The Tolman-Regge *Antitelephone* Paradox: Its Solution by Tachyon Mechanics (*).

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Summary. – The possibility of solving (at least « in microphysics ») all the ordinary causal paradoxes devised for tachyons is not yet widely recognized; on the contrary, the effectiveness of the Stuckelberg-Feynman « switching principle » is often misunderstood. We want, therefore, to show in detail and rigorously how to solve the oldest causal paradox, originally proposed by Tolman, which is the kernel of so many further tachyon paradoxes. The key to the solution is a careful application of *tachyon kinematics*, which can be unambiguously derived from special relativity. A systematic, thorough analysis of all tachyon paradoxes is going to appear elsewhere.

Introduction. – It has been claimed since long (1) that all the ordinary causal paradoxes proposed for tachyons can be solved (at least « in microphysics » (2)) on the basis of the « *switching procedure* » (SWP ») by STÜCKELBERG (3) and FEYNMAN (3), also known as the « reinterpretation principle »: a principle which has been given the status of a

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(1) O. M. B. BILANIUK, V. K. DESHPANDE and E. C. G. SUDARSHAN: *Am. J. Phys.*, **30**, 718 (1962); R. G. ROOT and J. S. TREFIL: *Lett. Nuovo Cimento*, **3**, 412 (1970); J. A. PARMENTOLA and D. D. H. YEE: *Phys. Rev. D*, **4**, 1912 (1971); E. RECAMI and R. MIGNANI: *Lett. Nuovo Cimento*, **7**, 388 (1973); E. RECAMI: in *Enciclopedia EST-Mondadori, Annuario 73*, edited by E. MACORINI (Mondadori, Milano, 1973), p. 85; E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209, 398 (1974); E. RECAMI: *Lett. Nuovo Cimento*, **18**, 501 (1977); *Found. Phys.*, **8**, 329 (1978); G. D. MACCARRONE and E. RECAMI: *Found. Phys.*, **10**, 949 (1980); A. GARUCCIO, G. D. MACCARRONE, E. RECAMI and J. P. VIGIER: *Lett. Nuovo Cimento*, **27**, 60 (1980); P. CALDIROLA and E. RECAMI: in *Italian Studies in the Philosophy of Science*, edited by M. DALLA CHIARA (Reidel, Boston, Mass., 1980), p. 249.
(2) E. RECAMI: *Classical tachyons and possible applications: a review*, Report INFN/AE-84/8 (Frascati, August 1984), to appear in *Riv. Nuovo Cimento.*
(3) E. C. G. STÜCKELBERG: *Helv. Phys. Acta*, **14**, 321, 588 (1941); R. P. FEYNMAN: *Phys. Rev.*, **76**, 749, 769 (1949). See also O. KLEIN: *Z. Phys.*, **53**, 157 (1929).

fundamental postulate [4] of special relativity (SR), both for bradyons [5] and for tachyons. Recently, SCHWARTZ gave the « SWP » a formalization in which it becomes « automatic » [6].

Most of the authors, however, seem still to ignore the effectiveness of those solutions; sometimes showing to be aware only of some older—and, therefore, preliminary, incomplete—papers, while being unaware of the more recent and complete literature. Some authors appear, moreover, to misunderstand even the literature known to them: for a late, remarkable example see, *e.g.*, GIRARD and MARCHILDON [7] [*].

We want, therefore, to show *here* (in detail and rigorously) how to solve the oldest « paradox », *i.e.* the *antitelephone* one, originally proposed by TOLMAN [8] and then reproposed by many authors. We shall refer to its most recent formulation, by REGGE [9], and spend some care in solving it, since it is the kernel of many other paradoxes. Let us tress that: i) any careful solution of the tachyon causal « paradoxes » must make recourse to explicit calculations based on the mechanics of tachyons; ii) such tachyon mechanics can be unambiguously and univocally derived from SR, by referring the space-like objects to the class of the ordinary, subluminal observers *only* (*i.e.*, without any need of introducing Superluminal reference frames); iii) the reader will be helped a lot, moreover, if he will first refer himself to the (subluminal, ordinary) SR based on the *whole* proper Lorentz group $\mathscr{L}_+ \equiv \mathscr{L}_+^\uparrow \cup \mathscr{L}_+^\downarrow$, rather than on its orthochronous subgroup $\mathscr{L}_+^\uparrow$ only (see ref. [5], and references therein).

A systematic, thorough analysis of the tachyon causal problems can be found in ref. [2,10], which will appear elsewhere.

Before going on, let us refer the reader—for a *modern* approach to the classical theory of tachyons—to ref. [2,10,11].

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(4) See *e.g.* E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209 398 (1974); E. RECAMI (editor): *Tachyons, monopoles, and Related Topics* (North-Holland, Amsterdam, 1978); E. RECAMI: in *Astrofizika, Kvanti i Teorya Otnositelnosti*, edited by F. I. FEDOROV (MIR, Moscow, 1982), p. 53; E. RECAMI: in *Relativity, Quanta and Cosmology*, Vol. **2**, Chapt. 16, edited by F. DE FINIS and M. PANTALEO (Johnson Rep. Co., New York, N. Y., 1979), p. 537.

(5) See *e.g.* M. PAVŠIČ and E. RECAMI: *Lett. Nuovo Cimento*, **34**, 357 (1982); **35**, 354 (1982); E. RECAMI and W. A. RODRIGUES Jr.: *Found. Phys.*, **12**, 709 (1982) (in connection with the last paper, let us take the opportunity for correcting some important misprints: at p. 741, correct « nonnegative energy objects exist » into « *no* negative-energy objects exist »; at p. 742, correct $\overline{PT} = +1$ into $\overline{PT} = -1$, and $\Lambda_+^\uparrow$ into $\Lambda_+^\downarrow$).

(6) C. SCHWARTZ: *Phys. Rev. D*, **25**, 356 (1982).

(7) R. GIRARD and L. MARCHILDON: *Found. Phys.*, **14**, 535 (1984).

(*) Let us mention, incidentally, that during the last few years we decided to not answer comments referring to old papers of ours, when the matter had *already* been clarified in more recent publications of ours: for that reason, we did not answer L. MARCHILDON, A. F. ANTIPPA and A. E. EVERETT's considerations appeared in *Phys. Rev. D*, **27**, 1740 (1983).

(8) R. C. TOLMAN: *The Theory of Relativity of Motion* (Berkeley, Cal., 1917), p. 54.

(9) T. REGGE: *Cronache dell'universo* (Boringhieri, Torino, 1981), p. 21. See also D. BOHM: *The Special Theory of Relativity* (New York, N. Y., 1965).

(10) E. RECAMI: *Tachyon kinematics and causality: a systematic, thorough analysis*, Internal Report No. 308 (IMECC, State University of Campinas, S.P., Brazil, March 1985), in press also as INFN Report (Frascati, Italy): to be published.

(11) See, besides ref. [2,10], E. RECAMI and W. A. RODRIGUES Jr.: *A model-theory for tachyons in two dimensions*, Report INFN/AE-85/2 (Frascati, March 1985), to appear in *Gravitational Radiation and Relativity*: *vol.* 3 *of the Sir Arthur Eddington Centenary Symposium Proceedings*, edited by T. M. KARADE (World Scient. Publ. Co., Singapore); E. RECAMI, A. CASTELLINO, G. D. MACCARRONE and M. RODONÒ: *Considerations about the apparent Superluminal expansions in astrophysics*, preprint PP/761 (Phys. Dept., State University of Catania, Italy, May 1984), in press also as INFN Report (Frascati): to be published; E. RECAMI and W. A. RODRIGUES Jr.: *Tachyons: may they have a role in elementary particle physics?*, Internal Report No. 316 (IMECC, State University of Campinas, S.P., Brazil, June 1985), to appear in vol. **15** of *Progress in Particle and Nuclear Physics* (« *Nucleus-Nucleus Collisions from the Coulomb Barrier up to the Quark-Gluon Plasma* »), edited by

Tachyon kinematics. – In ref. [12] there can be found exploited the basic « tachyon kinematics » related to the processes: *a*) proper (or « intrinsic ») emission of a tachyon T by an ordinary body A; *b*) « intrinsic » absorption of a tachyon T by an ordinary body A; *c*) exchange of a tachyon T between two ordinary bodies A and B. The word « intrinsic » refers to the fact that those processes (emission, absorption) are as described in the rest-frame of the body A; particle T can represent both a tachyon and an antitachyon. Let us recall only the following results.

Let us first consider a tachyon moving with velocity $\boldsymbol{V}$ in the frame s_0. If we pass to a second frame s', endowed with velocity $\boldsymbol{u}$ w.r.t. (with respect to) frame s_0, then the new observer s' will see—instead of the initial tachyon T—an antitachyon $\overline{\mathrm{T}}$ travelling the opposite way in space (due to the « SWP »), if and only if

$$\boldsymbol{u}\cdot\boldsymbol{V} > c^2\,. \tag{1}$$

Remember in particular that, if $\boldsymbol{u}\cdot\boldsymbol{V} < 0$, the « switching » does *never* come into play.

Now, let us explore some of the unusual and unexpected consequences of the trivial fact that in the case of tachyons it is

$$|E| = +\sqrt{\boldsymbol{p}^2 - m_0^2} \qquad (m_0 \text{ real};\; V^2 > 1)\,, \tag{2}$$

where we chose units so that, numerically, $c = 1$.

Let us, *e.g.*, describe the phenomenon of « intrinsic emission » of a tachyon, as seen in the rest frame of the emitting body: Namely, let us consider *in its rest frame* an ordinary body A, with initial rest mass M, which emits a tachyon (or antitachyon) T endowed with (real) rest mass $m \equiv m_0$, four-momentum $p^\mu \equiv (E_\mathrm{T}, \boldsymbol{p})$, and speed V along the x-axis. Let M' be the final rest mass of body A. The four-momentum conservation requires

$$M = \sqrt{\boldsymbol{p}^2 - m^2} + \sqrt{\boldsymbol{p}^2 + M'^2} \qquad \text{(rest frame)} \tag{3}$$

that is to say

$$2M|\boldsymbol{p}| = [(m^2 + \Delta)^2 + 4m^2M^2]^{\frac{1}{2}}, \qquad V = [1 + 4m^2M^2/(m+\Delta)^2]^{\frac{1}{2}}\,, \tag{4}$$

where we put $(E_\mathrm{T} \equiv +\sqrt{\boldsymbol{p}^2 - m^2})$

$$\Delta \equiv M'^2 - M^2 = -m^2 - 2ME_\mathrm{T} \qquad \text{(emission)} \tag{5}$$

so that

$$-M^2 < \Delta \leqslant -|\boldsymbol{p}|^2 \leqslant -m^2 \qquad \text{(emission)}\,. \tag{6}$$

It is essential to notice that Δ is, of course, an *invariant* quantity, that in a generic frame s writes

$$\Delta = -m^2 - 2p_\mu P^\mu\,, \tag{7}$$

where p^μ is the initial four-momentum of body A w.r.t. frame s.

A. FAESSLER (Pergamon Press, Oxford, U.K.). See also E. RECAMI and E. GIANNETTO: *Lett. Nuovo Cimento*, **43**, 267 (1985); P. SMRZ: *Lett. Nuovo Cimento*, **41**, 327 (1984); A. BARUT, G. D. MACCARRONE and E. RECAMI: *Nuovo Cimento A*, **71**, 509 (1982); E. RECAMI and G. D. MACCARRONE: *Lett. Nuovo Cimento*, **37**, 345 (1983); G. D. MACCARRONE, M. PAVŠIČ and E. RECAMI: *Nuovo Cimento B*, **73**, 91 (1983); G. D. MACCARRONE and E. RECAMI: *Found. Phys.*, **14**, 367 (1984); *Lett. Nuovo Cimento*, **34**, 251 (1982).

[12] G. D. MACCARRONE and E. RECAMI: *Nuovo Cimento A*, **57**, 85 (1980).

Notice that in the generic frame s the process of (intrinsic) emission can appear both as a T emission and as a $\bar{T}$ absorption (due to a possible « switching ») by body A. It holds, however, the theorem [2,10,12]:

Theorem 1. « Necessary and sufficient condition for a process to be a tachyon emission in the A rest-frame (*i.e.*, to be an *intrinsic emission*) is that during the process the body A *lowers* its rest-mass (invariant statement!) in such a way that $-M^2 < \Delta \leqslant -m^2$. »

Let us now describe the process of « intrinsic absorption » of a tachyon by body A; *i.e.* let us consider an ordinary body A to absorb *in its rest frame* a tachyon (or anti-tachyon) T, travelling again with speed V along the x-direction. The four-momentum conservation now requires

$$M + \sqrt{\boldsymbol{p}^2 - m^2} = \sqrt{\boldsymbol{p}^2 + M'^2} \qquad \text{(rest-frame)}, \tag{8}$$

which corresponds to

$$\Delta \equiv M'^2 - M^2 = -m^2 + 2ME_{\mathrm{T}} \qquad \text{(absorption)}, \tag{9}$$

so that

$$-m^2 \leqslant \Delta < +\infty \qquad \text{(absorption)}. \tag{10}$$

In a generic frame s, the quantity Δ takes the invariant form

$$\Delta = -m^2 + 2p_\mu P^\mu . \tag{11}$$

It follows the theorem [2,10,12]:

Theorem 2. « Necessary and sufficient condition for a process (observed either as the emission or as the absorption of a tachyon T by an ordinary body A) to be a tachyon absorption in the A–rest-frame—*i.e.*, to be an *intrinsic absorption*—is that $\Delta \geqslant -m^2$. »

We have now to describe the *tachyon exchange* between two ordinary bodies A and B. We have to consider the four-momentum conservation at A *and* at B; we need to choose a (unique) frame wherefrom to describe the whole interaction; let us choose the rest-frame of A. Let us explicitly remark, however, that—when bodies A and B exchange one tachyon T—the tachyon kinematics is such that the « intrinsic descriptions » of the processes at A and at B can *a priori* correspond to one of the following four cases [12]:

(12)
1) emission-absorption,
2) absorption-emission,
3) emission-emission,
4) absorption-absorption.

Case 3) can happen, of course, only when the tachyon exchange takes place in the receding phase (*i.e.*, while A, B are receding from each other); case 4) can happen, on the contrary, only in the approaching phase.

Let us consider, here, only the particular tachyon exchanges in which we have an « intrinsic emission» at A, and moreover the velocities $\boldsymbol{u}$ of B and $\boldsymbol{V}$ of T w.r.t. body A are such that $\boldsymbol{u}\cdot\boldsymbol{V}>1$. Due to the last condition and to the consequent « switching » (cf. eq. (1)), in the rest-frame of B it is then observed an antitachyon T *emitted* by B and absorbed by A. (*Necessary* condition for this to happen, let us recall, being that A, B by *receding* from each other.)

More in general, the kinematical conditions for a tachyon to be exchangeable between A and B can be shown (12) to be the following.

A) Case of « intrinsic emission » at A:

$$\text{(13)}\qquad \begin{cases} \text{if } \boldsymbol{u}\cdot\boldsymbol{V}<1\,, & \text{then } \Delta_{\text{B}}>-m^2 \quad (\Rightarrow \text{intrinsic absorption at B});\\ \text{if } \boldsymbol{u}\cdot\boldsymbol{V}>1\,, & \text{then } \Delta_{\text{B}}<-m^2 \quad (\Rightarrow \text{intrinsic emission at B}). \end{cases}$$

B) Case of « intrinsic absorption » at A:

$$\text{(14)}\qquad \begin{cases} \text{if } \boldsymbol{u}\cdot\boldsymbol{V}<1\,, & \text{then } \Delta_{\text{B}}<-m^2 \quad (\Rightarrow \text{intrinsic emission at B});\\ \text{if } \boldsymbol{u}\cdot\boldsymbol{V}>1\,, & \text{then } \Delta_{\text{B}}>-m^2 \quad (\Rightarrow \text{intrinsic absorption at B}). \end{cases}$$

Now, let us finally pass to examine the Tolman-Regge paradox.

The paradox. – In fig. 1, 2 the axes t and t' are the world-lines of two devices A and B, respectively, able to exchange tachyons and moving with constant relative speed u, ($u^2<1$), along the x-axis. According to the terms of the paradox (fig. 1), body A sends

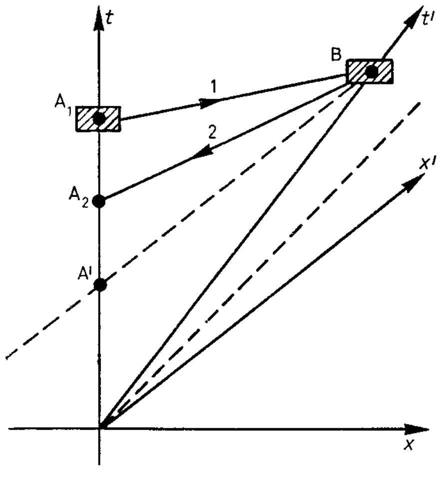

Fig. 1. – The apparent chain of events, according to the terms of the paradox.

tachyon 1 to B (in other words, tachyon 1 is supposed to move forward in time w.r.t. body A). The apparatus B is constructed so to send back a tachyon 2 to A as soon as it receives a tachyon 1 from A. If B has to *emit* (in its rest-frame) tachyon 2, then 2 must move forward in time w.r.t. body B, that is to say its world-line BA_2 must have a slope *lower* than the slope BA′ of the x'-axis (where $\text{BA}'//x'$); this means that A_2 must stay above A′. If the speed of tachyon 2 is such that A_2 falls between A′ and A_1, it seems that 2 reaches back to A (event A_2) *before* the emission of 1 (event A_1). This appears to realize an *antitelephone*.

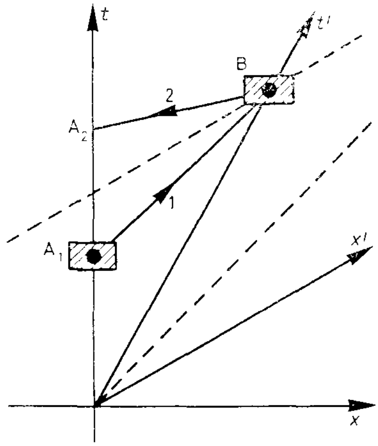

Fig. 2. – See the text.

The solution. – First of all, since tachyon 2 moves backwards in time w.r.t. body A, the event A_2 will appear to A as the emission of an antitachyon $\bar{2}$. The observer « t » will see his apparatus A (able to exchange tachyons) emit successively towards B the antitachyon $\bar{2}$ and the tachyon 1.

At this point, some supporters of the paradox (overlooking tachyon kinematics, as well as relations (12)) would say that, well, the description forwarded by the observer « t » can be orthodox, but then the device B is no longer working according to the premises, because B is no longer emitting a tachyon 2 on receipt of tachyon 1. Such a statement would be wrong, however, since the fact that « t » sees an « intrinsic emission » at A_2 *does not mean* that « t' » will see an « intrinsic absorption » at B! On the contrary, we are just in the case considered above, between eqs. (12) and (13): intrinsic emission by A, at A_2, with $\boldsymbol{u}\cdot\boldsymbol{V}_2 > c^2$, where $\boldsymbol{u}$ and $\boldsymbol{V}_2$ are the velocities of B and $\bar{2}$ w.r.t. body A, respectively; so that *both* A *and* B suffer an intrinsic *emission* (of tachyon 2 or of antitachyon $\bar{2}$) in their own rest frame.

But the terms of the « paradox » were cheating us even more, and *ab initio*. In fact fig. 1 makes it clear that, if $\boldsymbol{u}\cdot\boldsymbol{V}_{\bar{2}} > c^2$, then for tachyon 1 *a fortiori* $\boldsymbol{u}\cdot\boldsymbol{V}_1 > c^2$, where $\boldsymbol{u}$ and $\boldsymbol{V}_1$ are the velocities of B and 1 w.r.t. body A. Due to the above-seen tachyon kinematics, therefore, observer « t' » will see B intrinsically *emit* also tachyon 1 (or, rather, antitachyon $\bar{1}$). In conclusion, the proposed chain of events does *not* include any tachyon absorption by B (in its rest frame).

For body B to *absorb* tachyon 1 (in its own rest frame), the world-line of 1 ought to have a slope *higher* than the slope of the x'-axis (see fig. 2). Moreover, for body B to *emit* (« intrinsically ») tachyon 2, the slope of 2 should be lower than the x'-axis. In other words, when the body B, programmed to emit 2 as soon as it receives 1, does actually do so, the event A_2 does regularly happen *after* A_1 (cf. fig. 2).

The moral. – The moral of the story is twofold: i) one should never *mix together* the descriptions of one phenomenon yielded by different observers, otherwise—even in ordinary physics—one would immediately meet contradictions: in fig 1, *e.g.*, the motion direction of 1 is assigned by A and the motion-direction of 2 is assigned by B; this is illegal; ii) when proposing a problem about tachyons, one must comply ([1]) with the rules of tachyon mechanics ([12]), just as when formulating the text of an ordinary problem one must comply with the laws of ordinary physics (otherwise the problem in itself is « wrong »).

Most of the paradoxes proposed in the literature suffered the above shortcomings (cf. *e.g.* ref. [7]).

Notice that, in the case of fig. 1, neither A nor B regard event A_1 as the cause of event A_2 (or *vice-versa*). In the case of fig. 2, on the contrary, both A and B consider event A_1 to be the cause of event A_2: but in this case A_1 does chronologically *precede* A_2 according to both observers, in agreement with the relativistic covariance of the law of retarded causality.

For a systematic, thorough analysis of the tachyon causal problems we refer once more the interested reader to ref. [2,10].

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