```
"-----------------------------------------------------------------------";
"SUL METODO DI RITZ NEL CALCOLO DELLE TRAVI ELASTICHE
 File Mathematica

 Corso di Meccanica Computazionale dei Solidi e delle Strutture
 Universita' di Bergamo, Facolta' di Ingegneria, Dalmine
 prof. Giuseppe Cocchetti
 Marzo 2012";
"-----------------------------------------------------------------------";

"Analisi elastica di travi:
 - v[x] spostamento trasversale (positivo verso il basso),
 - F carico concentrato (positivo verso il basso),
 - q carico distribuito (positivo verso il basso),
 - L lunghezza dell'asta,
 - EI rigidezza flessionale";

"Istruzioni d'uso:
 Ogni cella di comandi puo' essere eseguita in Mathematica
 cliccando col mouse nello spazio all'interno dei delimitatori
 visibili a destra e agendo sulla tastiera con sfhit+enter";

"Disabilita la segnalazione di spelling errors";
Off[General::spell]
Off[General::spell1]


(* ################################################
 ############################################################ *)


"Mensola con incastro in (x=0), soggetta
  a carico concentrato F sull'estremità libera (x=L)"
ClearAll[v, EPT, Ris]
v[x_] = C2 * x^2
EPT = 1/2 ∫_0^L EI * (v''[x])^2 dx - F * v[L]
Ris = Solve[∂_C2 EPT == 0, C2][[1]]
v[x] /. Ris
{v[L], v'[L]} /. Ris
```

```
Mensola con incastro in (x=0), soggetta
  a carico concentrato F sull'estremità libera (x=L)
```

$$C2\, x^2$$

$$2\, C2^2\, EI\, L - C2\, F\, L^2$$

$$\left\{C2 \to \frac{F\, L}{4\, EI}\right\}$$

$$\frac{F\, L\, x^2}{4\, EI}$$

$$\left\{\frac{F\, L^3}{4\, EI}, \frac{F\, L^2}{2\, EI}\right\}$$

```
"Mensola con incastro in (x=0), soggetta
  a carico concentrato F sull'estremità libera (x=L)"
ClearAll[v, EPT, Ris]
v[x_] = C2 * x^2 + C3 * x^3
EPT = 1/2 ∫_0^L EI * (v''[x])^2 dx - F * v[L]
Ris = Solve[{∂_C2 EPT == 0, ∂_C3 EPT == 0}, {C2, C3}][[1]]
v[x] /. Ris
{v[L], v'[L]} /. Ris
```

```
Mensola con incastro in (x=0), soggetta
  a carico concentrato F sull'estremità libera (x=L)
```

$$C2\,x^2 + C3\,x^3$$

$$-F\left(C2\,L^2 + C3\,L^3\right) + \frac{1}{2}\left(4\,C2^2\,EI\,L + 12\,C2\,C3\,EI\,L^2 + 12\,C3^2\,EI\,L^3\right)$$

$$\left\{C2 \to \frac{F\,L}{2\,EI},\ C3 \to -\frac{F}{6\,EI}\right\}$$

$$\frac{F\,L\,x^2}{2\,EI} - \frac{F\,x^3}{6\,EI}$$

$$\left\{\frac{F\,L^3}{3\,EI},\ \frac{F\,L^2}{2\,EI}\right\}$$

```
"Mensola con incastro in (x=0), soggetta
  a carico concentrato F sull'estremità libera (x=L)"
ClearAll[v, EPT, Ris]
v[x_] = C2 * x^2 + C3 * x^3 + C4 * x^4 + C5 * x^5 + C6 * x^6 + C7 * x^7 + C8 * x^8 + C9 * x^9 + C10 * x^10
EPT = 1/2 ∫_0^L EI * (v''[x])^2 dx - F * v[L];
Ris = Solve[{∂_C2 EPT == 0, ∂_C3 EPT == 0, ∂_C4 EPT == 0, ∂_C5 EPT == 0, ∂_C6 EPT == 0, ∂_C7 EPT == 0,
    ∂_C8 EPT == 0, ∂_C9 EPT == 0, ∂_C10 EPT == 0}, {C2, C3, C4, C5, C6, C7, C8, C9, C10}][[1]]
v[x] /. Ris
{v[L], v'[L]} /. Ris
```

```
Mensola con incastro in (x=0), soggetta
  a carico concentrato F sull'estremità libera (x=L)
```

$$C2\,x^2 + C3\,x^3 + C4\,x^4 + C5\,x^5 + C6\,x^6 + C7\,x^7 + C8\,x^8 + C9\,x^9 + C10\,x^{10}$$

$$\left\{C2 \to \frac{F\,L}{2\,EI},\ C3 \to -\frac{F}{6\,EI},\ C4 \to 0,\ C5 \to 0,\ C6 \to 0,\ C7 \to 0,\ C8 \to 0,\ C9 \to 0,\ C10 \to 0\right\}$$

$$\frac{F\,L\,x^2}{2\,EI} - \frac{F\,x^3}{6\,EI}$$

$$\left\{\frac{F\,L^3}{3\,EI},\ \frac{F\,L^2}{2\,EI}\right\}$$

```
(* ################################################
 ############################################################ *)
```

```
"Trave cerniera-carrello con carico distribuito uniforme (q)"
ClearAll[w, v, EPT, Ris]
w[x_] = C0 + C1 * x + C2 * x^2
RisC = Solve[{w[0] == 0, w[L] == 0}, {C0, C1}][[1]]
v[x_] = w[x] /. RisC
```

$$\text{EPT} = \frac{1}{2}\int_0^{\text{L}} \text{EI} * (\text{v}''[\text{x}])^2 \, d\text{x} - \int_0^{\text{L}} \text{q} * \text{v}[\text{x}] \, d\text{x}$$

```
Ris = Solve[∂C2 EPT == 0, C2][[1]]
```

$$\left\{\text{v}\left[\frac{\text{L}}{2}\right], -\text{v}'[\text{L}]\right\} /. \text{Ris}$$

Trave cerniera-carrello con carico distribuito uniforme (q)

$$\text{C0} + \text{C1}\,\text{x} + \text{C2}\,\text{x}^2$$

$$\{\text{C1} \to -\text{C2}\,\text{L},\ \text{C0} \to 0\}$$

$$-\text{C2}\,\text{L}\,\text{x} + \text{C2}\,\text{x}^2$$

$$2\,\text{C2}^2\,\text{EI}\,\text{L} + \frac{1}{6}\,\text{C2}\,\text{L}^3\,\text{q}$$

$$\left\{\text{C2} \to -\frac{\text{L}^2\,\text{q}}{24\,\text{EI}}\right\}$$

$$\left\{\frac{\text{L}^4\,\text{q}}{96\,\text{EI}},\ \frac{\text{L}^3\,\text{q}}{24\,\text{EI}}\right\}$$

```
"Trave cerniera-carrello con carico distribuito uniforme (q)"
ClearAll[w, v, EPT, Ris]
w[x_] = C0 + C1 * x + C2 * x^2 + C3 * x^3
RisC = Solve[{w[0] == 0, w[L] == 0}, {C0, C1}][[1]]
v[x_] = w[x] /. RisC
```

$$\text{EPT} = \frac{1}{2}\int_0^{L} \text{EI} * (v''[x])^2 \, dx - \int_0^{L} q * v[x] \, dx$$

```
Ris = Solve[{∂C2 EPT == 0, ∂C3 EPT == 0}, {C2, C3}][[1]]
```

$$\left\{v\left[\frac{L}{2}\right], -v'[L]\right\} /. \text{Ris}$$

Trave cerniera-carrello con carico distribuito uniforme (q)

$$C0 + C1\, x + C2\, x^2 + C3\, x^3$$

$$\{C1 \to -C2\, L - C3\, L^2,\ C0 \to 0\}$$

$$\left(-C2\, L - C3\, L^2\right) x + C2\, x^2 + C3\, x^3$$

$$\frac{1}{2}\left(4\, C2^2\, EI\, L + 12\, C2\, C3\, EI\, L^2 + 12\, C3^2\, EI\, L^3\right) + \frac{1}{6} C2\, L^3\, q + \frac{1}{4} C3\, L^4\, q$$

$$\left\{C2 \to -\frac{L^2\, q}{24\, EI},\ C3 \to 0\right\}$$

$$\left\{\frac{L^4\, q}{96\, EI},\ \frac{L^3\, q}{24\, EI}\right\}$$

```
"Trave cerniera-carrello con carico distribuito uniforme (q)"
ClearAll[w, v, EPT, Ris]
w[x_] = C0 + C1 * x + C2 * x^2 + C3 * x^3 + C4 * x^4
RisC = Solve[{w[0] == 0, w[L] == 0}, {C0, C1}][[1]]
v[x_] = w[x] /. RisC
```

$$\text{EPT} = \frac{1}{2}\int_0^{L} \text{EI} * (v''[x])^2 \, dx - \int_0^{L} q * v[x] \, dx$$

$$\text{Ris} = \text{Solve}[\{\partial_{C2}\text{EPT} == 0,\ \partial_{C3}\text{EPT} == 0,\ \partial_{C4}\text{EPT} == 0\}, \{C2, C3, C4\}][[1]]$$

$$\left\{v\left[\frac{L}{2}\right],\ -v'[L]\right\} /. \text{Ris}$$

Trave cerniera-carrello con carico distribuito uniforme (q)

$$C0 + C1\,x + C2\,x^2 + C3\,x^3 + C4\,x^4$$

$$\left\{C1 \to -C2\,L - C3\,L^2 - C4\,L^3,\ C0 \to 0\right\}$$

$$\left(-C2\,L - C3\,L^2 - C4\,L^3\right)x + C2\,x^2 + C3\,x^3 + C4\,x^4$$

$$\frac{1}{2}\left(4\,C2^2\,EI\,L + 12\,C2\,C3\,EI\,L^2 + 12\,C3^2\,EI\,L^3 + 16\,C2\,C4\,EI\,L^3 + 36\,C3\,C4\,EI\,L^4 + \frac{144}{5}\,C4^2\,EI\,L^5\right) + \frac{1}{6}\,C2\,L^3\,q + \frac{1}{4}\,C3\,L^4\,q + \frac{3}{10}\,C4\,L^5\,q$$

$$\left\{C2 \to 0,\ C3 \to -\frac{L\,q}{12\,EI},\ C4 \to \frac{q}{24\,EI}\right\}$$

$$\left\{\frac{5\,L^4\,q}{384\,EI},\ \frac{L^3\,q}{24\,EI}\right\}$$

```
"Trave cerniera-carrello con carico distribuito uniforme (q)"
ClearAll[w, v, EPT, Ris]
w[x_] = C0 + C1 * x + C2 * x^2 + C3 * x^3 + C4 * x^4 + C5 * x^5 + C6 * x^6 + C7 * x^7 + C8 * x^8 + C9 * x^9 + C10 * x^10
RisC = Solve[{w[0] == 0, w[L] == 0}, {C0, C1}][[1]]
v[x_] = w[x] /. RisC
```

$$\text{EPT} = \frac{1}{2}\int_0^{L} \text{EI} * (v''[x])^2 \, dx - \int_0^{L} q * v[x] \, dx;$$

```
Ris = Solve[{∂C2 EPT == 0, ∂C3 EPT == 0, ∂C4 EPT == 0, ∂C5 EPT == 0, ∂C6 EPT == 0, ∂C7 EPT == 0,
    ∂C8 EPT == 0, ∂C9 EPT == 0, ∂C10 EPT == 0}, {C2, C3, C4, C5, C6, C7, C8, C9, C10}][[1]]
{v[L/2], -v'[L]} /.
 Ris
```

Trave cerniera-carrello con carico distribuito uniforme (q)

$$C0 + C1\,x + C2\,x^2 + C3\,x^3 + C4\,x^4 + C5\,x^5 + C6\,x^6 + C7\,x^7 + C8\,x^8 + C9\,x^9 + C10\,x^{10}$$

$$\{C1 \to -C2\,L - C3\,L^2 - C4\,L^3 - C5\,L^4 - C6\,L^5 - C7\,L^6 - C8\,L^7 - C9\,L^8 - C10\,L^9,\ C0 \to 0\}$$

$$\left(-C2\,L - C3\,L^2 - C4\,L^3 - C5\,L^4 - C6\,L^5 - C7\,L^6 - C8\,L^7 - C9\,L^8 - C10\,L^9\right) x + C2\,x^2 + C3\,x^3 + C4\,x^4 + C5\,x^5 + C6\,x^6 + C7\,x^7 + C8\,x^8 + C9\,x^9 + C10\,x^{10}$$

$$\left\{C2 \to 0,\ C3 \to -\frac{L\,q}{12\,EI},\ C4 \to \frac{q}{24\,EI},\ C5 \to 0,\ C6 \to 0,\ C7 \to 0,\ C8 \to 0,\ C9 \to 0,\ C10 \to 0\right\}$$

$$\left\{\frac{5\,L^4\,q}{384\,EI},\ \frac{L^3\,q}{24\,EI}\right\}$$

```
(* ################################################
 ################################################################ *)
```

```
"Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)"
ClearAll[v, EPT, Ris]
v[x_] = C2 * x^2
```

$$\text{EPT} = \frac{1}{2}\int_0^{2L} \text{EI} * (v''[x])^2 \, dx - F * v[L]$$

```
Ris = Solve[∂C2 EPT == 0, {C2}][[1]]
{{v[L], v'[L]}, {v[2 L], v'[2 L]}} /. Ris
N[%]
```

Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)

$$C2\,x^2$$

$$4\,C2^2\,EI\,L - C2\,F\,L^2$$

$$\left\{C2 \to \frac{F\,L}{8\,EI}\right\}$$

$$\left\{\left\{\frac{F\,L^3}{8\,EI},\ \frac{F\,L^2}{4\,EI}\right\},\ \left\{\frac{F\,L^3}{2\,EI},\ \frac{F\,L^2}{2\,EI}\right\}\right\}$$

$$\left\{\left\{\frac{0.125\,F\,L^3}{EI},\ \frac{0.25\,F\,L^2}{EI}\right\},\ \left\{\frac{0.5\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\}\right\}$$

```
"Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)"
ClearAll[v, EPT, Ris]
v[x_] = C2 * x^2 + C3 * x^3
EPT = 1/2 ∫_0^(2 L) EI * (v''[x])^2 dx - F * v[L]
Ris = Solve[{∂_C2 EPT == 0, ∂_C3 EPT == 0}, {C2, C3}][[1]]
{{v[L], v'[L]}, {v[2 L], v'[2 L]}} /. Ris
N[%]
```

Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)

$$C2\,x^2 + C3\,x^3$$

$$-F\left(C2\,L^2 + C3\,L^3\right) + \frac{1}{2}\left(8\,C2^2\,EI\,L + 48\,C2\,C3\,EI\,L^2 + 96\,C3^2\,EI\,L^3\right)$$

$$\left\{C2 \to \frac{3\,F\,L}{8\,EI},\ C3 \to -\frac{F}{12\,EI}\right\}$$

$$\left\{\left\{\frac{7\,F\,L^3}{24\,EI},\ \frac{F\,L^2}{2\,EI}\right\},\ \left\{\frac{5\,F\,L^3}{6\,EI},\ \frac{F\,L^2}{2\,EI}\right\}\right\}$$

$$\left\{\left\{\frac{0.291667\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\},\ \left\{\frac{0.833333\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\}\right\}$$

```
"Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)"
ClearAll[v, EPT, Ris]
v[x_] = C2 * x^2 + C3 * x^3 + C4 * x^4
EPT = 1/2 ∫_0^(2 L) EI * (v''[x])^2 dx - F * v[L]
Ris = Solve[{∂_C2 EPT == 0, ∂_C3 EPT == 0, ∂_C4 EPT == 0}, {C2, C3, C4}][[1]]
{{v[L], v'[L]}, {v[2 L], v'[2 L]}} /. Ris
N[%]
```

Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)

$$C2\,x^2 + C3\,x^3 + C4\,x^4$$

$$-F\left(C2\,L^2 + C3\,L^3 + C4\,L^4\right) + \frac{1}{2}\left(8\,C2^2\,EI\,L + 48\,C2\,C3\,EI\,L^2 + 96\,C3^2\,EI\,L^3 + 128\,C2\,C4\,EI\,L^3 + 576\,C3\,C4\,EI\,L^4 + \frac{4608}{5}\,C4^2\,EI\,L^5\right)$$

$$\left\{C2 \to \frac{17\,F\,L}{32\,EI},\ C3 \to -\frac{23\,F}{96\,EI},\ C4 \to \frac{5\,F}{128\,EI\,L}\right\}$$

$$\left\{\left\{\frac{127\,F\,L^3}{384\,EI},\ \frac{F\,L^2}{2\,EI}\right\},\ \left\{\frac{5\,F\,L^3}{6\,EI},\ \frac{F\,L^2}{2\,EI}\right\}\right\}$$

$$\left\{\left\{\frac{0.330729\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\},\ \left\{\frac{0.833333\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\}\right\}$$

```
"Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)"
ClearAll[v, EPT, Ris]
v[x_] = C2 * x^2 + C3 * x^3 + C4 * x^4 + C5 * x^5
EPT = 1/2 ∫_0^(2 L) EI * (v''[x])^2 dx - F * v[L];
Ris = Solve[{∂_C2 EPT == 0, ∂_C3 EPT == 0, ∂_C4 EPT == 0, ∂_C5 EPT == 0}, {C2, C3, C4, C5}][[1]]
{{v[L], v'[L]}, {v[2 L], v'[2 L]}} /. Ris
N[%]
```

Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)

$$C2\,x^2 + C3\,x^3 + C4\,x^4 + C5\,x^5$$

$$\left\{C2 \to \frac{17\,F\,L}{32\,EI},\ C3 \to -\frac{23\,F}{96\,EI},\ C4 \to \frac{5\,F}{128\,EI\,L},\ C5 \to 0\right\}$$

$$\left\{\left\{\frac{127\,F\,L^3}{384\,EI},\ \frac{F\,L^2}{2\,EI}\right\},\ \left\{\frac{5\,F\,L^3}{6\,EI},\ \frac{F\,L^2}{2\,EI}\right\}\right\}$$

$$\left\{\left\{\frac{0.330729\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\},\ \left\{\frac{0.833333\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\}\right\}$$

```
"Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)"
ClearAll[v, EPT, Ris]
v[x_] = C2 * x^2 + C3 * x^3 + C4 * x^4 + C5 * x^5 + C6 * x^6
EPT = 1/2 ∫_0^(2 L) EI * (v''[x])^2 dx - F * v[L];
Ris = Solve[
    {∂_C2 EPT == 0, ∂_C3 EPT == 0, ∂_C4 EPT == 0, ∂_C5 EPT == 0, ∂_C6 EPT == 0}, {C2, C3, C4, C5, C6}][[1]]
{{v[L], v'[L]}, {v[2 L], v'[2 L]}} /. Ris
N[%]
```

Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)

$$C2\,x^2 + C3\,x^3 + C4\,x^4 + C5\,x^5 + C6\,x^6$$

$$\left\{C2 \to \frac{31\,F\,L}{64\,EI},\ C3 \to -\frac{F}{12\,EI},\ C4 \to -\frac{35\,F}{256\,EI\,L},\ C5 \to \frac{21\,F}{256\,EI\,L^2},\ C6 \to -\frac{7\,F}{512\,EI\,L^3}\right\}$$

$$\left\{\left\{\frac{511\,F\,L^3}{1536\,EI},\ \frac{F\,L^2}{2\,EI}\right\},\ \left\{\frac{5\,F\,L^3}{6\,EI},\ \frac{F\,L^2}{2\,EI}\right\}\right\}$$

$$\left\{\left\{\frac{0.332682\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\},\ \left\{\frac{0.833333\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\}\right\}$$

```
"Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)"
ClearAll[v, EPT, Ris]
v[x_] = C2 * x^2 + C3 * x^3 + C4 * x^4 + C5 * x^5 + C6 * x^6 + C7 * x^7
EPT = 1/2 ∫_0^(2 L) EI * (v''[x])^2 dx - F * v[L];
Ris = Solve[{∂_C2 EPT == 0, ∂_C3 EPT == 0, ∂_C4 EPT == 0, ∂_C5 EPT == 0, ∂_C6 EPT == 0, ∂_C7 EPT == 0},
    {C2, C3, C4, C5, C6, C7}][[1]]
{{v[L], v'[L]}, {v[2 L], v'[2 L]}} /. Ris
N[%]
```

Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)

$$C2\,x^2 + C3\,x^3 + C4\,x^4 + C5\,x^5 + C6\,x^6 + C7\,x^7$$

$$\left\{C2 \to \frac{31\,F\,L}{64\,EI},\ C3 \to -\frac{F}{12\,EI},\ C4 \to -\frac{35\,F}{256\,EI\,L},\ C5 \to \frac{21\,F}{256\,EI\,L^2},\ C6 \to -\frac{7\,F}{512\,EI\,L^3},\ C7 \to 0\right\}$$

$$\left\{\left\{\frac{511\,F\,L^3}{1536\,EI},\ \frac{F\,L^2}{2\,EI}\right\},\ \left\{\frac{5\,F\,L^3}{6\,EI},\ \frac{F\,L^2}{2\,EI}\right\}\right\}$$

$$\left\{\left\{\frac{0.332682\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\},\ \left\{\frac{0.833333\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\}\right\}$$

```
"Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)"
ClearAll[v, EPT, Ris]
v[x_] = C2 * x^2 + C3 * x^3 + C4 * x^4 + C5 * x^5 + C6 * x^6 + C7 * x^7 + C8 * x^8
EPT = 1/2 ∫_0^(2 L) EI * (v''[x])^2 dx - F * v[L];
Ris =
 Solve[{∂_C2 EPT == 0, ∂_C3 EPT == 0, ∂_C4 EPT == 0, ∂_C5 EPT == 0, ∂_C6 EPT == 0, ∂_C7 EPT == 0, ∂_C8 EPT == 0},
    {C2, C3, C4, C5, C6, C7, C8}][[1]]
{{v[L], v'[L]}, {v[2 L], v'[2 L]}} /. Ris
N[%]
```

Mensola di lunghezza 2L con forza concentrata F applicata a metà (in x=L)

$$C2\,x^2 + C3\,x^3 + C4\,x^4 + C5\,x^5 + C6\,x^6 + C7\,x^7 + C8\,x^8$$

$$\left\{C2 \to \frac{261\,F\,L}{512\,EI},\ C3 \to -\frac{401\,F}{1536\,EI},\ C4 \to \frac{315\,F}{1024\,EI\,L},\right.$$
$$\left. C5 \to -\frac{231\,F}{512\,EI\,L^2},\ C6 \to \frac{1309\,F}{4096\,EI\,L^3},\ C7 \to -\frac{429\,F}{4096\,EI\,L^4},\ C8 \to \frac{429\,F}{32\,768\,EI\,L^5}\right\}$$

$$\left\{\left\{\frac{32\,743\,F\,L^3}{98\,304\,EI},\ \frac{F\,L^2}{2\,EI}\right\},\ \left\{\frac{5\,F\,L^3}{6\,EI},\ \frac{F\,L^2}{2\,EI}\right\}\right\}$$

$$\left\{\left\{\frac{0.333079\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\},\ \left\{\frac{0.833333\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\}\right\}$$

```
"Mensola di lunghezza 2L con forza concentrata F applicata a metà (in B, x=L)"
ClearAll[wAB, wBC, vAB, vBC, EPT, Ris]
wAB[x_] = C00 + C01 * x + C02 * x^2 + C03 * x^3
wBC[x_] = C10 + C11 * x + C12 * x^2 + C13 * x^3

RisC = Solve[{wAB[0] == 0, wAB'[0] == 0, wAB[L] == wBC[L], wAB'[L] == wBC'[L]},
    {C00, C01, C10, C11}][[1]]
vAB[x_] = wAB[x] /. RisC
vBC[x_] = wBC[x] /. RisC

EPT = 1/2 ∫_0^L EI * (vAB''[x])^2 dx + 1/2 ∫_L^(2 L) EI * (vBC''[x])^2 dx - F * vAB[L];
Ris = Solve[{∂_C02 EPT == 0, ∂_C03 EPT == 0, ∂_C12 EPT == 0, ∂_C13 EPT == 0}, {C02, C03, C12, C13}][[1]]
{{vAB[L], vAB'[L]}, {vBC[2 L], vBC'[2 L]}} /. Ris
N[%]
```

Mensola di lunghezza 2L con forza concentrata F applicata a metà (in B, x=L)

$$C00 + C01\,x + C02\,x^2 + C03\,x^3$$

$$C10 + C11\,x + C12\,x^2 + C13\,x^3$$

$$\left\{C10 \to -C02\,L^2 + C12\,L^2 - 2\,C03\,L^3 + 2\,C13\,L^3,\ C11 \to 2\,C02\,L - 2\,C12\,L + 3\,C03\,L^2 - 3\,C13\,L^2,\ C00 \to 0,\ C01 \to 0\right\}$$

$$C02\,x^2 + C03\,x^3$$

$$-C02\,L^2 + C12\,L^2 - 2\,C03\,L^3 + 2\,C13\,L^3 + \left(2\,C02\,L - 2\,C12\,L + 3\,C03\,L^2 - 3\,C13\,L^2\right)x + C12\,x^2 + C13\,x^3$$

$$\left\{C13 \to 0,\ C12 \to 0,\ C02 \to \frac{F\,L}{2\,EI},\ C03 \to -\frac{F}{6\,EI}\right\}$$

$$\left\{\left\{\frac{F\,L^3}{3\,EI},\ \frac{F\,L^2}{2\,EI}\right\},\ \left\{\frac{5\,F\,L^3}{6\,EI},\ \frac{F\,L^2}{2\,EI}\right\}\right\}$$

$$\left\{\left\{\frac{0.333333\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\},\ \left\{\frac{0.833333\,F\,L^3}{EI},\ \frac{0.5\,F\,L^2}{EI}\right\}\right\}$$

```
(* ################################################
 ############################################################ *)

"Determinazione delle funzioni di forma flessionali (cubiche) dell'elemento trave"
ClearAll[PP, QQ, psi, B, k]
PP[x_] = {1, x, x^2, x^3}
QQ = {PP[0], PP'[0], PP[L], PP'[L]}
psi[x_] = {Expand[Simplify[PP[x].Inverse[QQ]]]}
```

Determinazione delle funzioni di forma flessionali (cubiche) dell'elemento trave

$$\left\{1,\ x,\ x^2,\ x^3\right\}$$

$$\left\{\{1,\ 0,\ 0,\ 0\},\ \{0,\ 1,\ 0,\ 0\},\ \left\{1,\ L,\ L^2,\ L^3\right\},\ \left\{0,\ 1,\ 2\,L,\ 3\,L^2\right\}\right\}$$

$$\left\{\left\{1 - \frac{3\,x^2}{L^2} + \frac{2\,x^3}{L^3},\ x - \frac{2\,x^2}{L} + \frac{x^3}{L^2},\ \frac{3\,x^2}{L^2} - \frac{2\,x^3}{L^3},\ -\frac{x^2}{L} + \frac{x^3}{L^2}\right\}\right\}$$

```
"Calcolo matrice di rigidezza flessionale dell'elemento trave"
B = psi''[x]
k = ∫_0^L EI * Transpose[B].B dx;
MatrixForm[k]
```

Calcolo matrice di rigidezza flessionale dell'elemento trave

$$\left\{\left\{-\frac{6}{L^2}+\frac{12\,x}{L^3},\ -\frac{4}{L}+\frac{6\,x}{L^2},\ \frac{6}{L^2}-\frac{12\,x}{L^3},\ -\frac{2}{L}+\frac{6\,x}{L^2}\right\}\right\}$$

$$\begin{pmatrix} \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ \frac{6\,EI}{L^2} & \frac{4\,EI}{L} & -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} \\ -\frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} \\ \frac{6\,EI}{L^2} & \frac{2\,EI}{L} & -\frac{6\,EI}{L^2} & \frac{4\,EI}{L} \end{pmatrix}$$

```
(* ##################################################
 ############################################################ *)
```

```
"Determinazione delle funzioni di forma assiali
  (lineari) e flessionali (cubiche) dell'elemento trave"
ClearAll[NN, SS, psiA, PP, QQ, psiF, B, EE, k]
NN[x_] = {1, x};
SS = {NN[0], NN[L]};
psiA[x_] = Expand[Simplify[NN[x].Inverse[SS]]]

PP[x_] = {1, x, x^2, x^3};
QQ = {PP[0], PP'[0], PP[L], PP'[L]};
psiF[x_] = Expand[Simplify[PP[x].Inverse[QQ]]]
```

Determinazione delle funzioni di forma
  assiali (lineari) e flessionali (cubiche) dell'elemento trave

$$\left\{1-\frac{x}{L},\ \frac{x}{L}\right\}$$

$$\left\{1-\frac{3\,x^2}{L^2}+\frac{2\,x^3}{L^3},\ x-\frac{2\,x^2}{L}+\frac{x^3}{L^2},\ \frac{3\,x^2}{L^2}-\frac{2\,x^3}{L^3},\ -\frac{x^2}{L}+\frac{x^3}{L^2}\right\}$$

```
"Calcolo matrice di rigidezza completa dell'elemento trave"
B = Table[0, {i, 2}, {j, 6}];
B[[{1}, {1, 4}]] = psiA'[x];
B[[{2}, {2, 3, 5, 6}]] = psiF''[x];
MatrixForm[B]

EE = DiagonalMatrix[{EA, EI}];
MatrixForm[EE]

k = ∫_0^L Transpose[B].EE.B dx;
MatrixForm[k]
```

Calcolo matrice di rigidezza completa dell'elemento trave

$$\begin{pmatrix} -\frac{1}{L} & 0 & 0 & \frac{1}{L} & 0 & 0 \\ 0 & -\frac{6}{L^2}+\frac{12\,x}{L^3} & -\frac{4}{L}+\frac{6\,x}{L^2} & 0 & \frac{6}{L^2}-\frac{12\,x}{L^3} & -\frac{2}{L}+\frac{6\,x}{L^2} \end{pmatrix}$$

$$\begin{pmatrix} EA & 0 \\ 0 & EI \end{pmatrix}$$

$$\begin{pmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ 0 & \frac{6\,EI}{L^2} & \frac{4\,EI}{L} & 0 & -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & 0 & \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} \\ 0 & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 & -\frac{6\,EI}{L^2} & \frac{4\,EI}{L} \end{pmatrix}$$