
Response in the Frequency Domain
g(t)
force
g(τ)dτ pulse
Periodic force
(T)
ωT = 2π
τ
dτ
t
Property of Fourier transform:
Proof
based
Thus:
Fourier series
Eq of motion
general non periodic function
discrete frequency instances
frequency axis
real axis
Response in the frequency domain
frequency response function
Fourier transform: G(ω)=F(g(t))
Dirichlet condition
Inverse Fourier transform
Check


Response in the Frequency Domain
Fourier series
Periodic force
Property of Fourier transform
Fourier transform
Inverse Fourier transform
Eq. of motion
$F(m\ddot{u} + c\dot{u} + Ku = g(t))$
$[m(i\omega)^2 + ci\omega + K] U(\omega) = G(\omega)$
$H(\omega)^{-1} = [K - m\omega^2 + i c\omega] U(\omega) = G(\omega)$
response in the frequency domain
$U(\omega) = H(\omega) \cdot G(\omega)$
frequency response function
Dirichlet condition
$G(\omega) = F(g(t))$
$g(t) = F^{-1}(G(\omega))$
In fact, recall that response to harmonic force
INPUT (force)
OUTPUT (response)
TIME DOMAIN
FREQUENCY DOMAIN
STRUCTURAL SYSTEM
unit pulse response
Check


Eq of motion:
F(mü + cu̇ + Ku = g(t))
Response in the frequency domain
Frequency response function
and represents the compliance to harmonic loading
Fourier transform
Inverse Fourier transform
Dirichlet condition
compliance in the frequency domain
is actually the Fourier transform of unit pulse response h(t)
Check:
(*) from D.I.
- In fact, recall that response to harmonic force:
Convolution integral (Duhamel Integral)
unit pulse response
INPUT
(force)
g(t)
OUTPUT
(response)
u(t)
h(t)
TIME DOMAIN
FREQUENCY DOMAIN
h * g
D.I.
STRUCTURAL SYSTEM
H · G
it could be proved that H(ω) = F(h(t))
otherwise
= gives H(ω) in (Δ)

Multiple degree of freedom systems (MDOF)
shear-type frames
beams
EJ
continuous system → discrete system
discretization (with lumped masses)
∞ dofs
finite dofs
EJ, GA, EA
dynamical
n: # of dofs
Vector of degrees of freedom (Lagrange variables)
q = [q1, q2, q3, q4, q5]
n×1
Force vector (external loading actions)
F(t) = [F, G, H, I, L]
n×1
Fe = K u SDOF
Inertia forces
stiffness coefficients
Elastic stiffness matrix sym and PD
= K q
n×n n×1
vector of elastic forces


Multiple degree of freedom systems (MDOF)
shear type frames
beams
continuous system → discrete system
∞ dofs
finite dofs
Inertia forces
SDOF
stiffness coefficients
elastic energy
Sym and PD
displacement method
inverse relation
positive definite
force method
compliance coeff
Maxwell theorem
mass matrix
compliance matrix


E_j = E_2 = 1
stiffness coefficients
(displacement method)
elastic energy
scalar
positive definite
compliance coeff.
(influence)
Maxwell Theorem
force method
Eqns. of motion (n = 5)
Inertia forces
SDOF
vector of elastic forces
(elastic) stiffness matrix
sym. and PD
inverse relation:
mass matrix (n×n)
compliance matrix
stiffness matrix