Università degli studi di Bergamo

Scuola di Ingegneria (Dolmine)

CCS Ingegneria Edile

L-23 Ingegneria delle Tecnologie per l'Edilizia

Scienza delle Costruzioni

(ICAR/08 - SdC ; 9 CFU)

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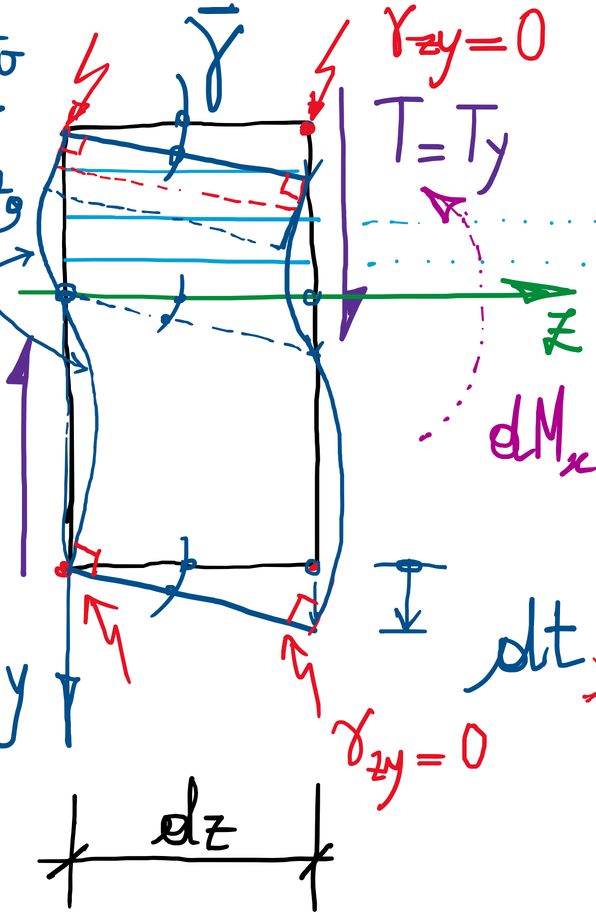
LEZIONE 23

- Deformazione del concio di trave dovuta al taglio

ingobbamento fuori piano della sezione, legato alle γ_z variabili sulle sez.



$dM_x = T_y\,dz$

$dt_y = \bar{\gamma}\,dz$

scorrimento angolare medio

$T=T_y$

↯

τ_z

legge costit. ↯

$\gamma_z = \dfrac{\tau_z}{G}$

G: modulo di elasticità tangenziale o modulo di taglio

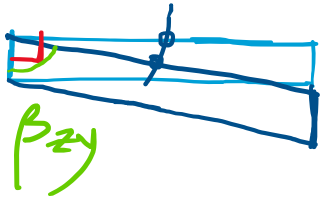
$$G = \frac{E}{2(1+\nu)}$$

materiale:
- elastico
- lineare
- isotropo → 2 parametri (es. E, ν)

variazione di angolo retto tra fibre inizialm. ⊥

$\gamma_{zy}(y)$ scorrimento angolare $\left(\gamma_{ij} = 2\varepsilon_{ij};\ \varepsilon_{ij} = \frac{1}{2}\gamma_{ij}\right)$

$$= \frac{\pi}{2} - \beta_{zy}$$



- Se γ_{zy} fosse costante in y, la deformazione del concio sarebbe "autosomigliante" a quella, costante, di ogni "striscolina" ideale, e si verrebbe a produrre solo uno scorrimento relativo dt della faccia di destra rispetto a quella di sinistra.

- Poichè $\gamma_{zy} = \gamma_{zy}(y)$ risulta variabile lungo la sezione, da $\gamma_{zy} = \frac{\tau_{zy}}{G}$, con $\tau_{zy}(y)$, secondo, ad es., la formula di Jourawsky, si valuta $\bar{\gamma}$

$\gamma_{zy}\uparrow$ (valor medio)

• Valutazione dello scorrimento medio via PLV : [effetti taglianti]

$$\frac{d\mathcal{L}_e}{dz} = T_y \frac{dt_y}{dz} = \int_A \underbrace{\tau_z \cdot \gamma_z}_{} \, dA \, \cancel{dz} = \frac{d\mathcal{L}_i}{dz} \qquad \gamma_z = \frac{\tau_z}{G}$$

$$\tau_z = \begin{Bmatrix} \tau_{zx} \\ \tau_{zy} \end{Bmatrix} ; \quad \gamma_z = \begin{Bmatrix} \gamma_{zx} \\ \gamma_{zy} \end{Bmatrix} = \begin{Bmatrix} \frac{\tau_{zx}}{G} \\ \frac{\tau_{zy}}{G} \end{Bmatrix} = \frac{1}{G}\tau_z$$

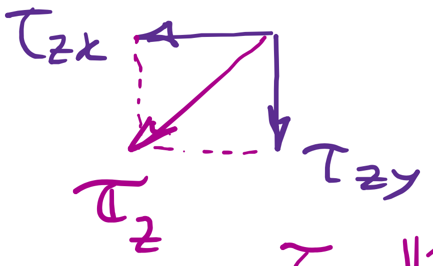
$$\bar{\tau}$$

$$\tau_{zx}\gamma_{zx} + \tau_{zy}\gamma_{zy} \Leftrightarrow \sigma_{ij}\varepsilon_{ij} = \sigma : \varepsilon$$

$$\tau_{zx}\frac{\tau_{zx}}{G} + \tau_{zy}\frac{\tau_{zy}}{G}$$

$$\bar{\gamma} = \frac{T_y}{GA^*} = \frac{dt}{dz}$$

$$\frac{1}{G}\underbrace{\left(\tau_{zx}^2 + \tau_{zy}^2\right)}_{\tau_z^2} = \frac{\tau_z^2}{G}$$



$$\tau_z = \|\tau_z\| = \sqrt{\tau_{zx}^2 + \tau_{zy}^2}$$

$$\tau_z^2 = \|\tau_z\|^2 = \tau_{zx}^2 + \tau_{zy}^2$$

Soluz. di Jourawsky

$$T_y \frac{dt}{dz} = \frac{1}{G}\int_A \tau_{zy}^{J\,2}\left[1 + \left(\frac{\tau_{zx}}{\tau_{zy}^J}\right)^2\right] dA$$

$$\begin{cases} \tau_{zy}^J(y) = \dfrac{T_y\, S_x^{**}(y)}{J_x\, b(y)} & \text{formula di Jourawsky} \\[2ex] \dfrac{\tau_{zx}(x;y)}{\tau_{zy}^J(y)} = -\dfrac{2\tan\alpha(y)}{b(y)} \cdot x \end{cases}$$

$$\cancel{T_y}\underbrace{\frac{dt}{dz}}_{\bar{\gamma}} = \frac{1}{G}\frac{T_y^{\cancel{2}}}{A}\underbrace{\frac{A}{J_x^2}\int_A \frac{S_x^{**}(y)^2}{b(y)^2}\left(1 + \frac{4\tan^2\alpha(y)}{b^2(y)}x^2\right) dA}_{}$$

μ : fattore di taglio della sez. trasv. ≥ 1

$$dt = \mu\frac{T_y}{GA}$$

GA rigidezza tagliante ;

$$A^* = \frac{A}{\mu} \text{ area ridotta} \Rightarrow dt = \frac{T_y}{GA^*}\,dz$$

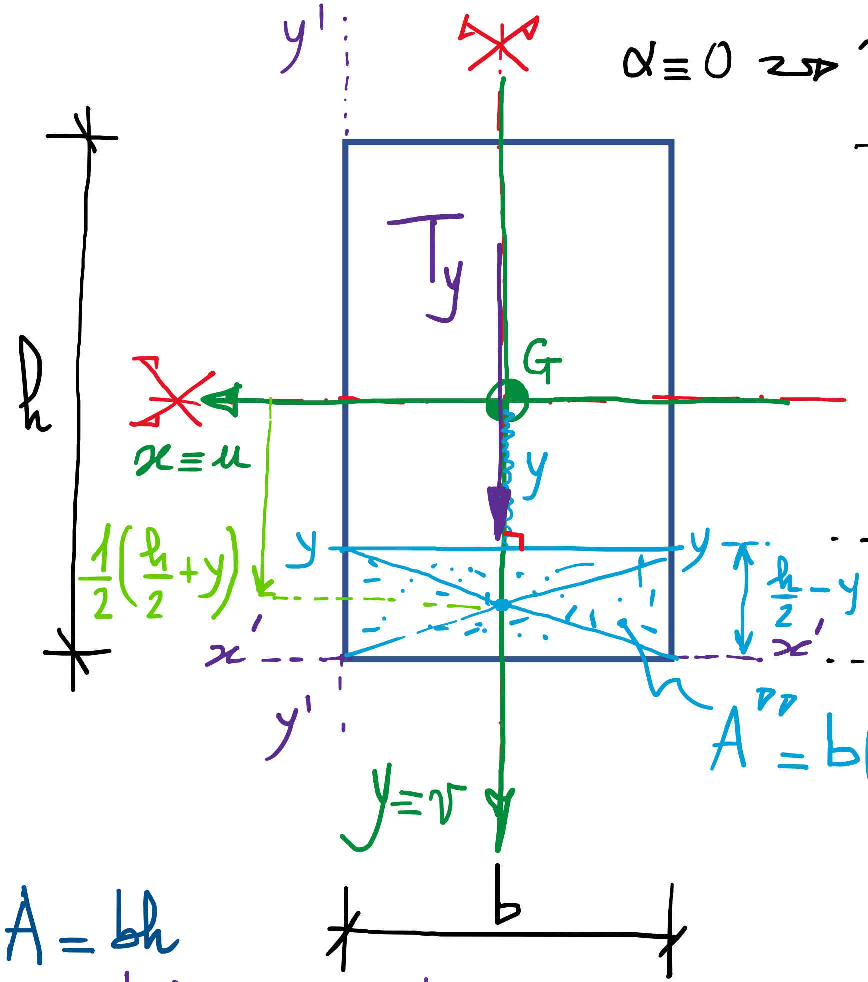
GA^* rigidezza tagliante efficace ... γ

intero prisma

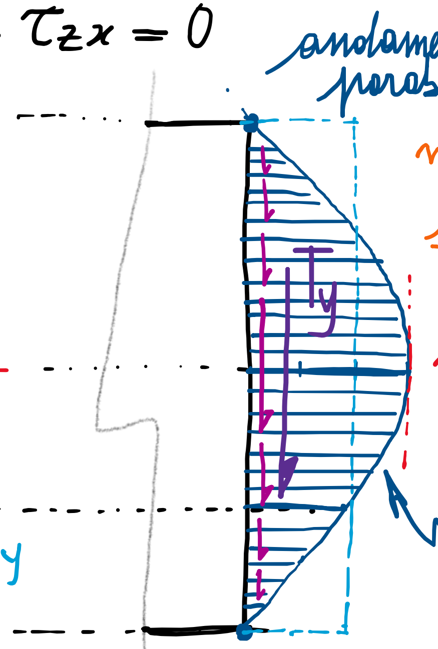
$$\Delta t = \frac{T_y\,\ell}{GA^*}$$

– Caso della sezione rettangolare: (sez. doppiamente simmetrica)

$\alpha \equiv 0 \Rightarrow \tau_{zx} = 0$



$A^{**} = b\left(\frac{h}{2}-y\right)$



max. rel.

$\frac{d\tau}{dy} \sim \frac{2y}{h^2} = 0$ in $y=0$

$\tau_{zy}^{max} = \frac{3}{2}\frac{T_y}{A} = 1.5\,\bar{\tau}_{zy}$

$\frac{T_y}{A} = \bar{\tau}_{zy}$ valor medio

$\bar{\tau}_{zy} = \frac{1}{\cancel{b}h}\int_{-h/2}^{h/2} \tau_{zy}\,\cancel{b}\,dy$

$$\chi = \frac{6}{5}$$

fattore di taglio

$$\tau_{zy}^{J}(y) = \frac{T_y\, S_x^{**}(y)}{J_x\, b} = \frac{T_y}{J_x\, b}\, S_x^{**}(y)$$

$$S_x^{**}(y) = A^{**}\cdot\frac{1}{2}\left(\frac{h}{2}+y\right) = b\left(\frac{h}{2}-y\right)\frac{1}{2}\left(\frac{h}{2}+y\right)$$

$$= \frac{b}{2}\left(\frac{h^2}{4}-y^2\right)$$

parabolico (0 per $y=\pm\frac{h}{2}$, f.ne pari in y, max rel. per $y=0$: $\frac{bh^2}{8}$)

$$\tau_{zy}^{J}(y) = \frac{T_y}{\frac{1}{12}bh^3}\,\frac{\frac{\cancel{b}}{2}\left(\frac{h^2}{4}-y^2\right)}{\cancel{b}}$$

$$= \frac{6\,T_y}{bh^3}\left(\frac{h^2}{4}-y^2\right) \quad \underbrace{(y/(h/2))^2}$$

$$= \frac{\cancel{6}^{\,3}\,T_y}{bh^3}\,\frac{h^2}{\cancel{4}_{\,2}}\left(1-4\frac{y^2}{h^2}\right)$$

$$= \frac{3}{2}\,\frac{T_y}{\underbrace{bh}_{A}}\,\underbrace{\left(1-4\frac{y^2}{h^2}\right)}$$

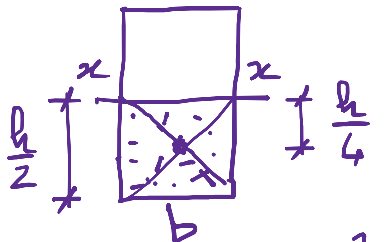
$\frac{d}{dy} = -4\cdot\frac{2y}{h^2}$ lin.

$A = bh$

$$J_{x'} = \int_0^b\int_0^h y'^2\, dA = \int_0^b \frac{y'^3}{3}\Big|_0^h dx', \quad J_x = \frac{1}{12}bh^3$$

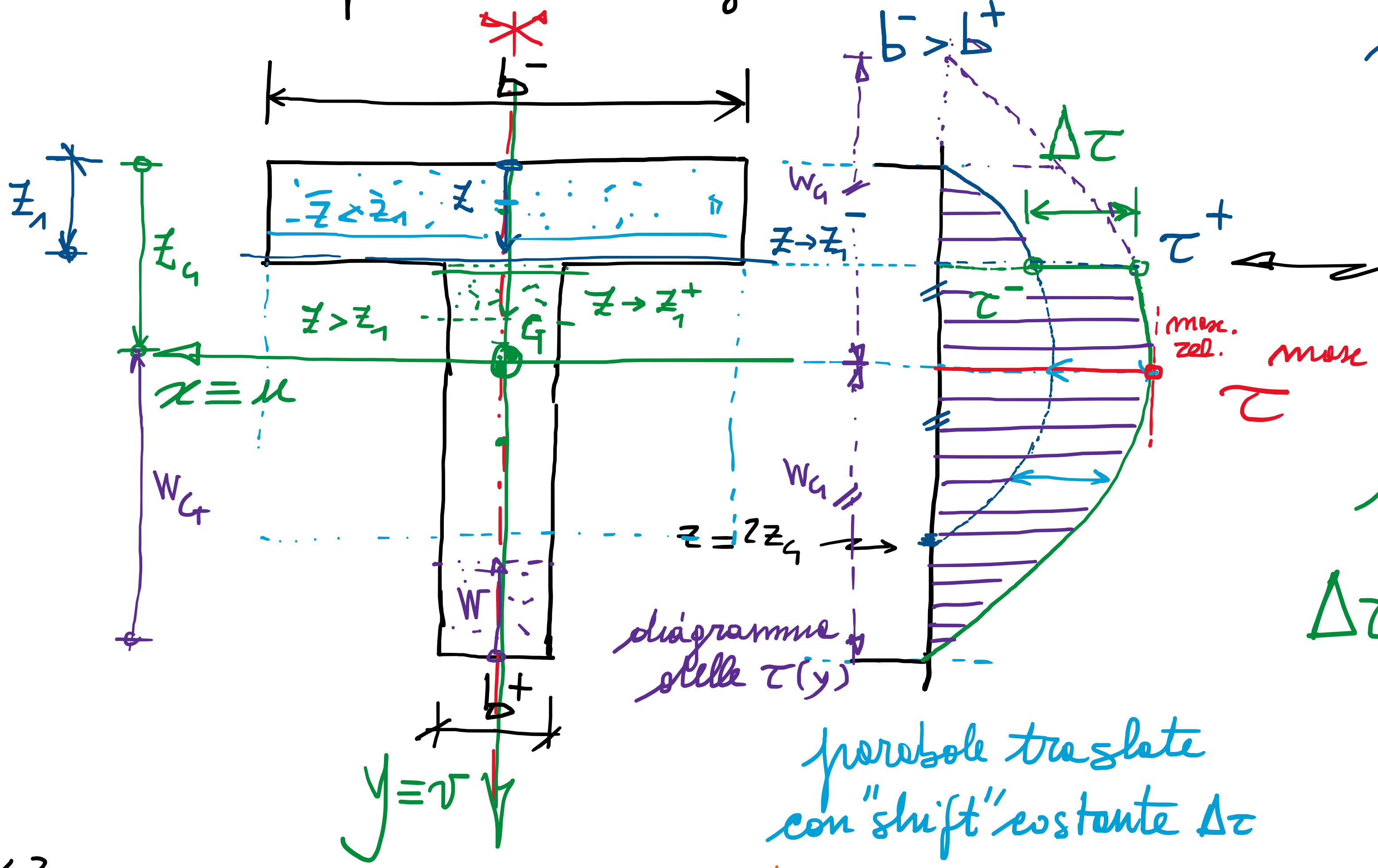
$$= \frac{1}{3}bh^3$$

$$J_x = J_{x'} - A\,d_G^2 = \frac{1}{3}bh^3 - bh\left(\frac{h}{2}\right)^2 = \left(\frac{1}{3}-\frac{1}{4}\right)bh^3 = \frac{4-3}{12}bh^3$$



$$S_x^{**} = b\,\frac{h}{2}\,\frac{h}{4} = \frac{bh^2}{8}$$

– Sezioni composte da rettangoli elementari:



$$z = z_1$$

$$\tau^- = \frac{T\,S_1}{J\,b^-}\;;\quad \tau^+ = \frac{T\,S_1}{J\,b^+}$$

discontinuità della larghezza della corda $\Delta b = b^- - b^+$

⇓

discontinuità "salto" della τ

$$\Delta\tau = \tau^+ - \tau^- = \frac{TS_1}{J}\left(\frac{1}{b^+} - \frac{1}{b^-}\right)$$

$$= \frac{TS_1}{J}\,\frac{b^- - b^+}{b^+ b^-} = \frac{TS_1}{J}\,\frac{\Delta b}{b^+ b^-}$$

$$= \tau^- \frac{\Delta b}{b^+} = \tau^+ \frac{\Delta b}{b^-}$$

$$\tau^- b^- = \tau^+ b^+\;;\quad \frac{\tau^+}{\tau^-} = \frac{b^-}{b^+}$$

$$\tau^+ = \frac{b^-}{b^+}\,\tau^-$$

parabole traslate con "shift" costante $\Delta\tau$

$z < z_1$

$$S_x = b^- \underbrace{z\left(z_G - \frac{z}{2}\right)}_{\text{stessa funzione}} \sim z^2$$

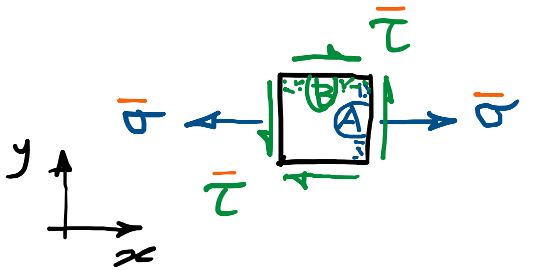
$$+ \overset{b^- - b^+}{\Delta b}\, z_1\left(z_G - \frac{z_1}{2}\right) \text{ "shift" costante}$$

$z > z_1$

$$S_x = \underbrace{b^- z_1\left(z_G - \frac{z_1}{2}\right)} + b^+ (z - z_1)\left(z_G - \frac{z + z_1}{2}\right) \sim z^2$$

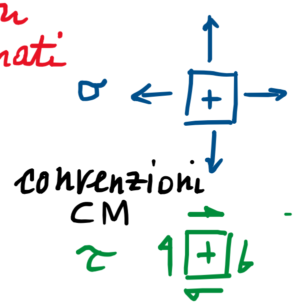
$$+ b^+ \underbrace{z\left(z_G - \frac{z}{2}\right)}_{\text{stessa funzione}} - \underbrace{b^+ z_1\left(z_G - \frac{z_1}{2}\right)}$$

• CM per stato di sforzo piano alla de Saint Venant (travi) (una componente di sforzo normale nulla)



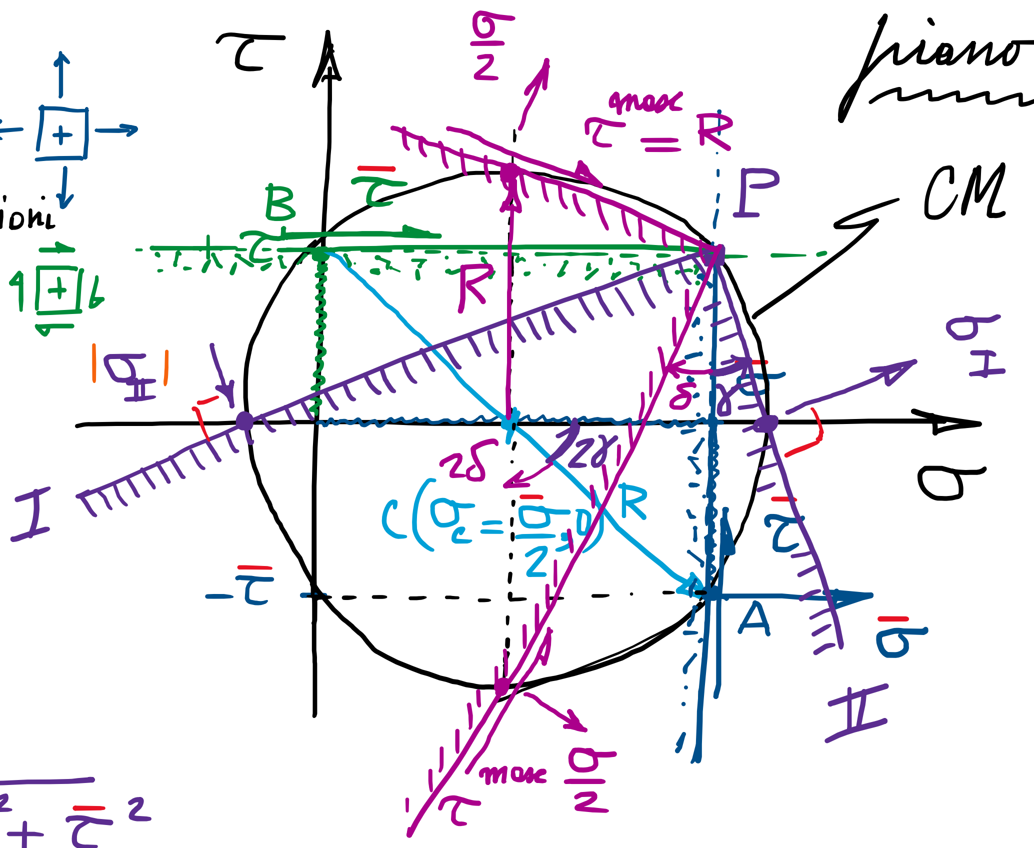
$$\begin{cases} \sigma_{xx} = \bar{\sigma} \\ \boxed{\sigma_{yy} = 0} \\ \tau_{xy} = \bar{\tau} \end{cases}$$

valori assegnati



particolare stato di sforzo piano con una componente di sforzo normale nulla

piano di Mohr



$$\sigma_c = \frac{\bar{\sigma}}{2}$$

$$R = \sqrt{\Big(\underbrace{\frac{\bar{\sigma}}{2}}_{\sigma_c}\Big)^2 + \bar{\tau}^2}$$

$$= \tau^{max}$$

tensioni principali

$$\sigma_{I,II} = \sigma_c \pm R$$

$$= \frac{\bar{\sigma}}{2} \pm \sqrt{\left(\frac{\bar{\sigma}}{2}\right)^2 + \bar{\tau}^2}$$

$$\sigma_I \cdot \sigma_{II} = (\sigma_c + R)(\sigma_c - R)$$

$$= \sigma_c^2 - R^2$$

$$= \cancel{\left(\frac{\bar{\sigma}}{2}\right)^2} - \cancel{\left(\frac{\bar{\sigma}}{2}\right)^2} - \bar{\tau}^2 \leq 0$$

$$\tan 2\gamma = \frac{2\bar{\tau}}{\bar{\sigma}}$$

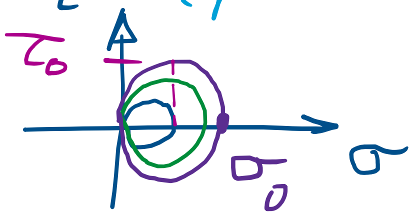
inclinaz. diz. princ. .... $\gamma = \frac{1}{2} \operatorname{atan}\left(\frac{2\bar{\tau}}{\bar{\sigma}}\right)$

inclinaz. diz. secondo cui agisce la τ^{max} .... $\delta = \frac{\pi}{4} - \gamma$ $\left(2\delta + 2\gamma = \frac{\pi}{2} \Rightarrow \delta + \gamma = \frac{\pi}{4}\right)$

τ^{max} nel piano $\Leftrightarrow$ tensioni principali di segno opposto

– Formule di verifica per stati di sforzo alla DSV

(parametrizzazione su prova di trazione)



– Criterio di Tresca : $\tau^{max} \leq \tau_0^T = \frac{\sigma_0}{2} = 0.5\,\sigma_0$

(è il più cautelativo) $2\tau^{max} \leq \sigma_0$

$$\sigma_{eq}^T = 2R = |\sigma_I - \sigma_{II}| = \sqrt{\bar{\sigma}^2 + \boxed{4}\bar{\tau}^2} \leq \sigma_0 \qquad \sigma_{eq}^T = \bar{\sigma}\sqrt{1+4\left(\frac{\bar{\tau}}{\bar{\sigma}}\right)^2} \leq \sigma_0$$

infatti: $\sigma_I - \sigma_{II} = \cancel{\sigma_c} + R - (\cancel{\sigma_c} - R) = 2R = 2\sqrt{\frac{\bar{\sigma}^2}{4} + \bar{\tau}^2} = \sqrt{\bar{\sigma}^2 + 4\bar{\tau}^2}$

– Criterio di von Mises :

$$\sigma_{eq}^{vM} = \sqrt{\sigma_I^2 + \sigma_{II}^2 - \sigma_I\sigma_{II}} = \sqrt{\bar{\sigma}^2 + \boxed{3}\bar{\tau}^2} \leq \sigma_0 \quad \Rightarrow \quad \tau_0^{vM} = \frac{1}{\sqrt{3}}\sigma_0 = \frac{\sqrt{3}}{3}\sigma_0 \simeq .577\,\sigma_0$$

($\sqrt{3} \sim 1.73$)

infatti: $\sigma_I^2 + \sigma_{II}^2 - \sigma_I\sigma_{II} = (\sigma_c + R)^2 + (\sigma_c - R)^2 - (\sigma_c + R)(\sigma_c - R)$

$= \sigma_c^2 + R^2 + 2\cancel{\sigma_c}R + \cancel{\sigma_c^2} + R^2 - 2\cancel{\sigma_c}R - \cancel{\sigma_c^2} + R^2 = \sigma_c^2 + 3R^2$

$= \frac{\bar{\sigma}^2}{4} + 3\left(\frac{\bar{\sigma}^2}{4} + \bar{\tau}^2\right) = \bar{\sigma}^2 + 3\bar{\tau}^2$

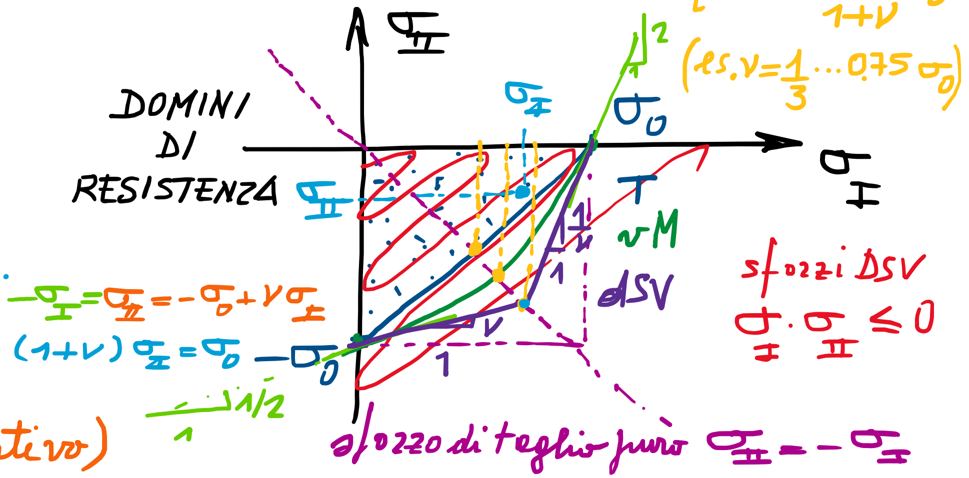
resist. a taglio $\begin{cases} \tau_0^T = 0.5\,\sigma_0 \\ \tau_0^{vM} = \frac{1}{\sqrt{3}}\sigma_0 \\ \tau_0^{dSV} = \frac{1}{1+\nu}\sigma_0 \end{cases}$ (es. $\nu = \frac{1}{3}$ ... $0.75\,\sigma_0$)

– Criterio di de Saint Venant :

$$\sigma_{eq}^{dSV} = \max\begin{Bmatrix} |\sigma_I - \nu\,\sigma_{II}| \\ |\sigma_{II} - \nu\,\sigma_I| \end{Bmatrix} \leq \sigma_0$$

(σ_{II} è < 0 ATT.!)

$\nu = \frac{E}{2G} - 1 < \frac{1}{2}$ coeff. di contraz. trasversale o di Poisson



– Ordine di conservatività : $\sigma_{eq}^T \geq \sigma_{eq}^{vM} \geq \sigma_{eq}^{dSV}$ (dSV è il meno cautelativo)