Astrophysics and Tachyons.

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Summary. — From the very special relativity theory, the existence of Superluminal reference frames and faster-than-light objects can be inferred, when its validity is not arbitrarily confined to subluminal velocities. Recently, in fact, special relativity has been extended to Superluminal inertial frames and to tachyons, thus allowing a self-consistent development of a classical theory of tachyons. In this paper, it is shown that the extended theory of relativity allows in particular extending the Doppler-effect formulae to Superluminal sources. The possibilities of radiocontact between subluminal and Superluminal objects are considered as well. The behaviour of tachyonic bodies in the gravitational field has been clarified. Tachyons are predicted to suffer a gravitational repulsion (so they might be considered as the antigravitational matter); however, owing to their dynamics, they happen finally to bend towards the gravitational source, as usual objects do (even if with different curvatures). At last the problem of gravitational Čerenkov radiation (GCR) is studied. Contrarily to the previous authors, extended relativity predicts tachyons not to emit GCR in vacuum or in bradyonic (subluminal) media.

1. — Introduction. Extended relativity in ten points.

Recently, special relativity has been extended (1-4) to Superluminal inertial frames and faster-than-light objects (tachyons). It yielded the framework for

(2) R. MIGNANI and E. RECAMI: Nuovo Cimento, 14 A, 169 (1973); Erratum, 16 A,
building up a self-consistent « classical theory » of tachyons (1-3) (where classical means relativistic but not yet quantum mechanical).

Let us here outline the logical scheme that guided that special relativity extension.

1) The very (Einsteinian) principle of relativity refers to inertial frames with constant relative velocity \( u \), without any \textit{a priori} restriction on the value of \( u \) (\( u \gtrless c \)). We wish to express the relativity principle (RP) in the form « physical laws of mechanics and electromagnetism are required to be covariant when passing from an inertial frame \( f_1 \) to another frame \( f_2 \) moving with constant relative velocity \( u \), where \(-\infty < u < +\infty \). We complete this postulate by adding the requirements that the vacuum is covariant when changing inertial frame, and that « physical signals are transported \textit{only} by positive-energy objects » (cf. ref. (1)).

2) We assume i) the RP and ii) the following postulate: « space-time is homogeneous and space is isotropic ».

3) From the previous assumptions 2), the existence of an invariant speed \( \bar{v} \) follows by generalizing the procedures in ref. (3); and experience shows that such a velocity is the speed of light: \( \bar{v} = c \).

4) From points 2) and 3) there follows a « duality principle » (DP) (1-4,6): «the terms bradyon (*) (B), tachyon (T), subluminal frame (s), Superluminal frame (S) do not have an absolute meaning, but only a \textit{relative} one. Light speed invariance allows an exhaustive partition (?) of all inertial \((u \leq c)\) frames in two sets \( \{s\}, \{S\} \), which are expected to be such that a (subluminal) Lorentz transformation (LT) maps \( \{s\}, \{S\} \) separately into themselves, and a Superluminal « Lorentz transformation » (SLT) maps \( \{s\} \) into \( \{S\} \) and \textit{vice versa}». A one-to-one correspondence may be set between frames \( s(u) \) and \( S(U) \), with \( u \parallel U \), where \( u \rhd U = c^2/u \), such a mapping being an « inversion » (i.e. a particular \textit{conformal mapping}).

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(*) Slower-than-light particle.

5) From RP, light speed invariance and DP it follows that transformations between two frames $f_1$, $f_2$ must be linear and such that, for every tetravector (four-position, four-momentum, four-velocity, ...),

$$e^2 t'^2 - x'^2 = \pm (e^2 t^2 - x^2) \quad (u \leq c).$$

The metric $(-\ldots)$ will be adopted throughout this work. Natural units $(c = 1)$ will be used, when convenient.

6) In eq. (1), the sign plus holds for $u < c$, and minus for $u > c$. In fact, when going from a frame $s$ to a frame $S$, the type of four-vectors associated with the same observed object changes from timelike to spacelike, or vice versa, as follows from point 4).

For instance, in the four-momentum space, since for bradyons

$$(2a) \quad E^2 - p^2 = p^2 = m_0^2 > 0 \quad (\beta^2 < 1),$$

then for a tachyon, i.e., after a SLT, we shall have

$$(2b) \quad E'^2 - p'^2 = p'^2 = -m_0^2 < 0 \quad (\beta^2 > 1),$$

where of course $m_0$ is always real.

7) It is easy to verify that, in the simple case of Superluminal, collinear, relative motion along the $x$-axis, eq. (1) breaks up into the two requirements

$$(1bis) \quad \begin{cases} e^2 t'^2 + (ix')^2 = (ict)^2 + x^2, \\
(ivy')^2 + (iz')^2 = y^2 + z^2 \end{cases} \quad (u^2 > c^2),$$

where use has been made of $g_{\mu\nu} = \delta_{\mu\nu}$ and of Einstein’s notation. By the way, we shall always avoid explicit use of a metric tensor, by writing the generical chronotopic vector as $x = (x_0, x_1, x_2, x_3) = (ct, ix, iy, iz)$.

8) Transformations satisfying eq. (1), in the case of collinear (subluminal or Superluminal) motion with velocity $u$ along the $x$-axis, can be shown

$$(^8) \quad \text{See, e.g., W. Rindler: Special Relativity (Edinburgh, 1966).}$$

$$(*) \quad \text{We always neglect space-time translations.}$$
to be the «generalized Lorentz transformations» (GLT) (*):

\[
\begin{align*}
    x' &= \pm \frac{x - ut}{\sqrt{1 - \beta^2}}, \\
    y' &= \pm y \sqrt{\frac{1 - \beta^2}{1 - \beta^2}}, \\
    t' &= \pm \frac{t - ux/\rho}{\sqrt{1 - \beta^2}}, \\
    z' &= \pm z \sqrt{\frac{1 - \beta^2}{1 - \beta^2}}
\end{align*}
\]

\((\infty < \beta = \frac{u}{\rho} < +\infty)\),

which obviously reduce to the standard (subluminal) ones when \(\beta^2 < 1\). All the GLT's form a new group \(G\) of transformation \(1-8.9\). All discussions relative to these GLT's (in particular, relative to the SLT's) may be found in ref. \((1.2)\). For instance, from eqs. (3) the «generalized velocity composition law» may be easily derived \((*)\).

In a more compact form, eqs. (3) may be rewritten as follows \((1.3)\):

\[
\begin{align*}
    x' &= \eta \gamma x - ct \tan \varphi, \\
    y' &= \eta \gamma y, \\
    t' &= \eta \gamma \left(t - \frac{x}{\rho} \tan \varphi\right), \\
    z' &= \eta \gamma z
\end{align*}
\]

\((\beta^2 \geq 1)\),

where we set

\[
\begin{align*}
    \beta &= \tan \varphi, \\
    \gamma &= +\left(1 - \tan^2 \varphi\right)^{1/2}, \\
    \eta &= \eta(\varphi) = \frac{\cos \varphi}{|\cos \varphi|} \delta^z, \\
    \delta &= +\left[(1 - \tan^2 \varphi)/(1 - \tan^2 \varphi)\right]^{1/2}
\end{align*}
\]

In eqs. (4), the angle \(\varphi\) runs over the whole round angle. For instance \((1)\), usual subluminal, homogeneous, orthochronous, proper LT's are got for \(\varphi\) running from \(-\pi/4\) to \(+\pi/4\); and the corresponding nonorthochronous ones for \(\frac{3}{4}\pi \leq \varphi \leq \frac{5}{4}\pi\).

9) Form (4) parametrizes the GLT's in a «continuous» fashion, where our parameter \(\varphi\) runs (with continuity) from 0 to \(2\pi\) rad. In particular, form (4) allows a straightforward (continuous) geometrical interpretation of generalized Lorentz transformations, for \(\beta^2 \geq 1\) (see, e.g., Fig. 5, 8 of ref. \((2)\)). For example, in the bidimensional case we have

\[
\begin{align*}
    x &= C(x' \cos \varphi + ct' \sin \varphi), \\
    t &= C\left(t' \cos \varphi + \frac{x'}{\rho} \sin \varphi\right)
\end{align*}
\]

\((0 \leq \varphi \leq 2\pi)\).


where

\[ \mathcal{O} = \pm \left( \frac{(1 + \tan^2 \varphi)}{(1 - \tan^2 \varphi)} \right)^{\frac{1}{2}}, \]

and where the physical meaning is quite clear. The SLT's corresponding to \( \varphi_1 = \pi/2 \) and \( \varphi_2 = (3/2)\pi \) will be called the «transcendent transformations» \(^{(1)}\) \( K_+ \) and \( K_- \), respectively.

10) Let us assume that we know, besides the class \( \mathcal{A} \) of usual physical laws (of mechanics and electromagnetism) for bradyons, also the class \( \mathcal{B} \) of the physical laws for tachyons (and antitachyons \(^{(1-3,10)}\)). When we pass from a subluminal frame \( s \) to a Superluminal one \( S \), class \( \mathcal{A} \) —because of point 4)—will transform into class \( \mathcal{B} \) and vice versa \(^{(3,5)}\). In this sense, the totality of physical laws \((\mathcal{A} \cup \mathcal{B})\) will be covariant under the whole group \( G \), i.e. «G-covariant» \(^{(8)}\). And in this sense inertial frames (with relative velocities \( |u| \geq c \)) are all equivalent \(^{(1)}\).

From the previous considerations, the «rule of tachyonization» (TR) immediately follows, which is just a consequence of point 4): «the relativistic laws (of mechanics and electromagnetism, at least) for tachyons follow by applying a SLT—e.g. the transcendent transformation \( K_+ \)—to the corresponding laws for bradyons».

Moreover, it resulted \(^{(1-3)}\) that physical laws (of special relativity) may be written in a (universal) form valid for both B's and T's, a form obviously coinciding with the usual one in the bradyonic case. For example, the \( G \)-covariant expression \(^{(1-3)}\)

\[ m = \frac{m_0}{\sqrt{1 - \beta^2}} \quad (\beta^2 \geq 1, \ m_0 \text{ real}) \]

in such a form has «universal» validity (see also the following).

2. — Superluminal sources and Doppler effect.

Once the «Superluminal Lorentz transformations» (SLT) are given for \( u^2 > c^2 \), it is immediate to get the generalization \(^{(1)}\) of the Doppler-effect formula from the time transformation law. Namely, in the case of relative col-


linear motion along the $x$-axis, one has \(^{(11)}\) for $u^2 \leq c^2$

\[
(8) \quad v = v_0\sqrt{(1 - \beta^2)/(1 + \beta \cos \alpha)} \quad (u^2 \leq c^2),
\]

where $u = \beta c$ is the relative speed and $\alpha = \vec{ui}$, the vector $\vec{l}$ being directed from the observer to the source.

In the particular case of relative motion (strictly) along the observation ray, since

\[
[\text{sign}(u)] \cdot [\text{sign}(\cos \alpha)] = - \quad \text{corresponds to approach,}
\]

\[
+ \quad \text{corresponds to recession,}
\]

we obtain the behaviour represented in Fig. 1, where the dashed curve refers to «approach» and the solid one to «recession».

![Fig. 1. - Doppler-effect extension: observed frequency vs. relative velocity for motion along the $x$-axis. The sign minus refers to approach (dashed line) and the plus to recession (solid line). The interpretation of the negative values appearing for the Superluminal approach is given in Fig. 2.](image)

It is interesting that, in the particular case $\sin \alpha = 0$, our formula (3) has been obtained—individually by us \(^{(2,n,11)}\)—also by Gregory \(^{(12)}\), through heuristic considerations.


The first important point we want to stress is the following. In Fig. 1, the two solid curves (recession) are one the conformal correspondent of the other, as expected, in the sense that the same frequency will be obtained both for 
\( u = \bar{v} < c \) and \( U = v^2/\bar{v} > c \). Therefore, an astrophysical source receding with Superluminal velocity \( U \) is expected to exhibit a Doppler effect identical to the one of a usual source travelling away at velocity \( u = c^2/U \).

The second result to be emphasized is the following. The above-mentioned conformal correspondence holds even for the two dashed curves (approach), except for the sign. Precisely, in the case of Superluminal approach, eq. (3) yields a negative sign (1,2,11,12) (Fig. 1). Such an occurrence represents the fact that a (subluminal) observer receives the radioemission of an approaching Superluminal source in the reversed chronological order, as is made clear in Fig. 2.

![Fig. 2. The radioemission of a Superluminal source, approaching the observer along the x-axis, will be received in reversed chronological order. This is the meaning of the negative frequencies entering Fig. 1. The line S is the Superluminal world-line.](image)

Therefore, if a macroscopic phenomenon is known to produce a radioemission obeying a certain chronological law, and one happens to detect the reversed radioemission, the observed source should be considered as a Superluminal, approaching object.

3. – Radiocontact possibilities.

Let us now examine an observer \( s \), and the Minkovsky space-time as «seen» by him. Namely, let us study (9) the relative position of the world-lines of both bradyons and tachyons, and of the light-cones associated with one or more events of their history. Notice (9) that—in a three-dimensional space-time, for simplicity—the light-cones springing from a bradyonic world-line are strictly one inside the other, since the locus of the vertices passes inside the cones. This is not true for the light-cones springing from a tachyonic world-line. In this second case, their envelope is constituted by two
planes: the «retarded light-cones» occupy entirely one dihedron with angle larger than 90°. We can easily get what follows (*):

A) Case of a subluminal receiver B. The detector B may receive radio-signals (RS) from both bradyonic and tachyonic emitters.

B) Case of a Superluminal receiver T.

1) The detector T will receive RS from a bradyon only until a certain instant (i.e. T will no longer receive any RS from a bradyon after a certain instant). Precisely, T will first receive the radiotransmission in the same sequence of emission, and afterwards in the reverse order.

2) T will receive RS from another tachyon t, moving along a parallel path (with respect to a subluminal frame), in the following way:

   a) If v(T) = v(t), then T and t will be either always, or never, in radio contact.

   b) If v(T) > v(t), then T will receive (retarded) RS’s until a certain instant; afterwards the contact will break.

   c) If v(T) < v(t), then the previous sequence will be reversed. For example, T will start receiving RS’s only after a certain instant.

3) T will receive, from a second, «unparallel» tachyon t, its RS’s either never, or until a certain instant, or after a certain instant, according to their positions and velocities (relative to the subluminal frame!). In general, the sequence of emission will be highly mixed up at the reception.

Causality problems connected with these situations have been solved, e.g., in ref. (1,2) and references therein.

4. – Faster-than-light objects in the gravitational field.

4'1. Introduction. – First of all, let us remember that, even in the Galilean relativity case, the universal-gravitation law may be written (13) in a form similar to that of Einstein’s gravitational equations. Namely, given a certain space-time, let us call \( I_{\alpha\beta}(x) \) the «affinity» (or affine connection) components defined on that space-time (13). The Galilei-Newton space-time has a very particular affine geometry: the «flat affine geometry», where \( I_{\alpha\beta}(x) = \tilde{I}_{\alpha\beta} \) vanishes at every point of the space-time manifold.

Then, the Newtonian equations of motion of a particle experiencing a

(13) J. L. ANDERSON: Principles of Relativity Physics, Chap. 1, 2, 5.2, 8.10, 10.6 (New York, N. Y., 1967).
gravitational force $F^\mu$ may read (13)

$$m_0 \frac{d^2 x^\mu}{dt^2} + \Gamma^\mu_{\nu\sigma} \frac{dx^\nu}{dt} \frac{dx^\sigma}{dt} = F^\mu \quad \text{(Newton’s case).} \tag{9}$$

4’2. Tachyons and gravitational field in special relativity. – Also in special-relativity theory (Minkovsky space-time), it is possible to introduce the gravitational field (15), and to build up a theory apparently explaining all observed gravitational phenomena (advances of planetary perihelia, bending of light rays, and so on), and deducible by a variational principle (16). We are referring essentially to the formulation by Belinfante (14) and by Fierz (14). By the way, such a theory may be improved (15), so as to lead to the same field equations of general relativity.

In Belinfante’s theory (13,14) (which, e.g., succeeds also in being associated with a definite spin value, namely 2), the gravitational field is described by a symmetric second-rank tensor $h_{\mu\nu}$. The equations of motion for a bradyon result essentially to be (13,14,16)

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\nu\sigma} \frac{dx^\nu}{ds} \frac{dx^\sigma}{ds} = 0 \quad \text{(ds timelike),} \tag{10}$$

where the quantities $\Gamma^\mu_{\nu\sigma}$ are the components of a certain «symmetric affinity» (13), i.e. are the Christoffel symbols formed from a certain tensor (15) $f_{\mu\nu}$ (built up by means of the Minkovskian metric tensor $g_{\mu\nu}$ and of $h_{\mu\nu}$). For our purposes, it is noticeable that Christoffel symbols behave as (third-rank) tensors with respect (only) to linear transformations of the co-ordinates. This is our case, since all GLT’s are linear transformations of Minkovsky space-time.

We are of course assuming that the gravitational interaction is relativistically covariant. Equation (10) tells us that the considered bradyon suffers the gravitational 4-force

$$F^\mu = -m_0 \Gamma^\mu_{\nu\sigma} \frac{dx^\nu}{ds} \frac{dx^\sigma}{ds} \quad \text{($\beta^2 < 1$).} \tag{10 bis}$$

From the «tachyonization rule» we may immediately derive the gravitational 4-force suffered by a tachyon, by applying a SLT (e.g. the transcendent transformation $K_+$) to eq. (10 bis). In the simple case of collinear motion along the $x$-axis, transformations $K_\pm$ have been shown in ref. (13) to be represented

by the $4 \times 4$ matrices

$$K_{\pm} = \begin{pmatrix} \pm \sigma_2 & 0 \\ 0 & -i\sigma_0 \end{pmatrix} = -(\mp \sigma_2 \mp i\sigma_0),$$

where

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

are Pauli matrices.

By the way, we eliminated the distinction between «covariant» and «contravariant» components, by means of our convention of suitably introducing imaginary units in the spatial-component definition. A summation is understood over repeated indices.

In conclusion, for $\beta^2 > 1$ we get

$$I^{\mu} = m_0 \Gamma_0^{\mu} \frac{dx'^{\mu}}{ds'} \frac{dx'^{\nu}}{ds'} \quad (\beta^2 > 1),$$

where now $m_0$ is the (real) proper mass of the tachyon considered.

In other words, a tachyon is seen to experience a gravitational repulsion (and not attraction!). However, since the fundamental equation of tachyon dynamics (see ref. (1))

$$I^{\mu} = -c \frac{d}{ds} \left( m_0 c \frac{dx'^{\nu}}{ds} \right) \quad (\beta^2 > 1)$$

brings about another sign change, the equations of motion for a tachyon in a gravitational field will still read

$$\frac{d^2 x'^{\mu}}{ds'^2} + \Gamma_0^{\mu \nu} \frac{dx'^{\nu}}{ds'} \frac{dx'^{\nu}}{ds'} = 0 \quad (ds' \text{ spacelike}).$$

Therefore, eqs. (10) and (10') are already $G$-covariant; they hold for both bradyons and tachyons. They may be (still $G$-covariantly) written

$$a^{\mu} + \Gamma_0^{\nu \sigma} u^\nu u^\sigma = 0 \quad (\beta^2 > 1),$$

where $a$ and $u$ are four-acceleration and four-velocity, respectively. Notice that, whilst $dx/ds$ is a four-vector only with respect to the proper Lorentz group, on the contrary $u = dx/d\tau_0$ is a $G$-four-vector.

As regards tachyons, let us, e.g., consider a tachyon going towards a gravita-
tional-field source. Since it will experience a repulsive force, its energy \( (1^3) \)
\[
E = \frac{m_0 \beta^2}{\sqrt{\beta^2 - 1}} \quad (\beta^2 > 1, \ m_0 \text{ real})
\]
will decrease (contrary to the bradyonic case); however (see ref. \( 1^3 \)) its velocity will increase. Therefore, we spoke about «gravitational repulsion» since the gravitational force applied to the tachyon is centrifugal (and not centripetal); however, as a result, the tachyon will accelerate towards \( (*) \) the gravitational-field source (similarly to a bradyon).

In conclusion, under our hypotheses it follows that:

a) From the dynamical (and energetical) point of view, tachyons appear as gravitationally repulsed. They might be considered to couple negatively to the gravitational field (the source velocity being inessential), i.e. to be the «antigravitational particles».

b) From the kinematical viewpoint, however, tachyons appear as «falling down» towards \( (*) \) the gravitational-field source (the source velocity being inessential, as well as for electromagnetic field).

Let us explicitly remember that usual antiparticles do couple positively to the gravitational field. There is no gravitational difference between objects and their antobjects (in accordance with the «reinterpretation principle»; see ref. \( 1^3 \) and the first point in Sect. 1).

For tachyons, in the «nonrelativistic limit» (speed near \( c\sqrt{2} \); cf. eq. (7)) we would find for the three-force magnitude no longer Newton's law \( F = -Gm_1 m_2 / r^2 \), but
\[
F = +G \frac{m_{01} m_{02}}{r^2} \quad (\beta^2 > 1; \ m_{01}, m_{02} \text{ real}),
\]

since when passing to tachyons \( G \) transforms into \( -G \).

All the foregoing accords with the principle of duality.

In fact, given an observer \( O \), all objects that appear as bradyons to him will also appear to couple positively to the gravitational field; on the contrary, all objects that appear as tachyons to him will also appear to couple negatively to the gravitational field. In any case, bradyons, tachyons and

\( (*) \) Note added in proof. – See also R. O. HETTEL and T. M. HELLIWELL: Nuovo Cimento, 13 B, 82 (1973). In this paper the problem of tachyon trajectories in a Schwartzschild field has been considered within general relativity. Cf. also R. W. FULLER and J. A. WHEELER: Phys. Rev., 128, 919 (1962).

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luxons (*) will all bend towards the gravitational-field sources, even if with different curvatures.

5. — On the so-called « gravitational Čerenkov radiation » and tachyons. Case of the vacuum.

All that was previously said holds under the assumption of negligible gravitational-wave radiation.

If we improved eq. (10), for bradyons in a gravitational field, by allowing gravitational-wave emission, then the tachyonization rule (in the spirit of the duality principle) would still permit us to derive immediately the corresponding law for tachyons (now allowed to emit gravitational waves when interacting with that gravitational field).

But here we want only to deal with the problem of the so-called « gravitational Čerenkov radiation » (GCR).

A usual bradyon (e.g. without charges, but of course massive) will generally radiate gravitational waves (\(^\star\)) as follows:

A) First of all, when (and if) it happens to enter a sensu lato medium (i.e. a force field), with speed larger than the gravitational-wave speed in that field, then it is expected to radiate a cone of gravitational radiation (somewhat similarly to the sound wave cone emitted by an ultrasonic airplane in air). Such a cone should not be confused with gravitational Čerenkov radiation, which on the contrary has the following features (\(^\star\)): i) it is emitted by the bodies (or particles) of the material medium and not by the travelling object itself, ii) it is a function of the velocity (and not of the acceleration) of the travelling object.

B) Secondly, when the considered bradyonic object (e.g. a galaxy) \(B\) happens for example to enter a cluster of uniformly distributed galaxies, with a speed larger than the gravitational-wave speed inside that « material medium » (the cluster), then \(B\) will cause the cluster galaxies to emit gravitational waves such that possibly they will coherently sum up to form a gravitational Čerenkov cone (\(^\star\)\(^{\star}\)).

Now, we want to investigate if and when tachyonic objects can emit cones of gravitational waves, and in particular cause emission of gravitational-radiation Čerenkov cones.

\(\star\) « Objects » travelling at the light speed, like photons and neutrinos.


The first result is that free tachyons in vacuum will not emit gravitational Čerenkov radiation, or gravitational-wave cones, or any gravitational radiation.

This result may be immediately obtained—see ref. (1.29)—by using the tachyonization rule, i.e. by applying a SLT to the behaviour of a free bradyon (e.g. at rest) in vacuum.

As regards the gravitational Čerenkov effect, let us consider an object that appears to us (frame $s_0$) as a T moving at constant velocity in vacuum. Such an object will appear as a B with respect (e.g.) to its rest frame $S$; according to frame $S$, therefore, the (bradyonic) differential energy loss through GCR in vacuum is obviously zero (13):

\[(14a) \quad \left( \frac{dE'}{ds'} \right)_0 = 0 \quad (\beta^2 < 1).\]

If we transform such a law by means of the SLT from $S$ to $s_0$ (and remember the vacuum-covariance postulate), we get the differential energy loss through GCR for a tachyon in vacuum (2.21):

\[(14b) \quad \left( \frac{dE'}{ds'} \right)_0 = 0 \quad (\beta^2 > 1).\]

Moreover, an object uniformly moving in vacuum does not emit any radiation both in the subluminal case and in the Superluminal one, as required by (extended) relativity.

Let us repeat that it is necessary not to confuse GCR with the radiation that an object can indeed emit in vacuum when it is accelerated.

Lastly, let us again emphasize that GCR comes out from the objects of the material medium, and not from the «radiating» object itself (19.28). Therefore, the very expression «GCR in vacuum» would have been meaningless, unless one simultaneously provided a suitable theory about an improbable «vacuum structure».

6. – Case of material «media».

Since the previous case A), considered in Sect. 5, is substantially not different from case B) (13.22), we shall confine ourselves to the study of Čerenkov radiation cones caused by tachyonic objects. The principle of duality, together with the tachyonization rule, allows us to solve also the general problem of GCR from tachyons in a material medium (21).


(23) See, e.g., W. PAULI: Relativitätstheorie (Leipzig, 1921).
Let us first define the proper "gravitational refraction index" $N_0$ of a medium $M_1$ with respect to a second medium $M_2$ as the ratio of the gravitational-wave (GW) speed in $M_2$ over the GW speed in $M_1$. Let us now generalize (Fig. 3) the "drag effect" (8) for Superluminal velocities, i.e. extend (11):

\[ N(u) \approx N \left( \frac{N_0 c}{c + N_0 u} \right) \quad (u^2 > c^2); \]

the quantity $v = c/N$ will give the GW speed in the moving medium considered, with respect to the observer $O$. The GW speed in vacuum is of course assumed to be equal to the light speed $c$.

It is easy to observe that i) the GW speed $v$ in a bradyonic medium appears to a bradyonic observer $O$ always slower than $c$; ii) the GW speed $v$ in a hypothetical, ideal "luminal medium" ($u = c$) would appear to $O$ as always equal to $c$; iii) the GW speed in a tachyonic medium will always appear to $O$ as larger than $c$.

It is immediate to deduce that

1) a bradyon $B$ can emit gravitational Čerenkov only in a subluminal medium (when it happens to travel with speed $w$ larger than the apparent
GW speed $v$ in that bradyonic medium). From duality and the tachyonization rule it is then straightforward to get that

2) a tachyon $T$ can emit GCR only in a Superluminal medium, and namely when it happens to travel in the (tachyonic) medium with speed $v$ slower than the apparent GW speed $v$ in that medium. Therefore, a tachyon will never emit gravitational Čerenkov in usual, subluminal media. Conversely, it is possible that a tachyon appears to move in a tachyonic medium with velocity slower than the apparent GW velocity there: see Fig. 3.

From the analytical point of view, the formula (*) of the standard, differential energy loss (per unit path length) through GCR from a bradyon in a medium may be written (30)

\begin{equation}
\frac{dW}{dl} \propto \frac{4\pi^2 Gm^2}{c^2} \int \left(1 - \frac{1}{\beta^2 N_0^2}\right) \nu dv \quad \left(\beta = \frac{w}{c}, \ w^2 < c^2\right),
\end{equation}

where now $\beta = w/c$, the quantity $w$ being the bradyon speed relative to $O$, and $N_0$ is the proper «gravitational refraction index» of the medium. In particular, for $N_0 = 1$ (i.e. for the vacuum) one gets

\begin{equation}
\left(\frac{dW}{dl}\right)_0 = 0 \quad (w^2 < c^2).
\end{equation}

Then, by applying the transcendent Superluminal Lorentz transformation $K_+$ to eq. (16a), we find the analogous formula of the differential energy loss (per unit path length), through GCR, from a tachyon in a medium (21):

\begin{equation}
\frac{dW}{dl} \propto \frac{4\pi^2 Gm^2}{c^2} \int \left(1 - \frac{\beta^2}{N_0^2}\right) \nu dv \quad \left(\beta = \frac{w}{c}, \ w^2 > c^2\right),
\end{equation}

where $N_0$ is still the proper «gravitational refraction index» of the medium, and $w$ the tachyon speed (relative to $O$). In particular, for $N_0 > 1$ (i.e. for the vacuum and for bradyonic media), one gets

\begin{equation}
\left(\frac{dW}{dl}\right)_0 = 0 \quad (w^2 > c^2).
\end{equation}

The «Čerenkov relation» giving the cone angle in the bradyonic case (30)

\begin{equation}
\cos \theta = \frac{1}{\beta N_0} \quad (\beta^2 < 1)
\end{equation}

(*) As follows by analogy with the electromagnetic case. Cf. ref. (20).
transforms under the same SLT into the following relation, giving the cone angle in the tachyonic case:

\[(18b) \cos \theta = \frac{\beta}{N_o} \quad (\beta^2 > 1).\]

All the previous papers (23-25) dealing with GCR, in our opinion, are not correct, since they are based on an uncritical extension of the usual procedure for bradyons to the tachyonic case. They, e.g., concluded about a GCR from tachyons in vacuum. We have seen from (extended) relativity, on the contrary, that Superluminal objects do not emit GCR in bradyonic media (e.g. in bradyonic galaxy clusters).

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**RIASSUNTO**

Dalla teoria della relatività ristretta, quando non se ne restringa arbitrariamente la validità a velocità subluminali, è possibile dedurre l’esistenza di sistemi di riferimento Superluminali e di oggetti più veloci della luce. Di recente, invero, la relatività ristretta è stata estesa ai sistemi di riferimento inerziali Superluminali e ai tachioni, così da consentire lo sviluppo autoconsistente di una « teoria classica » dei tachioni. La « teoria della relatività estesa » consente in particolare di estendere le formule dell’effetto Doppler a sorgenti Superluminali. Si prendono in considerazione anche le varie possibilità di contatto radio tra gli oggetti subluminali e Superluminali. Si chiarisce il comportamento dei corpi tachionicì nel campo gravitazionale. Si predice che i tachioni subiscono una repulsione gravitazionale (così che potrebbero essere considerati la « materia antigravitazionale »); tuttavia, a causa della loro dinamica, essi finiscono con l’avere traiettorie che s’incurvano verso la sorgente del campo gravitazionale, come gli oggetti soliti (anche se con curvature diverse). Infine, si studia il problema della radiazione Čerenkov gravitazionale (GCR). In contrasto con quanto affermato dagli autori precedenti, la relatività estesa predice che i tachioni non emettono GCR nel vuoto o nei mezzi bradionici (subluminali).
Астрофизика и тахионы.

Резюме (*). — Исходя из специальной теории относительности, можно сделать заключение о существовании сверхлюминесцентных систем отсчета и объектов, движущихся быстрее скорости света, когда справедливость этого не подтверждается при сублюминесцентных скоростях. Недавно специальная теория относительности была непосредственным образом продолжена на сверхлюминесцентные инерциальные системы и на тахионы, тем самым, допуская звук, самосогласованное развитие «классической теории» тахионов. «Обобщенная теория относительности» позволяет, в частности, распространить формулы для эффекта Допплера на сверхлюминесцентные источники. Также рассматриваются возможности радиоконтакта между сублюминесцентными и сверхлюминесцентными объектами. Было исследовано поведение тахионных тел в гравитационном поле. Предсказывается, что тахионы испытывают гравитационное отталкивание (так что могут рассматриваться как «антиграницичное вещество»). Однако благодаря их динамике, они, в конце концов, ведут себя, с геометрической и кинематической точек зрения, как обыкновенные объекты. В заключение, исследуется проблема гравитационного излучения Черенкова. В противоположность предыдущим авторам, «обобщенная теория относительности» предсказывает, что тахионы не образуют гравитационного черенковского излучения ни в вакууме, ни в брадионной (сублюминесцентной) среде.

(*) Переведено редакцией.