ARE MUON NEUTRINOS FASTER-TNTH-LIGHT PARTICLES?

E. GIANNETTO a, G.D. MACCARRONE a, R. MIGNANI b and E. RECAMI a,c,1,2

a Dipartimento di Fisica, Università di Catania, 1-95129 Catania, Italy
b Dipartimento di Fisica, Università “La Sapienza”, Rome, Italy
c Department of Applied Mathematics, State University at Campinas, 13100 Campinas, S.P., Brazil

Received 6 February 1986

Some papers have appeared recently which are noticeable since they call attention to interesting experimental results referring to the old/dated question whether neutrinos are superluminal (or not). They are complemented therefore, from the theoretical point of view, and some experimental predictions are added that could be tested, especially in connection with neutrino oscillations.

In a recent paper by Chodos et al. [1] attention has been called to five experimental articles [2] indicating that the muon neutrino coming from pion decay may carry a negative four-momentum squared, in the sense that four of those papers seem to favour such a conclusion. The experimental data analysed by Chodos et al. are certainly worth of further experimental check; particularly interesting is the “note added” at page 434 of ref. [1] (a note, incidentally, that came to our attention only recently), in which those authors claim the world average for the four-momentum squared carried by the muon neutrino produced in the decay $\pi^+ \rightarrow \mu^+ \nu$ to be, with the metric signature $(+,+,+,−,−)$,

$$p^2 = p_\nu p^n = (-0.166 \pm 0.091) \text{ MeV}^2/c^2,$$  (1a)

this fact suggesting of course – even if by two standard deviations only – that such neutrinos $\nu$ could be tachyonic. More recent data [3], based on a new precision measurement of the $\pi^+$ mass (and the ordinary assumption $m_{\pi^+} = m_{\nu^+}$), yield the value

$$p^2 = p_\nu p^n = (-0.097 \pm 0.072) \text{ MeV}^2/c^2.$$  (1b)

To start with our comments, let us first mention that the upper limits for neutrino mass are usually evaluated by setting equal to zero the probability squared-mass function for $p^2 < 0$. If we want, on the contrary, to leave open the possibility for muon neutrinos to be tachyons, those upper limits are to be recalculated. Actually, when assuming neutrinos to be slower than light (= bradyons) and neglecting the tachyonic tail, eq. (1b) yields the “ordinary” limit

$$m_B(\nu) \leq 0.27 \text{ MeV}/c^2 \quad (90\% \text{ CL}).$$

However, if we do not disregard the “tachyonic tail”, we get from eq. (1b)

$$m_B(\nu) \leq 0.22 \text{ MeV}/c^2 \quad (95\% \text{ CL}).$$

Conversely, when we assume those muons neutrinos to be faster than light, then we get from eq. (1b) the following two upper limits:

$$m_T(\nu) \leq 0.49 \text{ MeV}/c \quad (95\% \text{ CL}),$$

if the whole gaussian area is considered (i.e., if one retains the “bradyonic tail”) and

$$m_T(\nu) \leq 0.44 \text{ MeV}/c^2 \quad (95\% \text{ CL}),$$

---

* Work supported in part by INFN, CIME/IILU, and IBM-
do-Brazil.
1 Also at CSFN e SdM, Catania, Italy.
2 Also at INFN, Sezione di Catania, 1-95129 Catania, Italy.

0370-2693/86/$03.50 © Elsevier Science Publishers B.V. (North-Holland Physics Publishing Division) 115
if one excludes the "bradionic tail".

The idea, however, that neutrinos can be tachyons, at least in some cases, has a long story, related to a theoretical background that does not show up in ref. [1]. We want therefore to complement the "introduction" to eqs. (1) which appeared therein, from the theoretical point of view, and add some experimental predictions that could be tested, especially with regard to neutrino oscillations. Incidentally, let us recall that the issue of tachyons, although unconventional, already attracted something like $10^3$ publications, about six hundred of which can be found quoted in ref. [4]; see also the list of references in the old review article [5] and in ref. [6], as well as in the bibliographies by Perepelitsa [7].

(i) Let us premise that a little of theory shows $(c = 1)$ the relation $p^2 = p_\mu p^\mu = E^2 - p^2 = +m_\nu^2 > 0$ to hold only for bradyons (slower than light particles), whilst for tachyons it generalizes into $p^2 = -m_\nu^2 < 0$. Analogously, for $V^2 > 1$, $m = m_0/(V^2 - 1)^{1/2}$ holds. In the case of tachyons, therefore, the four-momentum squared $p^2$ is negative, rather than the square of the proper mass $m_0$ (which can be regarded as real). For a modern view on tachyons, see e.g. the recent review [4], & also ref. [8], and refs. [9–13].

(ii) Moreover, since an ordinary Lorentz transformation may carry a positive-energy tachyon $T$ (travelling forward in time) into a "negative-energy tachyon $T'$ travelling backwards in time", it is necessary to introduce – as the Third Postulate of Special Relativity [4,10,14] – the Stückelberg–Feynman switching procedure (also known as "re-interpretation rule") in order to reinterpret $T'$ as the antiparticle $\bar{T}$ of $T$: so that $T' = \bar{T}$, the object $\bar{T}$ being obviously endowed with positive energy and motion forward in time. The first application of this "switching" appeared in ref. [15]. As a consequence, if $\nu$ is a tachyon neutrino with velocity $V$ in the pion rest-frame, then an observer $O'$ travelling with respect to the pion with (subluminal) velocity $u$ such that $u \cdot V > 1$ will not see the decay $\pi \to \mu + \nu$, but the process [16]
\begin{equation}
\pi + \nu \to \mu, \tag{2}
\end{equation}

with the experimental consequences exploited below (see points (iv)).

(iii) As is well known, if neutrino masses are exactly zero, we have a relativistically invariant distinction between (left-handed: $H = -1$) neutrinos and (right-handed: $H = +1$) antineutrinos, based on their helicity. If neutrinos have a finite mass and are slower than light, this is no longer true: in fact, reference frames always exist, travelling faster than the neutrino, wherefrom the sign of its helicity would appear reversed. On the contrary, if neutrinos possess a finite mass, but are faster than light, then the previous distinction between neutrinos and antineutrinos still holds, in the sense that the "switching procedure" (cf. point (ii)) does reverse the helicity together with the particle/antiparticle character [4–6,8,9,14,17] 11.

(iv) Astrophysical arguments are also known, setting stringent limits on the possible masses of bradionic neutrinos (in fact the neutrino "density" should be comparable with the one of photons, which is about $10^2$ times the neutron density; and a mass equal to or larger than about 100 eV would imply a deceleration of the expansion of the universe in conflict with the observations). Those arguments, however, would not hold good for tachyonic neutrinos, because of the fact that their total energy can always approach zero and that their gravitational potential energy has to be computed in a new, different way [12,13].

(v) By the way, let us briefly check whether also the electron-neutrino might be a tachyon or not. The most precise information come from nuclear $\beta$-decay measurements. In the working hypothesis of tachyonic electron-neutrinos, the number $N$ of final states should be given by the modified equation
\[ N(p) \, dp \, F(z, p) = \alpha \, p^2 (E_0 - E)^2 \big[ 1 + m^2_z/(E_0 - E)^2 \big]^{1/2} \, dp, \]
where $p$ and $E$ are now the momentum and energy of the electron; $E_0$ is the available energy of the final state, and $F(z, p)$ is a Coulomb correction factor which is important only for low-

11 See in particular pp. 282, 283 of ref. [17].
energy electrons and for nuclei with large $z$. Thus, plotting $N(p)/p^2$ versus $E$, in the tachyonic case we would have an intersection (at $E = E_0$) with the $E$-axis lying on the other side of the zero neutrino-mass intersection with respect to the bradyonic case. On the contrary, the experiments [18] do strongly favour a bradyonic electron-neutrino.

(vii) As to our main point, regarding neutrino oscillations, we may get interesting consequences if the muon-neutrino (but not the electron-neutrino) is regarded as tachyonic. In this case, one of the mass eigenstates has to be tachyonic. For simplicity's sake, let us consider Majorana neutrinos with a finite mass. In the standard formulae

$$P(\nu_\mu \rightarrow \nu_e) = P(\nu_e \rightarrow \nu_\mu) = 1 - P,$$

with [19]:

$$P = P(\nu_\mu \rightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2 \left[1.267 \delta m^2 L/E \right],$$

the mass squared “difference” is given by $\delta m^2 = m_2^2 - m_1^2$, where $m_1$ is the tachyonic mass. Let us recall that $E$, $L$, $\theta$ are the neutrino energy, the distance from the source, and the mixing angle, respectively. Since for a tachyonic neutrino the mass upper limit is about 0.49 MeV/c$^2$, the quantity $\delta m^2$ can be of the order of $2.5 \times 10^{-11}$ (eV)$^2$; whilst in the ordinary, bradyonic case it can reach only values of the order of $\sim 10^{-3}$ (eV)$^2$. As a consequence, if the muon-neutrino is a tachyon, the position of the first oscillation maximum corresponds to values of $L/E$ various orders of magnitude smaller than ordinarily expected.

Even more important: in the present case it is easily shown that the coherence between the two "mass eigenstates" of the muon-neutrino (a condition necessary to interference) is lost, both in the solar neutrino and in the reactor experiments. It is still expected to remain satisfied for the cosmic radiation only. At last, let us recall that values of $\delta m^2$ large with respect to $E/L_1$ – the quantity $L_1$ being the distance of the first detector from the source – make the electron-neutrino "disappearance" experiments insensitive to the oscillations [19].

(viii) Coming to kinetics, the natural idea that neutrinos could be superluminal started to be common among the tachyon theorists in Europe (Milan, Catania, Rome, Palermo, Pisa, Ljubljana, Kiev, etc.) since the late sixties. One of us has been "propagandizing" such a possibility in a number of seminars, and even university lectures, since that time (~ 1968); early mentions of it being due also to Cavley [20] and Edmonds [20]. For instance, when eventually publishing in 1980 detailed kinematical calculations for the processes of: (A) tachyon absorption, (B) tachyon emission, and (C) tachyon exchange between two ordinary particles, Maccarrone and Recami warned (see p. 99 of ref. [16]); cf. also p. 110 of ref. [4], and p. 506 of ref. [11]) that in the center-of-mass of the decay $\pi \rightarrow \mu + \nu$, in which $|p|'_\mu = |p|'_\nu$, it results that [21,3]

$$|p|'_\nu = 29.7901 \text{ MeV/c} \equiv |p|'_0 \text{ if } m_\nu = 0, \nu_c = c$$

in the case that $p$ is a "luxon", whilst

$$|p|'_\nu > |p|'_0 \text{ if } m_\nu \neq 0, \nu_c < c$$

in the case that $p$ is a bradyon, and

$$|p|'_\nu > |p|'_0 \text{ if } m_\nu \neq 0, \nu_c > c$$

in the case that $p$ is a tachyon. Quantities $m_\nu$ and $\nu_c$ are the muon-neutrino proper-mass and speed, respectively. In particular, for a bradyonic neutrino with $m_\nu = 0.5$ MeV/c$^2$ one would get [21,3]

$$|p|'_\nu = 29.7868 \text{ MeV/c}; \text{ but, by using eqs. (1'), (2) in ref. [16], for a tachyonic neutrino with the same proper-mass } m_\nu = 0.5 \text{ MeV/c}^2 \text{ one gets:}$$

$$|p|'_\nu = 29.7934 \text{ MeV/c}. \text{ The most recent experimental data appear compatible with a tachyon-neutrino with } m_\nu = 0.31 \text{ MeV/c}^2:$$

$$|p|'_\nu = 29.7914 \text{ MeV/c}$$

if $m_\nu = 0.31$ MeV/c$^2$ with $\nu_c > c$,

as it follows from the eq. (1') mentioned in ref. [16]:

$$2m_\nu |p|_\nu = \left[ (m_\nu^2 + m_e^2 - m_\nu^2)^2 + 4m_e^2m_\nu^2 \right]^{1/2},$$

where we wrote $\tau$ instead of $\nu$. By the further eq.
\[ V^2 = 1 + 4m_\mu^2m_\nu^2/(m_\mu^2 + m_\nu^2 - m_\nu^2)^2, \quad (6b) \]

we get that the case in our eq. (5) corresponds to a superluminal speed \[ V = \frac{v_\mu}{c}, \]

\[ V/c = 1.000055. \]

In a generic (subluminal) frame \( f \), in which \( P^\alpha \) and \( p^\alpha \) are the pion and neutrino four-momentum, respectively, we would get \[ m_\mu^2 - m_\nu^2 = 2p_\mu^2 - m_\nu^2 \]
in the bradyon case, \[ = 2p_\mu^2 \]
in the luxon case, \[ = 2p_\mu^2 + m_\nu^2 \]
in the tachyon case.

Before going on, let us also remind the reader that \[ (i) \] (a) an ordinary particle \( A \) cannot emit in its rest-frame any tachyon \( T \) (whatever be the tachyon proper-mass \( m \)), unless the rest-mass \( M \) of \( A \) jumps to a lower value \( M' \) such that \( M^2 - M'^2 = m^2 = 2ME_T \), with \( E_T = \sqrt{(p^2 - m^2)} \) and \( m \) a positive real quantity; in a generic frame it being \( M^2 - M'^2 = m^2 + 2p_\mu^2 - m_\nu^2 \). On the contrary: (b) an ordinary particle \( A \) at rest can a priori absorb (suitable) tachyons both when increasing or conserving its rest-mass, and when lowering it: in fact \( M^2 - M'^2 = m^2 - 2ME_T \); and, in a generic (subluminal) frame \( f \), it is \( M^2 - M'^2 = m^2 - 2p_\mu^2 \).

Let us observe – especially in connection with eq.(7) – that the quantity \( p_\mu^2 \) is a Lorentz-invariant, even if it depends on the nature of the muon-neutrino.

(viii) Let us go back to the consideration at the end of point (ii) above, assuming the tachyon-neutrino mass \( m_\nu \equiv m_\mu = 0.31 \text{ MeV}/c^2 \), so that in the pion rest-frame \( \nu_\mu = V = 1.000055c \). If we analyse the decay into muons of pions in flight, we shall start to observe processes of the type (2), besides the ordinary decays \( \pi \rightarrow \mu + \nu \), when the pion speed in the lab is \( v_\pi > c^2/V \approx 0.999945c \). Let us for instance take pions with a lab energy \( E_\pi = 31.2 \text{ GeV} \), so that \( v_\pi = 0.99999c \); in such a condition, due to Lorentz dilation, the pion mean life-time will be \( \Delta \tau' = \gamma \Delta \tau_0 = 5.82 \times 10^{-9} \text{ s} \). But the ordinary decays in flight will appear in the lab as processes (2), i.e. as processes \( \pi \rightarrow \nu \rightarrow \mu \), whenever \( -v_\pi \cdot V > c^2 \). Some trivial geometry tells us, therefore, that one “decay” event out of \( \sim 45000 \) decays will actually appear in the lab as a tachyon-absorption process (2), which corresponds, for the “partial mode” (2), to a mean life-time in flight of \[ \Delta \tau' = 0.26083 \text{ s}, \]

\( \Delta \tau' \) being the mentioned geometrical factor, \( R = 4.48116 \times 10^4 \).

In the case \( m_\nu = m_\mu = 0.4 \text{ MeV}/c^2 \), we would have got the partial mean life-time \( \Delta \tau' = 0.14525 \text{ s} \), corresponding to one positive event out of \( \sim 25000 \) decays.

Of course, if the muon-neutrino is superluminal, the mean life-time \( \Delta \tau_0 \) of the pion at rest is connected – via a Lorentz transformation – with the \( \pi \rightarrow \nu \) cross-section times the \( \nu \) “cosmic flux”; see e.g. ref. [17]. If electron-neutrinos coming from the neutron decay were superluminal too, then interesting analogous considerations could be developed with regard to the neutron mean life-time, and so on.

In relation to the fact that an “intrinsic” (rest-frame) tachyon emission can appear as an anti-tachyon absorption in another suitable frame \( f \), let us finally report here the following two clarifying theorems [10,4,16].

**Theorem 1.** Necessary and sufficient condition for a process, observed either as the emission or as the absorption of a tachyon \( T \) by a bradyon \( A \), to be a tachyon-emission in the \( A \) rest-frame – i.e., to be an “intrinsic emission” – is that during the process \( A \) lowers its rest-mass (invariant statement!) in such a way that \( m_1 < \Delta M^2 < m_2^2 \). \( m_1 \) is the tachyon proper-mass, \( \Delta M^2 \equiv M_1^2 - M_0^2 \); and \( M_0, M_f \) are the bradyon initial and final rest-mass, respectively.

**Theorem 2.** Necessary and sufficient condition for a process, observed either as the emission or as the absorption of a tachyon \( T \) by a bradyon \( A \), to be a tachyon-absorption in the \( A \) rest-frame – i.e., to be an intrinsic absorption – is that \( -\infty < \Delta M^2 < m_1^2 \). Notice that \( \Delta M^2 = \Delta (M^2) \) can be both positive and negative.
(ix) To complement what reported under point (vii) above, let us moreover mention that in 1976 Mignani and Recami [22] (especially, p. 149; see also p. 91 in ref. [4], and p. 507 in ref. [11]) observed, while considering e.g. the possible classical vacuum decays into tachyons, that: (a) the tachyon cosmic flux is expected to be close to that of neutrinos; (b) the tachyon cosmic flux is expected to have a Lorentz-invariant four-momentum distribution, so that the large majority of “cosmic” tachyons ought to appear to every observer as endowed with speed very close to that of light (ref. [22,4,11]; see also ref. [23]).

(x) As to the spin of tachyons, let us notice that – if the muon neutrinos from pion decay are superluminal – the usual spin-statistics theorem, holding for bradyons, appears to hold also for tachyons, so as maintained by Sudarshan et al. [24], even if contrary opinions were expressed [25].

In ref. [1], actually, Chodos et al. wrote down and studied a Dirac-type equation for tachyon fermions. Concerning this point, let us stress that investigations of such an equation are not new in the literature: we call the reader’s attention, e.g., to the papers listed in ref. [26].

The approach in ref. [1] has been criticized by Van Dam et al. [27], on the basis of the well-known fact [28] that, among the unitary representations of the Poincaré group, no finite-dimensional representations exist in correspondence with $p^2 < 0$, except for the trivial (spin-zero) one. This fact has been a problem for tachyons since long. However, there are reasons for tachyons (reasons summarized e.g. in sections 5, 9 and 11 of ref. [4]; see for instance section 5.17 therein; see also ref. [29]) just to choose non-unitary representations in the space-like case. Unless one decides to modify the (Hilbert) state-space [30] (see especially pp. 244, 245). And, by resorting to the non-unitary representations for space-like objects, also tachyons can be associated with ordinary integer or semi-integer spins. We limit ourselves, here, to quote the related literature, listed in ref. [31] 12

(xii) Two last remarks. According to the “duality principle” [5,4], if the existence of tachyon-neutrinos is confirmed, there should exist both tachyonic and bradyonic neutrinos: even if they will behave differently (see e.g. ref. [32]).

According to one of the existing theoretical approaches [33], an interesting link exists between tachyons and magnetic monopoles, in the sense – for instance – that superluminal electric charges would contribute to the field equations just as expected from magnetic monopoles [5,4,33]. Therefore, if a particle in its bradyonic state carries an electric charge (or dipole), then in the tachyonic state it ought to appear to carry a (superluminal!) magnetic pole (dipole).

The authors are grateful for stimulating discussions to R.L. Baldini, G. Barbiellini, A.O. Barut, E. Bellotti, N. Cabibbo, P. Caldirola, G. Giacomelli, G. Goggi, M. Pavičić, D.H. Perkins, M.V. Tenório and G. Zilio.

References

4. E. Recami, Classical tachyons and possible applications: a review, Report IFN/NE-84/8 (Frascati, August 1984), and references therein.
7. V.F. Perepelitsa, Reports ITEP-100, ITEP-165 (ITEP, Moscow, 1980).

12 See also section 13.7 in ref. [3].


see also J.E. Murphy, Tachyon fields and causality, Physics Department preprint (Louisiana State University, New Orleans, 1971).


D. Shay, Lett. Nuovo Cimento 19 (1977) 333; see also E.


Austriaca 38 (1973) 113;


[29] See also, e.g., A.L. Carey, C.M. Ey and C.A. Hurst, Hadronic J. 2 (1979) 1021.


see also M.A. de Faria-Rosa, E. Recami and W.A.

Rodrigues, in preparation:


Astrofizika, Kvanti i Teorija Otstotelnosti, ed. F.I.

Fedorov (MIR, Moscow, 1982) p. 53.