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Structural Analysis of the Paderno d'Adda Bridge (Italy, 1889)

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Abstract. The Paderno d'Adda Bridge is a marvellous riveted iron viaduct with a doubly-built-in parabolic arch that crosses the river Adda near Milano, between Paderno d'Adda (Lecco province) and Calusco d'Adda (Bergamo province), in Lombardia, northern Italy. It was completed in 1889 by the *"Società Nazionale delle Officine di Savigliano" (SNOS)*. In this work, following a previous contribution to the last SAHC08 Conference (Ferrari and Rizzi 2008), a complete FEM model of the bridge is presented, in the attempt of querying the performance of the structure at design stage. Several static loading conditions have been carried-out in the elastic range and results have been compared to those available in the original SNOS Report (1889), with remarkable correspondence.

Keywords: 19th-century arch iron bridge, railway engineering, FEM model, static elastic response.

Introduction

Towards the end of the 19th century, rapidly growing industrial activities in Lombardia required the further expansion of the existing railway network. In particular, it became necessary to acquire the elevated crossing on the river Adda, North-East from Milano. In 1889, the SNOS completed the construction of the Paderno d'Adda Bridge (Fig. 1), sometimes called San Michele Bridge (SNOS 1889, Nascè et al. 1984). It is one of the very first great iron constructions designed through the practical application of the so-called *"Theory of the ellipse of elasticity"* (a specific account on this aspect has been given in the previous SAHC08 paper, Ferrari and Rizzi 2008). This is a graphical-analytical method of structural analysis that was developed by Karl Culmann (1821-1881) and his pupil Wilhelm Ritter (1847-1906) at the Polytechnical School of Zürich, where the man whom the design of the bridge is normally attributed was formed (Jules Röthlisberger, 1851-1911, head of the SNOS Technical Office for 25 years, since 1885).

Left bank Calusco d'Adda (BG)

Right bank Paderno d'Adda (LC)

Figure 1: Present up-stream view from Paderno d'Adda of the Paderno d'Adda Bridge (1889).

The iron bridge crosses the river Adda to a height of approximately 85 m from water. The main upper continuous beam, 5 m wide, is formed by a 266 m long metallic box girder, supported by nine bearings. The girder hosts the railway track in the inner deck, while the road is located on the upper deck. Four of the supports of the continuous beam are provided by a marvellous doubly-built-in parabolic arch of about 150 m of horizontal span and 37.5 m of vertical rise, with trapezoidal cross section having width and height increasing from crown to shoulders and front faces laying into symmetric inclined planes, in view of counteracting transverse horizontal loads. Four bearings (two at the extremities and two near half-length of the upper beam, the latter symmetrically located around the crown) and five vertical metallic piers warrant the load transfer from the upper beam to the underneath arch or directly to the banks' ground. The bridge is made with a "wrought iron" material, with riveted connections. Details on the various characteristic features of the bridge are available in Refs. [1-7]; specifically, in English compact form in Ferrari and Rizzi (2008), where the reported technical descriptions have been very much taken from Nascè et al. (1984), that is still the most comprehensive publication on the bridge.

A study on the structural performance of the bridge has been started at the University of Bergamo since 2005 (see Refs. [3-7]). Now, a complete FEM model of the bridge has been put in place, taking into account the geometry of the structure, as it appears from the original design drawings. In this paper, such complete FEM model is presented for the first time, together with a study on the static elastic response of the bridge for several loading conditions. These correspond to the original loading configurations that were used for the design of the structural elements of the bridge and to the loading conditions that were implemented in the first try-outs in 1889. In both cases, the comparison to the results provided in the original Report SNOS (1889), shows a remarkable correspondence, in terms of both deformations (vertical deflections) and stresses (bar axial forces and relative normal stresses).

FEM Modelling of the Bridge

The FEM model of the bridge has been implemented within the commercial code ABAQUS, by assembling a true 3D truss frame with beam elements, mutually built-in at the nodes. The morphology of the bridge is quite intricate, with a level of detail that is truly available in the original technical drawings, which are guarded at the Archivio Storico Nazionale di Torino. For instance, the longitudinal structural members are often made with a variable number of riveted plates and also, at sub-structure junctions, there often appear additional reinforcing plates, conceived for local stiffening. Some simplifications have been necessarily considered: the model has been outlined by assigning always bars with constant average cross section between the truss nodes, endowed with equivalent geometrical characteristics (area, principal moments and torsional inertia). Also, the additional reinforcing plates have not been explicitly represented, whereas the cross sections of the limiting bars that contour such stiffening plates have been assumed with higher geometrical characteristics, i.e. one thousand times the values that should be directly assigned to them. This way to proceed was mentioned by Nascè et al. (1984), which have already presented a first, much stylised, FEM model of the bridge. Indeed, the driving idea in the present FEM assembly has been that of developing a true 3D truss structure that would resemble the real 3D design geometry of the viaduct.

The FEM model has been created in four main steps: 1) modelling of the parabolic arch (a partial account on this has been given in Ferrari and Rizzi 2008); 2) outline of the vertical piers; 3) realisation of the upper continuous box beam; 4) assembly of the whole model by tying together the three parts. In the first three phases, the different parts have been tested first as stand-alone structural elements. Later, numerical simulations have been developed for the entire structure, to appreciate the structural response of the bridge as a whole. The different parts and total assembly are shortly described as follows, with reference to the technical descriptions of the real structural members that has been advanced in details in Nascè et al. (1984) and briefly reported in Ferrari and Rizzi (2008).

Arch. The 3D truss frame of the arch consists of two planar parabolic trusses laying into two inclined planes (of an angle $\alpha \cong \pm 8.63°$ to the vertical, with $sin\alpha = 0.15$), symmetrically located with respect to the vertical longitudinal plane of the viaduct. The inclined planes are placed at a relative distance of 5.096 m at the keystone. A single arch profile is considered in each inclined plane, with an arch body that accounts for the true presence of two secondary twin inclined arches, on the two sides of such inclined plane. The truss nodes are linked to each other through a reticular system that corresponds to the true bracing system of the arch. As said above, the additional reinforcing plates between the vertical bars in each of the secondary twin inclined arches, placed at the locations of the connections arch/bearings have not been explicitly represented, whereas the cross sections of the limiting vertical bars contouring the stiffening plates have been endowed with larger geometrical characteristics. The model of the arch is comprised of 1051 beam elements and 342 nodes.

Vertical Piers. Of the nine bearings of the upper box beam, five are constituted by truss piers with appreciable vertical body (the two inner bearings on the arch symmetrically located by the crown lay directly on the arch extrados). Four of these piers are placed symmetrically, in couples, to the keystone: one couple of piers, of height near 14 m, rests on the haunches (*piers on the arch*); the other, of height 31.5 m (*big piers*), directly on built-in stone supports on the river banks, which host as well the arch shoulders. The other single pier, of height 11.1 m (*intermediate pier*) is located on the Calusco bank and constitutes an additional support, since the upper beam is not placed symmetrically to the crown (see Fig. 3 later shown). The morphological analysis of the piers and consequent FEM modelling has therefore focused on *three pier typologies*. They appear to have been derived from a unique generating pyramid, always sectioned first at the same height at the top and then at different heights at the bottom, depending on the relative distance between the upper truss beam and the piers/ground or piers/arch connections. The upper rectangular closing frame on top, which hosts as well the bearing devices, appears to be the same for all the piers, with longitudinal (in a front view of the longitudinal plane of the bridge) 1.6 m and transverse (in planes orthogonal to the longitudinal plane) 6.3 m widths. Of the four faces of the box profile of the piers, the front ones lay in the same α-inclined planes of the arch profile (thus, in a sense, the piers protrude from the arch towards the top); the lateral ones lay in β-inclined planes of a smaller inclination angle β of around $\beta \cong 2.8°$. The piers are made by a pair of front box trusses, each formed by four T-section columns linked longitudinally by short horizontal bars and St. Andrew's crosses and transversally by even shorter horizontal and inclined bars. The two box trusses are further connected transversally by a windbracing system with slender horizontal bars and St. Andrew's crosses. In the FEM model, each box truss has been described by a single planar frame laying in the α-inclined plane of the front face, with transverse mutual connections between the two box trusses that resemble the true transverse windbracing. Finally, concerning the stiffened connections beam/pier (and pier/arch, in the piers on the arch) the corner columns have been prolonged on top to the upper height of the reinforcing plates (and on bottom to the height of the arch extrados, in the piers on the arch) and their local sections treated with higher inertia, as above. The digits (elements, nodes) of the piers' model are as follows: big pier (226, 94), pier on the arch (124, 56), intermediate pier (110, 50), for a total of 810 elements and 350 nodes. Specific numerical simulations for the pier on the arch are reported in Ref. [7].

Upper Continuous Beam. The 266 m long upper continuous box beam is composed of eight spans, each 33.25 m long. The spans have similar morphologies but some of the cross sections of their structural elements slightly differ. Despite this, the structure of the upper beam is symmetric with respect to Bearing III at half length (which is on the side of the arch crown towards the Calusco bank, Fig. 3). Also, the first two spans, around half length, of the four spans of each symmetric part are identical (Spans 3−4 and 5−6, Fig. 3). Thus, only *three span typologies* of the upper continuous beam have been modelled. The FEM model of the beam has been obtained by assembling the models of the separate spans, by creating mutual built-in ties between them. The assembly of the model has been set-up also by taking into account a pre-imposed vertical counterslope of the upper beam, which is appearing in the drawings. In fact, the beam supports are located to a height that increases of 3 cm per span length, from the extremities towards the centre. The counterslope profile is symmetric with respect to the crown of the arch: Span 4, located just above the crown and Span 8, on the Calusco bank have no counterslope. The FEM model of the structure presents two vertical longitudinal truss girders that are 6.25 m high and placed at a respective transverse distance of 5 m. These are made by main longitudinal members on top and on bottom, connected by a series of front cross bars, inclined at around 45°. Between the two wall beams, transverse beam connections are provided every 3.325 m, both on top, which constitutes the support system of the road and on bottom, for the support of the railway deck. These transverse systems are further connected longitudinally by four beams placed every 1 m, on the upper level and by two beams right underneath the above rails, on the lower level. The digits (elements, nodes) of the beam model are: (356, 172) per span, for 8 spans, for a total of 2848 elements and 1292 tied nodes.

FEM Assembly. The complete FEM model of the bridge has been made by assembling the parts by coupling (mutual) constraints, with boundary conditions corresponding to what was designed by the SNOS: absolute built-in constraints are imposed at the nodes of the arch shoulders and at the bases of the piers bearing on the ground; absolute rollers with single free translation along the longitudinal plane of the bridge are imposed at the bottom nodes of the main longitudinal elements of the wall beams, at the extremities of the beam on the river banks; similar relative rollers are imposed between the upper truss beam and the piers or bearings, except for Bearing III at half length (Fig. 3), which is imposed as a relative hinge (these connections look immaterial in Fig. 2); mutual built-in constrains are imposed at the pier/arch interfaces. Fig. 2 shows the complete assembly of the FEM model of the Paderno d'Adda Bridge, which is finally endowed of 4709 beam elements and 1972 tied nodes. It is fully symmetric with respect to the vertical longitudinal plane of the viaduct.

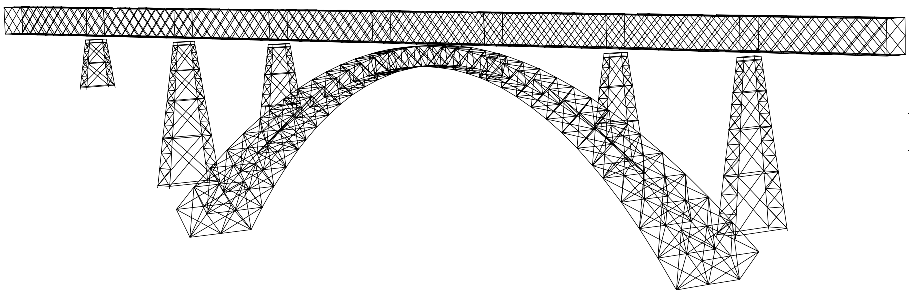

Figure 2: FEM model of the Paderno d'Adda Bridge (from a viewpoint similar to that in Fig. 1).

Structural Analyses of the Bridge

A study on the response of the assembled FEM model of the bridge has been attempted in the elastic range, for different static loading configurations. To the wrought iron material, the following nominal characteristics have been assigned, see Refs. [1-2]: $E = 17 \cdot 10^6$ t/m^2 for the Young's modulus, $G = 6.54 \cdot 10^6$ t/m^2 for the shear modulus (derived approximately from a Poisson's ratio of around $\nu = 0.3$); the admissible stresses in the structural members are indicated in $\sigma_a = 6.0$ kg/mm^2, with reduction to $\sigma_a = 4.2$ kg/mm^2 for the slender bars of the transverse windbracing systems.

FEM Results. First, the SNOS Report (1889) analyses independently, one by one, different design loadings on the arch, for subsequent superposition of effects: permanent weight of the arch; permanent weight of the upper beam, of the bridge piers and vertical actions induced by the wind acting on the girder beam; accidental vertical load on the upper beam, according to different distributions; temperature effects and compression on the arch due to the horizontal thrust; direct horizontal wind action on the arch. For each item, the SNOS reports the calculation of the axial forces in the various arch elements, as well as the final values that arise by the superposition of effects. Also, the corresponding deflections at the arch/bearings connections are reported as well. Most of the above loading configurations have been considered to act on the FEM model of the bridge. The obtained results have shown good correspondence with the original values reported by the SNOS.

Second, the viaduct tests are considered, which took place from 12th to 19th May 1889 (Fig. 3).

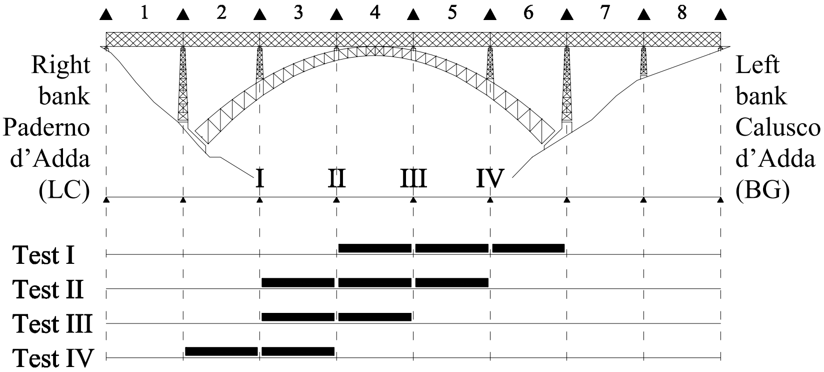

Figure 3: Scheme with four try-out configurations, with indication of the four beam bearings resting symmetrically on the arch (view from down-stream; Paderno left side, Calusco right side). Bearing III on the side of the left bank is at half length of the upper continuous beam.

The tests were carried-out in two moments: first, the different road loads were obtained by deposition of gravel on the upper deck; second, with a uniformly distributed gravel load of 3.9 t/m all over the road, 6 locomotives with tender, each of 83 t of weight, corresponding to a distributed load of 5.1 t/m, were displaced on the railway track according to four loading configurations (Fig. 3).

The results reported in SNOS (1889) seem to have regarded only the vertical displacements caused by the locomotives. Thus, the same loading configurations have been considered in the FEM model, with distributed loads of 5.1 t/m as reported in Fig. 3 and consequent output on the vertical deflections. Table 1 below lists the comparison on the observed and calculated vertical deflections of the arch at Bearings I-IV. The FEM outcomes confirm, to quite a good degree of accuracy, the values supplied in the original Report. In Fig. 4, the corresponding magnified deformed configurations of the bridge are reported as well.

Table 1: Arch vertical deflections [mm] observed and calculated for the four try-out tests in Fig. 3 (SNOS 1889, p. 71) vs. FEM results. Negative values indicate downward displacements.

Arch deflections [mm]	Bearing I			Bearing II			Bearing III			Bearing IV		
	SNOS		FEM	SNOS		FEM	SNOS		FEM	SNOS		FEM
	Obs.	Calc.		Obs.	Calc.		Obs.	Calc.		Obs.	Calc.	
Test I	+3.8	+3.3	+4.2	+0.1	−1.0	−0.9	−10.6	−10.8	−11.3	−5.6	−6.6	−6.6
Test II	+0.0	−1.6	−1.0	−7.9	−8.0	−8.6	−10.2	−8.0	−8.4	−1.2	−1.6	−0.9
Test III	−1.6	−4.0	−3.6	−10.2	−10.1	−11.1	−1.4	−2.2	−2.2	+2.5	+2.7	+3.5
Test IV	−6.3	−6.8	−7.6	−5.8	−6.4	−6.6	+4.2	+3.5	+4.0	+2.6	+3.1	+3.6

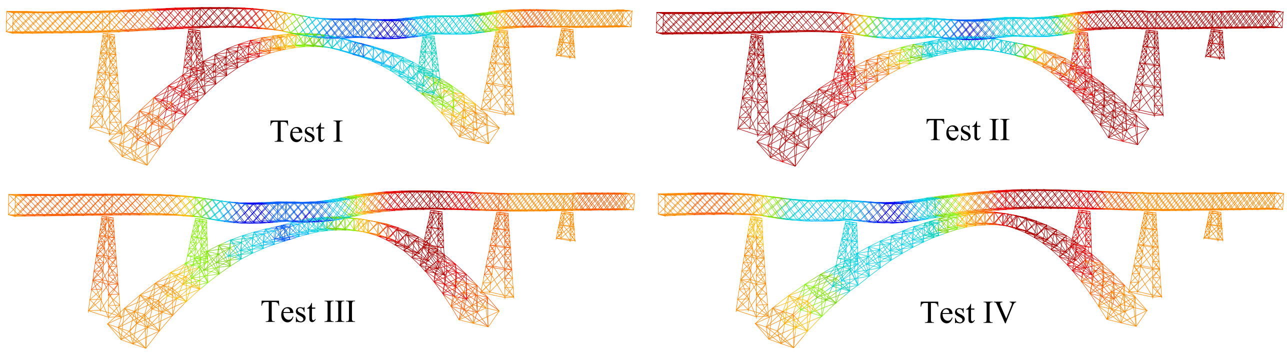

Figure 4: FEM deformed configurations of the try-out tests in Fig. 3 (amplification factor = 250).

For the same try-out loading configurations shown in Fig. 3, further FEM results have been inquired. In particular, the vertical displacements of the upper truss beam of the bridge have been read, at half length of each span and at the bearings in between them. These results could be compared to the target value of $l/1500 = 33250$ mm$/1500 = 22.17$ mm, that could be taken as a limit reference indication for bridges with railway traffic. The corresponding results are reported in Table 2, showing that the deflections are always less than that. Notice also that, due to the counterslope implemented at design stage, despite the negative inflections, the inner bearings in deformed configuration turn-out located always above the ground elevation of the bearings at the extremities of the continuous beam. Thus, the implemented counterslope appears effective in this sense.

Table 2: Beam FEM vertical deflections [mm] at half length of each numbered span (1-8) and inner bearings, for the four tests in Fig. 3. Negative values indicate downward displacements.

Beam defl. [mm]	1	▲	2	▲	3	▲	4	▲	5	▲	6	▲	7	▲	8
Test I	−0.4	+0.1	+2.2	+4.2	+4.0	−0.9	−11.0	−11.3	−13.6	−7.7	−8.1	−1.1	+0.9	+0.1	−0.2
Test II	−0.1	+0.1	+1.0	−1.5	−9.2	−8.6	−12.4	−8.4	−9.0	−1.5	+0.9	+0.1	−0.1	−0.0	+0.0
Test III	+0.1	+0.1	−0.2	−4.1	−12.6	−11.1	−11.7	−2.3	+2.8	+3.5	+1.9	+0.1	−0.3	−0.0	+0.1
Test IV	+1.1	−1.1	−8.7	−8.5	−13.1	−6.6	−0.1	+4.0	+4.7	+3.4	+1.5	+0.1	−0.2	−0.0	+0.1

In addition, axial forces in the bars and relevant normal stresses have been inquired at different locations of the FEM model of the bridge. Table 3 reports the values of such actions at the extrados and intrados of the arch shoulders, together with the minimum and maximum values in the various bridge elements (appearing in the arch). The stresses are all below the target admissible values.

Table 3: Bridge FEM axial forces N [t] and stresses σ [kg/mm²] at the arch shoulders and min/max values, for the test distributions shown in Fig. 3. Negative values indicate compression.

N [t] σ [kg/mm²]	Right-bank shoulder		Left-bank shoulder		Bridge elements (arch)			
	Extrados A_e = 159844 mm²	Intrados A_i	Intrados A_i = 243844 mm²	Extrados A_e	Min	A [mm²]	Max	A [mm²]
Test I	−157.542 −0.99	−0.949 −0.00	−231.236 −0.95	+50.421 +0.32	−231.236 −0.95	243844	+51.916 +1.00	51688
Test II	−79.128 −0.50	−138.811 −0.57	−137.517 −0.56	−80.401 −0.50	−155.707 −0.97	159844	+48.690 +1.11	44048
Test III	+1.000 +0.01	−163.615 −0.67	−11.339 −0.05	−137.167 −0.86	−169.752 −1.06	159844	+51.664 +0.44	117844
Test IV	+108.101 +0.68	−196.050 −0.80	+36.078 +0.15	−101.594 −0.64	−196.050 −0.80	243844	+108.101 +0.68	159844

Conclusions

This paper reports first results on the FEM modelling of the structure of the Paderno d'Adda Bridge. The outcomes appear to be consistent with what reported by SNOS (1889), showing that the FEM model seems to be able to provide a coherent account of the bridge structure at design stage. The present static elastic analyses show promise for further studies that might involve, to start with, the dynamic response of the bridge, which is intended to be considered next. Furthermore, the model could be taken as reference also for inquiring the present structural performance of the viaduct, in connection with non-linear and damage analyses, as applied to actual loadings and traffic conditions.

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References

[1] Società Nazionale delle Officine di Savigliano (1889). *"Viadotto di Paderno sull'Adda (ferrovia Ponte S. Pietro-Seregno)"*. Tip. e Lit. Camilla e Bertolero, Torino.

[2] Nascè, V., Zorgno, A.M., Bertolini, C., Carbone, V.I., Pistone, G., Roccati, R. (1984). *"Il ponte di Paderno: storia e struttura. Conservazione dell'architettura in ferro"*. Restauro, XIII(73−74).

[3] Ferrari, R. (2006). *"Sulla concezione strutturale ottocentesca del ponte in ferro di Paderno d'Adda"*. Laurea Thesis in Building Engineering, Advisor E. Rizzi, Università di Bergamo, Facoltà di Ingegneria, I-24044 Dalmine (BG), Italy, 228 pages, December 2006.

[4] Ferrari, R., Rizzi, E. (2008). *"On the theory of the ellipse of elasticity as a natural discretisation method in the design of Paderno d'Adda Bridge (Italy)"*, in Proc. of VI Int. Conf. on Structural Analysis of Historic Construction (SAHC08), Bath, UK, 2–4 July 2008, Vol. 1, 583−591.

[5] Facheris, M. (2009). *"Analisi morfologica e modellazione per elementi finiti della pila sull'arco del ponte di Paderno d'Adda"*. Laurea Thesis in Building Engineering, Advisor E. Rizzi, Co-Advisor R. Ferrari, Università di Bergamo, Facoltà di Ingegneria, I-24044 Dalmine (BG), Italy, 125 pages, September 2009.

[6] Ferrari, R. (2009). *"Analisi strutturale degli elementi portanti del ponte di Paderno d'Adda"*. Master Thesis in Building Engineering, Advisor E. Rizzi, Università di Bergamo, Facoltà di Ingegneria, I-24044 Dalmine (BG), Italy, 108 pages, September 2009.

[7] Ferrari, R., Facheris, M., Rizzi, E. (2010). *"Structural modelling of the piers of the Paderno d'Adda Bridge (1889, Italy)"*, in Proc. of 34th IABSE International Symposium on Bridge and Structural Engineering, Venice, Italy, 22−24 September 2010.