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FRACTURE-ENERGY-BASED REGULARIZATION OF A SCALAR DAMAGE MODEL

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SUMMARY. Finite element computations with the smeared approach give rise to mesh dependence when strain localization occurs. The inelastic process localizes in a narrow region of the structure diminishing to zero with further mesh refinement. The total energy dissipation vanishes and a full snap back of the solution is recovered when a mixed force/displacement control is provided. Regularization of a scalar damage constitutive description is introduced for obtaining objectivity of the results with mesh refinement of a plane strain concrete panel in axial extension.

1. INTRODUCTION

Strain localization occurs in bodies described with the smeared constitutive approaches which account for softening behavior, while the body is still treated as a continuum. Pointwise strain localization analysis for plasticity, smeared crack models or stiffness degradation models results in the prediction of failure bands with vanishing width. The onset of bifurcation is detected by singularity of the so-called localization tensor which characterizes fully the underlying bifurcation mode (see Rizzi [1] for a literature review).

For the post-bifurcation path the band width is not specified and a finite element computation is not objective since the failure prediction depends on mesh density and orientation. While directional bias can be reduced by resorting to enhanced finite element formulations (Steinmann and Willam [2]), independence of the response is achieved by regularizing the material description (de Borst [3], Willam and Dietsche [4]). Higher order continuum descriptions (non local models, gradient dependent models, polar Cosserat materials) prevent localization to appear in a discontinuous fashion since a characteristic length defining the region with smeared damage, is built in into the constitutive model. These models are not immediate though, thus it appears correct to regularize if possible the usual local constitutive descriptions. Since the failure process must be a surface driven phenomenon from an energetic view point, an intrinsic length which fills the gap between the material and the structural response must be introduced even in the local approach.

The fracture-energy based regularization introduces a modification of the softening parameters according to the mesh size, such that to impose the same energy dissipation per unit area even with mesh refinement (Willam and Dietsche [4], Feenstra [5]). Although this can be considered as a "trick", the approach seems to be completely legitimate since it's rather impossible to differentiate between material and structural response when localization occurs, resulting in non-homogeneous stress/strain states.

This contribution will introduce the fracture-energy regularization for the isotropic scalar damage model by Simo and Ju [6], which indeed originated from the formulation of Mazars [7] and that can be recast in the general framework for elastic-degradation provided recently by Carol et al. [8]. The model results in an exponential softening behavior in 1D-tension/extension problems, while the rapidity of the softening decay needs to be adjusted according to the finite element size. The model has been introduced into a research-oriented finite element code together with the appropriate techniques for tracing the post-peak and snap-back response beyond the limit/bifurcation point. A mixed force/displacement control in the form of the classical arc-length method by Wempner [9] and Riks [10] with the arc-length adaptation by Crisfield [11] and a simple back-tracking strategy (Dennis and Schnabel [12]) has been implemented.
The axial extension problem of a rectangular plane strain concrete panel shows the validity of the regularization procedure for Mode-I type failures and the robustness of the arc-length implementation. The overall force-displacement response shows a quite sharp regularized snap-back structural response, independent of the three discretization densities considered.

2. STRAIN LOCALIZATION FOR SCALAR DAMAGE MODELS

The constitutive behavior of concrete can be conveniently modeled with the smeared approach, which allows to define strain and stress at the local level. The constitutive relation for the overall macroscopic response can be given in a total or an incremental form (Feenstra [5]). A typical description which accounts for both strength as well as stiffness degradation, starting from a secant relation $\boldsymbol{\sigma}=\mathbf{E}:\boldsymbol{\epsilon}$, and a secant stiffness or compliance evolution law, can be recast in an incremental form $\dot{\boldsymbol{\sigma}}=\mathbf{E}_t:\dot{\boldsymbol{\epsilon}}$, after definition of the tangent stiffness operator relating stress and strain rates (Carol et al. [8]). Most of the Continuum Damage Mechanics models can be fitted in this framework.
Once the expression for the tangent operator is provided, it is possible to analyze the failure indicators for diffuse and discontinuous failure associated with limit and bifurcation points respectively (Rizzi [1]). The simplest scalar damage description, resorts to a single damage variable D which describes the stiffness degradation, whereby all the components of the initial secant stiffness tensor $\mathbf{E}_o$ are affected in the same manner as $\mathbf{E}=(1-D)\mathbf{E}_o$, and the stress/strain relation reads $\boldsymbol{\sigma}=(1-D)\mathbf{E}_o:\boldsymbol{\epsilon}$. The damage threshold is defined by a loading condition $F=0$, where $F=F(\tau, D)$ for an associated model (Carol et al. [8]), with the definition of the energy norm

$$\tau = \sqrt{\boldsymbol{\epsilon}:\mathbf{E}_o:\boldsymbol{\epsilon}} \tag{1}$$

For the plane strain axial extension case with $\epsilon_1 \geq 0$, $\sigma_2=0$, $\epsilon_3=0$, the scalar damage model results in onset of localization at the limit point with a critical direction of localization between loading axis and normal to the discontinuity surface depending on the Poisson ratio: $tan^2\theta_{cr}=\nu/(1-\nu)$ (Rizzi [1]). For $\nu=0$ Rankine type failure is recovered.
In this work, the elastic-damage part of the strain-based isotropic damage model presented by Simo and Ju [6], has been considered and implemented in a research oriented finite element code. The main features of the model can be summarized in

the following. The damage function and the tangent operator are given as

$$F = \tau - r = 0 \qquad \mathbf{E}_t = (1 - D)\ \mathbf{E}_o - \frac{\partial D}{\partial \tau}\ \frac{\boldsymbol{\sigma} \otimes \boldsymbol{\sigma}}{(1 - D)^2\ \tau} \tag{2}$$

where the damage evolution law is assigned by means of three parameters A, B, τ_o, according to

$$D(\tau) = 1 - (1 - A)\ \frac{\tau_o}{\tau} - A\ e^{B(\tau_o - \tau)} \tag{3}$$

The physical meaning of the model parameters is readily found considering the axial behavior, where $\tau{=}\sqrt{E_o}\epsilon$:

$$\sigma = (1 - D)\ E_o\,\epsilon = (1 - A)\ \sqrt{E_o}\ \tau_o + E_o\ \epsilon\ e^{B(\tau_o - \sqrt{E_o}\ \epsilon)} \tag{4}$$

Thus A defines the residual strength for full damage ($D{\rightarrow}1$), B defines the peak tensile strength f_t and the softening decay, and τ_o prescribes the initial elastic limit (Mazars [7]).
The algorithmic damage evolution is determined by the following procedure. During the incremental/iterative procedure the energy norm is computed from the current value of the total strain $\tau_{n+1}(\boldsymbol{\epsilon}_{n+1})$. Then, if τ_{n+1}-$r_n{\leq}0$, $D_{n+1}{=}D_n$, otherwise $D_{n+1}{=}D(\tau_{n+1})$. The damage threshold is updated as $r_{n+1}{=}max[r_n, \tau_{n+1}]$.

3. FRACTURE-ENERGY-BASED REGULARIZATION

In view of introducing a regularization into the constitutive description without resorting to more sophisticated material models, a possibility is to impose the same energy dissipation per unit area when fracture propagates with a relative opening displacement under a certain stress state. Then, the fracture energy G_f represents the area beneath the corresponding stress/displacement curve and the abscissa axis. When localization occurs the inelastic phenomena are highly localized in a narrow part of the discretized structure which depends from the element size, whereas the remaining part of the structure unloads elastically. Since the dissipation is vanishing while reducing the mesh density, it is necessary to change the material parameters such that the overall response is objective. In other words we have to impose that the same energy dissipation is always achieved. Indicating with g_f the area underneath the stress/strain diagram, and denoting with l_c a characteristic length, which fills the gap between structural and material response in the localization band, $\epsilon{=}u/l_c$, we have to set $g_f{=}G_f/l_c$.
The characteristic length must be directly related to the element size. In this analysis with quadrilateral four-noded elements, it has been assumed that

$$l_c = \sqrt{2}\ \sqrt{A_{el}} \tag{5}$$

where A_{el} is the area of the element evaluated numerically (Feenstra [5]).
From the 1D response considered above it is evident that for obtaining a bounded g_f it is necessary to set $A{=}1$, which means zero residual tensile strength. Further, for controlling the area g_f while keeping constant the peak stress f_t, the maximum stress must be reached at the initial elastic limit, which happens for $B{\geq}1/\tau_o$. In this case regularization is possible without changing the peak stress. Then the material description results in a linear elastic/exponential softening behavior and

the softening exponential decay parameter B can be adjusted for imposing the same energy dissipation:

$$\int_0^\infty \sigma(\epsilon) d\epsilon = \frac{1}{B^2} + \frac{\tau_o}{B} + \frac{1}{2}\tau_o^2 = \frac{G_f}{l_c} \qquad (6)$$

where G_f/l_c must be larger or equal to $\tau_o^2/2$ for avoiding a sharp snap-back at the constitutive level right after the peak at the initial elastic limit. This condition gives an upper bound to the characteristic length, while the condition $B \geq 1/\tau_o$ imposes a lower bound on l_c:

$$0.4 \, \frac{G_f}{\tau_o^2} \leq l_c \leq 2.0 \, \frac{G_f}{\tau_o^2} \qquad (7)$$

The admissible solution $B \geq 0$ of the second order equation (6) renders

$$B(l_c) = \frac{(\tau_o \, l_c + \sqrt{l_c \, (4 \, G_f - l_c \, \tau_o^2)}}{(2 \, G_f - l_c \, \tau_o^2)} \qquad (8)$$

which is the desired relation between the softening decay parameter and the characteristic length assumed within the bounds (7).

4. MIXED LOAD/DISPLACEMENT CONTROL

For the analysis of the post-peak and snap-back regimes the solution algorithm must introduce an additional constraint for allowing mixed load/displacement control. The classical arc-length technique originally proposed by Wempner [9] and Riks [10] is adopted. In this approach the tangent trial solution $\Delta\mathbf{u}_t$ in the displacement vector/load parameter plane $(\mathbf{u}, \lambda)$, is corrected along a path perpendicular to the tangent such that the final increment $\Delta\mathbf{u}$ satisfies the constraint condition

$$\Delta\mathbf{u} \cdot \Delta\mathbf{u} + (\Delta\lambda)^2 = (\Delta s)^2 \qquad (9)$$

where Δs indicates the arc-length in that plane. The sparsity of the tangent stiffness matrix $\mathbf{K}_t$ can be maintained by decomposing the displacement increment in two contributions $\Delta\mathbf{u} = \Delta\lambda \, \Delta\mathbf{u}_1 + \Delta\mathbf{u}_2$ and by solving two decoupled algebraic systems (de Borst [3])

$$\mathbf{K}_t \cdot \Delta\mathbf{u}_1 = \mathbf{P} \; ; \qquad\qquad \mathbf{K}_t \cdot \Delta\mathbf{u}_2 = \mathbf{R} \qquad (10)$$

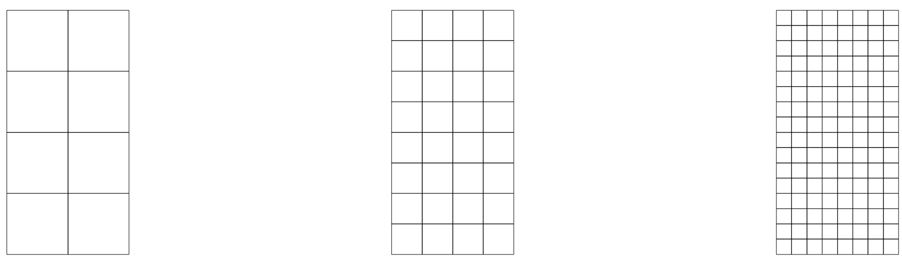
where $\mathbf{P}$ is the load vector and $\mathbf{R}$ the out-of-balance force vector.

For achieving convergence near limit points a backtracking algorithm along the line of the trust region methods for numerical unconstrained optimization problems (Dennis and Schnabel [12]) decreases conveniently the step size up to convergence: $\Delta s \leftarrow \Delta s/\alpha$, with $1 \leq \alpha \leq 2$. The backtracking parameter α is conveniently chosen after a preliminary calculation (α=1.2 has been adopted in the following analyses).

An automatic arc-length adjustment is necessary for accelerating the convergence if larger steps are allowed and for stabilizing the number of iterations from the previous increment n_{prev} through an optimal value $n^{opt} = 4 \div 5$. The arc-length adjustment by Crisfield [11], $\Delta s \leftarrow \Delta s \, n^{opt}/n_{prev}$ performed well with good stabilizing of the number of iterations after each perturbation of the step size. The arc-length might decrease dramatically near limit points or increase excessively loosing accuracy of the solution. Thus upper and lower bounds on the arc-length were established to assure accuracy and economy of the numerical solution.

5. PLANE STRAIN CONCRETE PANEL

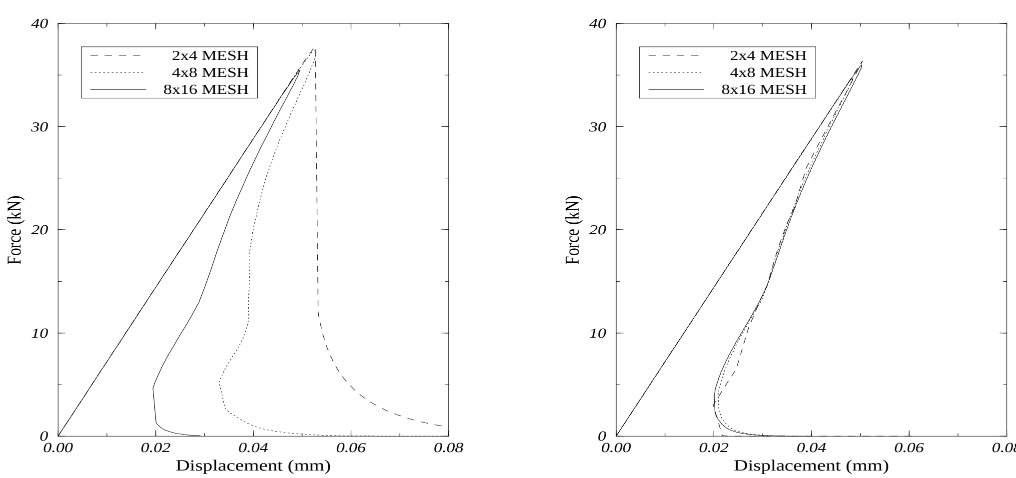
A plane strain concrete panel with aspect ratio 1:2 (8.5 *in* $\times$ 17 *in*) has been analyzed. Thickness is taken as 1 *in*. The following material parameters have been considered: E_o=4,000 *ksi*, ν=0.15, f_t=1 *ksi*, G_f=4.5 10^{-4} *kip/in*.
A quarter of a specimen has been discretized with standard four-noded quadrilateral elements with three different mesh densifications (Fig. 1). The homogeneous state has been altered by imposing a displacement constraint at the top of the specimen, which means no relative displacement between the specimen and the loading apparatus. That's necessary for triggering localization, which appears in the row of elements close to the top. This is consistent with the pointwise localization analysis mentioned above only for ν=0. Same result is obtained even using enriched elements (Rizzi and Willam [13]) without alignment, which indicates still the need of a regularization of the directional mesh bias.



(2×4) (4×8) (8×16)

Figure 1. Different standard quadrilateral element discretizations.

The non-regularized load/displacement response is showed in Fig. 2(a), where the softening decay parameter has been set equal to B=70 $in/\sqrt{kip}$. As expected, more brittle behavior is recovered for finer meshes. The regularized response is depicted in Fig. 2(b). The structural response is quite independent on the mesh density and shows a pretty sharp snap-back, well described by the arc-length technique.



(a) Standard solution *(b) Regularized solution*

Figure 2. Plane strain axial extension problem with constraints at the top.

6. CONCLUSIONS

A fracture-energy-based scalar damage model has been presented and implemented in a research-oriented finite element code, together with a mixed load/displacement control necessary for overcoming limit and snap-back points in the solution space. Regularization in mode-I type failure has been achieved, while the robustness of the algorithm has been checked. The regularization approach appears to be perfectly legitimate for this simple loading condition. More complex constitutive formulations appear to be necessary for a global regularization effect in 3D loading.

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