



**UNIVERSITÀ DEGLI STUDI DI BERGAMO**

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**University of Bergamo**

**Doctoral Program in Engineering and Applied Sciences**

## **Some Notions on Contact Mechanics**

**Short Doctoral Course (12 h): 13-16 February 2018**

**Prof. António PINTO DA COSTA (IST Lisboa)**

**School of Engineering, viale G. Marconi 5, DALMINE (BG)**

### **Course calendar:**

#### **Day 1 – Tue. Feb. 13**

09:00 – 12:00 (room B004)

#### **Day 2 – Wed. Feb. 14**

09:00 – 12:00 (room B004)

#### **Day 3 – Thu. Feb. 15**

09:00 – 12:00 (room B004)

#### **Day 4 – Fri. Feb. 16**

09:00 – 12:00 (room B004)

*Ref. Prof. Egidio Rizzi (ICAR/08 - Scienza delle Costruzioni), [egidio.rizzi@unibg.it](mailto:egidio.rizzi@unibg.it)*

# SOME NOTIONS ON CONTACT MECHANICS

Prof. António Pinto da Costa

Instituto Superior Técnico, Lisboa, Portugal

PhD Course (Doctoral Programme in Engineering and Applied Sciences)

School of Engineering (Dalmine), University of Bergamo, Italy

## What is this course about?

This course is dedicated to the part of Mechanics known as “contact mechanics”. *Unilateral frictional contacts* are encountered in many branches of engineering simply because contact between rigid or deformable solid components is the most usual way of force transmission. Frictional contact may affect mechanical systems in a variety of ways: it may lead to

- (i) prohibitively high losses of energy or of precision during industrial operations,
- (ii) undesirable bifurcational or unstable behaviours,
- (iii) the reduction of the life expectancy of structural or machinery components due to wear or,
- (iv) it may have the beneficial effects of enabling locomotion or damping vibration amplitudes through energy dissipation in a cheap, efficient and environmentally friendly way.

## What to expect from it?

Special attention will be paid to:

- (a) *formulation*,
- (b) *modelling*
- (c) *numerical resolution*,

of contact problems (either static, quasi-static or dynamic). We intend to provide a tutorial lecture series that enables non-specialists such as graduate students from engineering, physics or mathematics, to get into the field of contact mechanics. We aim at giving participants

- (1) basic mathematical tools necessary to understand the intrinsic non-smooth character of two important set valued force laws (the unilateral contact law and Coulomb’s friction law),
- (2) numerical algorithms for the modelling of frictional contact,
- (3) the capacity to assess the conditions for the occurrence of detrimental or beneficial effects of friction in structural analysis,
- (4) a good physical insight into the nonlinear/non-smooth phenomena in frictional contact mechanics.

Model problems involving mechanical systems made of springs and rigid bodies, with a small number of degrees of freedom, are explored by hand. More complex systems associated to analytical treatments or from finite element discretizations are explained as well. Geometry will often be a permanent ally for the understanding of

- (i) many aspects of the analyzed mechanical systems,
- (ii) the frictional contact laws governing their interaction with obstacles,
- (iii) the various alternative formulations of those laws.

### **What prerequisites are needed to follow this course?**

The participants are expected to have a good background in linear algebra, matrix theory, calculus, analytical geometry ... and, mainly, an open mind to accept “unilateral analysis” in a natural way with its associated counterintuitive phenomena, paradoxical behaviours, absence of or infinitely many solutions arising when one dares to explore the unpredictable territories of non-smooth set-valued constitutive laws. If you feel challenged by being confronted with unusual behaviours, this is the course you should enroll!

### **What is the goal?**

It is manifold:

- (A) To use frictional contact as a mean to teach the complex phenomena behind non-smooth set-valued force laws in the simplest possible manner; this is why attention will be given to the exploration of mechanisms with a small number of degrees of freedom.
- (B) To antagonize the (classical) way of thinking of using linearization and classical differentiation theory and to promote “convexification” in opposition to “linearization” in order to deal with force laws with “kinks”.
- (C) To cover basic concepts of subdifferential calculus and convex analysis, two essential tools for those types of laws.

### **What are the topics to be covered?**

Frictional contact conditions: physical origin and models. The bilateral contact law, the unilateral contact law and the Coulomb friction law. Why Coulomb’s law? Discontinuities everywhere! Examples from everyday life. Set-valued constitutive laws. What is meant by “non-smooth mechanics”? Classical versus modern thinking: approximations based on tangent linear mappings versus one-sided approximations. Why to use non-smooth models for real systems? Classes of friction problems: divide to conquer! The one-particle model solved by hand: how friction affects the equilibrium trajectories, friction locking, and a first visit to instability and bifurcations. A short visit to non-smooth and convex analyses. Writing frictional contact laws in different ways (literature “oblige”!). Non-existence/non-uniqueness of solution to several frictional contact problems. Important applications of Coulomb’s friction model. Formulations of the basic frictional contact problems of algebraic type (dynamic incremental, quasi-static incremental, first order rate). Numerical methods for the simulation of frictional contact/impact systems. Some examples of larger systems: simulation of granular materials and an application to earthquake engineering. An analytic solution: flutter instability in elastic half-spaces with frictional contact.

**Who delivers the course?**

António Pinto da Costa received a PhD degree in civil engineering in 2002 from Instituto Superior Técnico, Lisboa, Portugal, on *Instabilities and bifurcations in non-smooth behaviour systems*, and has been doing research on the subject of frictional contact and its applications since then. He is currently an assistant professor at the Department of Civil Engineering of Técnico, where he has taught a wide variety of courses: Statics, Dynamics, Strength of Materials, Structural Analysis, Laminar Structures and Stability, Steel Structures, Seismic Engineering, and Structural Design. His research areas of interest are: dynamic stability, cable dynamics, contact mechanics, complementarity problems, friction induced instabilities, existence/non-uniqueness of solutions to frictional contact problems, unilateral buckling, and moving loads/vehicles on beams on foundation.

**What is the duration of the course?**

A total of twelve hours in four consecutive days are planned (three hours in each day).