Energy approach (to instability problems)

A, C stationary points of the pot. en.

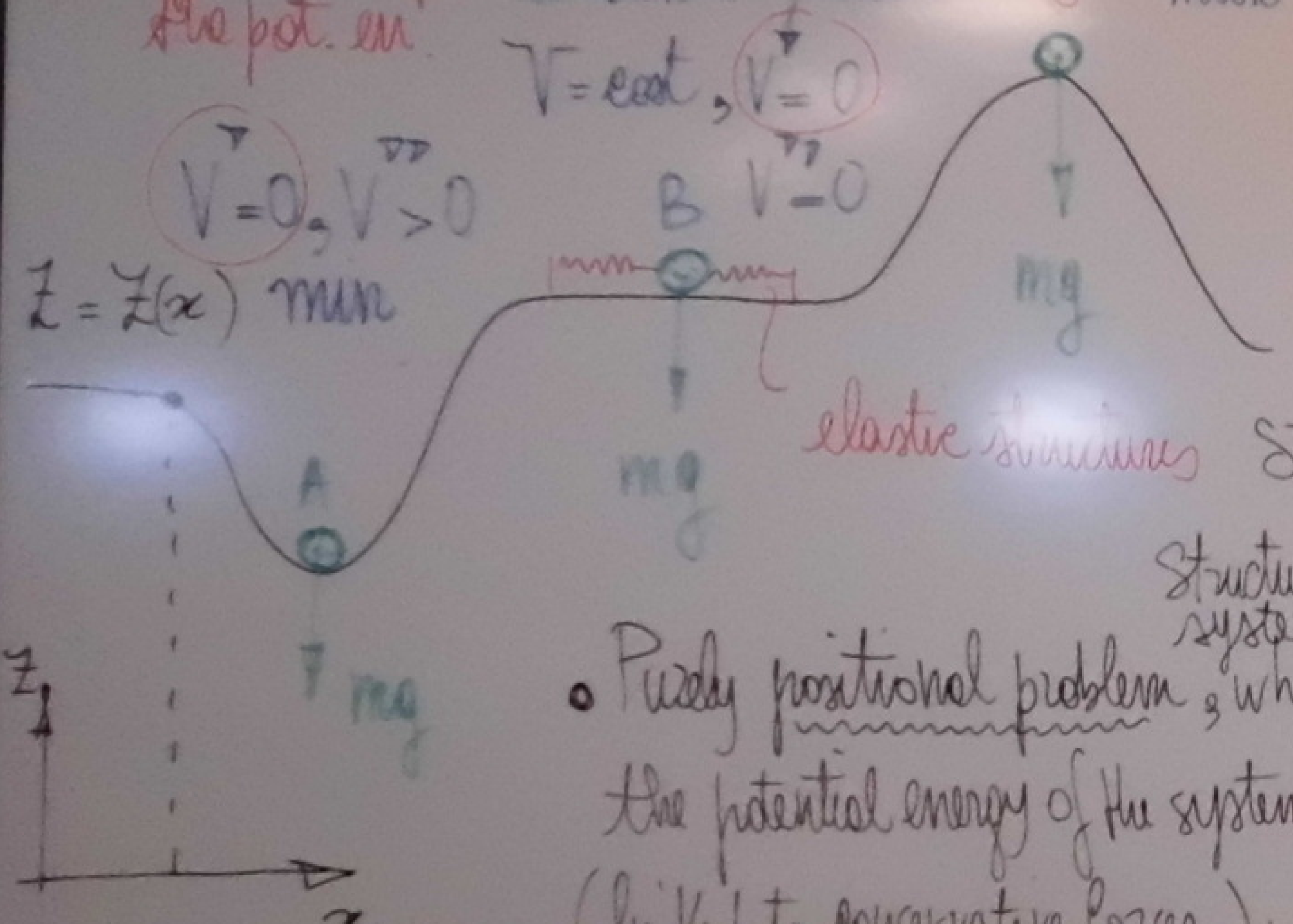
horizontal plateau

$V = \text{cost}$, $V' = 0$, $V'' = 0$

$V' = 0$, $V'' > 0$

$Z = Z(x)$ min

$V' = 0$, $V'' < 0$ max



elastic structures

STABLE

structural systems

- Purely positional problem, where the potential energy of the system (linked to conservative forces):

$$V(Z(x)) = mg\, z(x)$$

$( )' = \frac{\partial ( )}{\partial x}$

- From Liapunov dynamic stability criterion, states A, B and C look much different, in terms of potential effects of little perturbations of these equilibrium confn.

A: STABLE, after little perturbations the ball tries to get back to the original position. $V'' > 0$

B: INDIFFERENT; perturbations neither lead to system to leave nor to stay in the original configuration. $V'' = 0$ (separates stable for unstable)

C: UNSTABLE; little perturbations may let the system leave from the original position and acquire other ones (like A), that may be even far from the original. $V'' < 0$

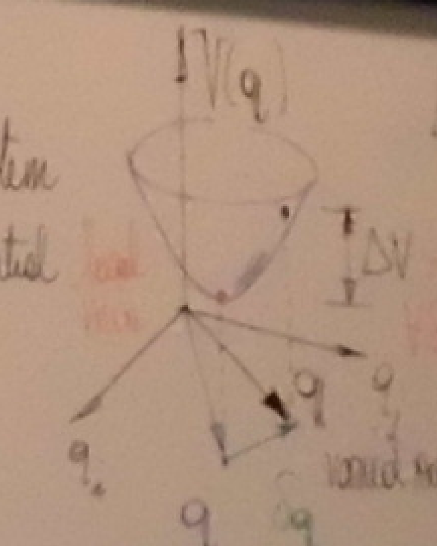
Theorem of Dirichlet (SC for stability)

The equilibrium confn. of a conservative, holonomous system with ideal constraints is stable if, there, the total potential energy of the system is at a (local) minimum

q_0 equil. confn.
$V(q_0)$ is min
$\Rightarrow q_0$ is stable

where: $V(q) = V_e(q) + V_f(q)$

$V_e(q)$: elastic energy; $V_f(q)$: potential energy of external conservative forces, $-L_e(q)$



$q = q_0 + \delta q$

$\delta q = q - q_0$

$\Delta V = V(q) - V(q_0)$

Energy approach to instability problems
From Liapunov dynamic stability criterion, states A, B and C look much different, in terms of potential effects of little perturbations of these equilibrium confn.
STABLE
elastic structures
structural systems
A STABLE, after little perturbations
B: INDIFFERENT; perturbations neither lead to system to leave nor to stay in the original configuration.
C: UNSTABLE; little perturbations might the system leave from the original position
Purely positional problem, where the potential energy of the system (linked to conservative forces)
Theorem of Dirichlet (SC for stability)
The equilibrium confn. of a conservative, holonomous system with ideal constraints is stable if, there, the total potential energy of the system is at a (local) minimum.
q0 equil. confn.
V(q0) is min
⇒ q0 is stable
where: V(q) = Ve(q) + Vf(q)
elastic energy
potential energy of external conservative forces
− Le(q)
V(q)
ΔV
local min
equil. confn.
varied confn.
variation in the neighborhood of q0
q = q0 + δq
δq = q − q0
ΔV = V(q) − V(q0) =
= δV + 1/2 δ²V
where:
δV = ∂V/∂q|q0 δq
1st order variation
+ Obviously, to be on a local minimum and then to achieve ΔV>0 ∀ δq, the 1st variation δV, which would be sensitive to the sign of δq, shall vanish, leading to the stationary condition:
δV = 0 ⇒ ∂V/∂q|q0 = 0
(NC for equilibrium configurations)


Theorem of Dirichlet (SC for stability)
The equilibrium confn of a conservative, holonomous system with ideal constraints is stable if, there, the total potential energy of the system is at a (local) minimum
q0 equil. confn. V(q0) is min ⇒ q0 is stable
where V(q) = Vε(q) + Vf(q)
elastic energy
potential energy of external conservative forces
−Lε(q)
V(q)
ΔV > 0 ∀δq
local min
varied confn.
q0 equil. confn.
δq variation in the neighborhood of q0
q = q0 + δq
δq = q − q0
ΔV = V(q) − V(q0) =
= δV + 1/2 δ²V
where:
− δV = ∂V/∂q |q0 T δq 1st-order variation
− 1/2 δ²V = 1/2 δqT ∂²V/∂q∂q |q0 δq = 1/2 δqT K δq 2nd-order variation
total stiffness matrix
+ Obviously, to be on a local minimum and then to achieve ΔV>0 ∀δq
the 1st-variation δV, which would be sensitive to the the sign of δq, shall vanish,
leading to the stationary condition: δV=0 ⇒ ∂V/∂q |q0 = 0
(NC for equilibrium configurations)


Example (SDOF discrete system, with concentrated elastic deformation)
Stability condition
In the equil. confn
load multiplier
Obviously
non-linear equation
same as from the static approach


Example (SDOF discrete system, with concentrated elastic deformation)
Stability condition
In the equil. confn
Stable
Dirichlet SC does not apply
INSTABILITY
STABILITY
Geometrically-small displacements
2nd-order theory
(2nd-order effects)
linearized equil. equation
as ϑ is small sin ϑ ≈ ϑ
and linearization of (*) leads to linearized equil. equation (•)
same as from the static approach

Stability condition
+ ϑ = 0
Stable P < P_cr = K/l
• 1 − p > 0 , p < 1
• 1 − p = 0 , p = 1
• 1 − p < 0 , p > 1
p = P/(K/l) = P/P_cr
p = ϑ / sin ϑ
(INSTABILITY)
(STABILITY)
"Geometrically-small" displacements
|ϑ| << 1
cos ϑ up to 2nd-order
V(ϑ) ≃ V_2(ϑ) = 1/2 K ϑ² − Pl (1 − (1 − 1/2 ϑ² + ...))
2nd-order theory
(2nd-order effects)
= 1/2 K ϑ² − Pl 1/2 ϑ²
= 1/2 (K − Pl) ϑ²
linearized equil. equation
(•) V_2'(ϑ) = (K − Pl) ϑ = 0
linear stat. cond.
K − Pl ≠ 0 , ϑ = 0
p ≠ 1
K − Pl = 0 , ϑ = arb.
p = 1
V_2''(ϑ) = K − Pl > 0 P < P_cr
const. = 0 P = P_cr
< 0 P > P_cr
as ϑ is small sin ϑ ≃ 0
and linearization of (*) leads to linearized equil. equation (•)