Recovering Causality for Taehyons even in Macrophysics (*).

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It was already shown (1) that "retarded causality" can be implemented in relativity by assuming the postulate that "negative-energy objects travelling forward in time do not exist" (third postulate of special relativity) (1-2). This "third postulate" is fundamentally equivalent to the well-known "reinterpretation principle" (RIP) by Dirac, Stuckelberg and Feynman (3). The "RIP" is quite enough (1) to ensure the validity of the law of retarded causality also for taehyons, even if many physicists are still unaware of this fact. In particular, it is generally believed that causal problems with taehyons are still present in macrophysics. That situation has been very recently emphasized, e.g., by Basano (4).

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We want here to stress, on the contrary, that our "third postulate" recovers causality for tachyons not only in the case of elementary processes—as mostly considered, till now—but also in macroscopic processes, where entropy consideration occurs.

Let us consider two bodies, A and B, for simplicity at rest one with respect to the other. Then, let us consider—in their rest frames (i.e., in the lab)—the exchange of a tachyon T along the positive x-direction, with speed \( +v \), from A (source) to B (detector). Now, the frame with speed \( u = \frac{c^2}{v} \) will see the tachyon T travelling at infinite speed \(^{(1)}\) and (as illustrated e.g. in fig. 12 of ref. \(^{(1)}\), p. 236) any frame with speed \( u > \frac{c^2}{v} \) would see T as going backwards in time and bearing negative energy; that is to say that it will actually observe an antitachyon \( \bar{T} \). In other words, if we consider a (subluminal) boost along the positive x-direction with speed \( \frac{c^2}{v} < u < c \), then the new observer \( s' \) will see \( (1,\vec{u}) \)—because of "RIP"—an antitachyon \( \bar{T} \) emitted by B (source) and absorbed by A (detector), and carrying, of course, positive energy forward in time. See fig. 1.

![Diagram](image)

**Fig. 1.** The figure represents the exchange from A to B of a particle \( P \) with negative energy (and charges *) and travelling backwards in time \((t_2 < t_1)\). From the Third Postulate of special relativity (*negative-energy particles travelling forward in time do not exist*) and from the fact that we necessarily *explore* Minkowski space-time going *forward in time*, it follows that such a process appears as the exchange from B to A of a particle \( Q \) endowed with positive energy (and *charges*) and moving forward in time. Particle \( Q \) results then to be—the antiparticle of the initial particle (except for the helicity, in this case): \( Q \to \bar{P} \). As is well-known, the *third postulate* of SR—while solving in *nuce* all the (apparent) paradoxes, which *seem* to imply an information—transfer into the past, both with usual particles (bradyons) and with tachyons—furthermore predicts, for any particle, the existence of its *antiparticle*.

The point is that, while \( s \) observes *e.g.* the energy of \( A \) decreasing in that process, on the contrary \( s' \) observes the energy of \( A \) increasing in the same process. However, for each observer, the *law* of energy conservation is *satisfied* in the observed process. Only the *description* *details* are not invariant, in full accord with special relativity (SR) that does not require their invariance. For suitable definition and analysis of *law*, *description*, etc., see *e.g.*, p. 248 of ref. \(^{(1)}\). To clarify the situation, let us remember that SR (and Doppler) taught us that, for example, the *colour* of an object is not Lorentz invariant (even if nothing prevents any observer, who knows SR, from *calculating* out the *real* colour, *i.e.*, the *intrinsic* colour displayed by the object in its rest frame). The necessary requirement of SR is that each observer forwards a *self-consistent* description containing the *same* (covariant) physical laws \(^{(1)}\).

\(^{(1)}\) E. ReCami and R. Mignani: *Riv. Nuovo Cimento*, 4, 209 (1974). See also ref. \(^{(1)}\).
At this point, we have nothing to do but notice that in fig. 1 we can have (instead of a tachyon exchange) a tachyon signal transmission by means of a modulated tachyon beam. Then, entropy changes and information exchanges behave—when passing from one frame to other ones—exactly as energy exchanges behaved in the previous example (fig. 1); so that, e.g., an information increase observed by s can become an information decrease as regarded by s', and so on.

We shall illustrate the arising, possible problems by considering two examples, i.e. by proposing two apparent paradoxes, and then by solving them.

First, let us consider the case of a subluminal frame \((x, t)\) and of a Superluminal frame \((x', t')\) boosted along the (positive) \(x\)-direction with relative speed \(U > c\). See fig. 2. And let us choose as common origin, \(O\), the event when both frames coincide. To \(s\), the \(x\)-axis represents all the events contemporaneous with \(O\); whilst, to \(s'\), all the events contemporaneous with \(O\) are represented by the \(x'\)-axis. If information can be carried by (almost \((^\ast)\)) infinite-speed tachyons, then observer \(s'\)—when is near \(O\)—can receive signals from event \(E\) (see fig. 2) and can tell such information to \(s\). It seems, therefore, that \(s'\) can communicate (when is near \(O\)) to \(s\) information about the possible future of \(s\).

![Fig. 2. The first apparent paradox here discussed. It seems that a Superluminal observer \((x', t')\), while overcoming at \(O\) the observer \((x, t)\) at rest, can tell the latter information about a future— for the latter—event \(E\), if tachyon signals exist travelling at (almost) infinite-speed. The paradox is immediately solved on the basis of the third postulate of SR, when one remembers that SR requires only laws (and not description details) to be invariant. The important point, here, is that causal paradoxes with tachyons can be easily solved even in macrophysics. \(tg \alpha = \beta = U/c\).](image)

However, it is clear that the \(\ast\) RIP \(\ast\) teaches us \((^\ast)\) the following: The signal \(E \rightarrow O\) will appear to observer \(s\) as a signal (or better \(\ast\) anti-signal \(\ast\) \((^\ast)\)) sent by \(s'\) from \(O\) towards \(E\) (since, with respect to \(s\), it would be a \(\ast\) negative signal going backwards in time \(\ast\), which must \((^\ast\ast)\) be reinterpreted according to \(\ast\) RIP \(\ast\)). In conclusion, according

\((^\ast)\) Infinite-speed tachyons carry zero-energy (but 3-momentum magnitude equal to \(m_e\)), and their direction along their motion-line has no definite sense. As \(\ast\) bradyons \(\ast\) at rest are points in space, extended in time along a line, so transcendent tachyons are \(\ast\) points \(\ast\) in time, extended in space along a line: cf., R. MIGNANI and E. RICAMI: Lett. Nuovo Cimento (in press). Infinite-speed tachyons will be dealt with elsewhere, also due to their possible rôle in hadron structure (remember, \(e.g.,\), Susskind's \(\ast\) instantons \(\ast\), Preparata's \(\ast\) bilocal functions \(\ast\), besides \(\ast\) virtual particles \(\ast\)). Cf., \(e.g.,\), R. MIGNANI and E. RICAMI: Nuovo Cimento, 30 A, 533 (1975); Phys. Lett. (to appear); and the quotations (87-89) in ref. \((^\ast)\).

\((^\ast\ast)\) See ref. \((^\ast)\) and p. 239 of ref. \((^\ast)\).

\((^\ast\ast\ast)\) An \(\ast\) anti-signal \(\ast\) is carried by anti-particles, so as the signal is carried by (their) particles. Of course, both carry positive energy. See also the following.
to \( s \), observer \( s' \) is predicting to him nothing but what can be actually "forecast"—on the basis of our physical operations and of the physical laws—i.e., observer \( s' \), to \( s \), is merely "predicting" the obvious effects on \( E \) of the (anti)-signals sent by himself from \( O \) towards \( E \), so to physically influence it.

Even in this case, each observer forwards a different description of the same chain of events, but the law of (retarded) causality holds in all descriptions, for all observers.

Let us now pass to a more subtle example, involving a little more deeply the tachyon mechanics, and spend some more words in solving the relative (apparent) paradox.

Let us now consider a process like that in fig. 1, but with the body \( B \) moving with respect to the body \( A \). Let us, therefore, consider two usual, inertial observers (fig. 3), moving along the \( x \)-direction relative to each other, with speed \( u < c \), and let us suppose that (according to the frame \( s \equiv (x, t) \)) observer \( s \) sends a signal to \( s' \equiv (x', t') \) by means of a tachyonic beam with speed \( V > c^2/u \).

According to \( s' \), however, the "tachyon beam" would actually appear as an antitachyon-beam emitted by \( s' \) himself towards \( s \).

Let us now pass to a more subtle example, involving a little more deeply the tachyon mechanics, and spend some more words in solving the relative (apparent) paradox.

Now, we can well imagine that—when overcoming \( s' \) at \( O \)—observer \( s \) told \( s' \) about his intention of transmitting to him (i.e., to \( s' \)) a tachyonic signal (with speed \( V > c^2/u \)) at time \( t \), whose Lorentz transformed is time \( t' \). Then, it seems that observer \( s' \) will be compelled—at a certain instant \( t' - \Delta t' \)—to emit an (anti)-signal towards \( s \), in order to save the validity of the classical theory of tachyons in four-dimensions. But this would, of course, violate the "free will" of observer \( s' \), who can, on the contrary, decide to send no signal (or anti-signal) towards \( s \). A fortiori, observer \( s' \) can be in the impossibility of sending that signal, lacking information about it (for instance, \( s \) told \( s' \)—at \( O \)—that at time \( t \) he, \( s \), will send to \( s' \) one piece of music by a tachyon beam with \( V > c^2/u \); and \( s' \) may have no record of that music, so that he cannot send to \( s \) any antimusic, i.e. any music by antitachyons beams). The situation seems self-contradictory. The solution of the apparent paradox is merely in the—previously ignored—fact that mechanical laws of tachyons do not allow observer \( s' \) (the moving body \( B \)) to absorb any tachyon with speed \( V > c^2/u \). In other words, \( s' \) can get information from \( s \) through such process (tachyon-beam emission by \( A \).
and tachyon-beam absorption by $B$) only by beams of tachyons with speed $V$ smaller than $c^2/u$ (in which case no causal problem arises).

Let us first remember that tachyon mechanics (1) is such that no body at rest (so as $A$ in its rest-frame) can emit tachyons—of any proper mass—without lowering its rest mass (*). On the contrary, a body at rest can absorb tachyons both when changing its rest mass and when conserving its rest mass. Here we are going to assume that a moving, macroscopic body $B$, when absorbs signal beam tachyons, will only either conserve its rest mass, or increase it. Here we are assuming, in other words, that the tachyons absorbed by $B$ have rest mass $m_0$ small in comparison with the rest mass $M$ of the macro-object $B$ and are endowed (with respect to $s$, that is to the rest frame of $A$) with not too small energies i.e. with speeds not almost infinite—so to be actually able to transmit information. Remember that for tachyons, in natural units,

\[ p_0 = (P^2 - m_0^2)^{1/2}. \]

Then, in the process of beam-tachyon absorption by $B$ we shall have

\[ \sqrt{P^2 - m_0^2} + \sqrt{P^2 + M^2} = \sqrt{(P + P')^2 + M^2}, \]

where $P$ is the body-$B$ initial 3-momentum. One gets immediately [$A^2 = M^2 - M^2$]:

\[ |P| = \frac{m_0}{2M^2} [m_0 |P| + \sqrt{(P^2 + M^2)(m_0^2 + 4M^2) + M^2} - \Delta^2], \]

so that, for every $m_0$, the body $B$ can absorb only tachyons with definite values of $|P|$ (which are discrete if $A^2 = M^2 - M^2$ can assume only discrete values, or it is in particular zero). From eq. (2) it is straightforward to derive the important relation

\[ V < c^2/u, \]

which allows us to solve the previous paradox too. In the four-dimensional case the relevant quantity will be, instead of $V$, the tachyon-speed component along the direction $AB$.

Further details will appear elsewhere. Here let us finally observe that one of us (M.P.) thinks that causal paradoxes could better be solved in the context of 5-dimensional relativity (15). In any case, considering SR in five dimensions already resulted to be a useful tool for attributing the correct rôle to proper mass and to proper time, and for giving extended relativity a more elegant form (11).

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(*) See p. 264 of ref. (1); and R. MIGNANI and E. RECAMI: Report IC/75/82 (Trieste), to appear in Phys. Lett.

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