	W_{DC} = -3/2Fb	W_{ED} = 2Fb	W_{FB} = 59/70Fb

H_{FG} = 7/6F	H_{GH} = 7/6F	H_{HD} = 7/6F
V_{FG} = 23/70F	V_{GH} = -3/2F	V_{HD} = -3/2F
W_{FG} = -59/70Fb	W_{GH} = -3/2Fb	W_{HD} = 3/2Fb
H_{GF} = -7/6F	H_{HG} = -7/6F	H_{DH} = -7/6F
V_{GF} = -23/70F	V_{HG} = 3/2F	V_{DH} = 3/2F
W_{GF} = 3/2Fb	W_{HG} = -3/2Fb	W_{DH} = 2Fb

SPOSTAMENTI NODALI

u_{AAB} = 0	u_B = 3/35δ	u_{CCB} = 3/35δ	u_D = 3/35δ	u_{EED} = 0
v_{AAB} = 0	v_B = 0	v_{CCB} = -2/35δ	v_D = 0	v_{EED} = 0
φ_{AAB} = -1/35δ/b	φ_B = -1/35δ/b	φ_{CCB} = -1/35δ/b	φ_D = 1/35δ/b	φ_E = -1/35δ/b

$u_F = 6/35\delta$ $u_{GGF} = 6/35\delta$ $u_{HHG} = 6/35\delta$
$v_F = 0$ $v_{GGF} = -2/35\delta$ $v_{HHG} = 0$
$\varphi_F = -1/35\delta/b$ $\varphi_{GGF} = -1/35\delta/b$ $\varphi_{HHD} = 1/35\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$ $u_{BBC} = 3/35\delta$ $u_{CCD} = 3/35\delta$ $u_{DDE} = 3/35\delta$ $u_{BBF} = 3/35\delta$
$v_{AAB} = 0$ $v_{BBC} = 0$ $v_{CCD} = -2/35\delta$ $v_{DDE} = 0$ $v_{BBF} = 0$
$\varphi_{AAB} = -1/35\delta/b$ $\varphi_{BBC} = -1/35\delta/b$ $\varphi_{CCD} = 1/35\delta/b$ $\varphi_{DDE} = -1/35\delta/b$ $\varphi_{BBF} = -1/35\delta/b$

$u_{FFG} = 6/35\delta$ $u_{GGH} = 6/35\delta$ $u_{HHD} = 6/35\delta$
$v_{FFG} = 0$ $v_{GGH} = -2/35\delta$ $v_{HHD} = 0$
$\varphi_{FFG} = -1/35\delta/b$ $\varphi_{GGH} = 1/35\delta/b$ $\varphi_{HHD} = -1/35\delta/b$

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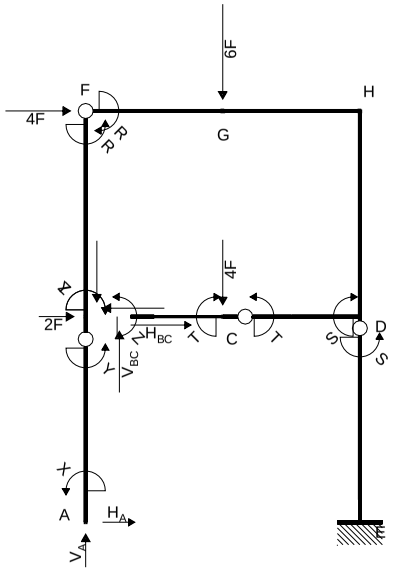


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

$3H_Ab -4V_Ab = -Xb +Sb -8Fb$

Rotazione intorno a C: aste CB

$-2V_{BC}b = -Zb +Tb$

Rotazione intorno a F: aste FB BA

$6H_Ab -3H_{BC}b = -Xb +Zb -Rb -6Fb$

Rotazione intorno a B: aste BA

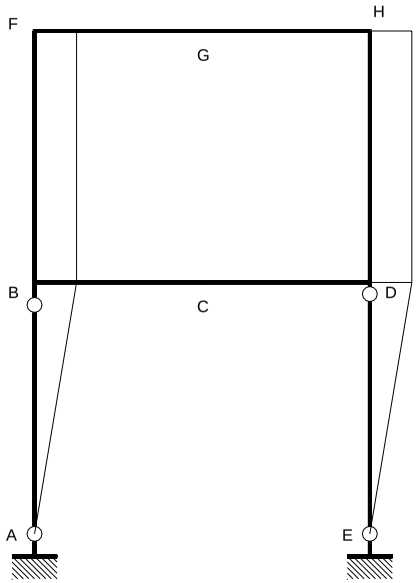
$3H_Ab = -Xb -Yb$

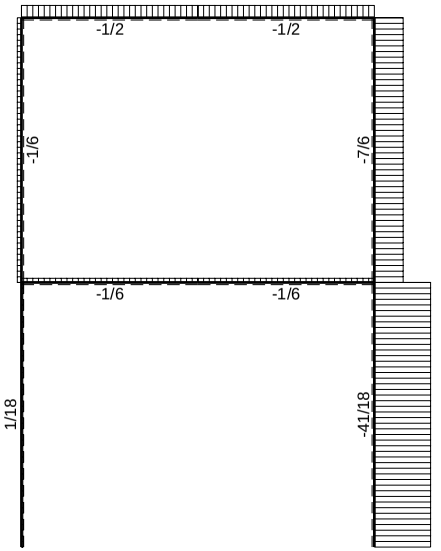
Matrice di equilibrio

	H_Ab	V_Ab	$H_{BC}b$	$V_{BC}b$		Xb	Yb	Zb	Tb	Sb	Rb	Fb
φ_{DE}	3	-4	0	0	=	-1	0	0	0	1	0	-8
φ_{CD}	0	0	0	-2		0	0	-1	1	0	0	0
φ_{FB}	6	0	-3	0		-1	0	1	0	0	-1	-6
φ_{BA}	3	0	0	0		-1	-1	0	0	0	0	0

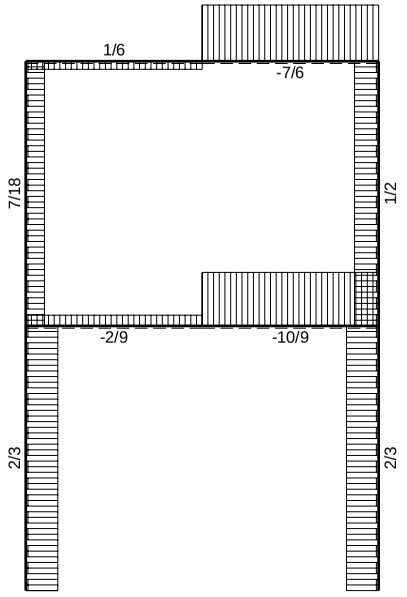
Soluzione del sistema

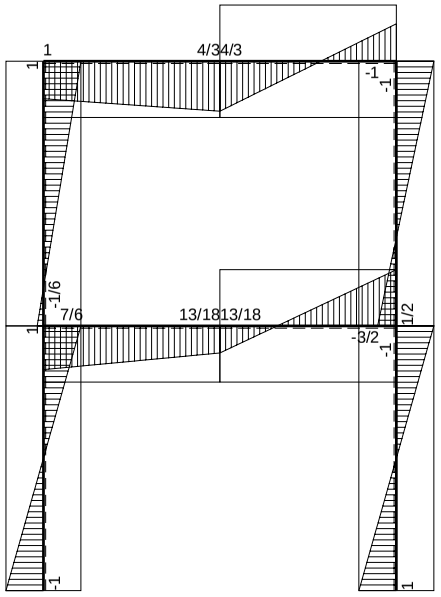
		Xb	Yb	Zb	Tb	Sb	Rb	Fb
H_Ab		-1/3	-1/3	0	0	0	0	0
$V_{BC}b$	=	0	0	1/2	-1/2	0	0	0
$H_{BC}b$		-1/3	-2/3	-1/3	0	0	1/3	2
V_Ab		0	-1/4	0	0	-1/4	0	2





F





F Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq} = 1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FB}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	1	0	0	0	0	≤	3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	1	0	0	0	≤	3/2
W_{DC}-	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC}+	0	0	1	-2	0	0	-8	≤	3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-1
W_{DE}+	0	0	0	0	1	0	0	≤	1
W_{ED}-	-1	-1	0	0	-1	0	18	≥	-1
W_{ED}+	-1	-1	0	0	-1	0	18	≤	1
W_{BF}-	0	-1	-1	0	0	0	0	≥	-1
W_{BF}+	0	-1	-1	0	0	0	0	≤	1
W_{FB}-	0	0	0	0	0	1	0	≥	-1
W_{FB}+	0	0	0	0	0	1	0	≤	1
W_{GH}-	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH}+	0	1/2	1	-1	1/2	-1	-4	≤	3/2
W_{HD}-	0	1	2	-2	1	-1	4	≥	-1
W_{HD}+	0	1	2	-2	1	-1	4	≤	1
W_{DH}-	0	0	-1	2	-1	0	8	≥	-1
W_{DH}+	0	0	-1	2	-1	0	8	≤	1
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	≥	-3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	≥	-3/2
W_{DC}-	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	8	≥	-3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-1
W_{DE}+	0	0	0	0	-1	0	0	≥	-1
W_{ED}-	-1	-1	0	0	-1	0	18	≥	-1
W_{ED}+	1	1	0	0	1	0	-18	≥	-1
W_{BF}-	0	-1	-1	0	0	0	0	≥	-1
W_{BF}+	0	1	1	0	0	0	0	≥	-1
W_{FB}-	0	0	0	0	0	1	0	≥	-1
W_{FB}+	0	0	0	0	0	-1	0	≥	-1
W_{GH}-	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	1	4	≥	-3/2
W_{HD}-	0	1	2	-2	1	-1	4	≥	-1
W_{HD}+	0	-1	-2	2	-1	1	-4	≥	-1
W_{DH}-	0	0	-1	2	-1	0	8	≥	-1
W_{DH}+	0	0	1	-2	1	0	-8	≥	-1
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{BC}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC}-	0	0	1	-2	0	0	0	0	-1	2	0	0	-8	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	0	0	1	-2	0	0	8	≤	-3/2
W_{DE}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{DE}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{ED}-	-1	-1	0	0	-1	0	1	1	0	0	1	0	18	≥	-1
W_{ED}+	1	1	0	0	1	0	-1	-1	0	0	-1	0	-18	≤	-1
W_{BF}-	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-1
W_{BF}+	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-1
W_{FB}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-1
W_{FB}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-1
W_{GH}-	0	1/2	1	-1	1/2	-1	0	-1/2	-1	1	-1/2	1	-4	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	1	0	1/2	1	-1	1/2	-1	4	≤	-3/2
W_{HD}-	0	1	2	-2	1	-1	0	-1	-2	2	-1	1	4	≥	-1
W_{HD}+	0	-1	-2	2	-1	1	0	1	2	-2	1	-1	-4	≤	-1
W_{DH}-	0	0	-1	2	-1	0	0	0	1	-2	1	0	8	≥	-1
W_{DH}+	0	0	1	-2	1	0	0	0	-1	2	-1	0	-8	≤	-1
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	1	-2	0	0	-8	1	≥	-3/2
φ_{DC}+	0	0	-1	2	0	0	8	-1	≥	-3/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	-1	-1	0	0	-1	0	18	3	≥	-1
φ_{ED}+	1	1	0	0	1	0	-18	-3	≥	-1
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	0	1/2	1	-1	1/2	-1	-4	0	≥	-3/2
φ_{GH}+	0	-1/2	-1	1	-1/2	1	4	0	≥	-3/2
φ_{HD}-	0	1	2	-2	1	-1	4	-1	≥	-1
φ_{HD}+	0	-1	-2	2	-1	1	-4	1	≥	-1
φ_{DH}-	0	0	-1	2	-1	0	8	0	≥	-1
φ_{DH}+	0	0	1	-2	1	0	-8	0	≥	-1
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 14-7

	X	Y	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	-4/9	-4/9	1	-2	-4/9	0	4/9	7/3	≥	-19/18
φ_{DC}+	4/9	4/9	-1	2	4/9	0	-4/9	-7/3	≥	-35/18
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	1/18	1/18	0	0	1/18	0	-1/18	-1/6	≥	-1/18
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	-2/9	5/18	1	-1	5/18	-1	2/9	2/3	≥	-23/18
φ_{GH}+	2/9	-5/18	-1	1	-5/18	1	-2/9	-2/3	≥	-31/18
φ_{HD}-	2/9	11/9	2	-2	11/9	-1	-2/9	-5/3	≥	-11/9
φ_{HD}+	-2/9	-11/9	-2	2	-11/9	1	2/9	5/3	≥	-7/9
φ_{DH}-	4/9	4/9	-1	2	-5/9	0	-4/9	-4/3	≥	-13/9
φ_{DH}+	-4/9	-4/9	1	-2	5/9	0	4/9	4/3	≥	-5/9
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	1/18	1/18	0	0	1/18	0	-1/18	-1/6	=	-1/18

Scambio pivotale 2-1

	φ_{AB}+	Y	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	4/9	-4/9	1	-2	-4/9	0	4/9	17/9	≥	-11/18
φ_{DC}+	-4/9	4/9	-1	2	4/9	0	-4/9	-17/9	≥	-43/18
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	≥	-1/9
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	2/9	5/18	1	-1	5/18	-1	2/9	4/9	≥	-19/18
φ_{GH}+	-2/9	-5/18	-1	1	-5/18	1	-2/9	-4/9	≥	-35/18
φ_{HD}-	-2/9	11/9	2	-2	11/9	-1	-2/9	-13/9	≥	-13/9
φ_{HD}+	2/9	-11/9	-2	2	-11/9	1	2/9	13/9	≥	-5/9
φ_{DH}-	-4/9	4/9	-1	2	-5/9	0	-4/9	-8/9	≥	-17/9
φ_{DH}+	4/9	-4/9	1	-2	5/9	0	4/9	8/9	≥	-1/9
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	=	-1/9

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Scambio pivotale 24-2

	φ_{AB}+	φ_{DH}+	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	1	-9/4	9/4	-9/2	5/4	0	1	1	≥	-5/4
φ_{BA}+	-1	9/4	-9/4	9/2	-5/4	0	-1	-1	≥	-3/4
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	1	0	0	-1	0	0	1	≥	-1/2
φ_{DC}+	0	-1	0	0	1	0	0	-1	≥	-5/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	-1/8	1/8	-1/4	1/8	0	0	0	≥	-1/8
φ_{BF}-	-1	9/4	-13/4	9/2	-5/4	0	-1	0	≥	-3/4
φ_{BF}+	1	-9/4	13/4	-9/2	5/4	0	1	0	≥	-5/4
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/2	-5/8	13/8	-9/4	5/8	-1	1/2	1	≥	-9/8
φ_{GH}+	-1/2	5/8	-13/8	9/4	-5/8	1	-1/2	-1	≥	-15/8
φ_{HD}-	1	-11/4	19/4	-15/2	11/4	-1	1	1	≥	-7/4
φ_{HD}+	-1	11/4	-19/4	15/2	-11/4	1	-1	-1	≥	-1/4
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	1	-9/4	9/4	-9/2	5/4	0	1	2	≥	-1/4
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	-1/8	1/8	-1/4	1/8	0	0	0	=	-1/8

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Scambio pivotale 22-3

	φ_{AB}+	φ_{DH}+	φ_{HD}+	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	10/19	≥	-26/19
φ_{BA}+	-10/19	18/19	9/19	18/19	1/19	-9/19	-10/19	-10/19	≥	-12/19
φ_{BC}-	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-23/19	≥	-59/38
φ_{BC}+	4/19	-11/19	4/19	-30/19	11/19	-4/19	4/19	23/19	≥	-55/38
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	1	0	0	-1	0	0	1	≥	-1/2
φ_{DC}+	0	-1	0	0	1	0	0	-1	≥	-5/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	≥	-5/38
φ_{BF}-	-6/19	7/19	13/19	-12/19	12/19	-13/19	-6/19	13/19	≥	-11/19
φ_{BF}+	6/19	-7/19	-13/19	12/19	-12/19	13/19	6/19	-13/19	≥	-27/19
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	3/19	6/19	-13/38	6/19	-6/19	-25/38	3/19	25/38	≥	-23/19
φ_{GH}+	-3/19	-6/19	13/38	-6/19	6/19	25/38	-3/19	-25/38	≥	-34/19
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-4/19	≥	-1/19
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	29/19	≥	-7/19
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	=	-5/38

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Scambio pivotale 22-5

	φ_{AB}+	φ_{DH}+	φ_{HD}+	T	Z	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	6/11	≥	-15/11
φ_{BA}+	-6/11	1	5/11	12/11	-1/11	-5/11	-6/11	-6/11	≥	-7/11
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	4/11	0	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-9/22
φ_{DC}+	-4/11	0	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-57/22
φ_{DE}-	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-12/11
φ_{DE}+	4/11	-1	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-10/11
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	≥	-3/22
φ_{BF}-	-6/11	1	5/11	12/11	-12/11	-5/11	-6/11	5/11	≥	-7/11
φ_{BF}+	6/11	-1	-5/11	-12/11	12/11	5/11	6/11	-5/11	≥	-15/11
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	3/11	0	-5/22	-6/11	6/11	-17/22	3/11	17/22	≥	-13/11
φ_{GH}+	-3/11	0	5/22	6/11	-6/11	17/22	-3/11	-17/22	≥	-20/11
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-4/11	≥	-1/11
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	17/11	≥	-4/11
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	=	-3/22

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Scambio pivotale 9-4

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	0	≥	-6/5
φ_{BA}+	-2/5	1	3/5	-2/5	3/5	-3/5	-2/5	0	≥	-4/5
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	2/15	0	2/15	-11/30	19/30	-2/15	2/15	-1/2	≥	-33/20
φ_{CD}+	-2/15	0	-2/15	11/30	-19/30	2/15	-2/15	1/2	≥	-27/20
T	2/15	0	2/15	-11/30	19/30	-2/15	2/15	1/2	≥	-3/20
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	≥	-3/20
φ_{BF}-	-2/5	1	3/5	-2/5	-2/5	-3/5	-2/5	1	≥	-4/5
φ_{BF}+	2/5	-1	-3/5	2/5	2/5	3/5	2/5	-1	≥	-6/5
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/5	0	-3/10	1/5	1/5	-7/10	1/5	1/2	≥	-11/10
φ_{GH}+	-1/5	0	3/10	-1/5	-1/5	7/10	-1/5	-1/2	≥	-19/10
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	1	≥	-1/5
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	=	-3/20

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Scambio pivotale 18-6

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	3/5	≥	-9/5
φ_{BA}+	-2/5	1	3/5	-2/5	3/5	3/5	-2/5	-3/5	≥	-1/5
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	2/15	0	2/15	-11/30	19/30	2/15	2/15	-19/30	≥	-91/60
φ_{CD}+	-2/15	0	-2/15	11/30	-19/30	-2/15	-2/15	19/30	≥	-89/60
T	2/15	0	2/15	-11/30	19/30	2/15	2/15	11/30	≥	-1/60
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	≥	-11/60
φ_{BF}-	-2/5	1	3/5	-2/5	-2/5	3/5	-2/5	2/5	≥	-1/5
φ_{BF}+	2/5	-1	-3/5	2/5	2/5	-3/5	2/5	-2/5	≥	-9/5
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/5	0	-3/10	1/5	1/5	7/10	1/5	-1/5	≥	-2/5
φ_{GH}+	-1/5	0	3/10	-1/5	-1/5	-7/10	-1/5	1/5	≥	-13/5
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	8/5	≥	-4/5
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	=	-11/60

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Scambio pivotale 4-8

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-4/3
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-1/3
φ_{BC}-	2/3	-5/3	-1	2/3	0	-1	2/3	5/3	≥	-7/6
φ_{BC}+	-2/3	5/3	1	-2/3	0	1	-2/3	-5/3	≥	-11/6
φ_{CD}-	5/9	-19/18	-1/2	1/18	0	-1/2	5/9	19/18	≥	-47/36
φ_{CD}+	-5/9	19/18	1/2	-1/18	0	1/2	-5/9	-19/18	≥	-61/36
T	-1/9	11/18	1/2	-11/18	1	1/2	-1/9	-11/18	≥	-5/36
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	≥	-7/36
φ_{BF}-	-2/3	5/3	1	-2/3	0	1	-2/3	-2/3	≥	-1/3
φ_{BF}+	2/3	-5/3	-1	2/3	0	-1	2/3	2/3	≥	-5/3
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	5/3	1	-2/3	1	0	-2/3	-5/3	≥	-4/3
φ_{GH}-	1/3	-1/3	-1/2	1/3	0	1/2	1/3	1/3	≥	-1/3
φ_{GH}+	-1/3	1/3	1/2	-1/3	0	-1/2	-1/3	-1/3	≥	-8/3
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	8/3	1	-5/3	1	1	-2/3	-5/3	≥	-5/6
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	-2/3	5/3	1	-2/3	1	1	-2/3	-8/3	≥	-4/3
L_X	2/3	-5/3	-1	2/3	-1	-1	2/3	5/3	≥	-7/6
Max	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	=	-7/36

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Scambio pivotale 12-2

	φ_{AB}+	φ_{DE}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
φ_{BC}-	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
φ_{BC}+	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
φ_{CD}-	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
φ_{CD}+	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
T	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	-1	0	0	0	0	0	0	≥	-2
φ_{DH}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
φ_{BF}-	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
φ_{BF}+	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
φ_{GH}-	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
φ_{GH}+	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{DH}-	0	1	0	-1	0	0	0	0	≥	-3/2
Y	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_X	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

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Tableau finale

	φ_{AB}+	φ_{DE}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
φ_{BC}-	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
φ_{BC}+	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
φ_{CD}-	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
φ_{CD}+	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
T	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	-1	0	0	0	0	0	0	≥	-2
φ_{DH}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
φ_{BF}-	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
φ_{BF}+	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
φ_{GH}-	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
φ_{GH}+	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{DH}-	0	1	0	-1	0	0	0	0	≥	-3/2
Y	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_X	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BC}-	0	0	0	0	0	0	0	0	≥	0
φ_{BC}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	0
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	0
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{DE}-	0	0	0	0	0	0	0	0	≥	0
φ_{DE}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{ED}-	0	0	0	0	0	0	0	0	≥	0
φ_{ED}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BF}-	0	0	0	0	0	0	0	0	≥	0
φ_{BF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FB}-	0	0	0	0	0	0	0	0	≥	0
φ_{FB}+	0	0	0	0	0	0	0	0	≥	0
φ_{GH}-	0	0	0	0	0	0	0	0	≥	0
φ_{GH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HD}-	0	0	0	0	0	0	0	0	≥	0
φ_{HD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	13/6	13/6	0	4/9	13/6	13/6	2/9	7/6	=	-2/9

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
1	1	-7/6	-13/18	1	1

Variabili soluzione differenza tra rotazioni

$$\begin{bmatrix} \varphi_{AB} & 1/18 \\ \varphi_{BA} & 1/18 \\ \varphi_{BC} & 0 \\ \varphi_{CD} & 0 \\ \varphi_{DC} & 0 \\ \varphi_{DE} & 1/18 \\ \varphi_{ED} & 1/18 \\ \varphi_{BF} & 0 \\ \varphi_{FB} & 0 \\ \varphi_{GH} & 0 \\ \varphi_{HD} & 0 \\ \varphi_{DH} & 0 \end{bmatrix}$$

REAZIONI Fattore di collasso = 2/9

$H_A = -2/3F$
$V_A = -1/18F$
$W_A = Fb$
$H_E = -2/3F$
$V_E = 41/18F$
$W_E = Fb$

$H_{AB} = -2/3F$	$H_{BC} = 1/6F$	$H_{CD} = 1/6F$	$H_{DE} = 2/3F$	$H_{BF} = -7/18F$
$V_{AB} = -1/18F$	$V_{BC} = -2/9F$	$V_{CD} = -10/9F$	$V_{DE} = -41/18F$	$V_{BF} = 1/6F$
$W_{AB} = Fb$	$W_{BC} = -7/6Fb$	$W_{CD} = -13/18Fb$	$W_{DE} = Fb$	$W_{BF} = 1/6Fb$
$H_{BA} = 2/3F$	$H_{CB} = -1/6F$	$H_{DC} = -1/6F$	$H_{ED} = -2/3F$	$H_{FB} = 7/18F$
$V_{BA} = 1/18F$	$V_{CB} = 2/9F$	$V_{DC} = 10/9F$	$V_{ED} = 41/18F$	$V_{FB} = -1/6F$
$W_{BA} = Fb$	$W_{CB} = 13/18Fb$	$W_{DC} = -3/2Fb$	$W_{ED} = Fb$	$W_{FB} = Fb$

$H_{FG} = 1/2F$	$H_{GH} = 1/2F$	$H_{HD} = 1/2F$
$V_{FG} = 1/6F$	$V_{GH} = -7/6F$	$V_{HD} = -7/6F$
$W_{FG} = -Fb$	$W_{GH} = -4/3Fb$	$W_{HD} = Fb$
$H_{GF} = -1/2F$	$H_{HG} = -1/2F$	$H_{DH} = -1/2F$
$V_{GF} = -1/6F$	$V_{HG} = 7/6F$	$V_{DH} = 7/6F$
$W_{GF} = 4/3Fb$	$W_{HG} = -Fb$	$W_{DH} = 1/2Fb$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 1/6\delta$	$u_C = 1/6\delta$	$u_D = 1/6\delta$	$u_{EED} = 0$
$v_{AAB} = 0$	$v_B = 0$	$v_C = 0$	$v_D = 0$	$v_{EED} = 0$
$\varphi_{AAB} = -1/18\delta/b$	$\varphi_B = -1/18\delta/b$	$\varphi_C = 0$	$\varphi_D = 0$	$\varphi_E = -1/18\delta/b$

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$u_F = 1/6\delta$	$u_G = 1/6\delta$	$u_H = 1/6\delta$
$v_F = 0$	$v_G = 0$	$v_H = 0$
$\varphi_F = 0$	$\varphi_G = 0$	$\varphi_H = 0$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{BBC} = 1/6\delta$	$u_{CCD} = 1/6\delta$	$u_{DDE} = 1/6\delta$	$u_{BBF} = 1/6\delta$
$v_{AAB} = 0$	$v_{BBC} = 0$	$v_{CCD} = 0$	$v_{DDE} = 0$	$v_{BBF} = 0$
$\varphi_{AAB} = -1/18\delta/b$	$\varphi_{BBC} = 0$	$\varphi_{CCD} = 0$	$\varphi_{DDE} = -1/18\delta/b$	$\varphi_{BBF} = 0$

$u_{FFG} = 1/6\delta$	$u_{GGH} = 1/6\delta$	$u_{HHD} = 1/6\delta$
$v_{FFG} = 0$	$v_{GGH} = 0$	$v_{HHD} = 0$
$\varphi_{FFG} = 0$	$\varphi_{GGH} = 0$	$\varphi_{HHD} = 0$

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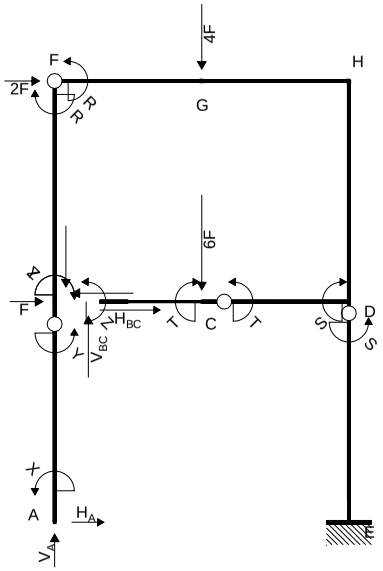


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

$3H_Ab - 4V_Ab = -Xb + Sb - 14Fb$

Rotazione intorno a C: aste CB

$-2V_{BC}b = -Zb + Tb$

Rotazione intorno a F: aste FB BA

$6H_Ab - 3H_{BC}b = -Xb + Zb + Rb - 3Fb$

Rotazione intorno a B: aste BA

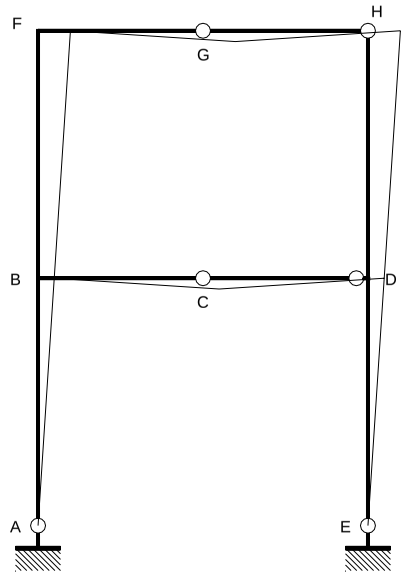
$3H_Ab = -Xb - Yb$

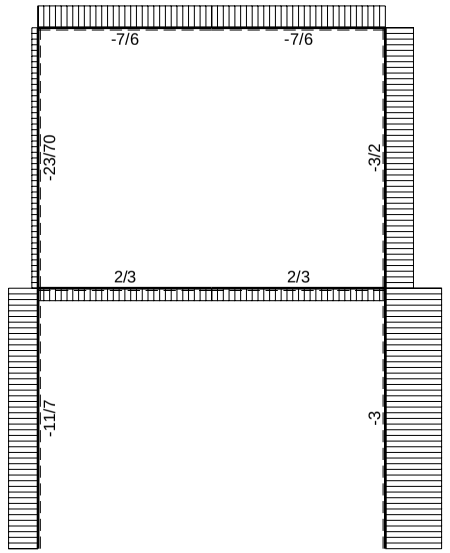
Matrice di equilibrio

$$\begin{array}{c} \\ \varphi_{DE} \\ \varphi_{CD} \\ \varphi_{FG} \\ \varphi_{BA} \end{array} \begin{bmatrix} H_Ab & V_Ab & H_{BC}b & V_{BC}b \\ 3 & -4 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 6 & 0 & -3 & 0 \\ 3 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1 & 0 & 0 & 0 & 1 & 0 & -14 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & -3 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

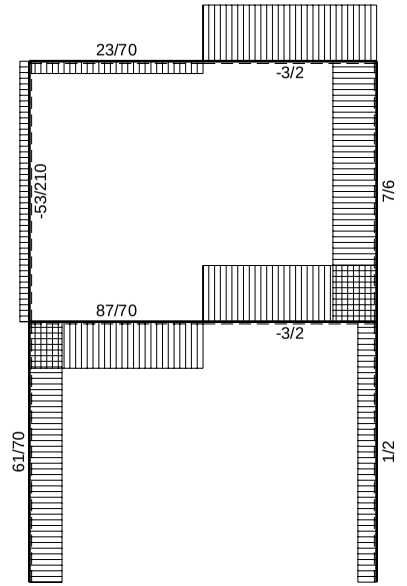
Soluzione del sistema

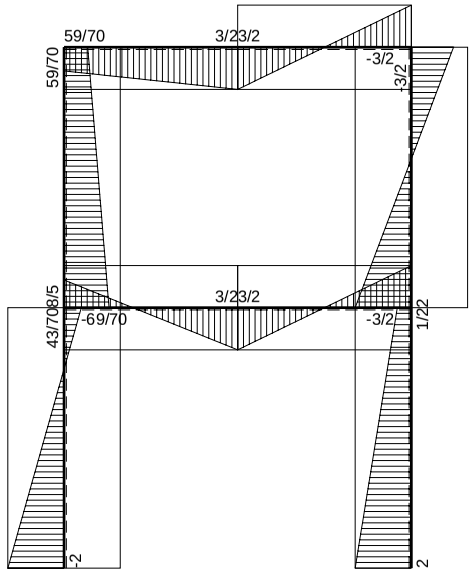
$$\begin{bmatrix} H_Ab \\ V_{BC}b \\ H_{BC}b \\ V_Ab \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ -1/3 & -2/3 & -1/3 & 0 & 0 & -1/3 & 1 \\ 0 & -1/4 & 0 & 0 & -1/4 & 0 & 7/2 \end{bmatrix}$$





F





F Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq} = 1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-2
W_{AB}+	1	0	0	0	0	0	0	≤	2
W_{BA}-	0	1	0	0	0	0	0	≥	-2
W_{BA}+	0	1	0	0	0	0	0	≤	2
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	1	0	0	0	0	≤	3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	1	0	0	0	≤	3/2
W_{DC}-	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC}+	0	0	1	-2	0	0	-12	≤	3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-2
W_{DE}+	0	0	0	0	1	0	0	≤	2
W_{ED}-	-1	-1	0	0	-1	0	9	≥	-2
W_{ED}+	-1	-1	0	0	-1	0	9	≤	2
W_{BF}-	0	-1	-1	0	0	0	0	≥	-2
W_{BF}+	0	-1	-1	0	0	0	0	≤	2
W_{FG}-	0	0	0	0	0	1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	1	0	≤	3/2
W_{GH}-	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH}+	0	1/2	1	-1	1/2	1	-7	≤	3/2
W_{HG}-	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG}+	0	-1	-2	2	-1	-1	6	≤	3/2
W_{DH}-	0	0	-1	2	-1	0	12	≥	-2
W_{DH}+	0	0	-1	2	-1	0	12	≤	2
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-2
W_{AB}+	-1	0	0	0	0	0	0	≥	-2
W_{BA}-	0	1	0	0	0	0	0	≥	-2
W_{BA}+	0	-1	0	0	0	0	0	≥	-2
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	≥	-3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	≥	-3/2
W_{DC}-	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	12	≥	-3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-2
W_{DE}+	0	0	0	0	-1	0	0	≥	-2
W_{ED}-	-1	-1	0	0	-1	0	9	≥	-2
W_{ED}+	1	1	0	0	1	0	-9	≥	-2
W_{BF}-	0	-1	-1	0	0	0	0	≥	-2
W_{BF}+	0	1	1	0	0	0	0	≥	-2
W_{FG}-	0	0	0	0	0	1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	-1	0	≥	-3/2
W_{GH}-	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	-1	7	≥	-3/2
W_{HG}-	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG}+	0	1	2	-2	1	1	-6	≥	-3/2
W_{DH}-	0	0	-1	2	-1	0	12	≥	-2
W_{DH}+	0	0	1	-2	1	0	-12	≥	-2
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-2
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-2
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-2
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-2
W_{BC}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC}-	0	0	1	-2	0	0	0	0	-1	2	0	0	-12	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	0	0	1	-2	0	0	12	≤	-3/2
W_{DE}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-2
W_{DE}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-2
W_{ED}-	-1	-1	0	0	-1	0	1	1	0	0	1	0	9	≥	-2
W_{ED}+	1	1	0	0	1	0	-1	-1	0	0	-1	0	-9	≤	-2
W_{BF}-	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-2
W_{BF}+	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-2
W_{FG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-3/2
W_{GH}-	0	1/2	1	-1	1/2	1	0	-1/2	-1	1	-1/2	-1	-7	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	-1	0	1/2	1	-1	1/2	1	7	≤	-3/2
W_{HG}-	0	-1	-2	2	-1	-1	0	1	2	-2	1	1	6	≥	-3/2
W_{HG}+	0	1	2	-2	1	1	0	-1	-2	2	-1	-1	-6	≤	-3/2
W_{DH}-	0	0	-1	2	-1	0	0	0	1	-2	1	0	12	≥	-2
W_{DH}+	0	0	1	-2	1	0	0	0	-1	2	-1	0	-12	≤	-2
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	1	-2	0	0	-12	1	≥	-3/2
φ_{DC}+	0	0	-1	2	0	0	12	-1	≥	-3/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	-1	-1	0	0	-1	0	9	3	≥	-2
φ_{ED}+	1	1	0	0	1	0	-9	-3	≥	-2
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-2
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	0	1/2	1	-1	1/2	1	-7	-2	≥	-3/2
φ_{GH}+	0	-1/2	-1	1	-1/2	-1	7	2	≥	-3/2
φ_{HG}-	0	-1	-2	2	-1	-1	6	3	≥	-3/2
φ_{HG}+	0	1	2	-2	1	1	-6	-3	≥	-3/2
φ_{DH}-	0	0	-1	2	-1	0	12	0	≥	-2
φ_{DH}+	0	0	1	-2	1	0	-12	0	≥	-2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 9-7

	X	Y	Z	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	0	0	1/12	-1/6	0	0	-1/12	1/12	≥	-1/8
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	-1	-1	3/4	-3/2	-1	0	-3/4	15/4	≥	-25/8
φ_{ED}+	1	1	-3/4	3/2	1	0	3/4	-15/4	≥	-7/8
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-2
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	0	1/2	5/12	1/6	1/2	1	7/12	-31/12	≥	-5/8
φ_{GH}+	0	-1/2	-5/12	-1/6	-1/2	-1	-7/12	31/12	≥	-19/8
φ_{HG}-	0	-1	-3/2	1	-1	-1	-1/2	7/2	≥	-9/4
φ_{HG}+	0	1	3/2	-1	1	1	1/2	-7/2	≥	-3/4
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	1/12	-1/6	0	0	-1/12	1/12	=	-1/8

Scambio pivotale 14-3

	X	Y	φ_{ED}+	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	4/3	4/3	-4/3	2	4/3	0	1	-6	≥	-8/3
φ_{BC}+	-4/3	-4/3	4/3	-2	-4/3	0	-1	6	≥	-1/3
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	1/9	1/9	-1/9	0	1/9	0	0	-1/3	≥	-2/9
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	4/3	4/3	-4/3	2	4/3	0	1	-5	≥	-7/6
φ_{BF}-	-4/3	-7/3	4/3	-2	-4/3	0	-1	7	≥	-5/6
φ_{BF}+	4/3	7/3	-4/3	2	4/3	0	1	-7	≥	-19/6
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	5/9	19/18	-5/9	1	19/18	1	1	-14/3	≥	-10/9
φ_{GH}+	-5/9	-19/18	5/9	-1	-19/18	-1	-1	14/3	≥	-17/9
φ_{HG}-	-2	-3	2	-2	-3	-1	-2	11	≥	-1/2
φ_{HG}+	2	3	-2	2	3	1	2	-11	≥	-5/2
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	1/9	1/9	-1/9	0	1/9	0	0	-1/3	=	-2/9

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Scambio pivotale 6-1

	φ_{BC}+	Y	φ_{ED}+	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	-3/4	-1	1	-3/2	-1	0	-3/4	7/2	≥	-9/4
φ_{AB}+	3/4	1	-1	3/2	1	0	3/4	-7/2	≥	-7/4
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	-3/4	-1	1	-3/2	-1	0	-3/4	9/2	≥	-1/4
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	-1/12	0	0	-1/6	0	0	-1/12	1/6	≥	-1/4
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	0	0	0	0	0	0	1	≥	-3/2
φ_{BF}-	1	-1	0	0	0	0	0	1	≥	-1/2
φ_{BF}+	-1	1	0	0	0	0	0	-1	≥	-7/2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	-5/12	1/2	0	1/6	1/2	1	7/12	-13/6	≥	-5/4
φ_{GH}+	5/12	-1/2	0	-1/6	-1/2	-1	-7/12	13/6	≥	-7/4
φ_{HG}-	3/2	-1	0	1	-1	-1	-1/2	2	≥	0
φ_{HG}+	-3/2	1	0	-1	1	1	1/2	-2	≥	-3
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	-1/12	0	0	-1/6	0	0	-1/12	1/6	=	-1/4

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Scambio pivotale 2-8

	φ_{BC}+	Y	φ_{ED}+	T	S	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	3/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1/2
φ_{BA}-	-3/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
φ_{BA}+	3/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-5/2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	3/14	2/7	-2/7	3/7	2/7	0	3/14	-9/7	≥	-5/2
φ_{CD}-	-3/14	-2/7	2/7	4/7	-2/7	0	-3/14	2/7	≥	-1
φ_{CD}+	3/14	2/7	-2/7	-4/7	2/7	0	3/14	-2/7	≥	-2
αbF	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	≥	-1/3
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	-3/14	-2/7	2/7	-3/7	5/7	0	-3/14	2/7	≥	-3/2
φ_{DE}+	3/14	2/7	-2/7	3/7	-5/7	0	3/14	-2/7	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-2
φ_{BF}-	17/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1
φ_{BF}+	-17/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3
φ_{FG}-	-3/14	-2/7	2/7	-3/7	-2/7	1	-3/14	2/7	≥	-1
φ_{FG}+	3/14	2/7	-2/7	3/7	2/7	-1	3/14	-2/7	≥	-2
φ_{GH}-	-37/42	-5/42	13/21	-16/21	-5/42	1	5/42	13/21	≥	-1/6
φ_{GH}+	37/42	5/42	-13/21	16/21	5/42	-1	-5/42	-13/21	≥	-17/6
φ_{HG}-	27/14	-3/7	-4/7	13/7	-3/7	-1	-1/14	-4/7	≥	-1
φ_{HG}+	-27/14	3/7	4/7	-13/7	3/7	1	1/14	4/7	≥	-2
φ_{DH}-	3/14	2/7	-2/7	3/7	-5/7	0	-11/14	-2/7	≥	-4
φ_{DH}+	-3/14	-2/7	2/7	-3/7	5/7	0	11/14	2/7	≥	0
L_X	-3/14	-2/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
Max	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	=	-1/3

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Scambio pivotale 24-2

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	S	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	0	-1	0	0	1	0	1	0	≥	-1/2
φ_{BA}-	-3/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3/2
φ_{BA}+	3/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-5/2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	0	-1	0	0	1	0	1	-1	≥	-5/2
φ_{CD}-	0	1	0	1	-1	0	-1	0	≥	-1
φ_{CD}+	0	-1	0	-1	1	0	1	0	≥	-2
αbF	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	≥	-1/3
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	-1	0	0	1	0	1	0	≥	-2
φ_{BF}-	7/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-1
φ_{BF}+	-7/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3
φ_{FG}-	0	1	0	0	-1	1	-1	0	≥	-1
φ_{FG}+	0	-1	0	0	1	-1	1	0	≥	-2
φ_{GH}-	-19/24	5/12	1/2	-7/12	-5/12	1	-5/24	1/2	≥	-1/6
φ_{GH}+	19/24	-5/12	-1/2	7/12	5/12	-1	5/24	-1/2	≥	-17/6
φ_{HG}-	9/4	3/2	-1	5/2	-3/2	-1	-5/4	-1	≥	-1
φ_{HG}+	-9/4	-3/2	1	-5/2	3/2	1	5/4	1	≥	-2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	-3/4	-7/2	1	-3/2	5/2	0	11/4	1	≥	0
L_X	0	1	0	0	-1	0	-1	0	≥	-3/2
Max	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	=	-1/3

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Scambio pivotale 19-5

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	φ_{GH}-	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-9/10
φ_{BA}-	-18/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-21/10
φ_{BA}+	18/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-19/10
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	1/5	≥	-29/10
φ_{CD}-	19/10	0	-6/5	12/5	12/5	-12/5	-1/2	-6/5	≥	-3/5
φ_{CD}+	-19/10	0	6/5	-12/5	-12/5	12/5	1/2	6/5	≥	-12/5
αbF	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	≥	-2/5
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-29/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-12/5
φ_{BF}-	23/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-2/5
φ_{BF}+	-23/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-18/5
φ_{FG}-	19/10	0	-6/5	7/5	12/5	-7/5	-1/2	-6/5	≥	-3/5
φ_{FG}+	-19/10	0	6/5	-7/5	-12/5	7/5	1/2	6/5	≥	-12/5
S	-19/10	1	6/5	-7/5	-12/5	12/5	-1/2	6/5	≥	-2/5
φ_{GH}+	0	0	0	0	-1	0	0	0	≥	-3
φ_{HG}-	51/10	0	-14/5	23/5	18/5	-23/5	-1/2	-14/5	≥	-2/5
φ_{HG}+	-51/10	0	14/5	-23/5	-18/5	23/5	1/2	14/5	≥	-13/5
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	-11/2	-1	4	-5	-6	6	3/2	4	≥	-1
L_X	19/10	0	-6/5	7/5	12/5	-12/5	-1/2	-6/5	≥	-11/10
Max	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	=	-2/5

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Scambio pivotale 21-6

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	φ_{GH}-	φ_{HG}-	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	35/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-51/46
φ_{BA}-	9/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-111/46
φ_{BA}+	-9/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-73/46
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	35/46	0	-6/23	1	-12/23	-12/23	11/46	-29/23	≥	-143/46
φ_{CD}-	-35/46	0	6/23	0	12/23	12/23	-11/46	6/23	≥	-9/23
φ_{CD}+	35/46	0	-6/23	0	-12/23	-12/23	11/46	-6/23	≥	-60/23
αbF	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	≥	-10/23
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-60/23
φ_{BF}-	14/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-2/23
φ_{BF}+	-14/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-90/23
φ_{FG}-	8/23	0	-8/23	0	30/23	7/23	-8/23	-8/23	≥	-11/23
φ_{FG}+	-8/23	0	8/23	0	-30/23	-7/23	8/23	8/23	≥	-58/23
S	35/46	1	-6/23	1	-12/23	-12/23	-35/46	-6/23	≥	-14/23
φ_{GH}+	0	0	0	0	-1	0	0	0	≥	-3
R	51/46	0	-14/23	1	18/23	-5/23	-5/46	-14/23	≥	-2/23
φ_{HG}+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	53/46	-1	8/23	1	-30/23	-30/23	39/46	8/23	≥	-35/23
L_X	-35/46	0	6/23	-1	12/23	12/23	-11/46	6/23	≥	-41/46
Max	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	=	-10/23

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Scambio pivotale 7-1

	φ_{CD}^-	φ_{DH}^+	φ_{ED}^+	T	φ_{GH}^-	φ_{HG}^-	φ_{DC}^-	φ_{AB}^+		Fb
φ_{AB}^-	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
φ_{BA}^-	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
φ_{BA}^+	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
φ_{BC}^-	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
X	-1	0	0	1	0	0	0	-1	≥	-7/2
φ_{BC}^+	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
φ_{CD}^+	-1	0	0	0	0	0	0	0	≥	-3
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
φ_{DC}^+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}^-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}^+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}^-	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
φ_{BF}^-	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
φ_{BF}^+	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
φ_{FG}^-	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{FG}^+	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
φ_{GH}^+	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{HG}^+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}^-	0	-1	0	0	0	0	0	0	≥	-4
Y	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_X	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35

PROCEDIMENTO E RISULTATI Nome: AL4.006

Tableau finale

	φ_{CD}^-	φ_{DH}^+	φ_{ED}^+	T	φ_{GH}^-	φ_{HG}^-	φ_{DC}^-	φ_{AB}^+		Fb
φ_{AB}^-	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
φ_{BA}^-	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
φ_{BA}^+	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
φ_{BC}^-	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
X	-1	0	0	1	0	0	0	-1	≥	-7/2
φ_{BC}^+	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
φ_{CD}^+	-1	0	0	0	0	0	0	0	≥	-3
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
φ_{DC}^+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}^-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}^+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}^-	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
φ_{BF}^-	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
φ_{BF}^+	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
φ_{FG}^-	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{FG}^+	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
φ_{GH}^+	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{HG}^+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}^-	0	-1	0	0	0	0	0	0	≥	-4
Y	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_X	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/35
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	0
φ_{BC}-	0	0	0	0	0	0	0	0	≥	0
φ_{BC}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{DE}-	0	0	0	0	0	0	0	0	≥	0
φ_{DE}+	0	0	0	0	0	0	0	0	≥	0
φ_{ED}-	0	0	0	0	0	0	0	0	≥	0
φ_{ED}+	0	0	0	0	0	0	0	0	≥	1/35
φ_{BF}-	0	0	0	0	0	0	0	0	≥	0
φ_{BF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FG}-	0	0	0	0	0	0	0	0	≥	0
φ_{FG}+	0	0	0	0	0	0	0	0	≥	0
φ_{GH}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{GH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HG}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{HG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	7/2	74/35	87/35	0	1	23/35	16/35	3/2	=	-16/35

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
2	43/70	69/70	-3/2	-1/2	-59/70

Variabili soluzione differenza tra rotazioni

φ_{AB}	1/35
φ_{BA}	0
φ_{BC}	0
φ_{CD}	-2/35
φ_{DC}	-2/35
φ_{DE}	0
φ_{ED}	1/35
φ_{BF}	0
φ_{FG}	0
φ_{GH}	-2/35
φ_{HG}	-2/35
φ_{DH}	0

REAZIONI Fattore di collasso = 16/35

H_A = -61/70F
V_A = 11/7F
W_A = 2Fb
H_E = -1/2F
V_E = 3F
W_E = 2Fb

H_{AB} = -61/70F	H_{BC} = -2/3F	H_{CD} = -2/3F	H_{DE} = 1/2F	H_{BF} = 53/210F
V_{AB} = 11/7F	V_{BC} = 87/70F	V_{CD} = -3/2F	V_{DE} = -3F	V_{BF} = 23/70F
W_{AB} = 2Fb	W_{BC} = 69/70Fb	W_{CD} = -3/2Fb	W_{DE} = -1/2Fb	W_{BF} = -8/5Fb
H_{BA} = 61/70F	H_{CB} = 2/3F	H_{DC} = 2/3F	H_{ED} = -1/2F	H_{FB} = -53/210F
V_{BA} = -11/7F	V_{CB} = -87/70F	V_{DC} = 3/2F	V_{ED} = 3F	V_{FB} = -23/70F
W_{BA} = 43/70Fb	W_{CB} = 3/2Fb	W_{DC} = -3/2Fb	W_{ED} = 2Fb	W_{FB} = 59/70Fb

H_{FG} = 7/6F	H_{GH} = 7/6F	H_{HD} = 7/6F
V_{FG} = 23/70F	V_{GH} = -3/2F	V_{HD} = -3/2F
W_{FG} = -59/70Fb	W_{GH} = -3/2Fb	W_{HD} = 3/2Fb
H_{GF} = -7/6F	H_{HG} = -7/6F	H_{DH} = -7/6F
V_{GF} = -23/70F	V_{HG} = 3/2F	V_{DH} = 3/2F
W_{GF} = 3/2Fb	W_{HG} = -3/2Fb	W_{DH} = 2Fb

SPOSTAMENTI NODALI

u_{AAB} = 0	u_B = 3/35δ	u_{CCB} = 3/35δ	u_D = 3/35δ	u_{EED} = 0
v_{AAB} = 0	v_B = 0	v_{CCB} = -2/35δ	v_D = 0	v_{EED} = 0
φ_{AAB} = -1/35δ/b	φ_B = -1/35δ/b	φ_{CCB} = -1/35δ/b	φ_D = 1/35δ/b	φ_E = -1/35δ/b

$u_F = 6/35\delta$ $u_{GGF} = 6/35\delta$ $u_{HHG} = 6/35\delta$
$v_F = 0$ $v_{GGF} = -2/35\delta$ $v_{HHG} = 0$
$\varphi_F = -1/35\delta/b$ $\varphi_{GGF} = -1/35\delta/b$ $\varphi_{HHD} = 1/35\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$ $u_{BBC} = 3/35\delta$ $u_{CCD} = 3/35\delta$ $u_{DDE} = 3/35\delta$ $u_{BBF} = 3/35\delta$
$v_{AAB} = 0$ $v_{BBC} = 0$ $v_{CCD} = -2/35\delta$ $v_{DDE} = 0$ $v_{BBF} = 0$
$\varphi_{AAB} = -1/35\delta/b$ $\varphi_{BBC} = -1/35\delta/b$ $\varphi_{CCD} = 1/35\delta/b$ $\varphi_{DDE} = -1/35\delta/b$ $\varphi_{BBF} = -1/35\delta/b$

$u_{FFG} = 6/35\delta$ $u_{GGH} = 6/35\delta$ $u_{HHD} = 6/35\delta$
$v_{FFG} = 0$ $v_{GGH} = -2/35\delta$ $v_{HHD} = 0$
$\varphi_{FFG} = -1/35\delta/b$ $\varphi_{GGH} = 1/35\delta/b$ $\varphi_{HHD} = -1/35\delta/b$

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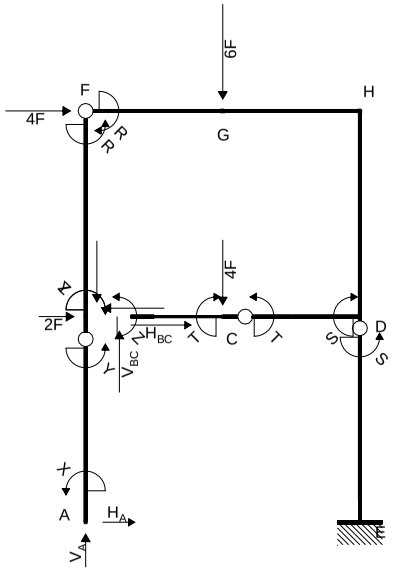


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

$3H_Ab - 4V_Ab = -Xb + Sb - 8Fb$

Rotazione intorno a C: aste CB

$-2V_{BC}b = -Zb + Tb$

Rotazione intorno a F: aste FB BA

$6H_Ab - 3H_{BC}b = -Xb + Zb - Rb - 6Fb$

Rotazione intorno a B: aste BA

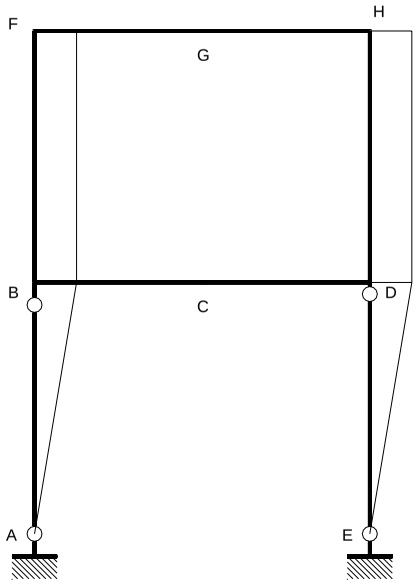
$3H_Ab = -Xb - Yb$

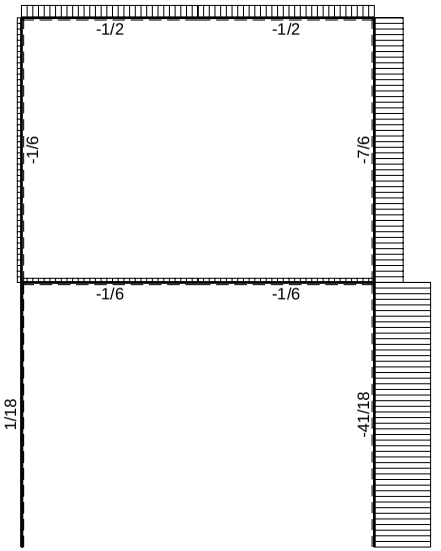
Matrice di equilibrio

	H_Ab	V_Ab	$H_{BC}b$	$V_{BC}b$		Xb	Yb	Zb	Tb	Sb	Rb	Fb
φ_{DE}	3	-4	0	0	=	-1	0	0	0	1	0	-8
φ_{CD}	0	0	0	-2		0	0	-1	1	0	0	0
φ_{FB}	6	0	-3	0		-1	0	1	0	0	-1	-6
φ_{BA}	3	0	0	0		-1	-1	0	0	0	0	0

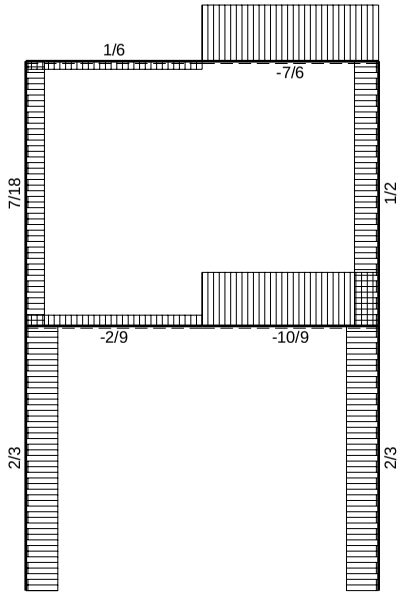
Soluzione del sistema

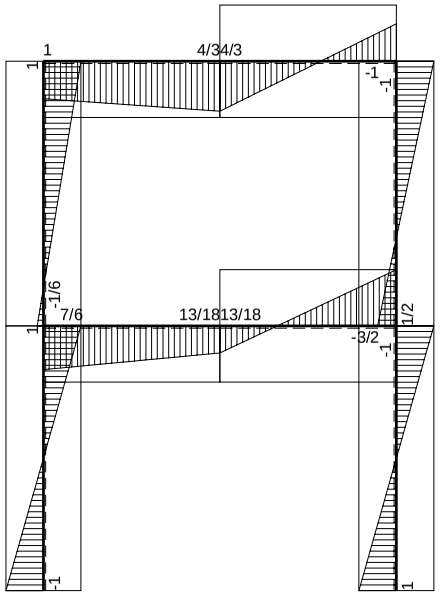
		Xb	Yb	Zb	Tb	Sb	Rb	Fb
H_Ab		-1/3	-1/3	0	0	0	0	0
$V_{BC}b$	=	0	0	1/2	-1/2	0	0	0
$H_{BC}b$		-1/3	-2/3	-1/3	0	0	1/3	2
V_Ab		0	-1/4	0	0	-1/4	0	2





F





F Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq} = 1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FB}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	1	0	0	0	0	≤	3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	1	0	0	0	≤	3/2
W_{DC}-	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC}+	0	0	1	-2	0	0	-8	≤	3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-1
W_{DE}+	0	0	0	0	1	0	0	≤	1
W_{ED}-	-1	-1	0	0	-1	0	18	≥	-1
W_{ED}+	-1	-1	0	0	-1	0	18	≤	1
W_{BF}-	0	-1	-1	0	0	0	0	≥	-1
W_{BF}+	0	-1	-1	0	0	0	0	≤	1
W_{FB}-	0	0	0	0	0	1	0	≥	-1
W_{FB}+	0	0	0	0	0	1	0	≤	1
W_{GH}-	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH}+	0	1/2	1	-1	1/2	-1	-4	≤	3/2
W_{HD}-	0	1	2	-2	1	-1	4	≥	-1
W_{HD}+	0	1	2	-2	1	-1	4	≤	1
W_{DH}-	0	0	-1	2	-1	0	8	≥	-1
W_{DH}+	0	0	-1	2	-1	0	8	≤	1
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	≥	-3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	≥	-3/2
W_{DC}-	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	8	≥	-3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-1
W_{DE}+	0	0	0	0	-1	0	0	≥	-1
W_{ED}-	-1	-1	0	0	-1	0	18	≥	-1
W_{ED}+	1	1	0	0	1	0	-18	≥	-1
W_{BF}-	0	-1	-1	0	0	0	0	≥	-1
W_{BF}+	0	1	1	0	0	0	0	≥	-1
W_{FB}-	0	0	0	0	0	1	0	≥	-1
W_{FB}+	0	0	0	0	0	-1	0	≥	-1
W_{GH}-	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	1	4	≥	-3/2
W_{HD}-	0	1	2	-2	1	-1	4	≥	-1
W_{HD}+	0	-1	-2	2	-1	1	-4	≥	-1
W_{DH}-	0	0	-1	2	-1	0	8	≥	-1
W_{DH}+	0	0	1	-2	1	0	-8	≥	-1
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{BC}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC}-	0	0	1	-2	0	0	0	0	-1	2	0	0	-8	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	0	0	1	-2	0	0	8	≤	-3/2
W_{DE}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{DE}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{ED}-	-1	-1	0	0	-1	0	1	1	0	0	1	0	18	≥	-1
W_{ED}+	1	1	0	0	1	0	-1	-1	0	0	-1	0	-18	≤	-1
W_{BF}-	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-1
W_{BF}+	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-1
W_{FB}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-1
W_{FB}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-1
W_{GH}-	0	1/2	1	-1	1/2	-1	0	-1/2	-1	1	-1/2	1	-4	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	1	0	1/2	1	-1	1/2	-1	4	≤	-3/2
W_{HD}-	0	1	2	-2	1	-1	0	-1	-2	2	-1	1	4	≥	-1
W_{HD}+	0	-1	-2	2	-1	1	0	1	2	-2	1	-1	-4	≤	-1
W_{DH}-	0	0	-1	2	-1	0	0	0	1	-2	1	0	8	≥	-1
W_{DH}+	0	0	1	-2	1	0	0	0	-1	2	-1	0	-8	≤	-1
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	1	-2	0	0	-8	1	≥	-3/2
φ_{DC}+	0	0	-1	2	0	0	8	-1	≥	-3/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	-1	-1	0	0	-1	0	18	3	≥	-1
φ_{ED}+	1	1	0	0	1	0	-18	-3	≥	-1
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	0	1/2	1	-1	1/2	-1	-4	0	≥	-3/2
φ_{GH}+	0	-1/2	-1	1	-1/2	1	4	0	≥	-3/2
φ_{HD}-	0	1	2	-2	1	-1	4	-1	≥	-1
φ_{HD}+	0	-1	-2	2	-1	1	-4	1	≥	-1
φ_{DH}-	0	0	-1	2	-1	0	8	0	≥	-1
φ_{DH}+	0	0	1	-2	1	0	-8	0	≥	-1
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 14-7

	X	Y	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	-4/9	-4/9	1	-2	-4/9	0	4/9	7/3	≥	-19/18
φ_{DC}+	4/9	4/9	-1	2	4/9	0	-4/9	-7/3	≥	-35/18
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	1/18	1/18	0	0	1/18	0	-1/18	-1/6	≥	-1/18
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	-2/9	5/18	1	-1	5/18	-1	2/9	2/3	≥	-23/18
φ_{GH}+	2/9	-5/18	-1	1	-5/18	1	-2/9	-2/3	≥	-31/18
φ_{HD}-	2/9	11/9	2	-2	11/9	-1	-2/9	-5/3	≥	-11/9
φ_{HD}+	-2/9	-11/9	-2	2	-11/9	1	2/9	5/3	≥	-7/9
φ_{DH}-	4/9	4/9	-1	2	-5/9	0	-4/9	-4/3	≥	-13/9
φ_{DH}+	-4/9	-4/9	1	-2	5/9	0	4/9	4/3	≥	-5/9
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	1/18	1/18	0	0	1/18	0	-1/18	-1/6	=	-1/18

Scambio pivotale 2-1

	φ_{AB}+	Y	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	4/9	-4/9	1	-2	-4/9	0	4/9	17/9	≥	-11/18
φ_{DC}+	-4/9	4/9	-1	2	4/9	0	-4/9	-17/9	≥	-43/18
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	≥	-1/9
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	2/9	5/18	1	-1	5/18	-1	2/9	4/9	≥	-19/18
φ_{GH}+	-2/9	-5/18	-1	1	-5/18	1	-2/9	-4/9	≥	-35/18
φ_{HD}-	-2/9	11/9	2	-2	11/9	-1	-2/9	-13/9	≥	-13/9
φ_{HD}+	2/9	-11/9	-2	2	-11/9	1	2/9	13/9	≥	-5/9
φ_{DH}-	-4/9	4/9	-1	2	-5/9	0	-4/9	-8/9	≥	-17/9
φ_{DH}+	4/9	-4/9	1	-2	5/9	0	4/9	8/9	≥	-1/9
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	=	-1/9

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Scambio pivotale 24-2

	φ_{AB}+	φ_{DH}+	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	1	-9/4	9/4	-9/2	5/4	0	1	1	≥	-5/4
φ_{BA}+	-1	9/4	-9/4	9/2	-5/4	0	-1	-1	≥	-3/4
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	1	0	0	-1	0	0	1	≥	-1/2
φ_{DC}+	0	-1	0	0	1	0	0	-1	≥	-5/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	-1/8	1/8	-1/4	1/8	0	0	0	≥	-1/8
φ_{BF}-	-1	9/4	-13/4	9/2	-5/4	0	-1	0	≥	-3/4
φ_{BF}+	1	-9/4	13/4	-9/2	5/4	0	1	0	≥	-5/4
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/2	-5/8	13/8	-9/4	5/8	-1	1/2	1	≥	-9/8
φ_{GH}+	-1/2	5/8	-13/8	9/4	-5/8	1	-1/2	-1	≥	-15/8
φ_{HD}-	1	-11/4	19/4	-15/2	11/4	-1	1	1	≥	-7/4
φ_{HD}+	-1	11/4	-19/4	15/2	-11/4	1	-1	-1	≥	-1/4
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	1	-9/4	9/4	-9/2	5/4	0	1	2	≥	-1/4
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	-1/8	1/8	-1/4	1/8	0	0	0	=	-1/8

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Scambio pivotale 22-3

	φ_{AB}+	φ_{DH}+	φ_{HD}+	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	10/19	≥	-26/19
φ_{BA}+	-10/19	18/19	9/19	18/19	1/19	-9/19	-10/19	-10/19	≥	-12/19
φ_{BC}-	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-23/19	≥	-59/38
φ_{BC}+	4/19	-11/19	4/19	-30/19	11/19	-4/19	4/19	23/19	≥	-55/38
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	1	0	0	-1	0	0	1	≥	-1/2
φ_{DC}+	0	-1	0	0	1	0	0	-1	≥	-5/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	≥	-5/38
φ_{BF}-	-6/19	7/19	13/19	-12/19	12/19	-13/19	-6/19	13/19	≥	-11/19
φ_{BF}+	6/19	-7/19	-13/19	12/19	-12/19	13/19	6/19	-13/19	≥	-27/19
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	3/19	6/19	-13/38	6/19	-6/19	-25/38	3/19	25/38	≥	-23/19
φ_{GH}+	-3/19	-6/19	13/38	-6/19	6/19	25/38	-3/19	-25/38	≥	-34/19
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-4/19	≥	-1/19
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	29/19	≥	-7/19
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	=	-5/38

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Scambio pivotale 22-5

	φ_{AB}+	φ_{DH}+	φ_{HD}+	T	Z	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	6/11	≥	-15/11
φ_{BA}+	-6/11	1	5/11	12/11	-1/11	-5/11	-6/11	-6/11	≥	-7/11
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	4/11	0	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-9/22
φ_{DC}+	-4/11	0	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-57/22
φ_{DE}-	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-12/11
φ_{DE}+	4/11	-1	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-10/11
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	≥	-3/22
φ_{BF}-	-6/11	1	5/11	12/11	-12/11	-5/11	-6/11	5/11	≥	-7/11
φ_{BF}+	6/11	-1	-5/11	-12/11	12/11	5/11	6/11	-5/11	≥	-15/11
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	3/11	0	-5/22	-6/11	6/11	-17/22	3/11	17/22	≥	-13/11
φ_{GH}+	-3/11	0	5/22	6/11	-6/11	17/22	-3/11	-17/22	≥	-20/11
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-4/11	≥	-1/11
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	17/11	≥	-4/11
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	=	-3/22

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Scambio pivotale 9-4

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	0	≥	-6/5
φ_{BA}+	-2/5	1	3/5	-2/5	3/5	-3/5	-2/5	0	≥	-4/5
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	2/15	0	2/15	-11/30	19/30	-2/15	2/15	-1/2	≥	-33/20
φ_{CD}+	-2/15	0	-2/15	11/30	-19/30	2/15	-2/15	1/2	≥	-27/20
T	2/15	0	2/15	-11/30	19/30	-2/15	2/15	1/2	≥	-3/20
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	≥	-3/20
φ_{BF}-	-2/5	1	3/5	-2/5	-2/5	-3/5	-2/5	1	≥	-4/5
φ_{BF}+	2/5	-1	-3/5	2/5	2/5	3/5	2/5	-1	≥	-6/5
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/5	0	-3/10	1/5	1/5	-7/10	1/5	1/2	≥	-11/10
φ_{GH}+	-1/5	0	3/10	-1/5	-1/5	7/10	-1/5	-1/2	≥	-19/10
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	1	≥	-1/5
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	=	-3/20

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Scambio pivotale 18-6

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	3/5	≥	-9/5
φ_{BA}+	-2/5	1	3/5	-2/5	3/5	3/5	-2/5	-3/5	≥	-1/5
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	2/15	0	2/15	-11/30	19/30	2/15	2/15	-19/30	≥	-91/60
φ_{CD}+	-2/15	0	-2/15	11/30	-19/30	-2/15	-2/15	19/30	≥	-89/60
T	2/15	0	2/15	-11/30	19/30	2/15	2/15	11/30	≥	-1/60
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	≥	-11/60
φ_{BF}-	-2/5	1	3/5	-2/5	-2/5	3/5	-2/5	2/5	≥	-1/5
φ_{BF}+	2/5	-1	-3/5	2/5	2/5	-3/5	2/5	-2/5	≥	-9/5
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/5	0	-3/10	1/5	1/5	7/10	1/5	-1/5	≥	-2/5
φ_{GH}+	-1/5	0	3/10	-1/5	-1/5	-7/10	-1/5	1/5	≥	-13/5
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	8/5	≥	-4/5
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	=	-11/60

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Scambio pivotale 4-8

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-4/3
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-1/3
φ_{BC}-	2/3	-5/3	-1	2/3	0	-1	2/3	5/3	≥	-7/6
φ_{BC}+	-2/3	5/3	1	-2/3	0	1	-2/3	-5/3	≥	-11/6
φ_{CD}-	5/9	-19/18	-1/2	1/18	0	-1/2	5/9	19/18	≥	-47/36
φ_{CD}+	-5/9	19/18	1/2	-1/18	0	1/2	-5/9	-19/18	≥	-61/36
T	-1/9	11/18	1/2	-11/18	1	1/2	-1/9	-11/18	≥	-5/36
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	≥	-7/36
φ_{BF}-	-2/3	5/3	1	-2/3	0	1	-2/3	-2/3	≥	-1/3
φ_{BF}+	2/3	-5/3	-1	2/3	0	-1	2/3	2/3	≥	-5/3
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	5/3	1	-2/3	1	0	-2/3	-5/3	≥	-4/3
φ_{GH}-	1/3	-1/3	-1/2	1/3	0	1/2	1/3	1/3	≥	-1/3
φ_{GH}+	-1/3	1/3	1/2	-1/3	0	-1/2	-1/3	-1/3	≥	-8/3
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	8/3	1	-5/3	1	1	-2/3	-5/3	≥	-5/6
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	-2/3	5/3	1	-2/3	1	1	-2/3	-8/3	≥	-4/3
L_X	2/3	-5/3	-1	2/3	-1	-1	2/3	5/3	≥	-7/6
Max	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	=	-7/36

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Scambio pivotale 12-2

	φ_{AB}+	φ_{DE}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
φ_{BC}-	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
φ_{BC}+	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
φ_{CD}-	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
φ_{CD}+	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
T	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	-1	0	0	0	0	0	0	≥	-2
φ_{DH}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
φ_{BF}-	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
φ_{BF}+	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
φ_{GH}-	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
φ_{GH}+	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{DH}-	0	1	0	-1	0	0	0	0	≥	-3/2
Y	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_X	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

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Tableau finale

	φ_{AB}+	φ_{DE}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
φ_{BC}-	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
φ_{BC}+	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
φ_{CD}-	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
φ_{CD}+	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
T	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	-1	0	0	0	0	0	0	≥	-2
φ_{DH}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
φ_{BF}-	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
φ_{BF}+	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
φ_{GH}-	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
φ_{GH}+	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{DH}-	0	1	0	-1	0	0	0	0	≥	-3/2
Y	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_X	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BC}-	0	0	0	0	0	0	0	0	≥	0
φ_{BC}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	0
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	0
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{DE}-	0	0	0	0	0	0	0	0	≥	0
φ_{DE}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{ED}-	0	0	0	0	0	0	0	0	≥	0
φ_{ED}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BF}-	0	0	0	0	0	0	0	0	≥	0
φ_{BF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FB}-	0	0	0	0	0	0	0	0	≥	0
φ_{FB}+	0	0	0	0	0	0	0	0	≥	0
φ_{GH}-	0	0	0	0	0	0	0	0	≥	0
φ_{GH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HD}-	0	0	0	0	0	0	0	0	≥	0
φ_{HD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	13/6	13/6	0	4/9	13/6	13/6	2/9	7/6	=	-2/9

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
1	1	-7/6	-13/18	1	1

Variabili soluzione differenza tra rotazioni

$$\begin{matrix}\varphi_{AB}\\ \varphi_{BA}\\ \varphi_{BC}\\ \varphi_{CD}\\ \varphi_{DC}\\ \varphi_{DE}\\ \varphi_{ED}\\ \varphi_{BF}\\ \varphi_{FB}\\ \varphi_{GH}\\ \varphi_{HD}\\ \varphi_{DH}\end{matrix}\begin{bmatrix}1/18\\ 1/18\\ 0\\ 0\\ 0\\ 1/18\\ 1/18\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

REAZIONI Fattore di collasso = 2/9

$H_A = -2/3F$
$V_A = -1/18F$
$W_A = Fb$
$H_E = -2/3F$
$V_E = 41/18F$
$W_E = Fb$

$H_{AB} = -2/3F$	$H_{BC} = 1/6F$	$H_{CD} = 1/6F$	$H_{DE} = 2/3F$	$H_{BF} = -7/18F$
$V_{AB} = -1/18F$	$V_{BC} = -2/9F$	$V_{CD} = -10/9F$	$V_{DE} = -41/18F$	$V_{BF} = 1/6F$
$W_{AB} = Fb$	$W_{BC} = -7/6Fb$	$W_{CD} = -13/18Fb$	$W_{DE} = Fb$	$W_{BF} = 1/6Fb$
$H_{BA} = 2/3F$	$H_{CB} = -1/6F$	$H_{DC} = -1/6F$	$H_{ED} = -2/3F$	$H_{FB} = 7/18F$
$V_{BA} = 1/18F$	$V_{CB} = 2/9F$	$V_{DC} = 10/9F$	$V_{ED} = 41/18F$	$V_{FB} = -1/6F$
$W_{BA} = Fb$	$W_{CB} = 13/18Fb$	$W_{DC} = -3/2Fb$	$W_{ED} = Fb$	$W_{FB} = Fb$

$H_{FG} = 1/2F$	$H_{GH} = 1/2F$	$H_{HD} = 1/2F$
$V_{FG} = 1/6F$	$V_{GH} = -7/6F$	$V_{HD} = -7/6F$
$W_{FG} = -Fb$	$W_{GH} = -4/3Fb$	$W_{HD} = Fb$
$H_{GF} = -1/2F$	$H_{HG} = -1/2F$	$H_{DH} = -1/2F$
$V_{GF} = -1/6F$	$V_{HG} = 7/6F$	$V_{DH} = 7/6F$
$W_{GF} = 4/3Fb$	$W_{HG} = -Fb$	$W_{DH} = 1/2Fb$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 1/6\delta$	$u_C = 1/6\delta$	$u_D = 1/6\delta$	$u_{EED} = 0$
$v_{AAB} = 0$	$v_B = 0$	$v_C = 0$	$v_D = 0$	$v_{EED} = 0$
$\varphi_{AAB} = -1/18\delta/b$	$\varphi_B = -1/18\delta/b$	$\varphi_C = 0$	$\varphi_D = 0$	$\varphi_E = -1/18\delta/b$

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$u_F = 1/6\delta$	$u_G = 1/6\delta$	$u_H = 1/6\delta$
$v_F = 0$	$v_G = 0$	$v_H = 0$
$\varphi_F = 0$	$\varphi_G = 0$	$\varphi_H = 0$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{BBC} = 1/6\delta$	$u_{CCD} = 1/6\delta$	$u_{DDE} = 1/6\delta$	$u_{BBF} = 1/6\delta$
$v_{AAB} = 0$	$v_{BBC} = 0$	$v_{CCD} = 0$	$v_{DDE} = 0$	$v_{BBF} = 0$
$\varphi_{AAB} = -1/18\delta/b$	$\varphi_{BBC} = 0$	$\varphi_{CCD} = 0$	$\varphi_{DDE} = -1/18\delta/b$	$\varphi_{BBF} = 0$

$u_{FFG} = 1/6\delta$	$u_{GGH} = 1/6\delta$	$u_{HHD} = 1/6\delta$
$v_{FFG} = 0$	$v_{GGH} = 0$	$v_{HHD} = 0$
$\varphi_{FFG} = 0$	$\varphi_{GGH} = 0$	$\varphi_{HHD} = 0$

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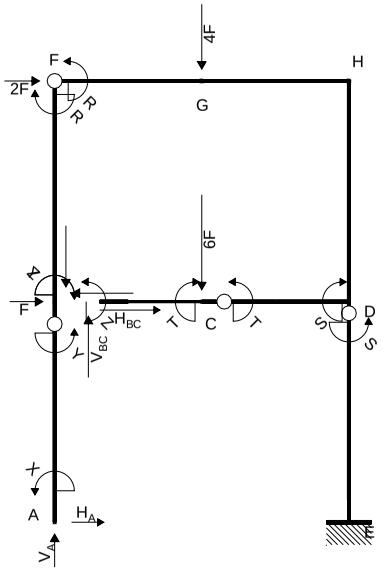


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

$3H_Ab - 4V_Ab = -Xb + Sb - 14Fb$

Rotazione intorno a C: aste CB

$-2V_{BC}b = -Zb + Tb$

Rotazione intorno a F: aste FB BA

$6H_Ab - 3H_{BC}b = -Xb + Zb + Rb - 3Fb$

Rotazione intorno a B: aste BA

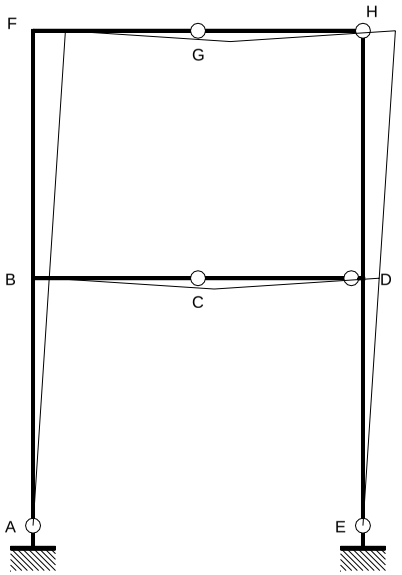
$3H_Ab = -Xb - Yb$

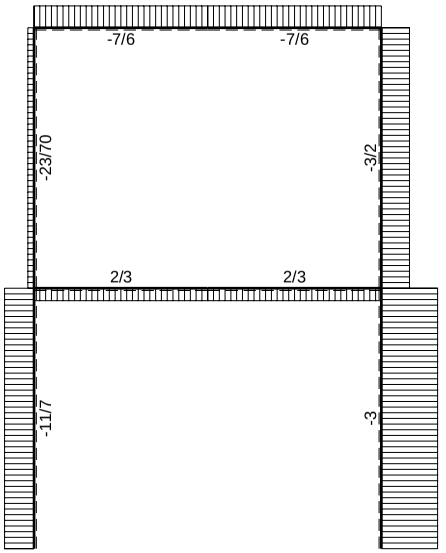
Matrice di equilibrio

$$\begin{array}{c} \\ \varphi_{DE} \\ \varphi_{CD} \\ \varphi_{FG} \\ \varphi_{BA} \end{array} \begin{bmatrix} H_Ab & V_Ab & H_{BC}b & V_{BC}b \\ 3 & -4 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 6 & 0 & -3 & 0 \\ 3 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1 & 0 & 0 & 0 & 1 & 0 & -14 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & -3 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

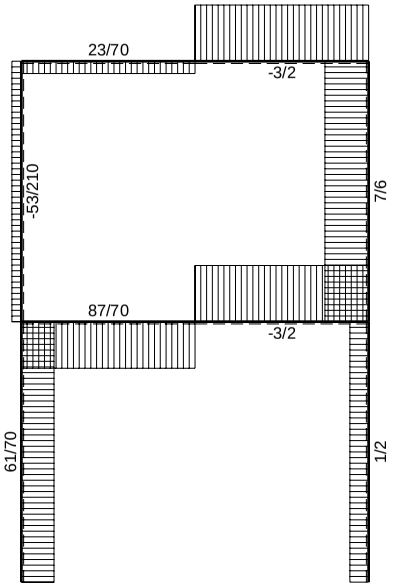
Soluzione del sistema

$$\begin{bmatrix} H_Ab \\ V_{BC}b \\ H_{BC}b \\ V_Ab \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ -1/3 & -2/3 & -1/3 & 0 & 0 & -1/3 & 1 \\ 0 & -1/4 & 0 & 0 & -1/4 & 0 & 7/2 \end{bmatrix}$$

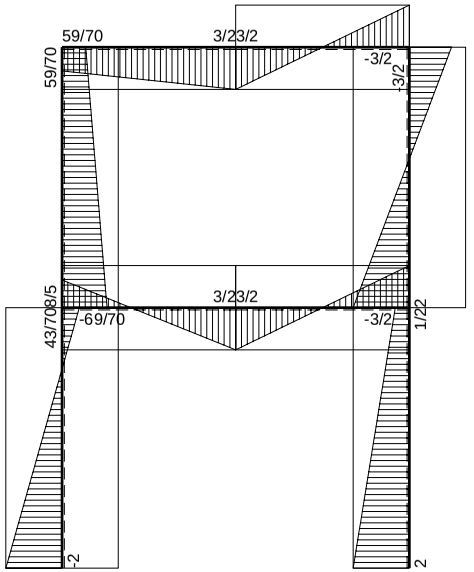




F



F



Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq} = 1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-2
W_{AB}+	1	0	0	0	0	0	0	≤	2
W_{BA}-	0	1	0	0	0	0	0	≥	-2
W_{BA}+	0	1	0	0	0	0	0	≤	2
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	1	0	0	0	0	≤	3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	1	0	0	0	≤	3/2
W_{DC}-	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC}+	0	0	1	-2	0	0	-12	≤	3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-2
W_{DE}+	0	0	0	0	1	0	0	≤	2
W_{ED}-	-1	-1	0	0	-1	0	9	≥	-2
W_{ED}+	-1	-1	0	0	-1	0	9	≤	2
W_{BF}-	0	-1	-1	0	0	0	0	≥	-2
W_{BF}+	0	-1	-1	0	0	0	0	≤	2
W_{FG}-	0	0	0	0	0	1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	1	0	≤	3/2
W_{GH}-	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH}+	0	1/2	1	-1	1/2	1	-7	≤	3/2
W_{HG}-	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG}+	0	-1	-2	2	-1	-1	6	≤	3/2
W_{DH}-	0	0	-1	2	-1	0	12	≥	-2
W_{DH}+	0	0	-1	2	-1	0	12	≤	2
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-2
W_{AB}+	-1	0	0	0	0	0	0	≥	-2
W_{BA}-	0	1	0	0	0	0	0	≥	-2
W_{BA}+	0	-1	0	0	0	0	0	≥	-2
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	≥	-3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	≥	-3/2
W_{DC}-	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	12	≥	-3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-2
W_{DE}+	0	0	0	0	-1	0	0	≥	-2
W_{ED}-	-1	-1	0	0	-1	0	9	≥	-2
W_{ED}+	1	1	0	0	1	0	-9	≥	-2
W_{BF}-	0	-1	-1	0	0	0	0	≥	-2
W_{BF}+	0	1	1	0	0	0	0	≥	-2
W_{FG}-	0	0	0	0	0	1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	-1	0	≥	-3/2
W_{GH}-	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	-1	7	≥	-3/2
W_{HG}-	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG}+	0	1	2	-2	1	1	-6	≥	-3/2
W_{DH}-	0	0	-1	2	-1	0	12	≥	-2
W_{DH}+	0	0	1	-2	1	0	-12	≥	-2
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-2
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-2
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-2
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-2
W_{BC}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC}-	0	0	1	-2	0	0	0	0	-1	2	0	0	-12	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	0	0	1	-2	0	0	12	≤	-3/2
W_{DE}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-2
W_{DE}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-2
W_{ED}-	-1	-1	0	0	-1	0	1	1	0	0	1	0	9	≥	-2
W_{ED}+	1	1	0	0	1	0	-1	-1	0	0	-1	0	-9	≤	-2
W_{BF}-	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-2
W_{BF}+	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-2
W_{FG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-3/2
W_{GH}-	0	1/2	1	-1	1/2	1	0	-1/2	-1	1	-1/2	-1	-7	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	-1	0	1/2	1	-1	1/2	1	7	≤	-3/2
W_{HG}-	0	-1	-2	2	-1	-1	0	1	2	-2	1	1	6	≥	-3/2
W_{HG}+	0	1	2	-2	1	1	0	-1	-2	2	-1	-1	-6	≤	-3/2
W_{DH}-	0	0	-1	2	-1	0	0	0	1	-2	1	0	12	≥	-2
W_{DH}+	0	0	1	-2	1	0	0	0	-1	2	-1	0	-12	≤	-2
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	1	-2	0	0	-12	1	≥	-3/2
φ_{DC}+	0	0	-1	2	0	0	12	-1	≥	-3/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	-1	-1	0	0	-1	0	9	3	≥	-2
φ_{ED}+	1	1	0	0	1	0	-9	-3	≥	-2
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-2
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	0	1/2	1	-1	1/2	1	-7	-2	≥	-3/2
φ_{GH}+	0	-1/2	-1	1	-1/2	-1	7	2	≥	-3/2
φ_{HG}-	0	-1	-2	2	-1	-1	6	3	≥	-3/2
φ_{HG}+	0	1	2	-2	1	1	-6	-3	≥	-3/2
φ_{DH}-	0	0	-1	2	-1	0	12	0	≥	-2
φ_{DH}+	0	0	1	-2	1	0	-12	0	≥	-2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 9-7

	X	Y	Z	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	0	0	1/12	-1/6	0	0	-1/12	1/12	≥	-1/8
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	-1	-1	3/4	-3/2	-1	0	-3/4	15/4	≥	-25/8
φ_{ED}+	1	1	-3/4	3/2	1	0	3/4	-15/4	≥	-7/8
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-2
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	0	1/2	5/12	1/6	1/2	1	7/12	-31/12	≥	-5/8
φ_{GH}+	0	-1/2	-5/12	-1/6	-1/2	-1	-7/12	31/12	≥	-19/8
φ_{HG}-	0	-1	-3/2	1	-1	-1	-1/2	7/2	≥	-9/4
φ_{HG}+	0	1	3/2	-1	1	1	1/2	-7/2	≥	-3/4
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	1/12	-1/6	0	0	-1/12	1/12	=	-1/8

Scambio pivotale 14-3

	X	Y	φ_{ED}+	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	4/3	4/3	-4/3	2	4/3	0	1	-6	≥	-8/3
φ_{BC}+	-4/3	-4/3	4/3	-2	-4/3	0	-1	6	≥	-1/3
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	1/9	1/9	-1/9	0	1/9	0	0	-1/3	≥	-2/9
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	4/3	4/3	-4/3	2	4/3	0	1	-5	≥	-7/6
φ_{BF}-	-4/3	-7/3	4/3	-2	-4/3	0	-1	7	≥	-5/6
φ_{BF}+	4/3	7/3	-4/3	2	4/3	0	1	-7	≥	-19/6
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	5/9	19/18	-5/9	1	19/18	1	1	-14/3	≥	-10/9
φ_{GH}+	-5/9	-19/18	5/9	-1	-19/18	-1	-1	14/3	≥	-17/9
φ_{HG}-	-2	-3	2	-2	-3	-1	-2	11	≥	-1/2
φ_{HG}+	2	3	-2	2	3	1	2	-11	≥	-5/2
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	1/9	1/9	-1/9	0	1/9	0	0	-1/3	=	-2/9

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Scambio pivotale 6-1

	φ_{BC}+	Y	φ_{ED}+	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	-3/4	-1	1	-3/2	-1	0	-3/4	7/2	≥	-9/4
φ_{AB}+	3/4	1	-1	3/2	1	0	3/4	-7/2	≥	-7/4
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	-3/4	-1	1	-3/2	-1	0	-3/4	9/2	≥	-1/4
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	-1/12	0	0	-1/6	0	0	-1/12	1/6	≥	-1/4
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	0	0	0	0	0	0	1	≥	-3/2
φ_{BF}-	1	-1	0	0	0	0	0	1	≥	-1/2
φ_{BF}+	-1	1	0	0	0	0	0	-1	≥	-7/2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	-5/12	1/2	0	1/6	1/2	1	7/12	-13/6	≥	-5/4
φ_{GH}+	5/12	-1/2	0	-1/6	-1/2	-1	-7/12	13/6	≥	-7/4
φ_{HG}-	3/2	-1	0	1	-1	-1	-1/2	2	≥	0
φ_{HG}+	-3/2	1	0	-1	1	1	1/2	-2	≥	-3
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	-1/12	0	0	-1/6	0	0	-1/12	1/6	=	-1/4

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Scambio pivotale 2-8

	φ_{BC}+	Y	φ_{ED}+	T	S	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	3/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1/2
φ_{BA}-	-3/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
φ_{BA}+	3/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-5/2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	3/14	2/7	-2/7	3/7	2/7	0	3/14	-9/7	≥	-5/2
φ_{CD}-	-3/14	-2/7	2/7	4/7	-2/7	0	-3/14	2/7	≥	-1
φ_{CD}+	3/14	2/7	-2/7	-4/7	2/7	0	3/14	-2/7	≥	-2
αbF	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	≥	-1/3
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	-3/14	-2/7	2/7	-3/7	5/7	0	-3/14	2/7	≥	-3/2
φ_{DE}+	3/14	2/7	-2/7	3/7	-5/7	0	3/14	-2/7	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-2
φ_{BF}-	17/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1
φ_{BF}+	-17/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3
φ_{FG}-	-3/14	-2/7	2/7	-3/7	-2/7	1	-3/14	2/7	≥	-1
φ_{FG}+	3/14	2/7	-2/7	3/7	2/7	-1	3/14	-2/7	≥	-2
φ_{GH}-	-37/42	-5/42	13/21	-16/21	-5/42	1	5/42	13/21	≥	-1/6
φ_{GH}+	37/42	5/42	-13/21	16/21	5/42	-1	-5/42	-13/21	≥	-17/6
φ_{HG}-	27/14	-3/7	-4/7	13/7	-3/7	-1	-1/14	-4/7	≥	-1
φ_{HG}+	-27/14	3/7	4/7	-13/7	3/7	1	1/14	4/7	≥	-2
φ_{DH}-	3/14	2/7	-2/7	3/7	-5/7	0	-11/14	-2/7	≥	-4
φ_{DH}+	-3/14	-2/7	2/7	-3/7	5/7	0	11/14	2/7	≥	0
L_X	-3/14	-2/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
Max	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	=	-1/3

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Scambio pivotale 24-2

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	S	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	0	-1	0	0	1	0	1	0	≥	-1/2
φ_{BA}-	-3/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3/2
φ_{BA}+	3/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-5/2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	0	-1	0	0	1	0	1	-1	≥	-5/2
φ_{CD}-	0	1	0	1	-1	0	-1	0	≥	-1
φ_{CD}+	0	-1	0	-1	1	0	1	0	≥	-2
αbF	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	≥	-1/3
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	-1	0	0	1	0	1	0	≥	-2
φ_{BF}-	7/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-1
φ_{BF}+	-7/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3
φ_{FG}-	0	1	0	0	-1	1	-1	0	≥	-1
φ_{FG}+	0	-1	0	0	1	-1	1	0	≥	-2
φ_{GH}-	-19/24	5/12	1/2	-7/12	-5/12	1	-5/24	1/2	≥	-1/6
φ_{GH}+	19/24	-5/12	-1/2	7/12	5/12	-1	5/24	-1/2	≥	-17/6
φ_{HG}-	9/4	3/2	-1	5/2	-3/2	-1	-5/4	-1	≥	-1
φ_{HG}+	-9/4	-3/2	1	-5/2	3/2	1	5/4	1	≥	-2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	-3/4	-7/2	1	-3/2	5/2	0	11/4	1	≥	0
L_X	0	1	0	0	-1	0	-1	0	≥	-3/2
Max	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	=	-1/3

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Scambio pivotale 19-5

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	φ_{GH}-	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-9/10
φ_{BA}-	-18/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-21/10
φ_{BA}+	18/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-19/10
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	1/5	≥	-29/10
φ_{CD}-	19/10	0	-6/5	12/5	12/5	-12/5	-1/2	-6/5	≥	-3/5
φ_{CD}+	-19/10	0	6/5	-12/5	-12/5	12/5	1/2	6/5	≥	-12/5
αbF	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	≥	-2/5
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-29/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-12/5
φ_{BF}-	23/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-2/5
φ_{BF}+	-23/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-18/5
φ_{FG}-	19/10	0	-6/5	7/5	12/5	-7/5	-1/2	-6/5	≥	-3/5
φ_{FG}+	-19/10	0	6/5	-7/5	-12/5	7/5	1/2	6/5	≥	-12/5
S	-19/10	1	6/5	-7/5	-12/5	12/5	-1/2	6/5	≥	-2/5
φ_{GH}+	0	0	0	0	-1	0	0	0	≥	-3
φ_{HG}-	51/10	0	-14/5	23/5	18/5	-23/5	-1/2	-14/5	≥	-2/5
φ_{HG}+	-51/10	0	14/5	-23/5	-18/5	23/5	1/2	14/5	≥	-13/5
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	-11/2	-1	4	-5	-6	6	3/2	4	≥	-1
L_X	19/10	0	-6/5	7/5	12/5	-12/5	-1/2	-6/5	≥	-11/10
Max	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	=	-2/5

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Scambio pivotale 21-6

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	φ_{GH}-	φ_{HG}-	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	35/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-51/46
φ_{BA}-	9/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-111/46
φ_{BA}+	-9/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-73/46
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	35/46	0	-6/23	1	-12/23	-12/23	11/46	-29/23	≥	-143/46
φ_{CD}-	-35/46	0	6/23	0	12/23	12/23	-11/46	6/23	≥	-9/23
φ_{CD}+	35/46	0	-6/23	0	-12/23	-12/23	11/46	-6/23	≥	-60/23
αbF	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	≥	-10/23
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-60/23
φ_{BF}-	14/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-2/23
φ_{BF}+	-14/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-90/23
φ_{FG}-	8/23	0	-8/23	0	30/23	7/23	-8/23	-8/23	≥	-11/23
φ_{FG}+	-8/23	0	8/23	0	-30/23	-7/23	8/23	8/23	≥	-58/23
S	35/46	1	-6/23	1	-12/23	-12/23	-35/46	-6/23	≥	-14/23
φ_{GH}+	0	0	0	0	-1	0	0	0	≥	-3
R	51/46	0	-14/23	1	18/23	-5/23	-5/46	-14/23	≥	-2/23
φ_{HG}+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	53/46	-1	8/23	1	-30/23	-30/23	39/46	8/23	≥	-35/23
L_X	-35/46	0	6/23	-1	12/23	12/23	-11/46	6/23	≥	-41/46
Max	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	=	-10/23

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Scambio pivotale 7-1

	φ_{CD}^-	φ_{DH}^+	φ_{ED}^+	T	φ_{GH}^-	φ_{HG}^-	φ_{DC}^-	φ_{AB}^+		Fb
φ_{AB}^-	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
φ_{BA}^-	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
φ_{BA}^+	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
φ_{BC}^-	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
X	-1	0	0	1	0	0	0	-1	≥	-7/2
φ_{BC}^+	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
φ_{CD}^+	-1	0	0	0	0	0	0	0	≥	-3
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
φ_{DC}^+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}^-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}^+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}^-	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
φ_{BF}^-	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
φ_{BF}^+	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
φ_{FG}^-	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{FG}^+	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
φ_{GH}^+	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{HG}^+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}^-	0	-1	0	0	0	0	0	0	≥	-4
Y	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_X	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35

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Tableau finale

	φ_{CD}^-	φ_{DH}^+	φ_{ED}^+	T	φ_{GH}^-	φ_{HG}^-	φ_{DC}^-	φ_{AB}^+		Fb
φ_{AB}^-	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
φ_{BA}^-	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
φ_{BA}^+	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
φ_{BC}^-	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
X	-1	0	0	1	0	0	0	-1	≥	-7/2
φ_{BC}^+	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
φ_{CD}^+	-1	0	0	0	0	0	0	0	≥	-3
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
φ_{DC}^+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}^-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}^+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}^-	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
φ_{BF}^-	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
φ_{BF}^+	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
φ_{FG}^-	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{FG}^+	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
φ_{GH}^+	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{HG}^+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}^-	0	-1	0	0	0	0	0	0	≥	-4
Y	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_X	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/35
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	0
φ_{BC}-	0	0	0	0	0	0	0	0	≥	0
φ_{BC}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{DE}-	0	0	0	0	0	0	0	0	≥	0
φ_{DE}+	0	0	0	0	0	0	0	0	≥	0
φ_{ED}-	0	0	0	0	0	0	0	0	≥	0
φ_{ED}+	0	0	0	0	0	0	0	0	≥	1/35
φ_{BF}-	0	0	0	0	0	0	0	0	≥	0
φ_{BF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FG}-	0	0	0	0	0	0	0	0	≥	0
φ_{FG}+	0	0	0	0	0	0	0	0	≥	0
φ_{GH}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{GH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HG}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{HG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	7/2	74/35	87/35	0	1	23/35	16/35	3/2	=	-16/35

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
2	43/70	69/70	-3/2	-1/2	-59/70

Variabili soluzione differenza tra rotazioni

φ_{AB}	1/35
φ_{BA}	0
φ_{BC}	0
φ_{CD}	-2/35
φ_{DC}	-2/35
φ_{DE}	0
φ_{ED}	1/35
φ_{BF}	0
φ_{FG}	0
φ_{GH}	-2/35
φ_{HG}	-2/35
φ_{DH}	0

REAZIONI Fattore di collasso = 16/35

H_A = -61/70F
V_A = 11/7F
W_A = 2Fb
H_E = -1/2F
V_E = 3F
W_E = 2Fb

H_{AB} = -61/70F	H_{BC} = -2/3F	H_{CD} = -2/3F	H_{DE} = 1/2F	H_{BF} = 53/210F
V_{AB} = 11/7F	V_{BC} = 87/70F	V_{CD} = -3/2F	V_{DE} = -3F	V_{BF} = 23/70F
W_{AB} = 2Fb	W_{BC} = 69/70Fb	W_{CD} = -3/2Fb	W_{DE} = -1/2Fb	W_{BF} = -8/5Fb
H_{BA} = 61/70F	H_{CB} = 2/3F	H_{DC} = 2/3F	H_{ED} = -1/2F	H_{FB} = -53/210F
V_{BA} = -11/7F	V_{CB} = -87/70F	V_{DC} = 3/2F	V_{ED} = 3F	V_{FB} = -23/70F
W_{BA} = 43/70Fb	W_{CB} = 3/2Fb	W_{DC} = -3/2Fb	W_{ED} = 2Fb	W_{FB} = 59/70Fb

H_{FG} = 7/6F	H_{GH} = 7/6F	H_{HD} = 7/6F
V_{FG} = 23/70F	V_{GH} = -3/2F	V_{HD} = -3/2F
W_{FG} = -59/70Fb	W_{GH} = -3/2Fb	W_{HD} = 3/2Fb
H_{GF} = -7/6F	H_{HG} = -7/6F	H_{DH} = -7/6F
V_{GF} = -23/70F	V_{HG} = 3/2F	V_{DH} = 3/2F
W_{GF} = 3/2Fb	W_{HG} = -3/2Fb	W_{DH} = 2Fb

SPOSTAMENTI NODALI

u_{AAB} = 0	u_B = 3/35δ	u_{CCB} = 3/35δ	u_D = 3/35δ	u_{EED} = 0
v_{AAB} = 0	v_B = 0	v_{CCB} = -2/35δ	v_D = 0	v_{EED} = 0
φ_{AAB} = -1/35δ/b	φ_B = -1/35δ/b	φ_{CCB} = -1/35δ/b	φ_D = 1/35δ/b	φ_E = -1/35δ/b

$u_F = 6/35\delta$ $u_{GGF} = 6/35\delta$ $u_{HHG} = 6/35\delta$
$v_F = 0$ $v_{GGF} = -2/35\delta$ $v_{HHG} = 0$
$\varphi_F = -1/35\delta/b$ $\varphi_{GGF} = -1/35\delta/b$ $\varphi_{HHD} = 1/35\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$ $u_{BBC} = 3/35\delta$ $u_{CCD} = 3/35\delta$ $u_{DDE} = 3/35\delta$ $u_{BBF} = 3/35\delta$
$v_{AAB} = 0$ $v_{BBC} = 0$ $v_{CCD} = -2/35\delta$ $v_{DDE} = 0$ $v_{BBF} = 0$
$\varphi_{AAB} = -1/35\delta/b$ $\varphi_{BBC} = -1/35\delta/b$ $\varphi_{CCD} = 1/35\delta/b$ $\varphi_{DDE} = -1/35\delta/b$ $\varphi_{BBF} = -1/35\delta/b$

$u_{FFG} = 6/35\delta$ $u_{GGH} = 6/35\delta$ $u_{HHD} = 6/35\delta$
$v_{FFG} = 0$ $v_{GGH} = -2/35\delta$ $v_{HHD} = 0$
$\varphi_{FFG} = -1/35\delta/b$ $\varphi_{GGH} = 1/35\delta/b$ $\varphi_{HHD} = -1/35\delta/b$

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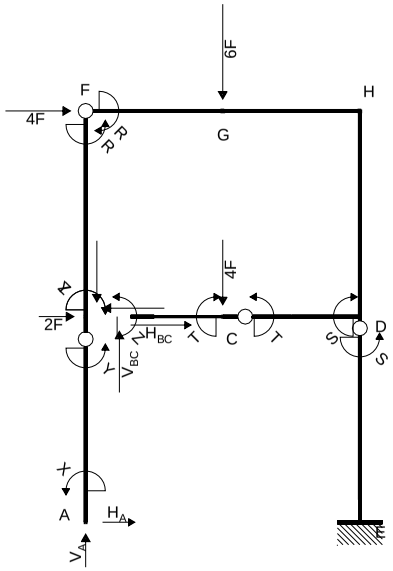


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

$3H_Ab - 4V_Ab = -Xb + Sb - 8Fb$

Rotazione intorno a C: aste CB

$-2V_{BC}b = -Zb + Tb$

Rotazione intorno a F: aste FB BA

$6H_Ab - 3H_{BC}b = -Xb + Zb - Rb - 6Fb$

Rotazione intorno a B: aste BA

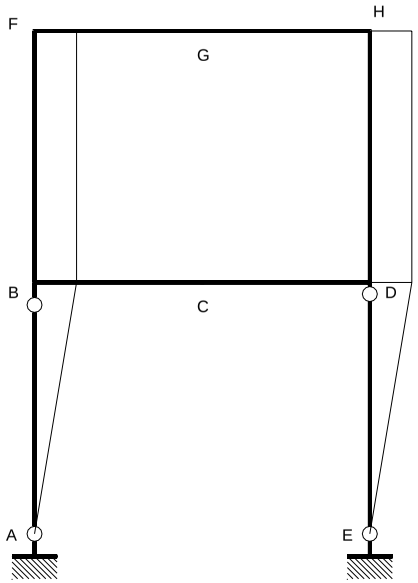
$3H_Ab = -Xb - Yb$

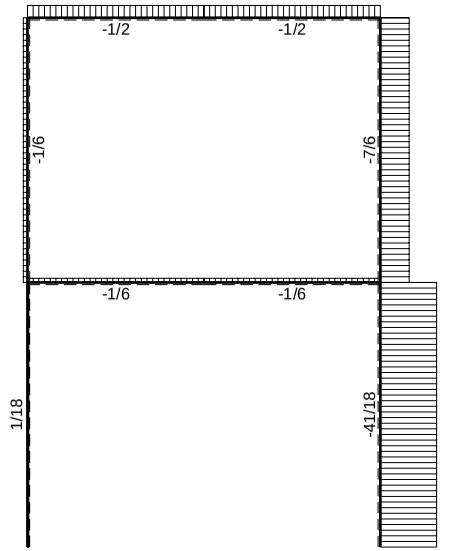
Matrice di equilibrio

	H_Ab	V_Ab	$H_{BC}b$	$V_{BC}b$		Xb	Yb	Zb	Tb	Sb	Rb	Fb
φ_{DE}	3	-4	0	0	=	-1	0	0	0	1	0	-8
φ_{CD}	0	0	0	-2		0	0	-1	1	0	0	0
φ_{FB}	6	0	-3	0		-1	0	1	0	0	-1	-6
φ_{BA}	3	0	0	0		-1	-1	0	0	0	0	0

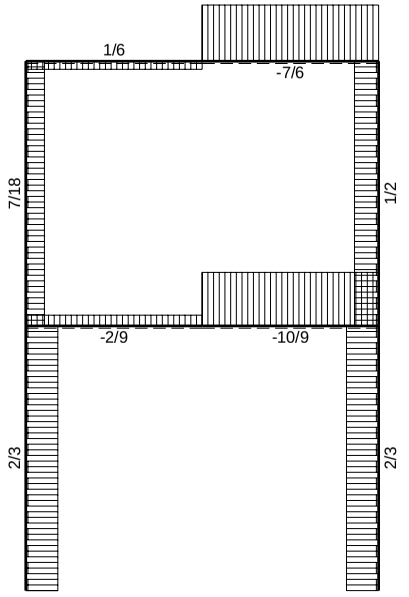
Soluzione del sistema

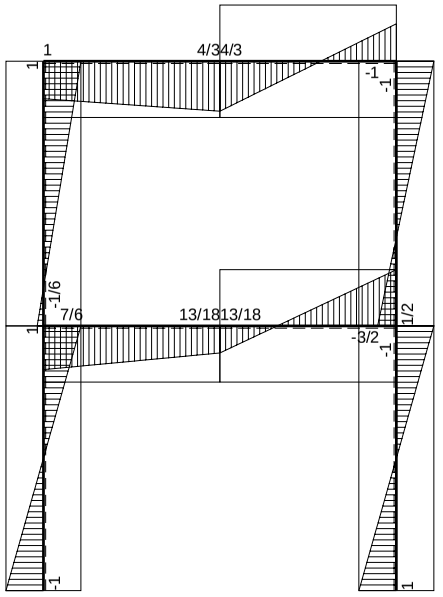
		Xb	Yb	Zb	Tb	Sb	Rb	Fb
H_Ab		-1/3	-1/3	0	0	0	0	0
$V_{BC}b$	=	0	0	1/2	-1/2	0	0	0
$H_{BC}b$		-1/3	-2/3	-1/3	0	0	1/3	2
V_Ab		0	-1/4	0	0	-1/4	0	2





F





F Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq} = 1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FB}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	1	0	0	0	0	≤	3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	1	0	0	0	≤	3/2
W_{DC}-	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC}+	0	0	1	-2	0	0	-8	≤	3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-1
W_{DE}+	0	0	0	0	1	0	0	≤	1
W_{ED}-	-1	-1	0	0	-1	0	18	≥	-1
W_{ED}+	-1	-1	0	0	-1	0	18	≤	1
W_{BF}-	0	-1	-1	0	0	0	0	≥	-1
W_{BF}+	0	-1	-1	0	0	0	0	≤	1
W_{FB}-	0	0	0	0	0	1	0	≥	-1
W_{FB}+	0	0	0	0	0	1	0	≤	1
W_{GH}-	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH}+	0	1/2	1	-1	1/2	-1	-4	≤	3/2
W_{HD}-	0	1	2	-2	1	-1	4	≥	-1
W_{HD}+	0	1	2	-2	1	-1	4	≤	1
W_{DH}-	0	0	-1	2	-1	0	8	≥	-1
W_{DH}+	0	0	-1	2	-1	0	8	≤	1
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	≥	-3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	≥	-3/2
W_{DC}-	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	8	≥	-3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-1
W_{DE}+	0	0	0	0	-1	0	0	≥	-1
W_{ED}-	-1	-1	0	0	-1	0	18	≥	-1
W_{ED}+	1	1	0	0	1	0	-18	≥	-1
W_{BF}-	0	-1	-1	0	0	0	0	≥	-1
W_{BF}+	0	1	1	0	0	0	0	≥	-1
W_{FB}-	0	0	0	0	0	1	0	≥	-1
W_{FB}+	0	0	0	0	0	-1	0	≥	-1
W_{GH}-	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	1	4	≥	-3/2
W_{HD}-	0	1	2	-2	1	-1	4	≥	-1
W_{HD}+	0	-1	-2	2	-1	1	-4	≥	-1
W_{DH}-	0	0	-1	2	-1	0	8	≥	-1
W_{DH}+	0	0	1	-2	1	0	-8	≥	-1
Max	0	0	0	0	0	0	1	=	0

PROCEDIMENTO E RISULTATI Nome: AL4.009

Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{BC}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC}-	0	0	1	-2	0	0	0	0	-1	2	0	0	-8	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	0	0	1	-2	0	0	8	≤	-3/2
W_{DE}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{DE}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{ED}-	-1	-1	0	0	-1	0	1	1	0	0	1	0	18	≥	-1
W_{ED}+	1	1	0	0	1	0	-1	-1	0	0	-1	0	-18	≤	-1
W_{BF}-	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-1
W_{BF}+	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-1
W_{FB}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-1
W_{FB}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-1
W_{GH}-	0	1/2	1	-1	1/2	-1	0	-1/2	-1	1	-1/2	1	-4	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	1	0	1/2	1	-1	1/2	-1	4	≤	-3/2
W_{HD}-	0	1	2	-2	1	-1	0	-1	-2	2	-1	1	4	≥	-1
W_{HD}+	0	-1	-2	2	-1	1	0	1	2	-2	1	-1	-4	≤	-1
W_{DH}-	0	0	-1	2	-1	0	0	0	1	-2	1	0	8	≥	-1
W_{DH}+	0	0	1	-2	1	0	0	0	-1	2	-1	0	-8	≤	-1
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11 08.06.11

Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	1	-2	0	0	-8	1	≥	-3/2
φ_{DC}+	0	0	-1	2	0	0	8	-1	≥	-3/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	-1	-1	0	0	-1	0	18	3	≥	-1
φ_{ED}+	1	1	0	0	1	0	-18	-3	≥	-1
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	0	1/2	1	-1	1/2	-1	-4	0	≥	-3/2
φ_{GH}+	0	-1/2	-1	1	-1/2	1	4	0	≥	-3/2
φ_{HD}-	0	1	2	-2	1	-1	4	-1	≥	-1
φ_{HD}+	0	-1	-2	2	-1	1	-4	1	≥	-1
φ_{DH}-	0	0	-1	2	-1	0	8	0	≥	-1
φ_{DH}+	0	0	1	-2	1	0	-8	0	≥	-1
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 14-7

	X	Y	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	-4/9	-4/9	1	-2	-4/9	0	4/9	7/3	≥	-19/18
φ_{DC}+	4/9	4/9	-1	2	4/9	0	-4/9	-7/3	≥	-35/18
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	1/18	1/18	0	0	1/18	0	-1/18	-1/6	≥	-1/18
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	-2/9	5/18	1	-1	5/18	-1	2/9	2/3	≥	-23/18
φ_{GH}+	2/9	-5/18	-1	1	-5/18	1	-2/9	-2/3	≥	-31/18
φ_{HD}-	2/9	11/9	2	-2	11/9	-1	-2/9	-5/3	≥	-11/9
φ_{HD}+	-2/9	-11/9	-2	2	-11/9	1	2/9	5/3	≥	-7/9
φ_{DH}-	4/9	4/9	-1	2	-5/9	0	-4/9	-4/3	≥	-13/9
φ_{DH}+	-4/9	-4/9	1	-2	5/9	0	4/9	4/3	≥	-5/9
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	1/18	1/18	0	0	1/18	0	-1/18	-1/6	=	-1/18

Scambio pivotale 2-1

	φ_{AB}+	Y	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	4/9	-4/9	1	-2	-4/9	0	4/9	17/9	≥	-11/18
φ_{DC}+	-4/9	4/9	-1	2	4/9	0	-4/9	-17/9	≥	-43/18
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	≥	-1/9
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-1
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-1
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	2/9	5/18	1	-1	5/18	-1	2/9	4/9	≥	-19/18
φ_{GH}+	-2/9	-5/18	-1	1	-5/18	1	-2/9	-4/9	≥	-35/18
φ_{HD}-	-2/9	11/9	2	-2	11/9	-1	-2/9	-13/9	≥	-13/9
φ_{HD}+	2/9	-11/9	-2	2	-11/9	1	2/9	13/9	≥	-5/9
φ_{DH}-	-4/9	4/9	-1	2	-5/9	0	-4/9	-8/9	≥	-17/9
φ_{DH}+	4/9	-4/9	1	-2	5/9	0	4/9	8/9	≥	-1/9
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	=	-1/9

Scambio pivotale 24-2

	φ_{AB}+	φ_{DH}+	Z	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	1	-9/4	9/4	-9/2	5/4	0	1	1	≥	-5/4
φ_{BA}+	-1	9/4	-9/4	9/2	-5/4	0	-1	-1	≥	-3/4
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	1	0	0	-1	0	0	1	≥	-1/2
φ_{DC}+	0	-1	0	0	1	0	0	-1	≥	-5/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	-1/8	1/8	-1/4	1/8	0	0	0	≥	-1/8
φ_{BF}-	-1	9/4	-13/4	9/2	-5/4	0	-1	0	≥	-3/4
φ_{BF}+	1	-9/4	13/4	-9/2	5/4	0	1	0	≥	-5/4
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/2	-5/8	13/8	-9/4	5/8	-1	1/2	1	≥	-9/8
φ_{GH}+	-1/2	5/8	-13/8	9/4	-5/8	1	-1/2	-1	≥	-15/8
φ_{HD}-	1	-11/4	19/4	-15/2	11/4	-1	1	1	≥	-7/4
φ_{HD}+	-1	11/4	-19/4	15/2	-11/4	1	-1	-1	≥	-1/4
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	1	-9/4	9/4	-9/2	5/4	0	1	2	≥	-1/4
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	-1/8	1/8	-1/4	1/8	0	0	0	=	-1/8

Scambio pivotale 22-3

	φ_{AB}+	φ_{DH}+	φ_{HD}+	T	S	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	10/19	≥	-26/19
φ_{BA}+	-10/19	18/19	9/19	18/19	1/19	-9/19	-10/19	-10/19	≥	-12/19
φ_{BC}-	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-23/19	≥	-59/38
φ_{BC}+	4/19	-11/19	4/19	-30/19	11/19	-4/19	4/19	23/19	≥	-55/38
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	1	0	0	-1	0	0	1	≥	-1/2
φ_{DC}+	0	-1	0	0	1	0	0	-1	≥	-5/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	≥	-5/38
φ_{BF}-	-6/19	7/19	13/19	-12/19	12/19	-13/19	-6/19	13/19	≥	-11/19
φ_{BF}+	6/19	-7/19	-13/19	12/19	-12/19	13/19	6/19	-13/19	≥	-27/19
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	3/19	6/19	-13/38	6/19	-6/19	-25/38	3/19	25/38	≥	-23/19
φ_{GH}+	-3/19	-6/19	13/38	-6/19	6/19	25/38	-3/19	-25/38	≥	-34/19
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-4/19	≥	-1/19
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	29/19	≥	-7/19
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	=	-5/38

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Scambio pivotale 22-5

	φ_{AB}+	φ_{DH}+	φ_{HD}+	T	Z	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	6/11	≥	-15/11
φ_{BA}+	-6/11	1	5/11	12/11	-1/11	-5/11	-6/11	-6/11	≥	-7/11
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	4/11	0	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-9/22
φ_{DC}+	-4/11	0	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-57/22
φ_{DE}-	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-12/11
φ_{DE}+	4/11	-1	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-10/11
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	≥	-3/22
φ_{BF}-	-6/11	1	5/11	12/11	-12/11	-5/11	-6/11	5/11	≥	-7/11
φ_{BF}+	6/11	-1	-5/11	-12/11	12/11	5/11	6/11	-5/11	≥	-15/11
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	3/11	0	-5/22	-6/11	6/11	-17/22	3/11	17/22	≥	-13/11
φ_{GH}+	-3/11	0	5/22	6/11	-6/11	17/22	-3/11	-17/22	≥	-20/11
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-4/11	≥	-1/11
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	17/11	≥	-4/11
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	=	-3/22

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Scambio pivotale 9-4

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	R	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	0	≥	-6/5
φ_{BA}+	-2/5	1	3/5	-2/5	3/5	-3/5	-2/5	0	≥	-4/5
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	2/15	0	2/15	-11/30	19/30	-2/15	2/15	-1/2	≥	-33/20
φ_{CD}+	-2/15	0	-2/15	11/30	-19/30	2/15	-2/15	1/2	≥	-27/20
T	2/15	0	2/15	-11/30	19/30	-2/15	2/15	1/2	≥	-3/20
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	≥	-3/20
φ_{BF}-	-2/5	1	3/5	-2/5	-2/5	-3/5	-2/5	1	≥	-4/5
φ_{BF}+	2/5	-1	-3/5	2/5	2/5	3/5	2/5	-1	≥	-6/5
φ_{FB}-	0	0	0	0	0	1	0	-1	≥	-1
φ_{FB}+	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/5	0	-3/10	1/5	1/5	-7/10	1/5	1/2	≥	-11/10
φ_{GH}+	-1/5	0	3/10	-1/5	-1/5	7/10	-1/5	-1/2	≥	-19/10
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	1	≥	-1/5
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	=	-3/20

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Scambio pivotale 18-6

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	3/5	≥	-9/5
φ_{BA}+	-2/5	1	3/5	-2/5	3/5	3/5	-2/5	-3/5	≥	-1/5
φ_{BC}-	0	0	0	0	1	0	0	-1	≥	-3/2
φ_{BC}+	0	0	0	0	-1	0	0	1	≥	-3/2
φ_{CD}-	2/15	0	2/15	-11/30	19/30	2/15	2/15	-19/30	≥	-91/60
φ_{CD}+	-2/15	0	-2/15	11/30	-19/30	-2/15	-2/15	19/30	≥	-89/60
T	2/15	0	2/15	-11/30	19/30	2/15	2/15	11/30	≥	-1/60
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	≥	-11/60
φ_{BF}-	-2/5	1	3/5	-2/5	-2/5	3/5	-2/5	2/5	≥	-1/5
φ_{BF}+	2/5	-1	-3/5	2/5	2/5	-3/5	2/5	-2/5	≥	-9/5
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	0	0	0	0	0	-1	0	1	≥	-1
φ_{GH}-	1/5	0	-3/10	1/5	1/5	7/10	1/5	-1/5	≥	-2/5
φ_{GH}+	-1/5	0	3/10	-1/5	-1/5	-7/10	-1/5	1/5	≥	-13/5
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	8/5	≥	-4/5
L_X	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	=	-11/60

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Scambio pivotale 4-8

	φ_{AB}+	φ_{DH}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-4/3
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-1/3
φ_{BC}-	2/3	-5/3	-1	2/3	0	-1	2/3	5/3	≥	-7/6
φ_{BC}+	-2/3	5/3	1	-2/3	0	1	-2/3	-5/3	≥	-11/6
φ_{CD}-	5/9	-19/18	-1/2	1/18	0	-1/2	5/9	19/18	≥	-47/36
φ_{CD}+	-5/9	19/18	1/2	-1/18	0	1/2	-5/9	-19/18	≥	-61/36
T	-1/9	11/18	1/2	-11/18	1	1/2	-1/9	-11/18	≥	-5/36
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	1	0	-1	0	0	0	0	≥	-3/2
φ_{DE}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	≥	-7/36
φ_{BF}-	-2/3	5/3	1	-2/3	0	1	-2/3	-2/3	≥	-1/3
φ_{BF}+	2/3	-5/3	-1	2/3	0	-1	2/3	2/3	≥	-5/3
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	5/3	1	-2/3	1	0	-2/3	-5/3	≥	-4/3
φ_{GH}-	1/3	-1/3	-1/2	1/3	0	1/2	1/3	1/3	≥	-1/3
φ_{GH}+	-1/3	1/3	1/2	-1/3	0	-1/2	-1/3	-1/3	≥	-8/3
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	8/3	1	-5/3	1	1	-2/3	-5/3	≥	-5/6
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-2
Y	-2/3	5/3	1	-2/3	1	1	-2/3	-8/3	≥	-4/3
L_X	2/3	-5/3	-1	2/3	-1	-1	2/3	5/3	≥	-7/6
Max	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	=	-7/36

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Scambio pivotale 12-2

	φ_{AB}+	φ_{DE}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
φ_{BC}-	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
φ_{BC}+	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
φ_{CD}-	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
φ_{CD}+	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
T	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	-1	0	0	0	0	0	0	≥	-2
φ_{DH}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
φ_{BF}-	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
φ_{BF}+	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
φ_{GH}-	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
φ_{GH}+	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{DH}-	0	1	0	-1	0	0	0	0	≥	-3/2
Y	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_X	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

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Tableau finale

	φ_{AB}+	φ_{DE}+	φ_{HD}+	φ_{DC}-	Z	φ_{FB}+	φ_{ED}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-5/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
φ_{BC}-	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
φ_{BC}+	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
φ_{CD}-	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
φ_{CD}+	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
T	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
φ_{DC}+	0	0	0	-1	0	0	0	0	≥	-3
φ_{DE}-	0	-1	0	0	0	0	0	0	≥	-2
φ_{DH}+	0	-1	0	1	0	0	0	0	≥	-1/2
φ_{ED}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
φ_{BF}-	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
φ_{BF}+	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ_{FB}-	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
φ_{GH}-	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
φ_{GH}+	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
φ_{HD}-	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
φ_{DH}-	0	1	0	-1	0	0	0	0	≥	-3/2
Y	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_X	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BC}-	0	0	0	0	0	0	0	0	≥	0
φ_{BC}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	0
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	0
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{DE}-	0	0	0	0	0	0	0	0	≥	0
φ_{DE}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{ED}-	0	0	0	0	0	0	0	0	≥	0
φ_{ED}+	0	0	0	0	0	0	0	0	≥	1/18
φ_{BF}-	0	0	0	0	0	0	0	0	≥	0
φ_{BF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FB}-	0	0	0	0	0	0	0	0	≥	0
φ_{FB}+	0	0	0	0	0	0	0	0	≥	0
φ_{GH}-	0	0	0	0	0	0	0	0	≥	0
φ_{GH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HD}-	0	0	0	0	0	0	0	0	≥	0
φ_{HD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	13/6	13/6	0	4/9	13/6	13/6	2/9	7/6	=	-2/9

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
1	1	-7/6	-13/18	1	1

Variabili soluzione differenza tra rotazioni

$$\begin{matrix}\varphi_{AB}\\ \varphi_{BA}\\ \varphi_{BC}\\ \varphi_{CD}\\ \varphi_{DC}\\ \varphi_{DE}\\ \varphi_{ED}\\ \varphi_{BF}\\ \varphi_{FB}\\ \varphi_{GH}\\ \varphi_{HD}\\ \varphi_{DH}\end{matrix}\begin{bmatrix}1/18\\ 1/18\\ 0\\ 0\\ 0\\ 1/18\\ 1/18\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

REAZIONI Fattore di collasso = 2/9

$H_A = -2/3F$
$V_A = -1/18F$
$W_A = Fb$
$H_E = -2/3F$
$V_E = 41/18F$
$W_E = Fb$

$H_{AB} = -2/3F$	$H_{BC} = 1/6F$	$H_{CD} = 1/6F$	$H_{DE} = 2/3F$	$H_{BF} = -7/18F$
$V_{AB} = -1/18F$	$V_{BC} = -2/9F$	$V_{CD} = -10/9F$	$V_{DE} = -41/18F$	$V_{BF} = 1/6F$
$W_{AB} = Fb$	$W_{BC} = -7/6Fb$	$W_{CD} = -13/18Fb$	$W_{DE} = Fb$	$W_{BF} = 1/6Fb$
$H_{BA} = 2/3F$	$H_{CB} = -1/6F$	$H_{DC} = -1/6F$	$H_{ED} = -2/3F$	$H_{FB} = 7/18F$
$V_{BA} = 1/18F$	$V_{CB} = 2/9F$	$V_{DC} = 10/9F$	$V_{ED} = 41/18F$	$V_{FB} = -1/6F$
$W_{BA} = Fb$	$W_{CB} = 13/18Fb$	$W_{DC} = -3/2Fb$	$W_{ED} = Fb$	$W_{FB} = Fb$

$H_{FG} = 1/2F$	$H_{GH} = 1/2F$	$H_{HD} = 1/2F$
$V_{FG} = 1/6F$	$V_{GH} = -7/6F$	$V_{HD} = -7/6F$
$W_{FG} = -Fb$	$W_{GH} = -4/3Fb$	$W_{HD} = Fb$
$H_{GF} = -1/2F$	$H_{HG} = -1/2F$	$H_{DH} = -1/2F$
$V_{GF} = -1/6F$	$V_{HG} = 7/6F$	$V_{DH} = 7/6F$
$W_{GF} = 4/3Fb$	$W_{HG} = -Fb$	$W_{DH} = 1/2Fb$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 1/6\delta$	$u_C = 1/6\delta$	$u_D = 1/6\delta$	$u_{EED} = 0$
$v_{AAB} = 0$	$v_B = 0$	$v_C = 0$	$v_D = 0$	$v_{EED} = 0$
$\varphi_{AAB} = -1/18\delta/b$	$\varphi_B = -1/18\delta/b$	$\varphi_C = 0$	$\varphi_D = 0$	$\varphi_E = -1/18\delta/b$

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$u_F = 1/6\delta$	$u_G = 1/6\delta$	$u_H = 1/6\delta$
$v_F = 0$	$v_G = 0$	$v_H = 0$
$\varphi_F = 0$	$\varphi_G = 0$	$\varphi_H = 0$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{BBC} = 1/6\delta$	$u_{CCD} = 1/6\delta$	$u_{DDE} = 1/6\delta$	$u_{BBF} = 1/6\delta$
$v_{AAB} = 0$	$v_{BBC} = 0$	$v_{CCD} = 0$	$v_{DDE} = 0$	$v_{BBF} = 0$
$\varphi_{AAB} = -1/18\delta/b$	$\varphi_{BBC} = 0$	$\varphi_{CCD} = 0$	$\varphi_{DDE} = -1/18\delta/b$	$\varphi_{BBF} = 0$

$u_{FFG} = 1/6\delta$	$u_{GGH} = 1/6\delta$	$u_{HHD} = 1/6\delta$
$v_{FFG} = 0$	$v_{GGH} = 0$	$v_{HHD} = 0$
$\varphi_{FFG} = 0$	$\varphi_{GGH} = 0$	$\varphi_{HHD} = 0$

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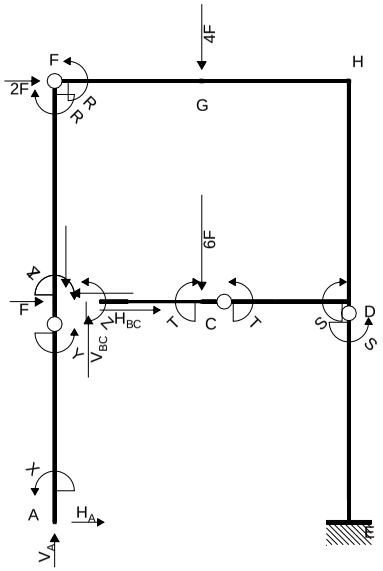


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

$3H_Ab -4V_Ab = -Xb +Sb -14Fb$

Rotazione intorno a C: aste CB

$-2V_{BC}b = -Zb +Tb$

Rotazione intorno a F: aste FB BA

$6H_Ab -3H_{BC}b = -Xb +Zb +Rb -3Fb$

Rotazione intorno a B: aste BA

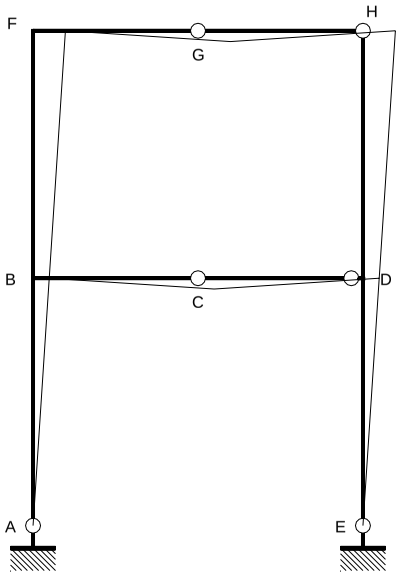
$3H_Ab = -Xb -Yb$

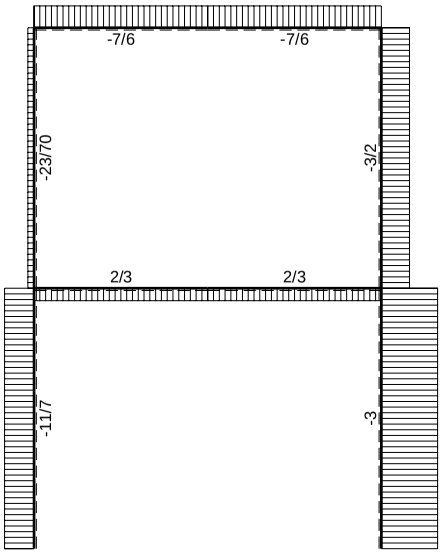
Matrice di equilibrio

	H_Ab	V_Ab	$H_{BC}b$	$V_{BC}b$		Xb	Yb	Zb	Tb	Sb	Rb	Fb
φ_{DE}	3	-4	0	0	=	-1	0	0	0	1	0	-14
φ_{CD}	0	0	0	-2		0	0	-1	1	0	0	0
φ_{FG}	6	0	-3	0		-1	0	1	0	0	1	-3
φ_{BA}	3	0	0	0		-1	-1	0	0	0	0	0

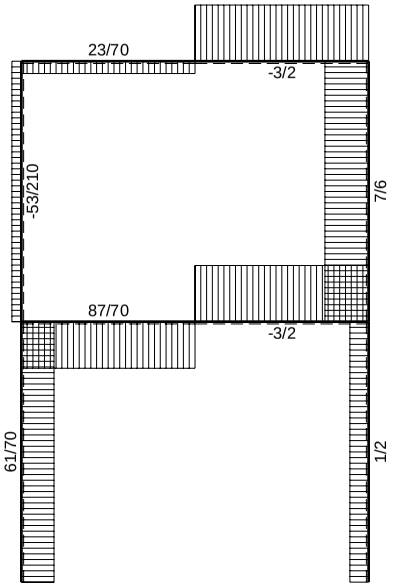
Soluzione del sistema

	Xb	Yb	Zb	Tb	Sb	Rb	Fb
H_Ab	-1/3	-1/3	0	0	0	0	0
$V_{BC}b$	0	0	1/2	-1/2	0	0	0
$H_{BC}b$	-1/3	-2/3	-1/3	0	0	-1/3	1
V_Ab	0	-1/4	0	0	-1/4	0	7/2

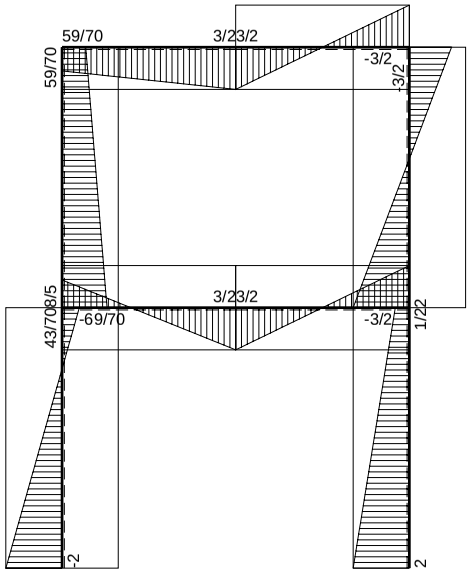




F



F



Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-2
W_{AB}+	1	0	0	0	0	0	0	≤	2
W_{BA}-	0	1	0	0	0	0	0	≥	-2
W_{BA}+	0	1	0	0	0	0	0	≤	2
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	1	0	0	0	0	≤	3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	1	0	0	0	≤	3/2
W_{DC}-	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC}+	0	0	1	-2	0	0	-12	≤	3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-2
W_{DE}+	0	0	0	0	1	0	0	≤	2
W_{ED}-	-1	-1	0	0	-1	0	9	≥	-2
W_{ED}+	-1	-1	0	0	-1	0	9	≤	2
W_{BF}-	0	-1	-1	0	0	0	0	≥	-2
W_{BF}+	0	-1	-1	0	0	0	0	≤	2
W_{FG}-	0	0	0	0	0	1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	1	0	≤	3/2
W_{GH}-	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH}+	0	1/2	1	-1	1/2	1	-7	≤	3/2
W_{HG}-	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG}+	0	-1	-2	2	-1	-1	6	≤	3/2
W_{DH}-	0	0	-1	2	-1	0	12	≥	-2
W_{DH}+	0	0	-1	2	-1	0	12	≤	2
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-2
W_{AB}+	-1	0	0	0	0	0	0	≥	-2
W_{BA}-	0	1	0	0	0	0	0	≥	-2
W_{BA}+	0	-1	0	0	0	0	0	≥	-2
W_{BC}-	0	0	1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	≥	-3/2
W_{CD}-	0	0	0	1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	≥	-3/2
W_{DC}-	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	12	≥	-3/2
W_{DE}-	0	0	0	0	1	0	0	≥	-2
W_{DE}+	0	0	0	0	-1	0	0	≥	-2
W_{ED}-	-1	-1	0	0	-1	0	9	≥	-2
W_{ED}+	1	1	0	0	1	0	-9	≥	-2
W_{BF}-	0	-1	-1	0	0	0	0	≥	-2
W_{BF}+	0	1	1	0	0	0	0	≥	-2
W_{FG}-	0	0	0	0	0	1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	-1	0	≥	-3/2
W_{GH}-	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	-1	7	≥	-3/2
W_{HG}-	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG}+	0	1	2	-2	1	1	-6	≥	-3/2
W_{DH}-	0	0	-1	2	-1	0	12	≥	-2
W_{DH}+	0	0	1	-2	1	0	-12	≥	-2
Max	0	0	0	0	0	0	1	=	0

PROCEDIMENTO E RISULTATI Nome: AL4.010

Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-2
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-2
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-2
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-2
W_{BC}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC}-	0	0	1	-2	0	0	0	0	-1	2	0	0	-12	≥	-3/2
W_{DC}+	0	0	-1	2	0	0	0	0	1	-2	0	0	12	≤	-3/2
W_{DE}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-2
W_{DE}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-2
W_{ED}-	-1	-1	0	0	-1	0	1	1	0	0	1	0	9	≥	-2
W_{ED}+	1	1	0	0	1	0	-1	-1	0	0	-1	0	-9	≤	-2
W_{BF}-	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-2
W_{BF}+	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-2
W_{FG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-3/2
W_{FG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-3/2
W_{GH}-	0	1/2	1	-1	1/2	1	0	-1/2	-1	1	-1/2	-1	-7	≥	-3/2
W_{GH}+	0	-1/2	-1	1	-1/2	-1	0	1/2	1	-1	1/2	1	7	≤	-3/2
W_{HG}-	0	-1	-2	2	-1	-1	0	1	2	-2	1	1	6	≥	-3/2
W_{HG}+	0	1	2	-2	1	1	0	-1	-2	2	-1	-1	-6	≤	-3/2
W_{DH}-	0	0	-1	2	-1	0	0	0	1	-2	1	0	12	≥	-2
W_{DH}+	0	0	1	-2	1	0	0	0	-1	2	-1	0	-12	≤	-2
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	1	-2	0	0	-12	1	≥	-3/2
φ_{DC}+	0	0	-1	2	0	0	12	-1	≥	-3/2
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	-1	-1	0	0	-1	0	9	3	≥	-2
φ_{ED}+	1	1	0	0	1	0	-9	-3	≥	-2
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-2
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	0	1/2	1	-1	1/2	1	-7	-2	≥	-3/2
φ_{GH}+	0	-1/2	-1	1	-1/2	-1	7	2	≥	-3/2
φ_{HG}-	0	-1	-2	2	-1	-1	6	3	≥	-3/2
φ_{HG}+	0	1	2	-2	1	1	-6	-3	≥	-3/2
φ_{DH}-	0	0	-1	2	-1	0	12	0	≥	-2
φ_{DH}+	0	0	1	-2	1	0	-12	0	≥	-2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 9-7

	X	Y	Z	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{BC}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	0	0	1/12	-1/6	0	0	-1/12	1/12	≥	-1/8
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	-1	-1	3/4	-3/2	-1	0	-3/4	15/4	≥	-25/8
φ_{ED}+	1	1	-3/4	3/2	1	0	3/4	-15/4	≥	-7/8
φ_{BF}-	0	-1	-1	0	0	0	0	2	≥	-2
φ_{BF}+	0	1	1	0	0	0	0	-2	≥	-2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	0	1/2	5/12	1/6	1/2	1	7/12	-31/12	≥	-5/8
φ_{GH}+	0	-1/2	-5/12	-1/6	-1/2	-1	-7/12	31/12	≥	-19/8
φ_{HG}-	0	-1	-3/2	1	-1	-1	-1/2	7/2	≥	-9/4
φ_{HG}+	0	1	3/2	-1	1	1	1/2	-7/2	≥	-3/4
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	1/12	-1/6	0	0	-1/12	1/12	=	-1/8

Scambio pivotale 14-3

	X	Y	φ_{ED}+	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-2
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-2
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	4/3	4/3	-4/3	2	4/3	0	1	-6	≥	-8/3
φ_{BC}+	-4/3	-4/3	4/3	-2	-4/3	0	-1	6	≥	-1/3
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	1/9	1/9	-1/9	0	1/9	0	0	-1/3	≥	-2/9
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	4/3	4/3	-4/3	2	4/3	0	1	-5	≥	-7/6
φ_{BF}-	-4/3	-7/3	4/3	-2	-4/3	0	-1	7	≥	-5/6
φ_{BF}+	4/3	7/3	-4/3	2	4/3	0	1	-7	≥	-19/6
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	5/9	19/18	-5/9	1	19/18	1	1	-14/3	≥	-10/9
φ_{GH}+	-5/9	-19/18	5/9	-1	-19/18	-1	-1	14/3	≥	-17/9
φ_{HG}-	-2	-3	2	-2	-3	-1	-2	11	≥	-1/2
φ_{HG}+	2	3	-2	2	3	1	2	-11	≥	-5/2
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	1/9	1/9	-1/9	0	1/9	0	0	-1/3	=	-2/9

PROCEDIMENTO E RISULTATI Nome: AL4.010

Scambio pivotale 6-1

	φ_{BC}+	Y	φ_{ED}+	T	S	R	φ_{DC}-	X-		Fb
φ_{AB}-	-3/4	-1	1	-3/2	-1	0	-3/4	7/2	≥	-9/4
φ_{AB}+	3/4	1	-1	3/2	1	0	3/4	-7/2	≥	-7/4
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-2
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	-3/4	-1	1	-3/2	-1	0	-3/4	9/2	≥	-1/4
φ_{CD}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	-1/12	0	0	-1/6	0	0	-1/12	1/6	≥	-1/4
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	0	0	0	1	0	0	-1	≥	-2
φ_{DE}+	0	0	0	0	-1	0	0	1	≥	-2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	0	0	0	0	0	0	1	≥	-3/2
φ_{BF}-	1	-1	0	0	0	0	0	1	≥	-1/2
φ_{BF}+	-1	1	0	0	0	0	0	-1	≥	-7/2
φ_{FG}-	0	0	0	0	0	1	0	-1	≥	-3/2
φ_{FG}+	0	0	0	0	0	-1	0	1	≥	-3/2
φ_{GH}-	-5/12	1/2	0	1/6	1/2	1	7/12	-13/6	≥	-5/4
φ_{GH}+	5/12	-1/2	0	-1/6	-1/2	-1	-7/12	13/6	≥	-7/4
φ_{HG}-	3/2	-1	0	1	-1	-1	-1/2	2	≥	0
φ_{HG}+	-3/2	1	0	-1	1	1	1/2	-2	≥	-3
φ_{DH}-	0	0	0	0	-1	0	-1	1	≥	-7/2
φ_{DH}+	0	0	0	0	1	0	1	-1	≥	-1/2
L_X	0	0	0	0	0	0	0	-1	≥	-2
Max	-1/12	0	0	-1/6	0	0	-1/12	1/6	=	-1/4

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Scambio pivotale 2-8

	φ_{BC}+	Y	φ_{ED}+	T	S	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	3/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1/2
φ_{BA}-	-3/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
φ_{BA}+	3/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-5/2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	3/14	2/7	-2/7	3/7	2/7	0	3/14	-9/7	≥	-5/2
φ_{CD}-	-3/14	-2/7	2/7	4/7	-2/7	0	-3/14	2/7	≥	-1
φ_{CD}+	3/14	2/7	-2/7	-4/7	2/7	0	3/14	-2/7	≥	-2
αbF	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	≥	-1/3
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	-3/14	-2/7	2/7	-3/7	5/7	0	-3/14	2/7	≥	-3/2
φ_{DE}+	3/14	2/7	-2/7	3/7	-5/7	0	3/14	-2/7	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-2
φ_{BF}-	17/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1
φ_{BF}+	-17/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3
φ_{FG}-	-3/14	-2/7	2/7	-3/7	-2/7	1	-3/14	2/7	≥	-1
φ_{FG}+	3/14	2/7	-2/7	3/7	2/7	-1	3/14	-2/7	≥	-2
φ_{GH}-	-37/42	-5/42	13/21	-16/21	-5/42	1	5/42	13/21	≥	-1/6
φ_{GH}+	37/42	5/42	-13/21	16/21	5/42	-1	-5/42	-13/21	≥	-17/6
φ_{HG}-	27/14	-3/7	-4/7	13/7	-3/7	-1	-1/14	-4/7	≥	-1
φ_{HG}+	-27/14	3/7	4/7	-13/7	3/7	1	1/14	4/7	≥	-2
φ_{DH}-	3/14	2/7	-2/7	3/7	-5/7	0	-11/14	-2/7	≥	-4
φ_{DH}+	-3/14	-2/7	2/7	-3/7	5/7	0	11/14	2/7	≥	0
L_X	-3/14	-2/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
Max	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	=	-1/3

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Scambio pivotale 24-2

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	S	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	0	-1	0	0	1	0	1	0	≥	-1/2
φ_{BA}-	-3/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3/2
φ_{BA}+	3/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-5/2
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	0	-1	0	0	1	0	1	-1	≥	-5/2
φ_{CD}-	0	1	0	1	-1	0	-1	0	≥	-1
φ_{CD}+	0	-1	0	-1	1	0	1	0	≥	-2
αbF	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	≥	-1/3
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	-1	0	0	1	0	1	0	≥	-2
φ_{BF}-	7/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-1
φ_{BF}+	-7/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3
φ_{FG}-	0	1	0	0	-1	1	-1	0	≥	-1
φ_{FG}+	0	-1	0	0	1	-1	1	0	≥	-2
φ_{GH}-	-19/24	5/12	1/2	-7/12	-5/12	1	-5/24	1/2	≥	-1/6
φ_{GH}+	19/24	-5/12	-1/2	7/12	5/12	-1	5/24	-1/2	≥	-17/6
φ_{HG}-	9/4	3/2	-1	5/2	-3/2	-1	-5/4	-1	≥	-1
φ_{HG}+	-9/4	-3/2	1	-5/2	3/2	1	5/4	1	≥	-2
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	-3/4	-7/2	1	-3/2	5/2	0	11/4	1	≥	0
L_X	0	1	0	0	-1	0	-1	0	≥	-3/2
Max	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	=	-1/3

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Scambio pivotale 19-5

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	φ_{GH}-	R	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-9/10
φ_{BA}-	-18/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-21/10
φ_{BA}+	18/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-19/10
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	1/5	≥	-29/10
φ_{CD}-	19/10	0	-6/5	12/5	12/5	-12/5	-1/2	-6/5	≥	-3/5
φ_{CD}+	-19/10	0	6/5	-12/5	-12/5	12/5	1/2	6/5	≥	-12/5
αbF	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	≥	-2/5
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-29/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-12/5
φ_{BF}-	23/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-2/5
φ_{BF}+	-23/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-18/5
φ_{FG}-	19/10	0	-6/5	7/5	12/5	-7/5	-1/2	-6/5	≥	-3/5
φ_{FG}+	-19/10	0	6/5	-7/5	-12/5	7/5	1/2	6/5	≥	-12/5
S	-19/10	1	6/5	-7/5	-12/5	12/5	-1/2	6/5	≥	-2/5
φ_{GH}+	0	0	0	0	-1	0	0	0	≥	-3
φ_{HG}-	51/10	0	-14/5	23/5	18/5	-23/5	-1/2	-14/5	≥	-2/5
φ_{HG}+	-51/10	0	14/5	-23/5	-18/5	23/5	1/2	14/5	≥	-13/5
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	-11/2	-1	4	-5	-6	6	3/2	4	≥	-1
L_X	19/10	0	-6/5	7/5	12/5	-12/5	-1/2	-6/5	≥	-11/10
Max	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	=	-2/5

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Scambio pivotale 21-6

	φ_{BC}+	φ_{DH}+	φ_{ED}+	T	φ_{GH}-	φ_{HG}-	φ_{DC}-	φ_{AB}+		Fb
φ_{AB}-	0	0	0	0	0	0	0	-1	≥	-4
X-	35/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-51/46
φ_{BA}-	9/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-111/46
φ_{BA}+	-9/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-73/46
φ_{BC}-	-1	0	0	0	0	0	0	0	≥	-3
X	35/46	0	-6/23	1	-12/23	-12/23	11/46	-29/23	≥	-143/46
φ_{CD}-	-35/46	0	6/23	0	12/23	12/23	-11/46	6/23	≥	-9/23
φ_{CD}+	35/46	0	-6/23	0	-12/23	-12/23	11/46	-6/23	≥	-60/23
αbF	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	≥	-10/23
φ_{DC}+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}-	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-60/23
φ_{BF}-	14/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-2/23
φ_{BF}+	-14/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-90/23
φ_{FG}-	8/23	0	-8/23	0	30/23	7/23	-8/23	-8/23	≥	-11/23
φ_{FG}+	-8/23	0	8/23	0	-30/23	-7/23	8/23	8/23	≥	-58/23
S	35/46	1	-6/23	1	-12/23	-12/23	-35/46	-6/23	≥	-14/23
φ_{GH}+	0	0	0	0	-1	0	0	0	≥	-3
R	51/46	0	-14/23	1	18/23	-5/23	-5/46	-14/23	≥	-2/23
φ_{HG}+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}-	0	-1	0	0	0	0	0	0	≥	-4
Y	53/46	-1	8/23	1	-30/23	-30/23	39/46	8/23	≥	-35/23
L_X	-35/46	0	6/23	-1	12/23	12/23	-11/46	6/23	≥	-41/46
Max	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	=	-10/23

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Scambio pivotale 7-1

	φ_{CD}^-	φ_{DH}^+	φ_{ED}^+	T	φ_{GH}^-	φ_{HG}^-	φ_{DC}^-	φ_{AB}^+		Fb
φ_{AB}^-	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
φ_{BA}^-	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
φ_{BA}^+	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
φ_{BC}^-	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
X	-1	0	0	1	0	0	0	-1	≥	-7/2
φ_{BC}^+	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
φ_{CD}^+	-1	0	0	0	0	0	0	0	≥	-3
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
φ_{DC}^+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}^-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}^+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}^-	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
φ_{BF}^-	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
φ_{BF}^+	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
φ_{FG}^-	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{FG}^+	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
φ_{GH}^+	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{HG}^+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}^-	0	-1	0	0	0	0	0	0	≥	-4
Y	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_X	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35

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Tableau finale

	φ_{CD}^-	φ_{DH}^+	φ_{ED}^+	T	φ_{GH}^-	φ_{HG}^-	φ_{DC}^-	φ_{AB}^+		Fb
φ_{AB}^-	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
φ_{BA}^-	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
φ_{BA}^+	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
φ_{BC}^-	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
X	-1	0	0	1	0	0	0	-1	≥	-7/2
φ_{BC}^+	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
φ_{CD}^+	-1	0	0	0	0	0	0	0	≥	-3
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
φ_{DC}^+	0	0	0	0	0	0	-1	0	≥	-3
φ_{DE}^-	0	1	0	0	0	0	-1	0	≥	-3/2
φ_{DE}^+	0	-1	0	0	0	0	1	0	≥	-5/2
φ_{ED}^-	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
φ_{BF}^-	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
φ_{BF}^+	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
φ_{FG}^-	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{FG}^+	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
φ_{GH}^+	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
φ_{HG}^+	0	0	0	0	0	-1	0	0	≥	-3
φ_{DH}^-	0	-1	0	0	0	0	0	0	≥	-4
Y	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_X	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/35
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	0
φ_{BC}-	0	0	0	0	0	0	0	0	≥	0
φ_{BC}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{DE}-	0	0	0	0	0	0	0	0	≥	0
φ_{DE}+	0	0	0	0	0	0	0	0	≥	0
φ_{ED}-	0	0	0	0	0	0	0	0	≥	0
φ_{ED}+	0	0	0	0	0	0	0	0	≥	1/35
φ_{BF}-	0	0	0	0	0	0	0	0	≥	0
φ_{BF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FG}-	0	0	0	0	0	0	0	0	≥	0
φ_{FG}+	0	0	0	0	0	0	0	0	≥	0
φ_{GH}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{GH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HG}-	0	0	0	0	0	0	0	0	≥	2/35
φ_{HG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	7/2	74/35	87/35	0	1	23/35	16/35	3/2	=	-16/35

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
2	43/70	69/70	-3/2	-1/2	-59/70

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Variabili soluzione differenza tra rotazioni

φ_{AB}	1/35
φ_{BA}	0
φ_{BC}	0
φ_{CD}	-2/35
φ_{DC}	-2/35
φ_{DE}	0
φ_{ED}	1/35
φ_{BF}	0
φ_{FG}	0
φ_{GH}	-2/35
φ_{HG}	-2/35
φ_{DH}	0

REAZIONI Fattore di collasso = 16/35

H_A = -61/70F
V_A = 11/7F
W_A = 2Fb
H_E = -1/2F
V_E = 3F
W_E = 2Fb

H_{AB} = -61/70F	H_{BC} = -2/3F	H_{CD} = -2/3F	H_{DE} = 1/2F	H_{BF} = 53/210F
V_{AB} = 11/7F	V_{BC} = 87/70F	V_{CD} = -3/2F	V_{DE} = -3F	V_{BF} = 23/70F
W_{AB} = 2Fb	W_{BC} = 69/70Fb	W_{CD} = -3/2Fb	W_{DE} = -1/2Fb	W_{BF} = -8/5Fb
H_{BA} = 61/70F	H_{CB} = 2/3F	H_{DC} = 2/3F	H_{ED} = -1/2F	H_{FB} = -53/210F
V_{BA} = -11/7F	V_{CB} = -87/70F	V_{DC} = 3/2F	V_{ED} = 3F	V_{FB} = -23/70F
W_{BA} = 43/70Fb	W_{CB} = 3/2Fb	W_{DC} = -3/2Fb	W_{ED} = 2Fb	W_{FB} = 59/70Fb

H_{FG} = 7/6F	H_{GH} = 7/6F	H_{HD} = 7/6F
V_{FG} = 23/70F	V_{GH} = -3/2F	V_{HD} = -3/2F
W_{FG} = -59/70Fb	W_{GH} = -3/2Fb	W_{HD} = 3/2Fb
H_{GF} = -7/6F	H_{HG} = -7/6F	H_{DH} = -7/6F
V_{GF} = -23/70F	V_{HG} = 3/2F	V_{DH} = 3/2F
W_{GF} = 3/2Fb	W_{HG} = -3/2Fb	W_{DH} = 2Fb

SPOSTAMENTI NODALI

u_{AAB} = 0	u_B = 3/35δ	u_{CCB} = 3/35δ	u_D = 3/35δ	u_{EED} = 0
v_{AAB} = 0	v_B = 0	v_{CCB} = -2/35δ	v_D = 0	v_{EED} = 0
φ_{AAB} = -1/35δ/b	φ_B = -1/35δ/b	φ_{CCB} = -1/35δ/b	φ_D = 1/35δ/b	φ_E = -1/35δ/b

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$u_F = 6/35\delta$ $u_{GGF} = 6/35\delta$ $u_{HHG} = 6/35\delta$
$v_F = 0$ $v_{GGF} = -2/35\delta$ $v_{HHG} = 0$
$\varphi_F = -1/35\delta/b$ $\varphi_{GGF} = -1/35\delta/b$ $\varphi_{HHD} = 1/35\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$ $u_{BBC} = 3/35\delta$ $u_{CCD} = 3/35\delta$ $u_{DDE} = 3/35\delta$ $u_{BBF} = 3/35\delta$
$v_{AAB} = 0$ $v_{BBC} = 0$ $v_{CCD} = -2/35\delta$ $v_{DDE} = 0$ $v_{BBF} = 0$
$\varphi_{AAB} = -1/35\delta/b$ $\varphi_{BBC} = -1/35\delta/b$ $\varphi_{CCD} = 1/35\delta/b$ $\varphi_{DDE} = -1/35\delta/b$ $\varphi_{BBF} = -1/35\delta/b$

$u_{FFG} = 6/35\delta$ $u_{GGH} = 6/35\delta$ $u_{HHD} = 6/35\delta$
$v_{FFG} = 0$ $v_{GGH} = -2/35\delta$ $v_{HHD} = 0$
$\varphi_{FFG} = -1/35\delta/b$ $\varphi_{GGH} = 1/35\delta/b$ $\varphi_{HHD} = -1/35\delta/b$

AL4.010

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